Computational Finance with C++
Tutorial: The Newton-Raphson Method

Exercise 1. Let $f : \mathbb{R} \to \mathbb{R}$ be given by $f(x) = (x - x_0)^4$, where $x_0 \in \mathbb{R}$ is a constant. Suppose that we apply the Newton method to the problem of minimising f without using a step size strategy (i.e. we always select $\alpha_k = 1$.)

a. Write down the update equation for Newton's method applied to this problem.

b. Let $y_k = |x_k - x_0|$, where x_k is the kth iterate in Newton's method. Show that the sequence $\{y_k\}$ satisfies $y_{k+1} = \frac{2}{3}y_k$.

c. Show that $x_k \to x_0$ for any initial guess.

Exercise 2. Consider the following function $f(x_1, x_2) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2$, where $x = [x_1, x_2]^\top$

a. With a starting point of $[0, 0]^\top$ apply two iterations of the Newtown algorithm (use a constant step-size of 1). Remember that a 2×2 matrix has the inverse,

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

b. Repeat part (b) using a fixed step size $a_k = 0.05$ and the steepest descent algorithm.