23K-0030

Assignment III

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Date:

Q1 (a) $\frac{b}{a} = k$, $\frac{c}{a} = m$

~~add $\frac{c}{a}$ to $\frac{b}{a}$~~

$\therefore \frac{b+c}{a}$

$\frac{b}{a} + \frac{c}{a} = (k+m) \Rightarrow L$ Taking L as $(k+m)$ where L, k and m are all integers

$\therefore \frac{b+c}{a}$ is proved true as $\frac{b+c}{a} = L$

(b) $\frac{b}{a} = k$

mult. c both sides

$\frac{bc}{a} = kc$ where kc will result in an integer ~~if c is an integer~~ so $\frac{bc}{a}$ is true.

(c) $\frac{b}{a} = k$, $\frac{c}{b} = m$

$b = ka$, subs. into $b|c$

$\frac{c}{ka} = m$

$\frac{c}{a} = mk$

mk is an integer so $a|c$ is true

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(b) take n and n+1

$n(n+1)$

n^2 if n is odd then n+1 is even

$\therefore (2k+1)(2k)$

$2(k(2k+1)) \rightarrow$ take $k(2k+1)$ as m

$2m$, hence proved. The converse will also be true :-

$(2k)(2k+1) \Rightarrow 2m$

(c) $p = (2k+1)$, $q = 2m+1$, $r = 2l$

$(2k+1)(2m+1)(2l)$

$2[l(2k+1)(2m+1)] \rightarrow$ take as a

$2a$ hence proved

Q2(a) 'There are real numbers' implies that this expression does not necessarily have to hold for all real numbers.

if $a=0$ or $b=0$

$\sqrt{0+b} = \sqrt{0} + \sqrt{b} = \sqrt{b}$ ABSER

$\sqrt{a+0} = \sqrt{a} + \sqrt{0} = \sqrt{a}$

$\sqrt{0+0} = \sqrt{0} + \sqrt{0} = 0$ ∴ the statement is true when either a or b or both are zero

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(b) $r = \frac{a}{b}$

$r^2 = \frac{a^2}{b^2}$

$r^2 = \left(\frac{a}{b}\right)^2$ proved

(c) check 6

$2^6 - 1 = 63 \rightarrow$ not a prime

check 7

$2^7 - 1 = 127 \rightarrow$ is prime

therefore statement is true

Q3 (a) $n = 2k + 1$, $m = 2p + 1$

~~prove that n+m is odd~~ assume n+m = odd

$2k + 1 + 2p + 1 = 2a + 1$

$2k + 2p + 2 = 2a + 1$

$2(k + p + 1) = 2a + 1$

$2L = 2a + 1$ $\rightarrow$ n+m = odd disproved so n+m = even must be true

LHS ≠ RHS

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(b) ~~Proof by~~ Assume that $P|(a+1)$ is true

$\therefore \frac{a}{p} = k$

$a = kp$

$a+1 = kp+1$ → no way to attain form $\frac{a+1}{p}$ = integer

LHS ≠ RHS so statement is disproved therefore the original statement is proved

(c) Assume that ~~primes~~ are there is a Max prime and take is as any.

$primes = \{p_1, p_2, p_3 \ldots p_n\}$

make a new number N as:-

$N = p_1 p_2 p_3 \ldots p_n + 1$ (product of all primes + 1)

Division by any prime yields a remainder of 1 so it is not divisible by any prime leaving only two cases. Either N is prime or N is composite. ~~and~~ If it is composite, it must have a prime divisor which is not in the set therefore both cases ~~&~~ tell us that a separate prime exists disproving our assumption and proving the original statement

Q4(a) Prove that if $x \leq 1$ and $x \geq -1$ then $|x| \leq 1$

if $x \geq 0$ then ~~$|x| \leq 1$~~ $x \leq 1$ and $|x| \leq 1$
if $x < 0$ then $x \geq -1$ and $|x| \leq 1$

Proved in each case

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(b) Prove that if m or n is odd and m or n is even then m+n is odd.

if m or n is odd and m or n is even, this statement means that m and n are even and odd or odd and even.

therefore; $n = 2k$, $n = 2p+1$

$2k + 2p + 1$

$2(k+p) + 1$ $(k+p)$ is k

$\therefore 2L + 1$ odd sum so proven

(c) if a or b is negative, or odd then ab is not divisble by 4

~~consider b is negative while a is even and positive~~

$b = (2k+1)$, $a = 2k$ $\quad a = 2k+1$, $b = 2k+1$

$\frac{2k(2k+1)}{4} = p$ where p is an integer $\quad \frac{(2k+1)(2k+1)}{4} = p$

$\frac{2(2k^2+k)}{4} = p$ $\quad \frac{4k^2+2k+2k+1}{4} = p$

Not an integer $\leftarrow \frac{(2k^2+k)}{2} = p$ $\quad$ Not an integer $\leftarrow \frac{4k^2+4k+1}{4} = p$

Contraposition true in both cases so original statement is also true

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Q5(a) i Take the prime 7

7+2=9 and 9 is not prime

ii Take the even number 6

$\frac{6}{4} = \frac{2}{3}$ no integer answer so 6 is even but is not divisible by four.

(b) (i) take positive integer 4

4 is divisible by 1, 2 and 4 therefore it is not prime

(ii) Take the real number 0.5

$0.5^2 = 0.25$

$0.25 < 0.5$ so the statement is false

(c) Disprove by ~~contradiction~~ counter-example

~~therefore~~ take $\sqrt{2}$ and $1/\sqrt{2}$

$\sqrt{2} \times \frac{1}{\sqrt{2}} = 1 \rightarrow$ 1 is rational so statement is proven false.

Q6 (a) Consider $6-7\sqrt{2}$ to be rational

$\therefore 6-7\sqrt{2} = \frac{p}{q}$

$-7\sqrt{2} = \frac{p}{q} - 6$

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$7\sqrt{2} = \frac{b-p}{q}$

$\sqrt{2} = \frac{b-p}{7q}$

$\sqrt{2}$ is known to be irrational so LHS ≠ RHS and contradiction found so original statement is true

(b) Consider $\sqrt{2}+\sqrt{3}$ to be rational

$\sqrt{2}+\sqrt{3} = \frac{p}{q}$

$\sqrt{2} = \frac{p}{q} - \sqrt{3}$ take $\frac{p}{q}$ as r

$\sqrt{2} = r - \sqrt{3}$

square both sides

$2 = r^2 - 2r\sqrt{3} + 3$

$2r\sqrt{3} = r^2 + 1$

$\sqrt{3} = \frac{(r+1)(r-1)}{2r}$

LHS is irrational while RHS is rational so contradiction found and original statement proven.

(c) Prove by contradiction

Assume for all 2, the expression is not a perfect square

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false 2=1

- $4(1+1+1)+-3(1)$

$4(3)-3=9$

9 is a perfect square (3^2) so the contradictory statement is proved false and the original proven true.

Q7 Base case :- $n=1$

LHS $1^2.1$

RHS $\frac{1(1+1)(2(1)+1)}{6}$

$\frac{1(2)(3)}{6} \Rightarrow \frac{6}{6} \Rightarrow 1$ LHS=RHS

Inductive case :- $n=k+1$

$LHS \Rightarrow \frac{k(k+1)(2k+1)}{6} + (k+1)^2$

$\frac{k(k+1)(2k+1)+6(k+1)^2}{6}$

$(k+1)\left[\frac{k(2k+1)+6(k+1)}{6}\right]$

$(k+1)\left[\frac{2k^2+7k+6}{6}\right]$

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$$RHS \Rightarrow \frac{(k+1)((k+1)+1)(2(k+1)+2)}{6}$$

$$\frac{(k+1)(k+2)(2k+3)}{6}$$

$$(k+1)\left[\frac{2k^2+3k+4k+6}{6}\right]$$

$$(k+1)\left[\frac{2k^2+7k+6}{6}\right]$$

LHS = RHS hence proved

(b) Base case:- $n=1$

LHS $1^3 = 1$

RHS $\frac{1}{4}(1)^2(1+1)^2$

$\frac{1}{4}(1)(2)^2 \Rightarrow \frac{4}{4} \Rightarrow 1$ so LHS = RHS

Inductive case:- $n = k+1$

LHS $\frac{1}{4}(k)^2(k+1)^2 + (k+1)^3$

$$(k+1)^2\left[\frac{k^2(k+1)^2+4}{4}\right]$$

$$(k+1)^2\left[\frac{k^2+4(k+1)}{4}\right]$$

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RHS $\frac{1}{4}(k+1)^2(k+1+1)^2$

$\frac{1}{4}(k+1)^2(k+2)^2$

$(k+1)^2\left[\frac{k^2+4k+4}{4}\right]$

$(k+1)^2\left[\frac{k^2+4(k+1)}{4}\right]$

LHS=RHS hence proved

(c) Base case :- $n=1$

LHS $\frac{1}{1(1+1)} \Rightarrow \frac{1}{2} = \frac{1}{2}$

RHS $\frac{1}{1+1} \Rightarrow \frac{1}{2}$ LHS=RHS

Inductive case :- $n=k+1$

LHS $\frac{1}{k(k+2)} \Rightarrow \frac{k}{k+1} + \frac{1}{(k+1)(k+1+1)}$

$\frac{k}{k+1} + \frac{1}{(k+1)(k+2)}$

$\frac{k(k+2)+1}{(k+1)(k+2)}$

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RHS $\frac{k+1}{k+1+1}$

$\frac{k+1}{(k+2)^2}$ $\frac{k+1}{k+2}$ → This term seems simplified so revisit LHS and simplify that

