

Static Single Point Positioning using GPS Pseudoranges

Objective

This assignment focuses on implementing a **Static Single Point Positioning (SPP)** algorithm using GPS pseudo-range observations. The goal is to compute the receiver's position and clock bias at each epoch using **the least squares estimation**, evaluate solution quality via **DOP values** and **position errors**, and analyze uncertainty propagation.

Methodology Overview

1. Rinex Observation Parsing

Extracted C1 pseudo-ranges for visible GPS satellites using the provided RINEX observation file.

2. Satellite Position Matching

Loaded satellite coordinates and pseudorange corrections from satpos.txt. Matched based on epoch time and PRN.

3. Least Squares Estimation

- Used an initial guess of (0,0,0,0) for position and clock bias.
- Built the design matrix A and misclosure vector w .
- Solved for updates using the normal equation:

$$\Delta \hat{x} = -(A^T P A)^{-1} A^T P w$$

- Iterated Until Convergence

4. Accuracy Evaluation

Compared estimated positions with true coordinates to compute **ENU errors**. Also calculated **HDOP, VDOP, PDOP, GDOP** values per epoch using the geometry matrix.

5. Visualization

Generated time-series plots for:

- **Estimated vs true ECEF positions**
- **Clock bias evolution**
- **ENU position errors with DOP-based uncertainty bands**
- **DOP values over time**
- **Number of satellites used**

Position Estimation Results

The estimated receiver coordinates over time were plotted against the known true ECEF position. The results indicate that the estimated position generally stays close to the true position in all three axes (X, Y, Z), but with some fluctuations.

- X-coordinate: The solution remains relatively stable with an offset of around 1–2 meters and occasional brief drifts.
- Y-coordinate: Shows a clear bias of approximately 5–8 meters, which could be due to geometry or uncorrected errors.
- Z-coordinate: The vertical component tends to underestimate the true height consistently by about 20–30 meters, which is expected in GPS due to weaker vertical satellite geometry.

Overall, the horizontal components (X and Y) perform better than the vertical, which is typical in GPS SPP due to the geometry of satellite constellations. These deviations will be further quantified in the accuracy assessment.

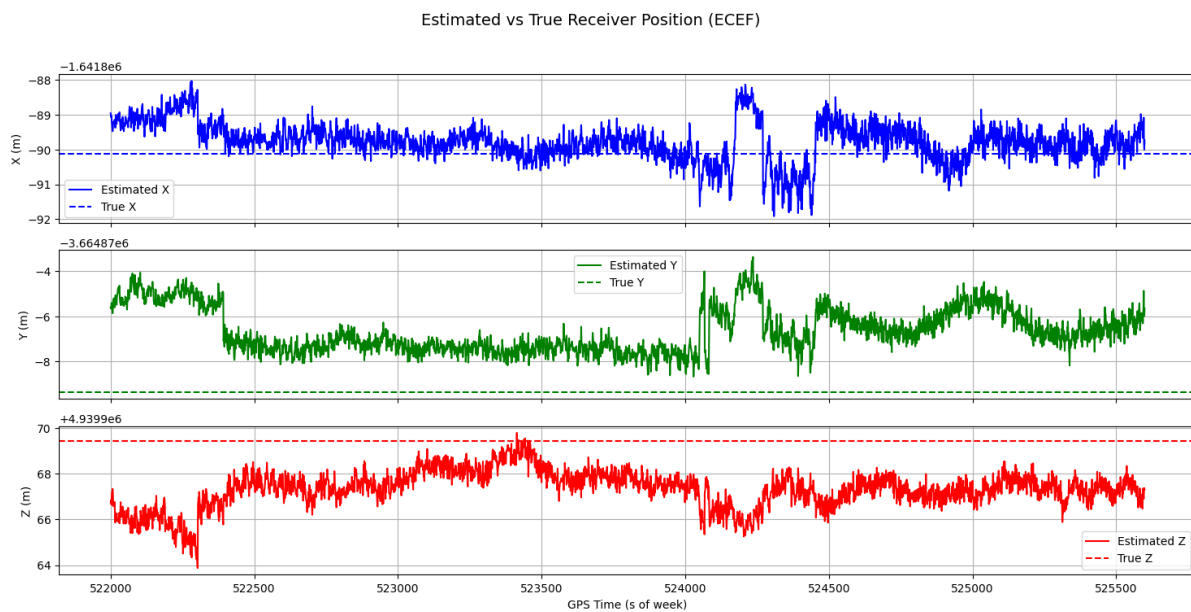


Figure 1: Estimated vs True Receiver Position (ECEF)

Receiver Clock Bias Analysis

The receiver clock bias represents the time offset between the satellite system time (GPS time) and the receiver's internal clock. Since even tiny timing errors can result in significant position errors (e.g., 1 microsecond \approx 300 meters), estimating and correcting for this bias is essential in GPS positioning.

