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A stable matching model with an entrance criterion applied to the assignment of students to dormitories at the technion

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Abstract This paper reports on a case study of the assignment of students to dormitories at the Technion–Israel Institute of Technology. Two criteria are used in considering applicants. The first criterion, determined by personal socio-economic characteristics, is used to make decisions about the privilege of getting on-campus housing. The second criterion is used for the actual assignment of the students who were found eligible for on-campus housing to specific dormitories—here the priority is determined by academic seniority and academic excellence. A modification of the classic stable matching model that allows for an "entrance criterion" is developed and analyzed. In particular, a new concept of quasi-stable outcomes is introduced and an algorithm that produces such an outcome with desirable properties is described. The algorithm was implemented successfully for the assignment of students to dormitories at the Technion toward the 2004/2005 academic year.

Keywords Dormitory placement · Matchings · Stable matchings · Deferred acceptance algorithm

1 Introduction

Housing is a major factor in determining the quality of life that students experience during their college years. In many colleges, on-campus housing in dormitories is available

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and desirable, but supply is limited. Further, on-campus rooms are not uniform and students have priorities over the different type of accommodation that they may get, if any. Thus, it becomes necessary to develop methods for assigning students to dormitories. In many campuses the implementation of the corresponding assignment mechanism is a very important event that generates a lot of anxiety among affected students.

Providing students with quality on-campus housing has been a long-term declared goal of the management at the Technion–Israel Institute of Technology (henceforth abbreviated Technion). In fact, adding beds and improving the quality of dormitories has been used as a strategy aimed at making the university a more attractive place for potential students.[1] Still, despite heavy investment, demand for on-campus housing consistently exceeds the supply and many students cannot get the privilege of living on-campus in the dormitory complexes they wish to live in. In particular, the specific dormitory assignments to students that gain dormitory-privilege does not always meet the students' desires.

Decisions about the assignment of students to dormitories at the Technion are based on a three-step process. The first step determines eligibility for on-campus housing. The second step allocates the students that were found eligible to dormitory-groups. Finally, the third step assigns students to specific rooms/apartments. Different criteria are used for these steps: socio-economic and personal data are the main factors for the decisions of the first step, academic seniority and academic excellence are the main factors for the decisions of the second step, and finally, student-priority over rooms and roommates are the main factors in third step.

In the current paper we develop a variant of the classic Gale-Shapley stable matching model that concerns the first two steps of the process that is described in the above paragraph—it incorporates two criteria, one for determining dormitory-eligibility and a second to determine priority in being assigned to particular dormitory-groups. We introduce a notion of quasi-stability and describe a modification of the Gale-Shapley Deferred Acceptance Algorithm that will produce a matching that satisfies this condition and has desirable properties. The new algorithm was adopted for assigning (single) students to dormitories at the Technion in the academic year 2004/2005. Herein, we do not analyze the third step of the assignment process—the one that concerns the assignment to specific rooms/apartments (though the project that was executed at the Technion did cover some aspects of this step).

The model we describe and analyze in the current paper is based on the current situation at the Technion where all dormitory-groups have the same priority over the students. Still, results we derive extend to situations where the dormitory-groups have individual preferences over the students, allowing the different dormitory-groups to express preference to certain fields of study, historical (parental) association, etc; this

[1] Over the years, the Technion Board of Governors repeatedly addressed the issue of dormitory construction and maintenance. For example, in its 2001 annual meeting the Board adopted (on 6/13/2001) the following Resolution: "**4. Dormitories:** (a) The Board notes with satisfaction the on-going construction of new dormitories in the Eastern Village and the plans for a new dormitory building in the Canada Village. (b) The Board asks the Technion to provide plans to continue dormitory construction in order to meet the goal of an increased number of available beds. (c) There is an urgent need to improve some of the buildings. Hence, the Board urges the Technion to continue to invest in extensive maintenance, renovation and the upgrading of existing dorms".

may include exclusion of some students from consideration by some dormitory-groups. (But, there is a potential efficiency loss in multiple priority models, see Abdulkadiroglu and Sönmez 2003; Balinski and Sönmez 1999).

The implementation we report is another demonstration of applicability of the Gale-Shapley matching model and its generalizations to practical problems. Classic applications include the assignment of medical students to their hospital residencies and internship (see Roth 1984, 1996, 2003) as well as other internship programs (see Avery et al. 2001); more recent applications include the assignment of students to schools in major metropolitan area like New York City and Boston (see Abdulkadiroglu et al. 2005a,b, 2006; Abdulkadiroglu and Sönmez 2003), and the development of programs to exchange kidneys that are donated for transplants in patients (see Roth et al. 2004; Roth et al. 2005). The distinction of our model is in the fact that it uses two criteria to express student-desirability - one determining students' eligibility for placement and the other for the actual assignment, once they are found eligible. These criteria are to be used in a nested (lexicographic) way, where the first criterion (called the merit-score) is uniform for all dormitory-groups. As a result we get a nested two-level lexicographic rule for assigning students to dormitory-groups. The first step qualifies the students for placement (namely getting out of the waiting list) and the second-step provides the actual assignment, with the possibility that a student remains unassigned (to which we refer as becoming a refugee).

The outline of this paper is as follows. Following the Introduction, Sect. 2 provides details about the dormitory situation at the Technion and about the method that was in use till 2003; in particular, we list limitations of that method and describe dimensions in which the project whose implementation is the subject of this paper attempted to improve. Next, a formal model is presented in Sect. 3 and an algorithm that determines an assignment with desirable properties is introduced and analyzed in Sect. 4. Finally, in Sect. 5, we describe the implementation of the new algorithm at the Technion.