LHS $\frac{k^2+2k+1}{(k+1)(k+2)}$

$\frac{k^2+k+k+1}{(k+1)(k+2)}$

$\frac{k(k+1)+1(k+1)}{(k+1)(k+2)}$

$\frac{(k+1)^2}{(k+1)(k+2)}$

$\frac{k+1}{k+2}$ so LHS = RHS hence proved

Q8 (a) $a = dq + r$

(i) $19 = 7q + r$

$\left[\frac{19}{7}\right] = 2$ → quotient

$\therefore 19 = 7 \times 2 + r$

$r = 19 - 14 = 5$ → remainder

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(ii) $-111 = 11q + r$

$$\left\lfloor \frac{-111}{11} \right\rfloor = -11$$

$-111 = 11(-11) + r$

$r = -111 + 121 = 10$

(iii) $789 = 23q + r$

$$\left\lfloor \frac{789}{23} \right\rfloor = 34$$

$784 = 23(34) + r$

$r = 2$

(iv) $1001 = 13q + r$

$$\left\lfloor \frac{1001}{13} \right\rfloor = 77$$

$1001 - 13(77) = r$

$r = 0$

(v) $10 = 19q + r$

$$\left\lfloor \frac{10}{19} \right\rfloor = 0$$

$r = 10$

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(vi) $3 = 5q + r$

$$\left\lfloor \frac{3}{5} \right\rfloor = 0$$

$r = 3$

(vii) $-1 = 3q + r$

$$\left\lfloor \frac{-1}{3} \right\rfloor = -1$$

$-1 = -3 + r$

$r = 2$

(viii) $4 = 1q + r$

$$\left\lfloor \frac{4}{1} \right\rfloor = 4$$

$r = 0$

(b) (i) a div m $\Rightarrow \left\lfloor \frac{a}{m} \right\rfloor$, a mod m $\Rightarrow a - m\left[\left\lfloor \frac{a}{m} \right\rfloor\right]$

a div m $\Rightarrow \left\lfloor \frac{-111}{99} \right\rfloor = -2$

a mod m $\Rightarrow -111 - 99(-2)$

$\Rightarrow 87$

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(ii) a div m => $\left\lfloor \frac{-9999}{101} \right\rfloor = -99$

a mod m => $-9999 - 101(-99)$

=> 0

(iii) a div m => $\left\lfloor \frac{10299}{999} \right\rfloor = 10$

a mod m => $10299 - 999(10)$

=> 309

(iv) a div m => $\left\lfloor \frac{123456}{1001} \right\rfloor = 123$

a mod m => $123456 - 1001(123)$

=> 333

(c)(i) For congruency :- a-b must be divisible by m

$\frac{80-5}{17}$

$\frac{75}{17}$ indivisible so 80 is not congruent

(ii) $\frac{103-5}{17}$ => $\frac{98}{17}$ indivisible so 103 is not congruent

(iii) $\frac{-29-5}{17} \Rightarrow \frac{-34}{17}$ is divisible so -23 is congruent

(iv) $\frac{-122-5}{17} \Rightarrow \frac{-127}{17}$ is not divisible so -122 is not cagruent

Q9 (x) (a) (i) For numbers to be relatively prime, their gcd must be 1

gcd (11, 15) = 1 since 11 is prime and 15 is not a multiple of 11 so they share no common factor other than 1

gcd (15, 19) = 1 since 19 is prime and 15 is not a multiple of 19

gcd (19, 11) = 1 both are prime

so the set {11, 15, 19} is relatively pairwise prime.

(ii) gcd (14, 15) = 1 since they don't have any common factors

gcd (14, 21) = 7 Common factor found so set is not pairwise relatively prime.

(iii) gcd (12, 31) = 1 since 31 is prime and 12 is less than 31.

gcd (12, 17) = 1 since 17 is prime and 12 is less than 17.

gcd (12, 37) = 1 since 37 is prime and 12 is less than 37

gcd (17, 31) = 1 since both are prime

gcd (17, 37) = 1 since both are prime

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gcd (31, 37) = 1 since both are prime so set is pairwise relatively prime

(iv) gcd (7,8) = 1 since 7 is prime and 7 is not a factor of 8

gcd (7,9) = 1 since 7 is prime and 9 is not a multiple of 7

gcd (7,11) = 1 since both are prime

gcd (8,9) = 1 since 8 and 9 have no common factors except 1

gcd (8,11) = 1 since 11 is prime and 8 is less than 11

gcd (9,11) = 1 since 11 is a prime and 9 is less than 11

So set is pairwise relatively prime

(b) (i) $88 \div 2 = 44$

$44 \div 2 = 22$

$22 \div 2 = 11$

$11 \div 11 = 1$

$\therefore 88 = 2 \times 2 \times 2 \times 11 = 2^3 \times 11$

(ii) $126 \div 2 = 63$

$63 \div 3 = 21$

$21 \div 3 = 7$

$7 \div 7 = 1$

$\therefore 126 = 2 \times 3 \times 3 \times 7 = 2 \times 3^2 \times 7$

(iii) $729 \div 3 = 243$

$243 \div 3 = 81$

$81 \div 3 = 27$

$27 \div 3 = 9$

$9 \div 3 = 3$

$3 \div 3 = 1$

$729 = 3 \times 3 \times 3 \times 3 \times 3 \times 3$

$= 3^6$

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(iv) $1001 \div 7 = 143$

$143 \div 11 = 13$

$13 \div 13 = 1$

$1001 = 7 \times 11 \times 13$

(v) $1111 \div 11 = 101$

$101 \div 101 = 1$

$1111 = 11 \times 101$

(vi) ~~$909 \div 303 =$~~ $909 \div 3 = 303$

$303 \div 3 = 101$

$101 \div 101 = 1$

$909 = 3 \times 3 \times 101 = 3^2 \times 101$

(c) (i) Prime Factorization:-

$11 \Rightarrow 11 \div 11 = 1$

$\therefore 11 = 11$

$15 \Rightarrow 15 \div 3 = 5$

$5 \div 5 = 1$

$15 = 3 \times 5$

$105 \Rightarrow 105 \div 3 = 35$

$35 \div 5 = 7$

$7 \div 7 = 1$

$105 = 3 \times 5 \times 7$

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gcd (11, 15, 105) = 1 since they have no common prime factors

LCM (11, 15, 105) :-

$11 = 11^1$, $15 = 3^1 \times 5^1$, $105 = 3^1 \times 5^1 \times 7^1$

multiply all the highest powers of the unique prime factors

$11^1 \times 3^1 \times 5^1 \times 7^1 = 1155$

(ii) Prime factorization :-

$14 \Rightarrow 14 \div 2 = 7$

$7 \div 7 = 1$

$14 = 2 \times 7$

$15 \Rightarrow 15 \div 3 = 5$

$5 \div 5 = 1$

$15 = 3 \times 5$

$21 \Rightarrow 21 \div 3 = 7$

$7 \div 7 = 1$

$21 = 3 \times 7$

~~No gcd since there a~~ gcd = 1 since there are no common prime factors

LCM = $2^1 \times 5^1 \times 3^1 \times 7^1 = 210$

(iii) Prime factorization :-

$12 \Rightarrow 12 \div 2 = 6$

$6 \div 2 = 3$

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$3 \div 3 = 1$

$12 = 2 \times 2 \times 3 = 2^2 \times 3$

$72 \Rightarrow 72 \div 2 = 36$
$36 \div 2 = 18$
$18 \div 2 = 9$
$9 \div 3 = 3$
$3 \div 3 = 1$

$72 = 2 \times 2 \times 2 \times 3 \times 3 = 2^3 \times 3^2$

$31 \Rightarrow 31 \div 31 = 1$

$31 = 31$

$32 \Rightarrow 32 \div 2 = 16$
$16 \div 2 = 8$
$8 \div 2 = 4$
$4 \div 2 = 2$
$2 \div 2 = 1$

$32 = 2 \times 2 \times 2 \times 2 \times 2 = 2^5$

(iv) ~~Prime factorization~~ ~~gcd = 2^2 = 4~~ gcd = 1 since no common prime factors

$17 \Rightarrow LCM = 2^5 \times 3^2 \times 31 = 8928$

(iv) Prime factorization:-

$17 \Rightarrow 17 \div 17 = 1$
$17 = 17$

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18 => 18 ÷ 2 = 9
9 ÷ 3 = 3
3 ÷ 3 = 1

$18 = 2 \times 3 \times 3 = 2 \times 3^2$

192 => 192 ÷ 2 = 96
96 ÷ 2 = 48
48 ÷ 2 = 24
24 ÷ 2 = 12
12 ÷ 2 = 6
6 ÷ 2 = 3
3 ÷ 3 = 1

$192 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 = 2^6 \times 3$

g 1021 => 1021 ÷ 1021 = 1

1021 = 1021

gcd = 1 since there are no common prime factors.