The figure below shows how the receiver clock bias varies over time. Although it fluctuates, the bias stays within a few meters throughout the observation period. These variations are expected due to thermal drift, oscillator instability, or environmental factors affecting the receiver.

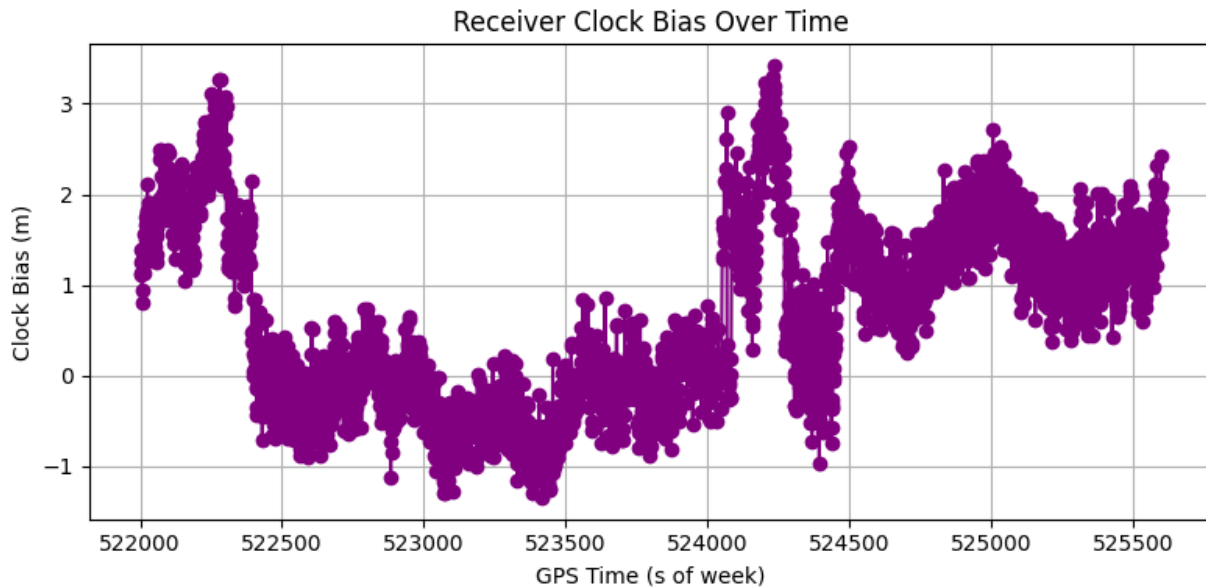


Figure 2: Receiver clock bias in meters as a function of GPS time. The bias fluctuates over time, highlighting the importance of estimating it during positioning.

DOP and Satellite Geometry Analysis

DOP (Dilution of Precision) values provide insight into how satellite geometry affects the accuracy of GPS positioning. Lower DOP values generally correspond to better satellite configurations and higher positional accuracy.

- **HDOP (Horizontal DOP)** and **VDOP (Vertical DOP)** reflect how geometry influences horizontal and vertical accuracy, respectively.
- **PDOP** combines both horizontal and vertical contributions.
- **GDOP** includes position and clock bias sensitivity.

In the plot, DOP values remain below 2 throughout the session — an indication of a strong, well-distributed satellite geometry. Occasional variations correspond to brief drops in satellite availability or geometry shifts, but the overall positioning environment remains favorable.