2 The method used in the past

In this section we provide a description of the situation of dormitory assignment at the Technion, as it existed in the fall of 2003. Dormitories at the Technion are divided into three major categories according to the personal status of potential occupants - single students, married students and families (married students with children). The study reported herein considered only dormitories for single students. Accommodations in these dormitories come in units, some of which are rooms that can house 2–3 students, while most are apartments that house 5–6 students who live in individual rooms. The occupants of each unit are expected to be of the same gender (no co-ed occupancy is allowed). But, units are not gender-specified a-priori – the gender of occupants is determined at the time that specific assignments are made. There are currently (2004) close to 3000 beds designated for single students. These are divided into 8 groups (also referred to as "villages"): (i) New East (360 beds) , (ii) Neve America (289 beds), (iii) Canada (471 beds), (iv) Old East (190 beds), (v) Renovated (120 beds), (vi) Senate (276 beds), (vii) Upper Dormitories (982 beds) and (viii) Lower Dormitories (278 beds). For example, Canada consists of 13 buildings, each having 6 apartments with five rooms and one apartment with six rooms; in addition, one building has an extra apartment with three rooms. We refer to the above eight groups as dormitory-groups. The units of each group differ in location, age (time of construction), number of occupants per unit/room, the convenience they provide and the rent that occupants pay (the latter serves as a moderator of the desirability of the more modern, renovated units). Technion management expects full occupancy in order to get maximal rent-revenue and have full utilization of a limited and valuable resource.

The head of the office of the dormitories reports to the Dean of Students. A director of operation and a director of maintenance report to him. The director of operation and her assistant are responsible for assigning students to dormitory-groups. Once these assignments are made, six dormitory managers, each responsible for 1-2 dormitory-groups, make the individual assignments of rooms. All units are reassigned each fall. A second round of reassigning rooms takes place at the beginning of the spring semester, but it is of limited scale and concerns only space that has been vacated at the end of the fall semester. There is also continuous ad-hoc treatment of occasional, unexpected vacancies.

The stages of the process that has been in use prior to the implementation of the work reported herein is next described. While the project eventually addressed only the problem of assigning single students (of the same gender) to dormitory-groups, the description includes details about the assignment of students to specific rooms.

Stage 1. Applications: Students submit applications for housing by completing a form in which they are required to supply personal details, including their income, the income of their parents, their home-town, ownership of a vehicle, details about social involvement in campus life and about their military service, number of siblings and their occupation (indicating if any of them are students), testimony of being a “lonely” student (which means that no parent lives in Israel), health and disabilities etc. The forms also require that students who are not freshmen specify a selection of a dormitory-group that is the student’s first-preference, given that he/she is found eligible for on-campus housing.[2] Once the forms are submitted, the number of academic credit points accumulated to date and information about academic excellence (e.g., being on the president’s list) is determined by the dormitory-administration and is added to the forms. The above information is used to determine two numbers for each student (usually integers). The first, referred to as the *merit-score* (ZAKAUT-score in Hebrew), ranges between 0 and 200 and is used to determine the student’s eligibility for on-campus housing. The second number, referred to as the *credit-score* (ZVIRA-score in Hebrew), is used to determine the priority that the student’s preferences receive in determining their specific dormitory-assignment, once the student is found eligible for on-campus housing. The merit-score is based on social-economic characteristics reported in the applications (income, personal status, distance of home-town from the Technion etc.). The credit-score normally equals the number of academic credits that the student has accumulated, but exceptional academic excellence (e.g., being on the president’s list) will increase the credit-score.

[2] With few exceptions, freshmen preferences are not considered in the assignment process – all those who are found eligible for on-campus housing are assigned to the (less desirable) Lower Dormitories or Upper Dormitories.

Stage 2. Determining availability and rank of dormitory-groups: The precise availability of beds in each dormitory-group is determined, after a certain percentage (usually 10–15%) of the beds is put aside as reserve (to be used for appeals, late-applications of freshmen and the ad-hoc solution of personal problems). Also, a ranking of the dormitory-groups is determined by using the data of the previous year—each dormitory-group being evaluated by the number of students that listed it in their application. Over the years, the ranking has been consistent, but not identical. The list of dormitory-groups that appears above [(i) New East, ..., (viii) Lower Dormitories] reflects the ranking in the fall of 2003, with "New East" as the most desirable and "Lower Dormitories" as the least desirable. Henceforth, we will assume that the resulting ranking is $t_1, \ldots, t_8$, with t_1 as the highest ranking and t_8 as the lowest, and that the corresponding number of available beds is $n_1, \ldots, n_8$.

Stage 3. The assignment: The total number of beds in all dormitory-groups is calculated and the corresponding number of students with the highest ZAKAUT-score are determined eligible for on-campus housing. Typically, no group of students with the same merit-score is split (this is achieved by rounding the cutting point upwards and adjusting the number of beds assigned for reserve). The next step is to rank the students who passed the eligibility-test by their credit-score. The top n_1 students that listed t_1 as their choice are assigned to t_1. The choice of students who listed t_1 and did not get it, is switched to t_2 (the local expression is "slided"). Using the modified student-choices, the process is re-applied for selecting n_2 students that will be assigned to t_2. This process is repeated, where at the beginning of each sub-stage $k = 2, \ldots, 8$, students whose (modified) choice in the previous stage was t_{k-1} and were not assigned to t_{k-1} are switched ("slided") to t_k and n_k of the remaining students who have the highest credit-score are assigned to t_k. Here again, the cutoff is adjusted upwards on account of beds assigned for reserve.

Stage 4. Announcement of the assignment to dormitory-groups and first round of appeals (Weeping I): Students are notified of the dormitory-group they were assigned to or of the fact that they did not get any. Those that were qualified for on-campus housing are requested to submit a new form in which they specify priority about specific rooms and roommates—they are entitled to specify up to 5 names of individuals they wish to be with, declare priority over smoking habits and religious practice of roommates (following Jewish kosher dietary practice and "observing Shabat" imposes restrictions on roommates), and request to stay in a room they have previously occupied. Students are also given the opportunity to appeal the initial assignment and quite a few exercise this option. Typical complaints include declared preference to live off-campus over accepting the assignment they received and concern about distance of the dormitory-group they were assigned from their academic department. Appeals are considered but rarely accommodated at this stage (see Stage 6).