$LCM = 2^6 \times 3^2 \times 17 \times 1021 = 999732$

Q1o (a)(i) gcd (144, 89)

144 = 89 x 1 + 55
89 = 55 x 1 + 34
55 = 34 x 1 + 21
34 = 21 x 1 + 13
21 = 13 x 1 + 8
13 = 8 x 1 + 5

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$8 = 5 \times 1 + 3$

$5 = 3 \times 1 + 2$

$3 = 2 \times 1 + 1$

Now work backwards

$1 = 3 - 1 \cdot 2$

$2 = 5 - 1 \cdot 3$

$3 = 8 - 1 \cdot 5$

$5 = 13 - 1 \cdot 8$

$8 = 21 - 1 \cdot 13$

$13 = 34 - 1 \cdot 21$

$21 = 55 - 1 \cdot 34$

$34 = 89 - 1 \cdot 55$

$55 = 144 - 1 \cdot 89$

$1 = 3 - 1 \cdot (5 - 1 \cdot 3)$

$1 = 3 - 1 \cdot 5 + 1 \cdot 3$

$1 = 2 \cdot 3 - 1 \cdot 5$

$1 = 2 \cdot (8 - 1 \cdot 5) - 1 \cdot 5$

$1 = 2 \cdot 8 - 2 \cdot 5 - 1 \cdot 5 = 2 \cdot 8 - 3 \cdot 5$

$1 = 2 \cdot 8 - 3 \cdot (13 - 1 \cdot 8)$

$1 = 2 \cdot 8 - 3 \cdot 13 + 3 \cdot 8 = 5 \cdot 8 - 3 \cdot 13$

$1 = 5 \cdot (21 - 1 \cdot 13) - 3 \cdot 13$

$1 = 5 \cdot 21 - 5 \cdot 13 - 3 \cdot 13 = 5 \cdot 21 - 8 \cdot 13$

$1 = 5 \cdot 21 - 8 \cdot (34 - 1 \cdot 21)$

$1 = 5 \cdot 21 - 8 \cdot 34 + 8 \cdot 21 = 13 \cdot 21 - 8 \cdot 34$

$1 = 13 \cdot (55 - 1 \cdot 34) - 8 \cdot 34$

$1 = 13 \cdot 55 - 13 \cdot 34 - 8 \cdot 34 = 13 \cdot 55 - 21 \cdot 34$

$1 = 13 \cdot 55 - 21 \cdot (89 - 1 \cdot 55)$

$1 = 13 \cdot 55 - 21 \cdot 89 + 21 \cdot 55 = 34 \cdot 55 - 21 \cdot 89$

$1 = 34 \cdot (144 - 1 \cdot 89) - 21 \cdot 89$

$1 = 34 \cdot 144 - 34 \cdot 89 - 21 \cdot 89 = \boxed{34 \cdot 144 - 55 \cdot 89}$ Answer

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(ii) GCD (100...01, 10000...01)

$10000...01 = 100...01 \times 99 + 902$
$100...01 = 902 \times 1 + 99$
$902 = 99 \times 9 + 11$
$99 = 11 \times 9 + 0 \rightarrow$ discard
work backwards

$11 = 902 - 9.99$
$99 = 100...01 - 1.902$
$902 = 10000...01 - 99.100...01$

$11 = 1.902 - 9.(100...01 - 1.902)$
$11 = 1.902 - 9.100...01 + 9.902 = 10.902 - 9.100...01$
$11 = 10.(10000...01 - 99.100...01) - 9.100...01$
$11 = 10.10000...01 - 990.100...01 - 9.100...01 = \boxed{10.10000...01 - 999.100...01}$

Answer

(b)(i) $55x \equiv 34 \pmod{89}$

Find modular inverse of 55 but check if 55 and 34 are pairwise primes

Prime Factorization:-

$55 \Rightarrow 55 \div 5 = 11$
$11 \div 11 = 1$
$55 = 5 \times 11$

$34 \Rightarrow 34 \div 2 = 17$
$17 \div 17 = 1$
$34 = 2 \times 17$

Now use Euclidean algorithm

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$55u \equiv 1 \pmod{89}$

$89 = 55 \times 1 + 34$
$55 = 34 \times 1 + 21$
$34 = 21 \times 1 + 13$
$21 = 13 \times 1 + 8$
$13 = 8 \times 1 + 5$
$8 = 5 \times 1 + 3$
$5 = 3 \times 1 + 2$
$3 = 2 \times 1 + 1$
Work backwards

$1 = 1 \cdot 3 - 1 \cdot 2$
$2 = 1 \cdot 5 - 1 \cdot 3$
$3 = 1 \cdot 8 - 1 \cdot 5$
$5 = 1 \cdot 13 - 1 \cdot 8$
$8 = 1 \cdot 21 - 1 \cdot 13$
$13 = 1 \cdot 34 - 1 \cdot 21$
$21 = 1 \cdot 55 - 1 \cdot 34$
$34 = 1 \cdot 89 - 1 \cdot 55$

$1 = 1 \cdot 3 - 1 \cdot (1 \cdot 5 - 1 \cdot 3)$
$1 = 1 \cdot 3 - 1 \cdot 5 + 1 \cdot 3 = 2 \cdot 3 - 1 \cdot 5$
$1 = 2 \cdot (1 \cdot 8 - 1 \cdot 5) - 1 \cdot 5$
$1 = 2 \cdot 8 - 2 \cdot 5 - 1 \cdot 5 = 2 \cdot 8 - 3 \cdot 5$
$1 = 2 \cdot 8 - 3 \cdot (1 \cdot 13 - 1 \cdot 8)$
$1 = 2 \cdot 8 - 3 \cdot 13 + 3 \cdot 8 = 5 \cdot 8 - 3 \cdot 13$
$1 = 5 \cdot (1 \cdot 21 - 1 \cdot 13) - 3 \cdot 13$
$1 = 5 \cdot 21 - 5 \cdot 13 - 3 \cdot 13 = 5 \cdot 21 - 8 \cdot 13$
$1 = 5 \cdot 21 - 8 \cdot (1 \cdot 34 - 1 \cdot 21)$
$1 = 5 \cdot 21 - 8 \cdot 34 + 8 \cdot 21 = 13 \cdot 21 - 8 \cdot 34$
$1 = 13 \cdot (1 \cdot 55 - 1 \cdot 34) - 8 \cdot 34$

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$1 = 13 \cdot 55 - 13 \cdot 34 - 8 \cdot 34 = 13 \cdot 55 - 21 \cdot 34$

$1 = 13 \cdot 55 - 21 \cdot (1 \cdot 89 - 1 \cdot 55)$

$1 = 13 \cdot 55 - 21 \cdot 89 + 21 \cdot 55 = \underbrace{34} \cdot 55 - 21 \cdot 89$

↓ modular inverse

$\therefore \quad 55x \equiv 34 \pmod{89}$

$34 \times 55x \equiv 34 \times 34 \pmod{89}$

$x \equiv 1156 \pmod{89}$

$x \equiv 88$ Ans

(ii) $89x \equiv 2 \pmod{232}$

Prime factorization:-

$89 \div 89 = 1$

$89 = 89$

$2 \div 2 = 1$

$2 = 2$

∴ They are pairwise prime

$89x \equiv 1 \pmod{232}$

$232 = 89 \times 2 + 54$

$89 = 54 \times 1 + 35$

$54 = 35 \times 1 + 19$

$35 = 19 \times 1 + 16$

$19 = 16 \times 1 + 3$

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$16 = 3 \times 5 + 1$

work backwards

$1 = 1 \cdot 16 - 5 \cdot 3$
$3 = 1 \cdot 19 - 1 \cdot 16$
$16 = 1 \cdot 35 - 1 \cdot 19$
$19 = 1 \cdot 54 - 1 \cdot 35$
$35 = 1 \cdot 89 - 1 \cdot 54$
$54 = 1 \cdot 232 - 2 \cdot 89$

$1 = 1 \cdot 16 - 5 \cdot (1 \cdot 19 - 1 \cdot 16)$
$1 = 1 \cdot 16 - 5 \cdot 19 + 5 \cdot 16 = 6 \cdot 16 - 5 \cdot 19$
$1 = 6 \cdot (1 \cdot 35 - 1 \cdot 19) - 5 \cdot 19$
$1 = 6 \cdot 35 - 6 \cdot 19 - 5 \cdot 19 = 6 \cdot 35 - 11 \cdot 19$
$1 = 6 \cdot 35 - 11 \cdot (1 \cdot 54 - 1 \cdot 35)$
$1 = 6 \cdot 35 - 11 \cdot 54 + 11 \cdot 35 = 17 \cdot 35 - 11 \cdot 54$
$1 = 17 \cdot (1 \cdot 89 - 1 \cdot 54) - 11 \cdot 54$
$1 = 17 \cdot 89 - 17 \cdot 54 - 11 \cdot 54 = 17 \cdot 89 - 28 \cdot 54$
$1 = 17 \cdot 89 - 28 \cdot (1 \cdot 232 - 2 \cdot 89)$
$1 = 17 \cdot 89 - 28 \cdot 232 + 56 \cdot 89 = \underbrace{73}_{\text{inverse}} \cdot 89 - 28 \cdot 232$

$\therefore 73 \times 89x \equiv 2 \times 73 \pmod{232}$

$x \equiv 146 \pmod{232}$

$x \equiv 146$

(c) (i) $\gcd(2, 17) = 1$

$\therefore 17 = 8 \times 2 + 1$
$1 = 1 \cdot 17 - 8 \cdot 2$

No need to check factorization as prime property of pair is already established in question.
so inverse of 2 is 8

