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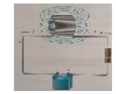
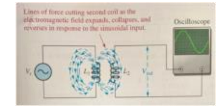
2. TRANSFORMERS

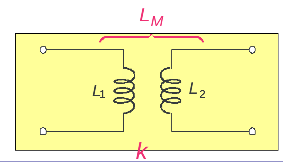
Mutual Inductance





When two coils are placed close to each other, a changing flux in one coil will cause an induced voltage in the second coil. The coils are said to have **mutual inductance** (L_M), which can either add or subtract from the total inductance depending on if the fields are aiding or opposing.

The coefficient of coupling is a measure of how well the coils are linked; it is a number between 0 and 1.



Mutual Inductance

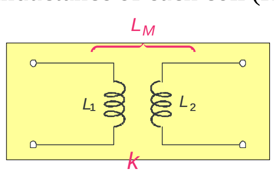
The formula for mutual inductance is

$$L_M = k\sqrt{L_1 L_2}$$

k = the coefficient of coupling (dimensionless)

L_1, L_2 = inductance of each coil (H)

The coefficient of coupling depends on factors such as the orientation of the coils to each other, their proximity, and if they are on a common core.



Mutual Inductance

1. What is the mutual inductance when k=0.75, L_1=1 µH and L_2= 4 µH.

2. Determine the coupling coefficient when L_M=1µH, L_1=8 µH and L_2= 2 µH.

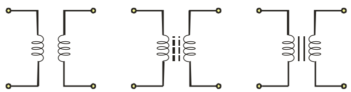
1. $L_M = k\sqrt{L_1 L_2} = 0.75\sqrt{(1\ \mu\text{H})(4\ \mu\text{H})} = \mathbf{1.5\ \mu H}$

2. $L_M = k\sqrt{L_1 L_2}$

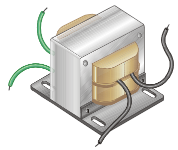
$$k = \frac{L_M}{\sqrt{L_1 L_2}} = \frac{1\ \mu\text{H}}{\sqrt{(8\ \mu\text{H})(2\ \mu\text{H})}} = \mathbf{0.25}$$

Basic Transformer

The basic transformer is formed from two coils that are usually wound on a common core to provide a path for the magnetic field lines.
Schematic symbols indicate the type of core.



Air core Ferrite core Iron core



Small power transformer

Turns ratio

A useful parameter for ideal transformers is the turns ratio defined* as

$$n = \frac{N_{sec}}{N_{pri}}$$

N_{sec} = number of secondary windings

N_{pri} = number of primary windings

* Based on the IEEE dictionary definition for electronics power transformers.

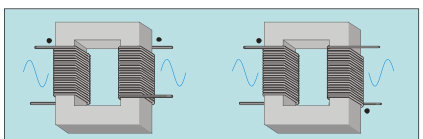
Most transformers are not marked with turns ratio, however it is a useful parameter for understanding transformer operation.

Example

A transformer has 800 turns on the primary and a turns ratio of 0.25. How many turns are on the secondary? 200

Direction of windings

The direction of the windings determines the polarity of the voltage across the secondary winding with respect to the voltage across the primary.
Phase dots are sometimes used to indicate polarities.



In phase Out of phase

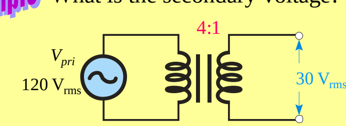
Step-up and step-down transformers

In a **step-up transformer**, the secondary voltage is greater than the primary voltage and $n > 1$.
In a **step-down transformer**, the secondary voltage is less than the primary voltage and $n < 1$.

$$n = \frac{V_{sec}}{V_{pri}}$$

Example

What is the secondary voltage?



What is the turns ratio? 0.25

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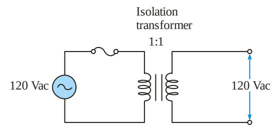
Isolation transformers

A special transformer with a turns ratio of 1 is called an **isolation transformer**. Because the turns ratio is 1, the secondary voltage is the same as the primary voltage, hence ac is passed from one circuit to another.