Stage 5. Assignment of rooms and appeals and second round of appeals (Weeping II): The managers of the dormitory-groups get lists of students that are assigned to the dormitory-groups for which they are responsible. They assign students to rooms manually, trying to respond to the preferences they expressed in the forms submitted in Stage 4. In this process, students get priority to stay in a unit they are occupying at the end of the summer (and have not vacated it during the summer break), conditioned on their regaining eligibility for on-campus housing in the corresponding dormitory-group. Students are notified of their assignment and their roommates. Students are allowed to appeal and many of them do. Frequent arguments in appeals request alternatives to assignments that were not the students' first choice. But, there are also appeals that reflect reconsideration of the priority expressed in the original forms.

Stage 6. Secondary round including responses to appeals: All appeals are considered together with late applications (e.g., students that were admitted late, including students whose military service was delayed by the army so that they can complete their studies first ("atudaim")). Decisions are communicated to students.

The method that is described above had some serious deficiencies. Most important, there was no channel for students to voice their full ranking of dormitory-groups. Students were only entitled to state their top priority and if an assignment in that dormitory-group was not possible, their first choice was (iteratively) replaced by the next dormitory-group according to a "global ranking" (determined by students' selections in the previous year). A typical example of how individual preferences could not be accounted for is a student who desires to get a dormitory that is close to his/her department, if his/her first choice (e.g., the most modern apartments) is not fulfilled. Further, the method created room for strategic manipulation in the students' statements about their top-choice—students may state as their first choice an "achievable" dormitory-group, so as to avoid getting into the position that their assignment is influenced by the "global ranking". Empirical observations of such strategic behavior by families facing the placement of their children in the Boston school system have been recorded in Abdulkadiroglu et al. (2006) (in fact, this is a restricted form of "truncation" manipulativity studied in Roth and Rothblum 1999). Another deficiency of the old system was the fact that students could not express a position that they find particular dormitory-groups unacceptable and prefer to live off-campus over living in those dormitories. This situation resulted a larger number of rejections (that could have been anticipated earlier) and thereby requiring more reassignments. Of course, as students could not given the option of recording their complete preferences, the system could not attend to their unexpressed desires. A survey that was conducted among the students revealed considerable concern about the performance of the assignment mechanism.

The project that was the basis of the current paper undertook the challenge of improving the mechanism that assigns students to dormitory-groups (the above stages 1–4). A new method was designed that allows students to express complete preferences over dormitory-groups. A modification of the classic stable-roommate model was developed and a variant of the Gale-Shapley Deferred Acceptance Algorithm was constructed. The output of the algorithm turns out to have desirable properties.

3 The stable assignment model with an entrance criterion

In this section we consider a modification of the classic stable matching model of Gale and Shapley (1962) (see also Gusfield and Irwin 1989; Knuth 1976; Roth and Sotomayor 1990) and introduce a new quasi-stability concept that incorporates an "entrance criterion". An algorithm that produces a corresponding stable outcome with desirable properties is described and analyzed in the next section.

The data for our model includes two disjoint finite sets S and T, to which we refer as the set of *students* and *dormitory-groups*, respectively. Without loss of generality, assume that $|S| = n$, $|T| = m$, $S = \{1, \ldots, n\}$ and $T = \{n + 1, \ldots, n + m\}$. Each $t \in T$ is associated with a positive integer q_t, representing the *capacity* (the number of available beds) of dormitory-group t. Also, each $s \in S$ is associated with two nonnegative numbers m_s and c_s, a subset T_s of T and a ranking (that is, a complete order) $\succ_s$ of $T_s \cup \{s\}$ which has s as its least element. We refer to m_s and c_s as the *merit-score* and the *credit-score* of student s (see the Introduction). Also, we refer to $\succ_s$ as the *preference of* student s over the set of dormitory-groups in T_s and over being left without an assignment, the latter represented by s; $t \notin T_s$ means that student s finds dormitory-group t unacceptable and prefers to live off-campus (outcome s) over living in dormitory-group t.

For simplicity, we assume throughout that $m_1, \ldots, m_n$ and $c_1, \ldots, c_n$ are, respectively, distinct. In reality, at the Technion, the m_s's take integer values ranging between 0 and 200. Also, with a limited number of exceptions, each c_s represents the number of academic credit-points that student s has accumulated, as such it is typically an integer or half-integer that ranges between 0 and 200 (generally, about 120 credits are required for graduating in a 3-year program, 160 in a 4-year program and 200 in a double-degree program). As there are annually about 4,000 students at the Technion that apply for on-campus housing, ties occur. In practice, ties in the $m_.$-score are broken by using the $c_.$-score, and ties in the $c_.$-score are broken by using the current GPA (grade-point-average) of the students.

An *assignment* is a list of pairs $\mu = \{(s_i, t_i) : i = 1, \ldots, p\}$ for some integer $p \geq 0$ with $s_1, \ldots, s_p$ distinct, with $t_i \in T_{s_i}$ for each $i = 1, \ldots, p$ and with $|\{s \in S : (s, t) \in \mu\}| \leq q_t$ for each $t \in T$. In this case, we identify μ with the map $\mu(\cdot)$ from S into $T \cup S$ having $\mu(s_i) = t_i$ for $i = 1, \ldots, p$ and $\mu(s) = s$ for $s \in S \setminus \{s_1, \ldots, s_p\}$. Under this interpretation, μ represents the assignment of students to dormitory-groups, with $\mu(s) = s$ indicating that student s is not assigned under μ to any dormitory-group. Further, the assumptions about an assignment translate to $\mu(s) \in T \cup \{s\}$ for each student s and $|\{s : \mu(s) = t\}| \leq q_t$ for each $t \in T$.

An *outcome* is a triplet (μ, W, R) where μ is an assignment and the pair W, R partitions $\{s \in S : \mu(s) = s\}$; here, the interpretation is that the students in W are in a waiting list while those in R are excluded from further consideration and we refer to such students as *refugees*.