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(ii) gcd(34, 89) = 1

$89 = 34 \times 2 + 21$

$34 = 21 \times 1 + 13$

$21 = 13 \times 1 + 8$

$13 = 8 \times 1 + 5$

$8 = 5 \times 1 + 3$

$5 = 3 \times 1 + 2$

$3 = 2 \times 1 + 1$

Work backwards

$1 = 1 \cdot 3 - 1 \cdot 2$

$2 = 1 \cdot 5 - 1 \cdot 3$

$3 = 1 \cdot 8 - 1 \cdot 5$

$5 = 1 \cdot 13 - 1 \cdot 8$

$8 = 1 \cdot 21 - 1 \cdot 13$

$13 = 1 \cdot 34 - 1 \cdot 21$

$21 = 1 \cdot 89 - 2 \cdot 34$

$1 = 1 \cdot 3 - 1 \cdot (1 \cdot 5 - 1 \cdot 3)$

$1 = 1 \cdot 3 - 1 \cdot 5 + 1 \cdot 3 = -1 \cdot 5 + 2 \cdot 3$

$1 = -1 \cdot 2 \cdot (1 \cdot 8 - 1 \cdot 5) - 1 \cdot 5$

$1 = 2 \cdot 8 - 2 \cdot 5 - 1 \cdot 5 = 2 \cdot 8 - 3 \cdot 5$

$1 = 2 \cdot 8 - 3 \cdot (1 \cdot 13 - 1 \cdot 8)$

$1 = 2 \cdot 8 - 3 \cdot 13 + 3 \cdot 8 = 5 \cdot 8 - 3 \cdot 13$

$1 = 5 \cdot (1 \cdot 21 - 1 \cdot 13) - 3 \cdot 13$

$1 = 5 \cdot 21 - 5 \cdot 13 - 3 \cdot 13 = 5 \cdot 21 - 8 \cdot 13$

$1 = 5 \cdot 21 - 8 \cdot (1 \cdot 34 - 1 \cdot 21)$

$1 = 5 \cdot 21 - 8 \cdot 34 + 8 \cdot 21 = 13 \cdot 21 - 8 \cdot 34$

$1 = 13 \cdot (1 \cdot 89 - 2 \cdot 34) - 8 \cdot 34$

$1 = 13 \cdot 89 - 26 \cdot 34 - 8 \cdot 34 = 13 \cdot 89 - 34 \cdot 34$

modular inverse of 34 is -34.

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Adjust to positive:- $89 - 34 = 55$

$\boxed{55}$ Answer

(iii) $\gcd(144, 233) = 1$

$233 = 144 \times 1 + 89$
$144 = 89 \times 1 + 55$
$89 = 55 \times 1 + 34$
$55 = 34 \times 1 + 21$
$34 = 21 \times 1 + 13$
$21 = 13 \times 1 + 8$
$13 = 8 \times 1 + 5$
$8 = 5 \times 1 + 3$
$5 = 3 \times 1 + 2$
$3 = 2 \times 1 + 1$

Work backwards

$1 = 1 \cdot 3 - 1 \cdot 2$
~~$1 = 1$~~ $2 = 1 \cdot 5 - 1 \cdot 3$
$3 = 1 \cdot 8 - 1 \cdot 5$
$5 = 1 \cdot 13 - 1 \cdot 8$
$8 = 1 \cdot 21 - 1 \cdot 13$
$13 = 1 \cdot 34 - 1 \cdot 21$
$21 = 1 \cdot 55 - 1 \cdot 34$
$34 = 1 \cdot 89 - 1 \cdot 55$
$55 = 1 \cdot 144 - 1 \cdot 89$
$89 = 1 \cdot 233 - 1 \cdot 144$

$1 = 1 \cdot 3 - 1 \cdot (1 \cdot 5 - 1 \cdot 3)$
$1 = 1 \cdot 3 - 1 \cdot 5 + 1 \cdot 3 = 2 \cdot 3 - 1 \cdot 5$
$1 = 2 \cdot (1 \cdot 8 - 1 \cdot 5) - 1 \cdot 5$
$1 = 2 \cdot 8 - 2 \cdot 5 - 1 \cdot 5 = 2 \cdot 8 - 3 \cdot 5$
$1 = 2 \cdot 8 - 3 \cdot (1 \cdot 13 - 1 \cdot 8)$
$1 = 2 \cdot 8 - 3 \cdot 13 + 3 \cdot 8 = 5 \cdot 8 - 3 \cdot 13$
$1 = 5 \cdot (1 \cdot 21 - 1 \cdot 13) - 3 \cdot 13$
$1 = 5 \cdot 21 - 5 \cdot 13 - 3 \cdot 13 = 5 \cdot 21 - 8 \cdot 13$
$1 = 5 \cdot 21 - 8 \cdot (1 \cdot 34 - 1 \cdot 21)$
$1 = 5 \cdot 21 - 8 \cdot 34 + 8 \cdot 21 = 13 \cdot 21 - 8 \cdot 34$
$1 = 13 \cdot (1 \cdot 55 - 1 \cdot 34) - 8 \cdot 34$
$1 = 13 \cdot 55 - 13 \cdot 34 - 8 \cdot 34 = 13 \cdot 55 - 21 \cdot 34$
$1 = 13 \cdot 55 - 21 \cdot (1 \cdot 89 - 1 \cdot 55)$
$1 = 13 \cdot 55 - 21 \cdot 89 + 21 \cdot 55 = 34 \cdot 55 - 21 \cdot 89$
$1 = 34 \cdot (1 \cdot 144 - 1 \cdot 89) - 21 \cdot 89$
$1 = 34 \cdot 144 - 34 \cdot 89 - 21 \cdot 89 = 34 \cdot 144 - 55 \cdot 89$
$1 = 34 \cdot 144 - 55 \cdot (1 \cdot 233 - 1 \cdot 144)$
$1 = 34 \cdot 144 - 55 \cdot 233 + 55 \cdot 144 = 89 \cdot 144 - 55 \cdot 233$

$\therefore$ 89 is the inverse of 144 modulo 233

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(iv) gcd(200, 1001) = 1

1001 = 200 × 5 + 1

1 = 1 · 1001 − 5 · 200

so −5 is the inverse

Adjust to positive = 1001 − 5
= 996 → Ans

Q11 (a) (i) $X_{10} \equiv \sum_{i=1}^{9} i x_i \pmod{11}$

$X_{10} \equiv 1\times1 + 2\times2 + 3\times5 + 4\times9 + 5\times7 + 6\times3 + 7\times1 + 8\times2 + 9\times8 \pmod{11}$

$X_{10} \equiv 264 \pmod{11}$

$X_{10} \equiv 6$

Checking validity:-

$264 + 6\times10 \equiv 0 \pmod{11}$

$264 \equiv 0 \pmod{11}$

$264 \div 11 = 24$

$\therefore 0 \equiv 0$ so book is valid

(ii) Find inverse of 7 modulo 26

check if they are pairwise prime

Prime factorization:-

$7 \div 7 = 1 \Rightarrow 7 = 7$

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$26 \Rightarrow 26 \div 2 = 13$
$13 \div 13 = 1$

$26 = 2 \times 13$
No common prime factors

$\gcd(7, 26) = 1$

$26 = 7 \times 3 + 5$
$7 = 5 \times 1 + 2$
$5 = 2 \times 2 + 1$
work backwards

$1 = 1 \cdot 5 - 2 \cdot 2$
$2 = 1 \cdot 7 - 1 \cdot 5$
$5 = 1 \cdot 26 - 3 \cdot 7$

$1 = 1 \cdot 5 - 2 \cdot (1 \cdot 7 - 1 \cdot 5)$
$1 = 1 \cdot 5 - 2 \cdot 7 + 2 \cdot 5 = 3 \cdot 5 - 2 \cdot 7$
$1 = 3 \cdot (1 \cdot 26 - 3 \cdot 7) - 2 \cdot 7$
$1 = 3 \cdot 26 - 9 \cdot 7 - 2 \cdot 7 = 3 \cdot 26 - 11 \cdot 7$

-11 is inverse

Positive adjustment :- $26 - 11 = 15$

$\therefore$ 15 is Answer.

(b) (i) 'exam' = {05, 23, 1}, {04, 23, 06, 12}
$c = 3, n = 11 \cdot 3$
$n = 33$

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$ed \equiv 1 \pmod{33}$

find inverse of e modulo 33 to find d

$33 = 3x$

$C = M^e \pmod{n}$

$C_1 = 4^3 \pmod{33}$, $C_2 = 23^3 \pmod{33}$, $C_3 = 0^3 \pmod{33}$, $C_4 = 12^3 \pmod{33}$

$C_1 = 31$, $C_2 = 23$, $C_3 = 0$, $C_4 = 12$

Encrypted message in integer form = {31, 23, 0, 12}

(ii) Shift encryption used.