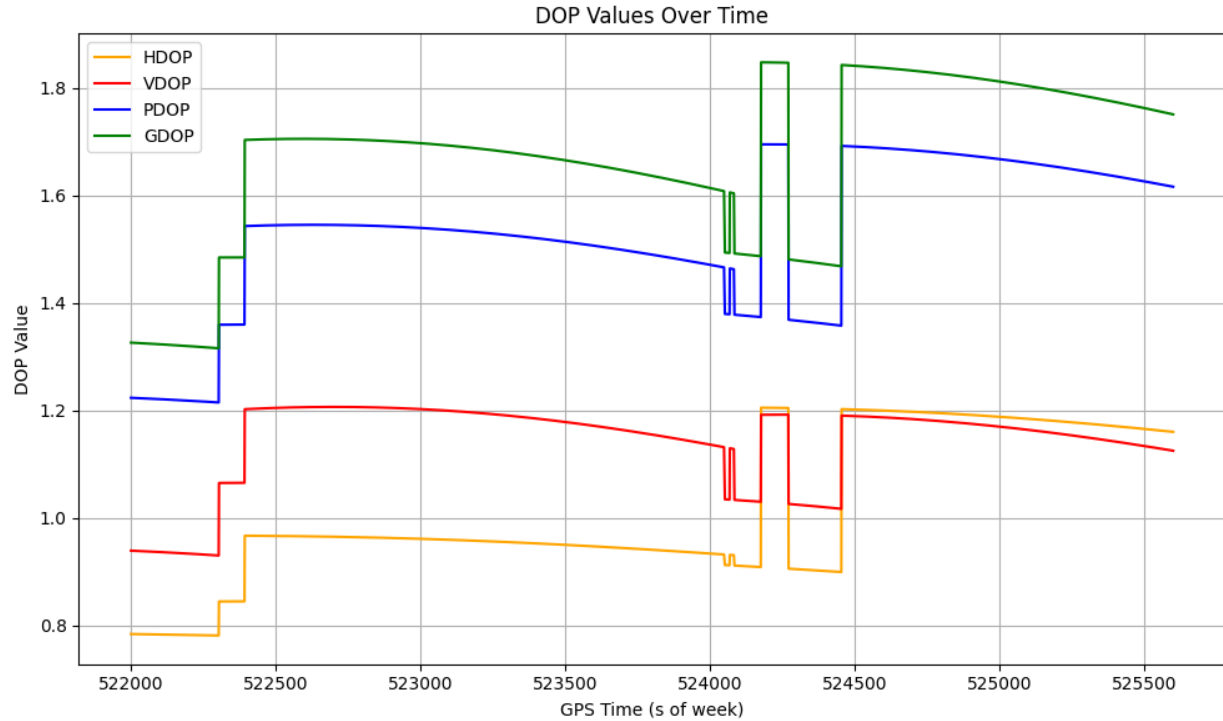


Figure 3: DOP values over time. All values are within acceptable ranges, indicating reliable positioning conditions throughout most of the observation window.

Uncertainty in Position Estimation

To assess how well the computed position errors match expected uncertainty, we visualize the ENU (East, North, Up) errors along with shaded uncertainty bands derived from DOP values multiplied by an assumed standard deviation (σ). This illustrates how satellite geometry and measurement noise propagate into the final position solution.

We evaluated two different assumptions:

1. Non-uniform standard deviations:

- $\sigma = 1.5$ m for horizontal components (East, North)
- $\sigma = 3.5$ m for the vertical component (Up)

This reflects the fact that vertical accuracy is generally worse due to poorer satellite geometry in the zenith direction.

2. Uniform standard deviation:

- $\sigma = 1$ m for all directions

This assumes measurement uncertainty is equal in all axes.

The first approach with different sigma better captures the true variation in errors — the actual East, North, and especially Up errors fall within the uncertainty bands for most epochs. In contrast, the uniform σ underestimates the uncertainty in the Up component, where true errors often exceed the predicted bounds.

This analysis shows that using **direction-specific σ values** offers a more accurate and realistic estimation of positioning uncertainty.