We say that an outcome (μ, W, R) is *acceptable* if

(a) $m_s < m_{s'}$ for each $s \in W$ and $s' \in S \setminus W$, and
(b) either $W = \emptyset$ or $|\{s : \mu(s) = t\}| = q_t$ for each $t \in T$.

Condition (a) of acceptability asserts that the $m_.$-score of each student in the waiting list is below that of every student that is not (namely, a student who has either been assigned or has been excluded from further consideration). It follows immediately that if (μ, W, R) and (μ', W', R') are two outcomes that satisfy Condition (a), then W and W' are ordered by set-inclusion. Condition (b) of acceptability asserts that either all students are processed (meaning assigned or determined to be refugees), or all dormitory-groups are at capacity.

We say that a pair $(s, t) \in S \times T$ is a *blocking pair* of an outcome (μ, W, R) if:

(a) $t \succ_s \mu(s)$ (which implies that $t \in T_s$), and
(b) either $|\{s : \mu(s) = t\}| < q_t$, or $c_s > c_{s'}$ for some $s' \in S$ with $\mu(s') = t$.

An outcome (μ, W, R) is called *internally stable* if it has no blocking pair (s, t) with $s \in S \backslash W$. An outcome (μ, W, R) is called *quasi-stable* if it is acceptable and internally stable.

An assignment μ is *stable* (under the regular notion of stability) if $(\mu, \emptyset, R)$ is internally stable, where $R = \{s \in S : \mu(s) = s\}$. We observe that an outcome (μ, W, R) is internally stable if and only if μ satisfies the regular notion of stability for the matching model with the student-set restricted to $S \setminus W$.

Intuitively, a distinction between refugees and students on the waiting list of an outcome is that the former are assumed to have been given a chance to be assigned while no such chance has yet been given to students on the waiting list. Still, in a dynamic environment, it is reasonable that when beds become available due to voluntary departures of students, refugees with high merit-score should get a chance for an assignment before those in the waiting list with lower merit score. So, the label "refugee" in not intended to reflect permanent status.

The following example demonstrates the above definitions.

Example 1 Consider a model with 6 students $s_1, s_2, s_3, s_4, s_5, s_6$ and 3 dormitory-groups t_1, t_2, t_3 with $q_{t_1} = q_{t_3} = 1$ and $q_{t_2} = 2$. The data about the students-scores and preferences (the latter listed in decreasing order from top to bottom) is provided in Table 1 below

There are 7 outcomes that are internally stable and satisfy condition (a) of acceptability; they are listed in Table 2 below, indicating those that satisfy condition (b) of acceptability (and are quasi-stable):

Table 2 contains two quasi-stable outcomes (μ^6, W^6, R^6) and (μ^7, W^7, R^7); we note that (s_6, t_1) is a blocking pair for (μ^6, W^6, R^6), but it does not violate the definition of internal-stability as $s_6 \in W^6$. Further, consider the modified example in which only students s_1, s_2, s_3 are present, with the same preferences as in Table 1. In this case, the assignment $\mu = \{(s_1, t_2), (s_2, t_1)\}$ is part of the quasi-stable outcome $(\mu, \emptyset, \{s_3\})$ and of the outcome $(\mu, \{s_3\}, \emptyset)$ which does not satisfy condition (b) of acceptability (and is not quasi-stable). We finally observe that the only (regularly) stable assignment in Table 2 is μ^7.

Table 1 Data for a example 1

	s_1	s_2	s_3	s_4	s_5	s_6
m_s	100	90	80	70	60	50
c_s	50	60	40	30	65	70
Preferences	t_1	t_1	t_1	t_3	t_2	t_1
	t_2	t_2		t_1	t_1	t_2
		t_3		t_2	t_3	t_3

Table 2 The internally stable outcomes that satisfy condition (a) of acceptability

j	μ^j	W^j	R^j	Acceptable (and quasi-stable)
1	$\emptyset$	$\{s_1, s_2, s_3, s_4, s_5, s_6\}$	$\emptyset$	No
2	$\{(s_1, t_1)\}$	$\{s_2, s_3, s_4, s_5, s_6\}$	$\emptyset$	No
3	$\{(s_1, t_2), (s_2, t_1)\}$	$\{s_3, s_4, s_5, s_6\}$	$\emptyset$	No
4	$\{(s_1, t_2), (s_2, t_1)\}$	$\{s_4, s_5, s_6\}$	$\{s_3\}$	No
5	$\{(s_1, t_2), (s_2, t_1), (s_4, t_3)\}$	$\{s_5, s_6\}$	$\{s_3\}$	No
6	$\{(s_1, t_2), (s_2, t_1), (s_4, t_3), (s_5, t_2)\}$	$\{s_6\}$	$\{s_3\}$	Yes
7	$\{(s_2, t_2), (s_4, t_3), (s_5, t_2), (s_6, t_1)\}$	$\emptyset$	$\{s_1, s_3\}$	Yes

The following Lemma and its three corollaries record the effect of the waiting lists of quasi-stable outcomes on the number of students that are matched to each dormitory-group and on the refugee lists.