$f(p) = (p-5) \bmod 26$ is decoding function

Message in integer form = {23, 24, 19, 20, 24, 5, 16, 15, 13, 18, 11}

$23-5 \bmod (26) = 18 \Rightarrow$ "S"
$24-5 \bmod (26) = 19 \Rightarrow$ "T"
$19-5 \bmod (26) = 14 \Rightarrow$ "O"
$20-5 \bmod (26) = 15 \Rightarrow$ "P"
$24-5 \bmod (26) = 19 \Rightarrow$ "T"
$5-5 \bmod (26) = 0 \Rightarrow$ "A"
$16-5 \bmod (26) = 11 \Rightarrow$ "L"
$15-5 \bmod (26) = 10 \Rightarrow$ "K"
$13-5 \bmod (26) = 8 \Rightarrow$ "I"
$18-5 \bmod (26) = 13 \Rightarrow$ "N"
$11-5 \bmod (26) = 6 \Rightarrow$ "G"

So decoded Message is:-

"STOP TALKING"

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(a) (i) $u \equiv -1 \pmod{25}$

$x = 25k - 1$ → eq (i)

$-100 \leq 25k - 1 \leq 100$

$-99 \leq 25k \leq 101$

$-3.96 \leq k \leq 4.04$

$k = -3, -2, -1, 0, 1, 2, 3, 4$ put values in equation (i) to find x

set of integers = $\{-76, -51, -26, -1, 24, 49, 74, 99\}$

(ii) Find $5^{119} \equiv 5^{119} \bmod 59$

use Fermat's theorem

$5^{58} \equiv 1 \pmod{59}$

$\therefore 5^{58\times 2+3} = (5^{58})^2 \cdot 5^3 \equiv (1)^2 \cdot 5^3 \equiv 125 \equiv 7 \bmod 59$

$\therefore 5^{119} \equiv 7 \bmod 59$

$5^{119} \bmod 59 = 7$

Q12 (a) (i) $x = a_1 M_1 y_1 + a_2 M_2 y_2 + a_3 M_3 y_3 \pmod{m}$

$m = m_1 \times m_2 \times m_3$ → $y_k M_k \equiv 1 \pmod{m_k}$

$M_k = m/m_k$

$m = 5 \times 6 \times 7 = 210$, 5, 6 and 7 are relatively Prime

$M_1 = \frac{210}{5}$, $M_2 = \frac{210}{6}$, $M_3 = \frac{210}{7}$

$M_1 = 42$, $M_2 = 35$, $M_3 = 36$

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Find inverses of $M_k \pmod{m_k}$ for y_k

$y_1 \cdot 2 \equiv 1 \pmod 5$

$y_1 2 \equiv y_1 2 \equiv 1 \pmod 5$ → 42 is converted to 2 without repercussions because they both have the same remainder when divided by 5 so this is a way to simplify the solution.

$\therefore$ ~~1~~ $5 = 2 \times 2 + 1$

$1 = 1 \cdot 5 - 2 \cdot 2$

-2 is inverse, adjust to positive $\Rightarrow 5 - 2 = 3$

$\therefore y_1 = 3$

$y_2 \cdot 35 \equiv 1 \pmod 6$

~~$y_2 \cdot 5 \equiv 1$~~ $y_2 \cdot 35 \equiv y_2 \cdot 5 \equiv 1 \pmod 6$

$6 = 5 \times 1 + 1$

$1 = 1 \cdot 6 - 1 \cdot 5$

-1 is inverse; adjust to positive $\Rightarrow 6 - 1 = 5$

$\therefore y_2 = 5$

$y_3 \cdot 30 \equiv 1 \pmod 7$

~~$y_3 \cdot 1 \equiv$~~ $y_3 \cdot 30 \equiv y_3 \cdot 2 \equiv 1 \pmod 7$

$7 = 2 \times 3 + 1$

$1 = 1 \cdot 7 - 3 \cdot 2$

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-3 is inverse; adjust to +ive => 7-3=4

$y_3 = 4$

~~$x = 2 \times 35 \times 1 \times 34$~~

$x \equiv 1 \times 42 \times 3 + 2 \times 35 \times 5 + 3 \times 30 \times 4 \pmod{210}$

$x \equiv 836 \pmod{210}$

$x \equiv 206$ Ans

(ii) 2, 3, 5 and 11 are coprimes

$m = 2 \times 3 \times 5 \times 11 = 330$

$M_1 = \frac{330}{2}$, $M_2 = \frac{330}{3}$, $M_3 = \frac{330}{5}$, $M_4 = \frac{330}{11}$

$M_1 = 165$, $M_2 = 110$, $M_3 = 66$, $M_4 = 30$

Find y_k by inverting $M_k \pmod{m_k}$

~~165~~ $y_1 \times$ $y_1 \times 165 \equiv 1 \pmod 2$

$y_1 \times 165 \equiv y_1 \times 1 \equiv 1 \pmod 2$

No working needed as 1 is ~~produced~~ already present so the inverse is 1

$y_1 = 1$

$y_2 \times 110 \equiv 1 \pmod 3$

$y_2 \times 110 \equiv y_2 \times 2 \equiv 1 \pmod 3$

$3 = 2 \times 1 + 1$

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$1 = 1 \cdot 3 - 1 \cdot 2$

-1 is inverse; adjust to +ive $\Rightarrow 3 - 1 = 2$

$\therefore y_2 = 2$

$y_3 \times 66 \equiv 1 \pmod 5$

$y_3 \times 66 \equiv y_3 \times 1 \equiv 1 \pmod 5$

1 is already present so inverse is 1

$y_3 = 1$

$y_4 \times 30 \equiv 1 \pmod{11}$

$y_4 \times 30 \equiv y_4 \times 8 \equiv 1 \pmod{11}$

$11 = 8 \times 1 + 3$
$8 = 3 \times 2 + 2$
$3 = 2 \times 1 + 1$
Work backwards

$1 = 1 \cdot 3 - 1 \cdot 2$
$2 = 1 \cdot 8 - 2 \cdot 3$
$3 = 1 \cdot 11 - 1 \cdot 8$

$1 = 1 \cdot 3 - 1 \cdot (1 \cdot 8 - 2 \cdot 3)$
$1 = 1 \cdot 3 - 1 \cdot 8 + 2 \cdot 3 = 3 \cdot 3 - 1 \cdot 8$
$1 = 3 \cdot (1 \cdot 11 - 1 \cdot 8) - 1 \cdot 8$
$1 = 3 \cdot 11 - 3 \cdot 8 - 1 \cdot 8 = 3 \cdot 11 - 4 \cdot 8$
-4 is inverse; adjust to +ive $\Rightarrow 11 - 4 = 7$

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$\therefore y_4 = 7$

$x \equiv 1 \times 165 \times 1 + 2 \times 110 \times 2 + 3 \times 66 \times 1 + 4 \times 30 \times 7 \pmod{330}$

$x \equiv 1643 \pmod{330}$

$x \equiv 323 \pmod{330}$

(b) $x \equiv 3 \pmod 5$
$x \equiv 3 \pmod 6$
$x \equiv 1 \pmod 7$
$x \equiv 0 \pmod{11}$
} All are coprimes

$m = 5 \times 6 \times 7 \times 11 = 2310$

$M_1 = \frac{2310}{5}, \quad M_2 = \frac{2310}{6}, \quad M_3 = \frac{2310}{7}, \quad M_4 = \frac{2310}{11}$

$M_1 = 462, \quad M_2 = 385, \quad M_3 = 330, \quad M_4 = 210$

Find y_k as the inverse of $M_k \pmod{m_k}$

$y_1 \times 462 \equiv 1 \pmod 5$

$y_1 \times 462 \equiv y_1 \times 2 \equiv 1 \pmod 5$

$5 = 2 \times 2 + 1$

$1 = 1 \cdot 5 - 2 \cdot 2$

-2 is inverse; adjust for +ive $\Rightarrow 5 - 2 = 3$

$y_2 = 3$

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$y_2 \times 385 \equiv 1 \pmod 6$

$y_2 \times 385 \equiv y_2 \times 1 \equiv 1 \pmod 6$

1 present so 1 is the inverse.

$y_2 = 1$

$y_3 \times 330 \equiv 1 \pmod 7$

$y_3 \times 330 \equiv y_3 \times 1 \equiv 1 \pmod 7$

1 present so inverse is 1

$y_3 = 1$

$y_4 \times 210 \equiv 1 \pmod{11}$

$y_4 \times 210 \equiv y_4 \times 1 \equiv 1 \pmod{11}$

1 is present so inverse is 1

$y_4 = 1$

$x \equiv 3 \times 462 \times 3 + 3 \times 385 \times 1 + 1 \times 330 \times 1 + 0 \times 210 \times 1 \pmod{2310}$

$x \equiv 3643 \pmod{2310}$

$x \equiv 1023 \pmod{2310}$ oranges

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(c) $x \equiv 3 \pmod 4$
$x \equiv 5 \pmod 9$
$x \equiv 8 \pmod{13}$
— coprimes

$m = 4 \times 9 \times 13 = 468$

$M_1 = \frac{468}{4}$, $M_2 = \frac{468}{9}$, $M_3 = \frac{468}{13}$

$M_1 = 117$, $M_2 = 52$, $M_3 = 36$

Find y_1, y_2, y_3

$y_1 \times 117 \equiv 1 \pmod 4$

$y_1 \times 117 \equiv y_1 \times 1 \equiv 1 \pmod 4$

$y_1 = 1$ since 1 is already present which means that inverse is also 1

$y_2 \times 52 \equiv 1 \pmod 9$

$y_2 \times 52 \equiv y_2 \times 7 \equiv 1 \pmod 9$

$9 = 7 \times 1 + 2$
$7 = 2 \times 3 + 1$
work backwards

$1 = 1 \cdot 7 - 3 \cdot 2$
$2 = 1 \cdot 9 - 1 \cdot 7$

$1 = 1 \cdot 7 - 3 \cdot (1 \cdot 9 - 1 \cdot 7)$
$1 = 1 \cdot 7 - 3 \cdot 9 + 3 \cdot 7 = 4 \cdot 7 - 3 \cdot 9$
so inverse is 4 ; $y_2 = 4$

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$y_3 \times 36 \equiv 1 \pmod{13}$

$y_3 \times 36 \equiv y_3 \times 10 \equiv 1 \pmod{13}$

$13 = 10 \times 1 + 3$

$10 = 3 \times 3 + 1$

Work backwards

$1 = 1 \cdot 10 - 3 \cdot 3$

$3 \cdot 3 = 1 \cdot 13 - 1 \cdot 10$

$1 = 1 \cdot 10 - 3 \cdot (1 \cdot 13 - 1 \cdot 10)$

$1 = 1 \cdot 10 - 3 \cdot 13 + 3 \cdot 10 = 4 \cdot 10 - 3 \cdot 13$

so inverse is 4 ; $y_3 = 4$

$x \equiv 3 \times 117 \times 1 + 5 \times 52 \times 4 + 8 \times 36 \times 4 \pmod{468}$