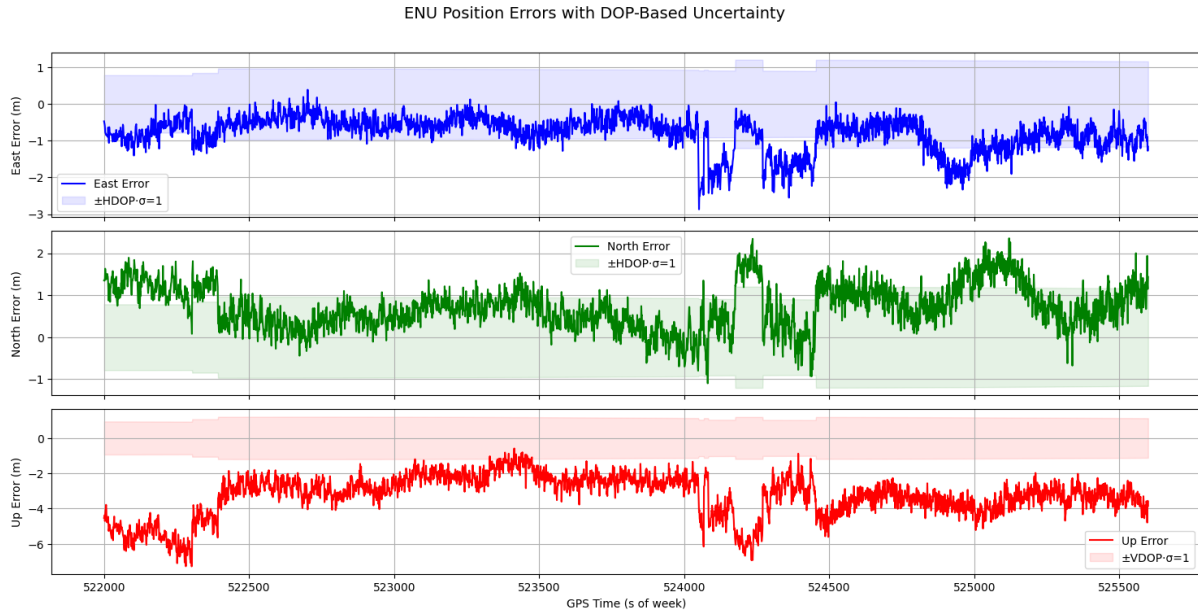


Figure 4: ENU Position Errors with DOP-Based Uncertainty ($\sigma = 1$ m uniform)

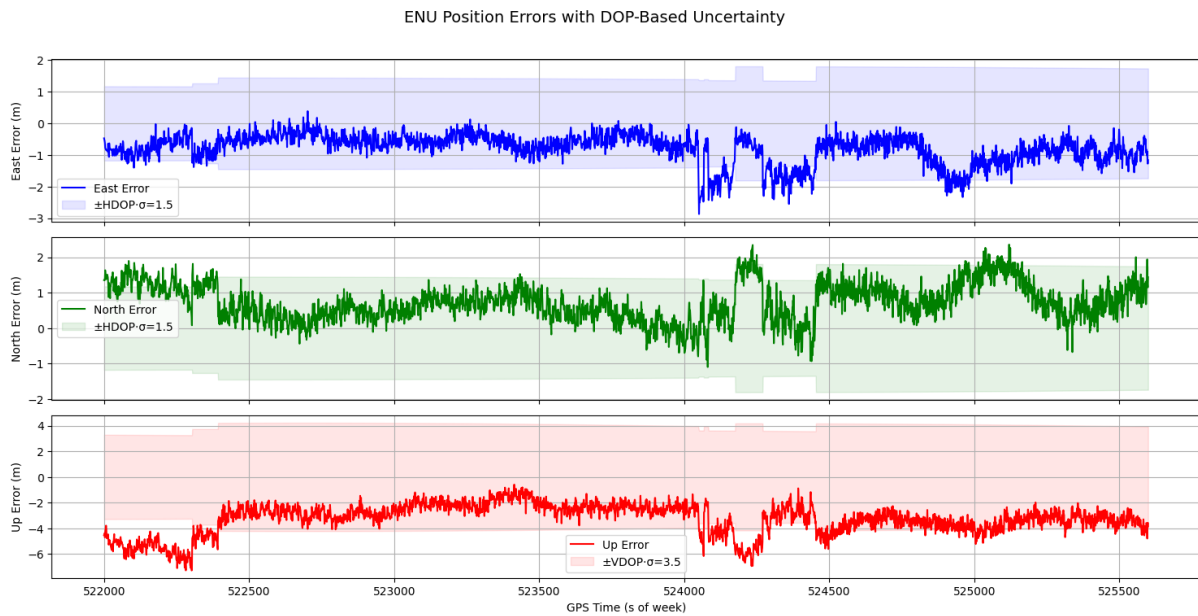


Figure 5: ENU Position Errors with DOP-Based Uncertainty ($\sigma = 1.5$ m horizontal, 3.5 m vertical)

Accuracy Assessment

To evaluate the overall positioning accuracy, we analyzed the distribution of ENU position errors and computed key statistical measures: maximum absolute error, mean error, standard deviation (STD), and root mean square error (RMS) for each axis.

Figure 6 shows the error histograms for East, North, and Up directions. The distributions are relatively centered for East and North, but the Up error is notably biased and has a larger spread.

The computed error statistics are summarized below:

Figure 6: Histograms of position errors in East, North, and Up directions.

Axis	Max Abs Error (m)	Mean Error (m)	STD (m)	RMS (m)
East	2.869	-0.819	0.470	0.945
North	2.356	0.738	0.533	0.910
Up	7.283	-3.295	1.142	3.487