Lemma 1 *If (μ, W, R) and (μ', W', R') are two quasi-stable outcomes with $W' \subseteq W$, then $R \subseteq R'$ and $|\{s : \mu'(s) = t\}| \geq |\{s : \mu(s) = t\}|$ for each $t \in T$.*

Proof As (μ, W, R) and (μ', W', R') are internally stable, μ and μ' are (regularly) stable in the models where the set of students is restricted, respectively, to $S \setminus W$ and $S \setminus W'$. As $W' \subseteq W$, we have that $S \setminus W' \supseteq S \setminus W$ and results about the regular stability (e.g., Roth and Sotomayor 1990, Theorems 5.12 and 5.35, pages 144 and 166, respectively) assure that $R = \{s \in S : \mu(s) = s\} \subseteq \{s \in S : \mu'(s) = s\} = R'$. To verify the second conclusion of the lemma apply the McVite–Wilson Algorithm (McVitie and Wilson 1970) (nondeterministic, student-courting, one-proposal at-a-time) variant of the classic Deferred Acceptance Algorithm of Gale–Shapley Gale and Shapley (1962) with the set of students restricted to those in $S \setminus W$ and with an arbitrary sequence of (feasible) proposals. Next, apply that algorithm with the larger set of students $S \setminus W'$, starting with the previous sequence of proposals and continuing arbitrarily. The resulting matchings, say $\bar{\mu}$ and $\bar{\mu}'$, are (regularly) stable for the corresponding problems and the construction assures that $|\{s : \bar{\mu}'(s) = t\}| \geq |\{s : \bar{\mu}(s) = t\}|$. By (Roth and Sotomayor 1990, Theorems 5.12, pp. 144), the same inequality also holds when the bars are removed. □

Define the *occupancy* of an assignment μ *at* dormitory-group t to be the cardinality of $\{s : \mu(s) = t\}$, and let the *occupancy* of μ be its cardinality $|\mu|$. The first corollary of Lemma 1 shows invariance of the occupancies at each of the dormitory-groups under assignments that are part of quasi-stable outcomes.

Corollary 1 *If (μ, W, R) and (μ', W', R') are two quasi-stable outcomes, then $|\{s : \mu(s) = t\}| = |\{s : \mu'(s) = t\}|$ for each $t \in T$.*

Proof Let (μ, W, R) and (μ', W', R') be two quasi-stable outcomes. If $W = W'$, Lemma 1 assures that $|\{s : \mu(s) = t\}| = |\{s : \mu'(s) = t\}|$ for each $t \in T$. So, assume that $W \neq W'$. As condition (a) of acceptability assures that W and W' are ordered by set-inclusion, one can assume without loss of generality that $W \supset W'$. In this case, Lemma 1 assures that $|\{s : \mu(s) = t\}| \leq |\{s : \mu'(s) = t\}|$ for each $t \in T$. Further, as $W \neq \emptyset$, condition (b) of acceptability assures that for each $t \in T$, $|\{s : \mu(s) = t\}| = q_t$ and therefore $q_t = |\{s : \mu(s) = t\}| \leq |\{s : \mu'(s) = t\}| \leq q_t$ (the last inequality following from the fact that μ' is an assignment); the conclusion of the corollary now follows. □

The second corollary of lemma 1 shows that the waiting lists of quasi-stable outcomes are in one-to-one correspondence with the refugee lists.

Corollary 2 *If* (μ, W, R) *and* (μ', W', R') *are two quasi-stable outcomes, then the following are equivalent: (i)* $W = W'$*, (ii),* $|W| = |W'|$*, (iii)* $R = R'$ *and (iv)* $|R| = |R'|$.

Proof Condition (a) of acceptability assures that (i) $\Leftrightarrow$ (ii). Lemma 1 implies that (i) $\Rightarrow$ (iii). Next, Corollary 1 assures that $|\mu| = |\mu'|$ and therefore, as $|\mu| + |W| + |R| = |\mu'| + |W'| + |R'| = n$ and the equivalence of (ii) $\Leftrightarrow$(iv) follows. Finally, the implication (iii)$\Rightarrow$(iv) is trite. □

The third corollary of lemma 1 demonstrates that quasi-stable outcomes with larger waiting lists result in assignments under which students that get dormitories have higher merit-scores. To express this last property formally we need some further definitions. We say that student *s is matched under* an assignment μ, if $(s, t) \in \mu$ for some $t \in T$. We say that assignment μ *elig-dominates* assignment μ' if the merit-score of every student s who is matched under μ but not under μ' is higher than the merit-score of any student s' who is matched under μ' but not under μ.

Corollary 3 *Let* (μ, W, R) *and* (μ', W', R') *be two quasi-stable outcomes with* $W \supseteq W'$*. Then* μ *elig-dominates* μ'.

Proof Let A and A' be the sets of students who are assigned under μ and μ', respectively. Also, let m^* be the highest merit-score of a student in W. We will show that $m_s > m^* \geq m_{s'}$ for every $s \in A \setminus A'$ and $s' \in A' \setminus A$. Indeed, for $s \in A \setminus A' \subseteq S \setminus W$, condition (a) of acceptability implies that $m_s > m^*$. Next, assume that there exists a student s' in $S \setminus A$ with $m_{s'} > m^*$. It will then follow that $s' \in R \subseteq R'$ (the inclusion following from Lemma 1); so, $s' \notin A'$. This proves that $m_{s'} \leq m^*$ for every $s' \in A' \setminus A$. □

The Gale–Shapley Theorem (Gale and Shapley 1962) guarantees existence of (regularly) stable assignments and a standard result (e.g., Roth and Sotomayor 1990, Theorem 2.22, p. 42) assures that R' is common for all internally stable outcomes $(\mu', \emptyset, R')$. In particular, it follows that the occupancy rates of all (regularly) stable assignments coincide and we shall denote its common value by $\bar{q}'$. Also, let $\bar{q}$ be the joint occupancy of the assignments of the quasi-stable outcomes (the forthcoming Theorem 1 guarantees that the set of quasi-stable outcome is not empty). Now, let (μ, W, R) be a quasi-stable outcome. If $|\mu| = \bar{q} < \sum_t q_t$, condition (b) of acceptability implies that $W = \emptyset$; thus, μ is (regularly) stable and $\bar{q} = |\mu| = \bar{q}'$. Alternatively, assume $\bar{q} = \sum_t q_t$; as the occupancy of no assignment can exceed $\sum_t q_t$, we have that $\bar{q}' \leq \sum_t q_t = \bar{q}$. We conclude that $\bar{q}$, the joint occupancy of the assignments associated with quasi-stable outcomes, is an upper bound on the (joint) occupancy of the set of all (regularly) stable assignments.