$x \equiv 2543 \pmod{468}$

$x \equiv 203 \pmod{468}$ presents

Q13(a)(i) I LOVE DISCRETE MATHEMATICS

~~8, 10, 13, 19, 4, 3, 8~~

I => {8}

LOVE => {11, 14, 21, 4}

DISCRETE => {3, 8, 18, 2, 17, 4, 19, 4}

MATHEMATICS => {12, 0, 19, 7, 4, 12, 0, 19, 8, 2, 18}

Caesar Cipher :- $f(p) = (p+3) \pmod{26}$

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∴ {8} => {11} => L

{11, 14, 21, 4} => {14, 17, 24, 7} => ORYH

{3, 8, 18, 2, 17, 4, 19, 4} => {6, 11, 21, 5, 20, 7, 22, 7} =>

=> GLVFUHWH

{12, 0, 19, 7, 4, 12, 0, 19, 8, 2, 18} => {15, 3, 22, 10, 7, 15, 3, 22, 11, 5, 21} => PDWKHPDWLFV

(ii) STOP => {18, 19, 14, 15}

POLLUTION => {15, 14, 11, 11, 20, 19, 8, 14, 13}

(i) f(p) = (p+4) mod 26

∴ {18, 19, 14, 15} => {21, 22, 17, 18} => VWRS

{15, 14, 11, 11, 20, 19, 8, 14, 13} => {18, 17, 14, 14, 23, 22, 11, 17, 16} =>

=> SROOXWLRQ

(ii) f(p) = (p+22) mod 26

(i) f(p) = (p+4) mod 26

{18, 19, 14, 15} => {22, 23, 18, 19} => WXST

{15, 14, 11, 11, 20, 19, 8, 14, 13} => {19, 18, 15, 15, 24, 23, 12, 18, 17} =>

=> TSPPYXMSR

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$f(p) = (p+21) \bmod 26$

~~{19, 20}~~, {18, 19, 14, 15} => {13, 14, 9, 10} => NOJK

{15, 14, 11, 11, 20, 19, 8, 14, 13} => {10, 9, 6, 6, 15, 14, 3, 9, 8} =>

=> KJFFPODJI

(b)(i) Caesar cypher decryption algorithm;

$f(p) = (p-3) \bmod 26$

PLG => {15, 11, 6} => {12, 8, 3} => MID

WZR => {22, 25, 17} => {19, 22, 14} => TWO

DVVLJQPHQW => {3, 21, 21, 11, 9, 16, 15, 7, 16, 22} =>

{0, 18, 18, 8, 6, 13, 12, 4, 13, 19} => ASSIGNMENT

(ii) IDVW => {8, 3, 21, 22} => {5, 0, 18, 19} => FAST

QXFHV => {16, 23, 5, 7, 21} => {13, 20, 2, 4, 18} =>

=> ~~NRXES~~ NUCES

XQLYHUVLWB => {23, 16, 11, 24, 7, 20, 21, 11, 22, 1} =>

=> {20, 13, 8, 21, 4, 17, 18, 8, 19, 24} => UNIVERSITY

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(e) (i) Decryption will be $t(p) = (p - 10) \bmod 26$

CEBBOXNOB => {2, 4, 1, 1, 14, 23, 13, 14, 1} => {18, 20, 17, 17, 17, 4, 13, 3, 4, 17}

=> SURRENDER

XYG => {23, 24, 6} => {13, 14, 22} => NOW

(ii) LO => {11, 14} => {1, 4} => BE

WI => {22, 8} => {12, 24} => MY

PBSOXN = {15, 1, 18, 14, 23, 13} => {5, 17, 8, 4, 13, 3}

=> FRIEND

Q14 (a) (i) 034567981

$h(k) = k \bmod 97$

$h(034567981) = 034567981 \bmod 97$

$= 91$

(ii) $h(18321232) = 18321232 \bmod 97$

$= 57$

(iii) $h(220195744) = 220195744 \bmod 97$

$= 21$

(iv) $h(987255335) = 987255335 \bmod 97$

$= 5$

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Q9 (i) $h(16457890) = 16457890 \bmod 101$
$= 58$

(ii) $h(43222187) = 43222187 \bmod 101$
$= 60$

(iii) $h(37220919) = 37220919 \bmod 101$
$= 52$

(iv) $h(50338753) = 50338753 \bmod 101$
$= 3$

(c) (i) $x_1 = (4 \times 3 + 1) \bmod 7 = 6$
$x_2 = (4 \times 6 + 1) \bmod 7 = 4$
$x_3 = (4 \times 4 + 1) \bmod 7 = 3$
$x_4 = (4 \times 3 + 1) \bmod 7 = 6$
$x_5 = (4 \times 6 + 1) \bmod 7 = 4$

(ii) $x_1 = (7 \times 5 + 2) \bmod 9 = 1$
$x_2 = (7 \times 1 + 2) \bmod 9 = 0$
$x_3 = (7 \times 0 + 2) \bmod 9 = 2$
$x_4 = (7 \times 2 + 2) \bmod 9 = 7$
$x_5 = (7 \times 7 + 2) \bmod 9 = 6$

Q15 (a) (i) For $x_{12} \Rightarrow$

$2 \times 3 + 3 + 2 \times 3 + 3 + 2 \times 3 + 1 + 8 + 3 + 4 + 4 \times 3 + 3 + 4 \times 3 + x_{12} \equiv 0 \pmod{10}$

$95 + x_{12} \equiv 0 \pmod{10}$

$95 \bmod 10 = x_{12} = x_{12}$

$\therefore x_{12} = 5$

$x_{12} = 5$
$95 + 5 \bmod (10) = 0$
$\therefore 0 \equiv 0 \bmod (10)$

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(ii) $3\times6+3+6\times3+3+6\times3+2+3\times3+9+9\times3+1+3\times3+4+6\times3 \equiv x_{12} \pmod{10}$

$x_{12} = 118 \bmod 10 = 8$ $\quad 118+2 \bmod 10 = 0$

$0 \equiv 0 \therefore x_{12} = 2$

(b)(i) $0\times3+3+6\times3+0+0\times3+0+2\times3+9+1\times3+4+5\times3+2 \equiv 0 \bmod 10$

$60 \equiv 0 \pmod{10}$

$0 \equiv 0 \pmod{10}$ so it is valid

(ii) $0\times3+1+2\times3+3+4\times3+5+6\times3+7+8\times3+9+0\times3+3 \equiv 0 \pmod{10}$

$88 \equiv 0 \pmod{10}$

$8 \neq 0 \pmod{10}$ so invalid

(c)(i) $x_{10} = \sum_{i=1}^{9} i x_i \bmod (11)$

$x_{10} = 1\times0 + 2\times0 + 3\times7 + 4\times1 + 5\times1 + 6\times9 + 7\times8 + 8\times8 + 9\times1 \bmod 11$

$= 213 \bmod 11 = 4$

(ii) For valid ISBN-10 code;

$\sum_{i=1}^{10} i x_i \equiv 0 \pmod{11}$

$0\times1 + 3\times2 + 2\times3 + 1\times4 + 5\times5 + 0\times6 + 0\times7 + Q\times8 + 9\times1 + 8\times10 \equiv 0 \pmod{11}$

$8Q + 130 \equiv 0 \pmod{11}$

$130 \bmod 11 = 9$

$\therefore 8Q + 9 \equiv 0 \pmod{11}$

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$9Q \equiv -9 \pmod{11}$
$8Q \equiv 2 \pmod{11}$

now find modular inverse of 8

$11 = 8 \times 1 + 3$
$8 = 3 \times 2 + 2$
$3 = 2 \times 1 + 1$

$1 = 1 \cdot 3 - 1 \cdot 2$
$2 = 1 \cdot 8 - 2 \cdot 3$
$3 = 1 \cdot 11 - 1 \cdot 8$

$1 = 1 \cdot 3 - 1(1 \cdot 8 - 2 \cdot 3)$
$1 = 1 \cdot 3 - 1 \cdot 8 + 2 \cdot 3 = 3 \cdot 3 - 1 \cdot 8$
$1 = 3 \cdot (1 \cdot 11 - 1 \cdot 8) - 1 \cdot 8$
$1 = 3 \cdot 11 - 3 \cdot 8 - 1 \cdot 8 = 3 \cdot 11 - 4 \cdot 8$

-4 is inverse, adjust to +ive $\Rightarrow -4 + 11 = 7$

$\therefore 7 \times 8Q \equiv 2 \times 7$
$Q \equiv 14 \pmod{11}$
$Q \equiv 3 \pmod{11}$ so $Q = 3$

Q16 (a) "ATTACK" = {0, 19, 19, 0, 2, 10}

$n = 43 \cdot 59 = 2537$, $e = 13$

Group into pairs

{(0, 19), (19, 0), (2, 10)}

$C_1 = 019^{13} \bmod 2537$, $C_2 = 190^{13} \bmod 2537$, $C_3 = 210^{13} \bmod 2537$

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"EXAMINATION" = {4, 23, 0, 12, 8, 13, 0, 19, 8, 14, 13}

Group in pairs:-

{(4, 23), (0, 12), (8, 13), (0, 19), (8, 14), (13, 0)}

Padding 'A' to complete pair

$C_1 = 423^{13} \bmod 2537$, $C_2 = 012^{13} \bmod 2537$, $C_3 = 813^{13} \bmod 2537$,

$C_4 = 019^{13} \bmod 2537$, $C_5 = 814^{13} \bmod 2537$, $C_6 = 130^{13} \bmod 2537$

(b) "ASSIGNMENT" = {0, 18, 18, 8, 6, 13, 12, 4, 13, 19}

$n = 13 \cdot 19 = 247$, $e = 47$

$C_1 = 0^{47} \bmod 247$, $C_2 = 18^{47} \bmod 247$, $C_3 = 18^{47} \bmod 247$, $C_4 = 8^{47} \bmod 247$,

$C_5 = 6^{47} \bmod 247$, $C_6 = 13^{47} \bmod 247$, $C_7 = 12^{47} \bmod 247$, $C_8 = 4^{47} \bmod 247$,

$C_9 = 13^{47} \bmod 247$, $C_{10} = 19^{47} \bmod 247$.

"SEMESTER" = {18, 4, 12, 4, 18, 19, 4, 17}

$C_1 = 18^{47} \bmod 247$, $C_2 = 4^{47} \bmod 247$, $C_3 = 812^{47} \bmod 247$, $C_4 = 4^{47} \bmod 247$,

$C_5 = 18^{47} \bmod 247$, $C_6 = 19^{47} \bmod 247$, $C_7 = 4^{47} \bmod 247$, $C_8 = 17^{47} \bmod 247$

(c)(i) $5^{2003} \bmod 7$

$5^6 \equiv 1 \bmod 7$ → Fermat's theorem

$(5^6)^{333+5} \bmod 7$

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$(5^6)^{333} \cdot 5^5 \bmod 7$

$(1)^{333} \cdot 3125 \bmod 7$

$3125 \bmod 7 = 3$

(ii) $5^{2003} \bmod 11$

$5^{10} \equiv 1 \bmod 11$

$(5^{10})^{200} \cdot 3 \bmod 11$

$(1)^{200} \cdot 5^3 \bmod 11$

$125 \bmod 11 = 4$

(iii) $5^{2003} \bmod 13$

$5^{12} \equiv 1 \bmod 13$

$(5^{12})^{166} \cdot 11 \bmod 13$

$(1)^{166} \cdot 5^{11} \bmod 13$

$5^{11} \bmod 13 = 8$

(iv) $5^{2003} \bmod 17$

$5^{16} \equiv 1 \bmod 17$

$(5^{16})^{125} \cdot 3 \bmod 17$

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(1) $(1)^{125} \cdot 5^3 \bmod 17$

$125 \bmod 17 = 6$

Q17(a) $x \equiv 2 \pmod 7$
$x \equiv 3 \pmod{17}$
$x \equiv 5 \pmod{19}$
} coprimes

$m = 7 \times 17 \times 19 = 2261$

$M_k = \frac{m}{m_k}$ ∴ $M_1 = \frac{2261}{7}$, $M_2 = \frac{2261}{17}$, $M_3 = \frac{2261}{19}$

$M_1 = 323$, $M_2 = 133$, $M_3 = 119$

y_k is modular inverse of $M_k \pmod{m_k}$

∴ $y_1 \Rightarrow 323 \pmod 7$

$\Rightarrow 1 \pmod 7$ → remainder of 323 when divided by 7 is 1 and modular inverse of 1 is always 1

$y_1 = 1$

$y_2 \Rightarrow 133 \pmod{17}$

modular inverses of remainder are always equal to the inverse of the original number since in the context of this operation both numbers are equal

$y_2 \Rightarrow 14 \pmod{17}$

$17 = 1 \times 14 + 3$
$14 = 3 \times 4 + 2$
$3 = 2 \times 1 + 1$

$1 = 1 \cdot 3 - 1 \cdot 2$
$2 = 1 \cdot 14 - 4 \cdot 3$
$3 = 1 \cdot 17 - 1 \cdot 14$

$1 = 1 \cdot 3 - 1 \cdot (1 \cdot 14 - 4 \cdot 3)$
$1 = 1 \cdot 3 - 1 \cdot 14 + 4 \cdot 3 = 5 \cdot 3 - 1 \cdot 14$
$1 = 5 \cdot (1 \cdot 17 - 1 \cdot 14) - 1 \cdot 14$
$1 = 5 \cdot 17 - 5 \cdot 14 - 1 \cdot 14 = 5 \cdot 17 - 6 \cdot 14$

-6 is inverse; adjust to +ive
$-6 + 17 = 11$
∴ $y_2 = 11$

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$y_3 \Rightarrow 119 \bmod 19$

$5 \bmod 19$

$19 = 5 \times 3 + 4$
$5 = 4 \times 1 + 1$

$1 = 1 \cdot 5 - 1 \cdot 4$
$4 = 1 \cdot 19 - 3 \cdot 5$

$1 = 1 \cdot 5 - 1 \cdot (1 \cdot 19 - 3 \cdot 5)$
$1 = 1 \cdot 5 - 1 \cdot 19 + 3 \cdot 5 = 4 \cdot 5 - 1 \cdot 19$

4 is inverse so $y_3 = 1$

$x = 2 \times 323 \times 1 + 3 \times 133 \times 11 + 5 \times 119 \times 4 \pmod{2261}$

$x = 7415 \pmod{2261} = 632$

(b) Convert answer to text $\Rightarrow$ "Six Hundred And Thirty Two"

Encrypt with Caesar's Cipher $f(p) = (p+3) \bmod 26$

~~"Six" = {18, 8, 23}~~ "Six" = {18, 8, 23}

~~Encrypt $\Rightarrow$ Encrypted = {22, 12, 1}~~ Encrypted = {21, 11, 0}

~~"Hundred" = {8, 21, 14, 4, 18, 5, 4}~~ "Hundred" = {7, 20, 13, 3, 17, 4, 3}

~~Encrypted = {11, 24, 17, 7, 21, 8, 7}~~ Encrypted = {10, 23, 16, 6, 20, 7, 6}

~~"And" = {~~ "And" = {0, 13, 3}

Encrypted = {3, 16, 6}

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"Thirty" = {19, 7, 8, 17, 19, 24}

Encrypted = {22, 10, 11, 20, 22, 13}

"Two" = {19, 22, 14}

Encrypted = {22, 25, 17}

(c) $n = 6+3+2$

$= 11$

$11^{3\cdot2} \pmod 7$

$11^6 \equiv 1 \pmod 7$

$(11^6)^{5\cdot0+2} \pmod 7$

~~$(1)^5 \cdot b \pmod 7$~~ $1^{50} \cdot 11^2 \pmod 7$

~~$121 \pmod 7 \equiv 2$~~ $121 \pmod 7 = 2$

Q18(a) ~~27 offices, each of~~ 37 offices on each of the 27 floors

$\therefore$ Offices $= 37 \times 27 = 999$

(b) Types = colors x gender x sizes

$= 12 \times 2 \times 3$

$= 72$

(c) 3 letter-initials = no. of letters^3

$= 26^3$ → 26 for each initial since repetition is not

$= 17576$ disallowed

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Q19(a) $^{13}C_5 = \frac{13!}{(13-5)!5!} = 1287$

(b) $^{75}P_5 = \frac{75!}{(75-5)!}$

$= \frac{75!}{70!}$

$= \frac{75 \times 74 \times 73 \times 72 \times 71 \times 70!}{70!} = 2071126800$

(c) $^{27}C_4 = \frac{27!}{(27-4)!4!} = 17550$

Q20(a) Total values for Hexadecimal number system = 16

∴ 10 digit WEP Key = 16^{10}

Total Keys = $16^{10} + 16^{28} + 16^{58} = 6.9 \times 10^{69}$

~~(b) Total lowercase letters = 26~~

~~4-6 4-length string where one letter must be x ⇒ "xxxx"~~

~~∴ No. of strings = 26^3 → To fill the three spaces without an x~~

~~= 17576~~

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(b) Total 4-letter strings = 26^4

Total 4-letter strings without 'x' = 25^4

Total 4-letter strings with at least one 'x' = $26^4 - 25^4$
$= 66351$

(c) Three-letter initials without repetition = $26 \times 25 \times 24$ → chosen letter is removed from next selection pool.
$= 15600$

Q11 (a) m^2 in for elements in first set and 2 since the other set has only 2 elements

(b) ${}^5P_5 = \frac{5!}{(5-5)!} = 5! = 120$

~~(c) Number of different faces = $15 \times 48 \times 2 \times 24 \times 34 \times 28 \times 28$~~

There are 2 characteristics that have the same number of elements

(c) Number of different faces = $15 \times 48 \times 24 \times 34 \times 28 \times 28$