4 The dormitory assignment algorithm

In this section we describe an algorithm that will be shown to produce a quasi-stable outcome (as defined in the previous section) with distinctive desirable properties. The new algorithm is a dynamic application of the McVite–Wilson Algorithm (McVitie and Wilson 1970) which is a (nondeterministic, student-courting, one-proposal at-a-time) variant of the classic Deferred Acceptance Algorithm of Gale and Shapley (1962).

We start by describing the McVite–Wilson Algorithm in the context of our (many-to-one) student versus dormitory-group problem. The algorithm starts with the empty assignment with all students *unassigned* and the beds in all the dormitories *available*. At each stage a student who has not yet been rejected by all dormitory-groups on his list is picked. The student proposes to a dormitory-group he prefers most among those that have not yet rejected him/her. A dormitory-group that receives a proposal from a student accepts it (that is, assigns a bed to the student) if it is not at capacity, or if it is at capacity and one of the students that is currently assigned to it has a lower credit-score than the proposing student; in the latter case the proposer replaces the student with low credit-score who becomes unassigned. If a proposal of a student is not accepted, then the student remains unassigned and another "proposer" is selected from the set of unassigned students. It is well known (e.g., McVitie and Wilson 1970) that McVite–Wilson Algorithm terminates and the constructed assignment at termination is independent of the non-deterministic selection of the unassigned students and is (regularly) stable; in fact, the joint outcome coincides with the outcome of the classic Student-Courting Deferred Acceptance Algorithm of Gale–Shapley.

The dormitory assignment algorithm (DorAA):

Initiation: Initially, let ProcessList consist of the $\min\{q = \sum_t q_t, |S|\}$ students having the highest merit-score, let WaitList be the remaining students, and let RefugeeList be the empty set.

Iterative step: Recursively apply steps of the McVite–Wilson Algorithm on data which considers only students who are in ProcessList.
If the current application of the McVite–Wilson Algorithm terminates, STOP.
If a student has completed his list of dormitory-groups (having proposed to all of them and having been rejected by all of them), do the following: (i) move that student from ProcessList to RefugeeList, (ii) if WaitList is not empty, move the student with the highest merit-score in WaitList to ProcessList, and (iii) restart a new iterative step.

Output: The outcome (μ, W, R) that is available when stopping, where μ is the set of pairs $(s, t) \in S \times T$ that are matched when stopping and W and R are, respectively, WaitList and RefugeeList at that time.

Table 3 Assignments reached in executing DorAA

WaitList	The assignment	RefugeeList
$\{s_1, s_2, s_3, s_4, s_5, s_6\}$	$\emptyset$	$\emptyset$
s_1, s_2, s_3, s_4 are admitted to ProcessList		
$\{s_5, s_6\}$	$\{(s_2, t_1)\}$	$\emptyset$
$\{s_5, s_6\}$	$\{(s_2, t_1), (s_1, t_2)\}$	$\emptyset$
s_3 is removed from ProcessList into RefugeeList		
s_5 is admitted into ProcessList from WaitList		
$\{s_6\}$	$\{(s_5, t_2), (s_2, t_1), (s_1, t_2)\}$	$\{s_3\}$
$\{s_6\}$	$\{(s_5, t_2), (s_2, t_1), (s_1, t_2), (s_4, t_3)\}$	$\{s_3\}$

The Dormitory Assignment Algorithm (DorAA) is nondeterministic (as the selection of "proposing" students from ProcessList in any iteration step is unrestricted). But, we will see (in Theorem 1 below) that the output of DorAA is independent of the specific selections. We next demonstrate the execution of DorAA on the data of Example 1 with a specific rule for selecting the proposing student

Example 1 (Continued) Consider an application of DorAA on the data of Example 1 with the following rule: at each iteration step, the student who will next proposed is the unassigned student in ProcessList with the highest credit-score. Table 3 lists the assignment that are generated after chain reactions that follow a proposal of unmatched student in ProcessList makes a proposal. Pairs are ordered by the merit-score of the students.

Some further definitions will be useful for analyzing DorAA. A pair $(s, t) \in S \times T$ is called *possible* if there exists a quasi-stable outcome (μ, W, R) with $(s, t) \in \mu$, in this case we say that dormitory-group t and student s *are possible for each other*. We say that a student s is a *potential waiter* if $s \in W$ for some quasi-stable outcome (μ, W, R). We say that a student s is an *ultimate refugee* if $s \in R$ for every quasi-stable outcome (μ, W, R). Let W^* be the set of all potential waiters and let R_* be the set of all ultimate refugees.

The next theorem shows that DorAA produces a quasi-stable outcome having the largest waiting list and the smallest refugee list among all quasi-stable outcomes, that it is the best quasi-stable outcome for all the students that are not in its waiting list and that its assignment elig-dominates the assignment of any other quasi-stable outcome. Further, under this outcome, students with a complete preference lists who pass the eligibility test (that is, they are not in the waiting list) will actually get a dormitory-assignment; in different words, a student who does not exclude any dormitory-group will not be a refugee.

Below, we use the standard notation $|L|$ to denote the length of a list L.

Theorem 1 *DorAA terminates and its output is independent of the nondeterministic selections of the proposing students. Further, the joint output of all executions of DorAA, say* (μ, W, R)*, is a quasi-stable outcome with the following properties*:

(a) $W = W^*$.
(b) $R = R_*$.
(c) *μ elig-dominates the assignment associated with any quasi-stable outcome.*
(d) *$\mu(s) \succeq_s \mu'(s)$ for each quasi-stable outcome (μ', W', R') and $s \in S \setminus W^*$.*
(e) *R contains no student for whom all dormitory-groups are acceptable.*

Proof As the McVite–Wilson Algorithm (MV–W) is guaranteed to terminate (when not interrupted), DorAA must terminate. We next verify that each output of DorAA is quasi-stable. First observe that an execution of DorAA that results in an output (μ, W, R) can be identified with an execution of MV–W with the set of students restricted to $S \setminus W$. It then follows from results about MV–W that all outputs are internally-stable and all outputs (μ, W, R) with the same W coincide. Next, condition (a) of acceptability is trivially satisfied because the order at which students are moved out of WaitList is by their merit-score. Finally, note that along each execution of DorAA, the number of students assigned to dormitory-group t cannot exceed q_t. Further, as long as WaitList is not empty, the combined number of students in ProcessList is maintained to be $\sum_t q_t$. Now, if termination occurs with WaitList not empty, the termination condition assures that all students in ProcessList at that stage are assigned; as there are $\sum_t q_t$ such students, we have that for each t the number of students assigned to dormitory-group t is q_t. So, condition (a) of acceptability is verified, completing the proof that each output of DorAA is quasi-stable.