Q11(a)
$= 46061568$

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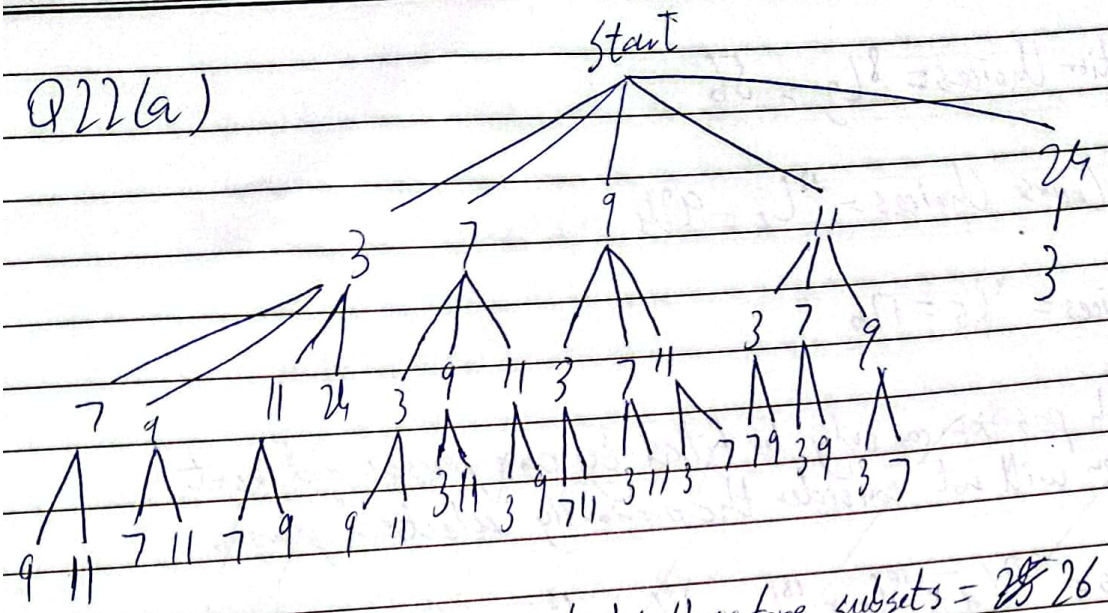
Q22(a)



No. of leaf nodes are the subsets therefore subsets = ~~25~~ 26

Remove duplicates by calculating distinct valid sums

∴ No. of subsets = 14

(b) Calculate bit strings of 4 length without consecutive 1s.

Total length 4 bit strings = 16

Total length 4 bit strings with consecutive 1s = 3P3 = 6

3P3 since the string will be of the form "1122"

∴ 4 length bit strings without consecutive 1s = 16 - 6 = 10

(c) No. of choices = ${}^{4}C_{3}+1$ * ${}^{3}C_{1}$ + ${}^{4}C_{1}$ + ${}^{2}C_{1}$ = 9

${}^{3}C_{1}$ is for president selection since A is ineligible

${}^{4}C_{1}$ is for Treasurer. Since all are eligible

${}^{2}C_{1}$ is for Secretary since only C and D are eligible

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Q23 (a) Position Choices = $^8C_3 = 56$

(b) Course Choices = $^{12}C_6 = 924$

(c) Player choices = $^9C_5 = 126$

Q24 (a) Each position can only be filled by one person and next selection will not consider the previously selected person

$\therefore$ Choices = $^{20}C_1 * {}^{19}C_2 * {}^{18}C_1 * {}^{17}C_1 * {}^{16}C_1 = 90$

Q24 (a) Order Matters so; choices = $^{20}P_5 = 1860480$

(b) Selections = $^{16}P_4 = 43630$

(c) Selections = $^{15}P_2 = 216$

Q25 (a) Different Types = $^5C_1 \times {}^3C_2 \times {}^4C_1 \times {}^6C_3 = 1200$

(b) Distinct toppings needed

$\therefore$ Unique Ice Creams = $12P5 = 95040$

(c) (i) Postal zip codes with 4 at the beginning = "4 ? ? ? ?"

$\therefore$ no. of codes = 10^4 → 10 digits can be chosen 4 times with repetition

= 10000

(ii) Without repetition;

codes = $10 \times 9 \times 8 \times 7 \times 6 = 3024$

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Q26(a) (ii) With repetition;

First digit must be 3 or 1

∴ Codes = $2 \times 10^4 = 20000$

Without repetition;

Codes = $2 \times 9 \times 8 \times 7 \times 6 = 6048$

Q26(a) 10-length bit strings which start with 3 zeroes = 2^7
10-length bit strings which end with 2 zeroes = 2^8
10-length bit strings which start with 3 and end with 2 zeroes = 2^5

∴ 10-length bit strings with either 3 zeroes at the start or 2 zeroes at the end = $2^7 + 2^8 - 2^5 = 352$

(b) Begin with 0 = 2^4
End with 2 ones = 2^3
Begin with 0 and end with 2 ones = 2^2

either Begin with 0 or end with 2 ones = $2^4 + 2^3 - 2^2 = 20$

(c) 26 letters and 10 digits allowed so total characters = 36

(i) with Repitition :-

2 letters followed by 4 digits = $26^2 \times 10^4 = 6760000$

(ii) Without Repitition :-

2 letters followed by 4 digits = $26 \times 25 \times 10 \times 9 \times 8 \times 7 = 3276000$

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(ii) With Repitition:- 5 vowels and 5 odd digits

3 odd digits followed by 2 vowels = $5^3 \times 5^2 = 3125$

Without Repitition:-

- - - - - - - = $5 \times 4 \times 3 \times 5 \times 4 = 1200$

Q27 (a) Pigeon objects = 30, boxes = 26 (letters)

$\left\lceil \frac{30}{26} \right\rceil = 2$ hence proved by pigeonhole principle

(b) objects = 8008278, boxes = 1000 000

$\left\lceil \frac{8008278}{1000\,000} \right\rceil = 9$ hence proved

(c) objects = 677, boxes = 38

$\left\lceil \frac{677}{38} \right\rceil = 18$ classrooms

Q28(a)(i) $(1+x)^{11} = {}^{11}C_r (1)^{11-r} (x)^r$

for x^5, $r = 5$

$\therefore {}^{11}C_5 (1)^{11-5} (x)^5 \Rightarrow 462 x^5$

co-eff. of x^5 is 462

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(ii) a^7b^{17} in $(2a-b)^{24}$

~~$4-r=7$, $r=17$~~ $(2a-b)^{24} \Rightarrow {}^{24}C_r (2a)^{24-r}(-b)^r$

$r=17$ for a^7b^{17}

$\therefore {}^{24}C_{17}(2a)^7(-b)^{17} \Rightarrow -443013128\,a^7b^{17}$

co-eff. found

(b)(i) First six rows of Pascal Triangle :-

1

1 2 1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

(ii) $(3u-2)^4$ Expand using Pascal Triangle

co-eff. of $(3u)$ and (-2) are 1 4 6 4 1

$\therefore 1(3u)^4 + 4(3u)^3(-2)^1 + 6(3u)^2(-2)^2 + 4(3u)^1(-2)^3 + 1(-2)^4$

The powers are evaluated according to binomial expansion

$$(a+b)^n = \sum_{r=0}^{n} \binom{n}{r} a^{n-r} \sum_{r=0}^{n} {}^nC_r\, a^{n-r} b^r$$

$\therefore 81u^4 - 216u^3 + 216u^2 - 96u + 16$ Ans

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(a)(i) Use binomial Expansion;

$${}^6C_0(2x)^{6-0}(y)^0 + {}^6C_1(2x)^5(y)^1 + {}^6C_2(2x)^4(y)^2 + {}^6C_3(2x)^3(y)^3$$

$$+ {}^6C_4(2x)^2(y)^4 + {}^6C_5(2x)(y)^5 + {}^6C_6(2x)^0(y)^6$$

$$64x^6 + 192x^5y + 240x^4y^2 + 160x^3y^3 + 60x^2y^4 + 12xy^5 + y^6$$

(ii) ${}^7C_0(3a)^7(-2b)^0 + {}^7C_1(3a)^6(-2b)^2 + {}^7C_2(3a)^5(-2b)^2 +$

$${}^7C_3(3a)^4(-2b)^3 + {}^7C_4(3a)^3(-2b)^4 + {}^7C_5(3a)^2(-2b)^5 + {}^7C_6(3a)(-2b)^6 +$$

$${}^7C_7(3a)^0(-2b)^7$$

$$2187a^7 - 10206a^6b + 20412a^5b^2 - 22680a^4b^3 + 15120a^3b^4 - 6048a^2b^5 + 1344ab^6 - 128b^7$$

Q29(a) Arrangements needed

(i) All in one row $= (20+16)! = 3.72\times10^{41}$

(ii) 7 in one row $= {}^{36}P_7 = 4.20\times10^{10}$

(iii) All women on left $= 20!$

All men on right $= 16!$

$\therefore$ Women on left and men on right $= 20! \times 16! = 5.09\times10^{31}$

(b) Decide the first four spots without the pitcher first then decide the rest of the spots without the 4 that have already been arranged

$\therefore$ First four spots $= {}^8P_4 = 1680$

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the rest = $(9-4)! = 5! = 120$

∴ Total arrangements = ~~1200×168~~ 1680×120

$= 201600$

(e) Distinct Pizzas with 3 or less toppings = ${}^{12}C_0 + {}^{12}C_1 + {}^{12}C_2 + {}^{12}C_3$

$= 299$

Q30 (a) Choices = ${}^{15}C_3 \times {}^{12}C_3 \times {}^{7}C_3 \times {}^{5}C_3 \times {}^{4}C_3 \times {}^{1}C_3$

$= 3.53 \times 10^{11}$

(b) Objects = 97, boxes = 10

$\left\lceil \frac{97}{10} \right\rceil = 10$ students

(c) ~~Choose captain~~ Captain choices = ${}^{15}C_1$

Vice-Captain choices = ${}^{14}C_1$

wicket-keeper choices = ${}^{2}C_1$

Combined choices = ${}^{2}C_1 \times {}^{14}C_2 \times {}^{13}C_1$

$= 364$

Wicket-keeper chosen first since there is a chance that captain and vice-captain would both be wicket-keepers.

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