We next prove that if (μ, W, R) is an output of DorAA and (μ', W', R') is a quasi-stable outcome, then $W' \subseteq W$. As the above paragraph assures quasi-stability of all outputs of DorAA, (μ, W, R) is quasi-stable. Now, if $|\{s : \mu'(s) = t\}| < q_t$ for some $t \in T$, Corollary 1 implies that $|\{s : \mu(s) = t\}| = |\{s : \mu'(s) = t\}| < q_t$ and Condition (b) of acceptability assures that $W = W' = \emptyset$. Next assume that $|\{s : \mu'(s) = t\}| = q_t$ for all $t \in T$, in which case $n = |W'| + \sum_t q_t + |R'|$. Condition (a) of acceptability assures that W and W' are ordered by set-inclusion. Thus, the negation of the claim $W' \subseteq W$ asserts that $W \subset W'$, which is next shown to lead to a contradiction. So, assume that $W \subset W'$.

By Corollary 1 $|\{s : \mu(s) = t\}| = |\{s : \mu'(s) = t\}| = q_t$ for all $t \in T$, consequently, ProcessList has $\sum_t q_t$ students throughout any execution of DorAA that results in output (μ, W, R). Let $s^\#$ be the student in $W' \supset W$ with the highest merit-score. Consider an execution of DorAA that produces (μ, W, R) and let $W^\#$, $R^\#$ and $P^\#$ be, respectively, the set of students in WaitList, RefugeeList and ProcessList just after the move of $s^\#$ from WaitList to ProcessList, in particular, $W^\# = W' \setminus \{s^\#\}$, $|P^\#| = \sum_t q_t$ and $|R^\#| = n - |W^\#| - |P^\#| = n - |W'| - \sum_t q_t + 1 > |R'|$. The steps of DorAA before the move of $s^\#$ from WaitList to ProcessList can be identified with the beginning of an execution of MV–W with the set of students restricted to $S \setminus W'$; one can continue this algorithm (with the set of students restricted to $S \setminus W'$) and produce an internally stable outcome, say (μ'', W', R''). As there is no exit from RefugeeList while the algorithm progresses, we have that $R^\# \subseteq R''$. Also, the internal-stability of (μ'', W', R'') and (μ', W', R') and Lemma 1 assure that $R' = R''$. We conclude that $|R'| = |R''| \geq |R^\#| > |R'|$, establishing the desired contradiction.

The conclusion of the above paragraph and the quasi-stability of all outputs of DorAA, implies that if (μ, W, R) and (μ', W', R') are two outputs of DorAA, then

$W = W'$. But, we already observed in the first paragraph of this proof that $W = W'$ assures $(\mu, W, R) = (\mu', W', R')$. This completes the proof that all outputs of DorAA coincide. Also, the established conclusion that if (μ, W, R) is an output of DorAA and (μ', W', R') is a quasi-stable outcome, then $W' \subseteq W$ verifies (a). Further, combining that conclusion with Lemma 1, we conclude that $R \subseteq R'$ whenever (μ, W, R) is an output of DorAA and (μ', W', R') is quasi-stable, verifying (b).

Part (c) is immediate from part (b) and Corollary 3.

Next, to verify part (d), let (μ, W, R) be the joint outcome of the executions of DorAA and let (μ', W', R') be any other quasi-stable outcome. By part (b), $W = W^* \supseteq W'$, implying that $S \setminus W \subseteq S \setminus W$'. As the execution of DorAA can be identified with the executing the MV–W with the set of students restricted to $S \setminus W$; so, μ coincides with the assignment associated with the outcomes obtained from applying the Gale-Shapley Student-Courting Deferred-Acceptance-Algorithm with students restricted to $S \setminus W$. Let μ'' be the assignment associated with the (joint) outcomes obtained from applying that algorithm with students restricted to $S \setminus W' \supseteq S \setminus W$. It then follows from classic results (e.g., Roth and Sotomayor 1990, Theorem 2.25, p. 44) that for every $s \in S \setminus W$, $\mu(s) \succeq_s \mu''(s) \succeq_s \mu'_s$, verifying (d).

Finally, to verify part (e), let (μ, W, R) continue to be the joint outcome of the executions of DorAA and let s be a student who finds all dormitory-groups acceptable, that is, $T_s = T$. We will assume that $s \in R$ and derive a contradiction. Consider any specific execution of DorAA. At the time s is removed from ProcessList, he/she has been rejected by all dormitory-groups; this has happened at a stage where the dormitory-group must have been at capacity. As a dormitory-group that reaches capacity must stay that way, we have that just before the removal of s from ProcessList, all dormitory-groups are at capacity. In addition, there is a student at that stage in ProcessList who is not assigned and caused the removal of s from his current dormitory-group; so, $|\text{ProcessList}| \geq \sum_t q_t + 1 > \sum_t q_t$, contradicting the fact ProcessList cannot have more than $\sum_t q_t$ students. This contradiction verifies (e). □

5 Implementation of DorAA

DorAA, the algorithm that is described and analyzed in Sect. 4, was developed in the winter of 2004 and was implemented at the Technion the following year. Before its implementation, the algorithm was simulated in two stages. A preliminary simulation used data that was collected during the fall of 2003 in a limited-scope survey conducted among residents of dormitories (with 69 responses). As one of the questions in that survey asked students to rank all dormitory-groups, the survey provided the first access to a (small) sample of complete preferences of students over dormitory-groups. The outcome of the simulation of the new algorithm was compared with the outcome of a simulation of the old algorithm on the same data. In both runs, the capacity on the dormitory-groups was taken as the number of students who responded to the survey and were actually living in that dormitory-group at the time. The outcome of the simulation of the new algorithm yielded improvement for 22.5% of the students while 6% received a less desirable outcome than what they had. Typically, the students in the second group were students who preferred dormitory-groups that were "down the ladder" and received housing "up the ladder" due to higher credit-scores (though these students ranked these outcomes as less desirable). The simulation did not leave any of the students without housing (that is, there were no refugees). It is noted that there was apparently a bias in the sample as all surveyed students had on-campus housing and the survey did not include students that were found eligible and decided to live off-campus).

In the spring of 2004, the Dean of Students decided to adopt the new method and implement it for the fall-assignment of the 2004/2005 academic year. As a result, forms that students were supposed to fill were augmented with a question requesting that they list complete preferences over dormitory-groups rather than listing only their top-choice. Not all students responded to that question—the preferences of those that did not were assumed to coincide with the previous year (joint) “ladder” ranking. Freshmen were asked not to respond to the new question and their preferences were taken as “Upper Dormitories” being their top choice, “Lower Dormitories” being their second choice and no other alternatives (the Dean of Students decided that freshmen will continue to live together in these dormitory-groups). A simulation was run on two groups of applicants - men and women, each consisting of about 2000 students. Some adjustments were made after the simulation was run—for example, freshmen from Architecture were assigned to the Upper Dormitories which have larger rooms and joint assignment of freshmen from other academic units took place (a principle that had been applied in the past). A concern that a relatively large group of eligible students will be qualified as refugees did not materialize and there were only 30 such students. Following the simulation, the algorithm was implemented successfully and the assignment of students to dormitory-groups in the fall of 2004 was base on the output of the new algorithm.

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Appendix

In this Appendix, we provide a detailed description of DorAA that includes specifics about the data structure that is used for its implementation (so as to ease potential programming).

The dormitory assignment algorithm (DorAA):

Input:

1. A list T (dormitory-groups), with each $t \in T$ associated with an integer (capacity) q_t.

2. A list S (students), with each $s \in S$ associated with two numbers m_s and c_s (the merit-score and the credit-score of s), and with a sorted list of a subset T_s of T, where the notation $t \succ_s t'$ is used if s ranks t above t'.

Data structures:

1. WaitList: A list of students, sorted by their merit-score from the highest to the lowest (those whose processing has not yet started).
2. ProcessList: A list of students (those who are currently processed).
3. RefugeeList: A list of students (those who will not get a dormitory-assignment)
4. DormStatus: An array of size $|T|$ that contains for each $t \in T$:
   (i) StudentList$_{[t]}$: A list of students (those currently matched to t), sorted by credit-score from the highest to the lowest
   (ii) NSpace$_{[t]}$: $\{q_t - |\text{StudentList}_{[t]}|\}$ (the current number of openings in t).
5. StudentStatus: An array of size $|S|$ that contains for each $s \in S$:
   (i) Status$_{[s]}$: A symbol (the current status of s) which belongs to one of the following: “W” (s is in WaitList), “P0” (s is in ProcessList but is not assigned), “P1” (s is in ProcessList and is assigned) or “R” (s is in RefugeeList).
   (ii) LeftList$_{[s]}$: A sorted sublist of T_s (the dormitory-groups for which s is still considered).

Initialization phase

1. For each student s, set [Status$_{[s]}$ $\leftarrow$ “W”] and [LeftList$_{[s]}$ $\leftarrow T_s$].
2. For each $t \in T$, set [StudentList$_{[t]}$ $\leftarrow \emptyset$] and [NSpace$_{[t]}$ $\leftarrow q_t$].
3. Set [RefugeeList$\leftarrow \emptyset$].
4. Move all students from S into WaitList and sort them by their merit-score.
5. Move the top $\min\{|S|, \sum_t q_t\}$ students from WaitList to ProcessList and change their status from “W” to “P0”.

Iteration phase:

1. If there is a student in ProcessList with status “P0”: Let s be any such student. Go to Step 2.
   If there is no student in ProcessList with status “P0”: Go to the Output Phase.
2. If LeftList$_{[s]}$ is not empty: Let t be the top element in LeftList$_{[s]}$. Go to Step 3.
   If LeftList$_{[s]}$ is empty: Move s from ProcessList into RefugeeList and change Status$_{[s]}$ from “P0” to “R”.
   If WaitList is not empty: Move the student from the top of WaitList, say student s^*, to ProcessList and change Status$_{[s^*]}$ from “W” to “P0”. Go to Step 1.
   If WaitList is empty: Go to Step 1.
3. If NSpace$_{[t]} > 0$: Add student s to the right position in StudentList$_{[t]}$ (according to his/her credit-score), change Status$_{[s]}$ from “P0” to “P1” and reduce NSpace$_{[t]}$ by 1. Go to 1.
   If NSpace$_{[t]} = 0$ and the credit-score of s is less than that of the last student in StudentList$_{[t]}$: Remove t from LeftList$_{[s]}$. Go to step 2.
   If NSpace$_{[t]} = 0$ and the credit-score of s is higher than that of the last student in StudentList$_{[t]}$ say s': Remove s' from StudentList$_{[t]}$ and change Status$_{[s']}$ from “P1” to “P0”. Add student s to the right position in StudentList$_{[t]}$ (according to s’s credit-score) and change Status$_{[s]}$ from “P0” to “P1”. Go to step 1.

Output phase:

The output is the triplet (μ, W, R) where $\mu = \{(s, t) : s$ appears in $\text{StudentList}_{[t]}\}$, W consists of the students in WaitList (those with $\text{Status}_{[s]} = W$) and R consists of the students in RefugeeList (those with $\text{Status}_{[s]} = R$) .

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