The isolation transformer breaks the dc path between two circuits while maintaining the ac path. The dc is blocked by the transformer, because magnetic flux does not change with dc.

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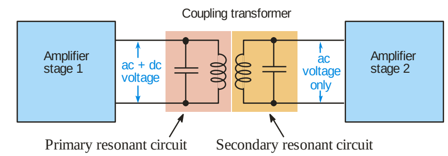
Coupling transformers

Coupling transformers are used to pass a higher frequency signal from one stage to another. Because they are high frequency transformers, they typically are configured with a resonant circuit on the primary and the secondary.

Some specialty isolation amplifiers use transformer coupling to isolate power.



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Current

Transformers cannot increase the applied power.
If the secondary voltage is higher than the primary voltage, then the secondary current must be lower than the primary current and vice-versa.

The ideal transformer turns ratio equation for current is

$$\boxed{n = \frac{I_{pri}}{I_{sec}}}$$

Notice that the primary current is in the numerator.

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Power

The *ideal* transformer does not dissipate power.
Power delivered from the source is passed on to the load by the transformer.
This important idea can be summarized as

$$P_{pri} = P_{sec}$$

$$V_{pri} I_{pri} = V_{sec} I_{sec}$$

$$\frac{V_{sec}}{V_{pri}} = \frac{I_{pri}}{I_{sec}}$$

These last ratios are, of course, the turns ratio, *n*.

Reflected resistance

A transformer changes both the voltage and current on the primary side to different values on the secondary side. This makes a load resistance appear to have a different value on the primary side.

From Ohm's law, $R_{pri} = \dfrac{V_{pri}}{I_{pri}}$ and $R_L = \dfrac{V_{sec}}{I_{sec}}$

Taking the ratio of R_{pri} to R_L,

$$\frac{R_{pri}}{R_L} = \left(\frac{V_{pri}}{V_{sec}}\right)\left(\frac{I_{sec}}{I_{pri}}\right) = \left(\frac{1}{n}\right)\left(\frac{1}{n}\right) = \frac{1}{n^2}$$

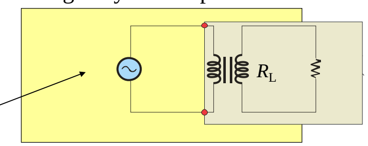
Reflected resistance

The resistance "seen" on the primary side is called the **reflected resistance.**

$$R_{pri} = \left(\frac{1}{n}\right)^2 R_L \qquad n = \sqrt{\frac{R_L}{R_{pri}}}$$

If you "look" into the primary side of the circuit, you see an effective load that is changed by the reciprocal of the turns ratio squared.

You see the primary side resistance, so the load resistance is effectively changed.



Problems

- 120 V ac is connected to the primary of a transformer with a turns ratio of 1.5, what is the secondary voltage

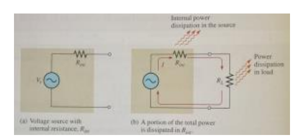
$$n = \frac{N_{sec}}{N_{pri}} = \frac{150}{100} = 1.5$$
$$V_{sec} = 1.5V_{pri} = 1.5(120\text{ V}) = \mathbf{180\ V}$$

- To step down 120V to 30V, what turns ratio is needed.

$$\frac{V_{sec}}{V_{pri}} = \frac{N_{sec}}{N_{pri}}$$
$$n = \frac{N_{sec}}{N_{pri}} = \frac{30\text{ V}}{120\text{ V}} = \mathbf{0.25}$$

Impedance matching

- The maximum power transfer states that maximum power is transferred from a source to a load when the load resistance is equal to the source resistance.
- Figure (a) shows an ac source with a fixed internal resistance - internal resistance is inherent in all sources due to their internal circuitry.
- (b) shows a load connected to the source. In this case, the objective is often to transfer as much power to the load as possible.
- A special type of wide-band transformer comes in handy. You can use the reflected-resistance characteristic provided by a transformer to make the load resistance appear to have the same resistance as the source resistance. This technique is called impedance matching, and the transformer is called an impedance matching transformer because it also transforms reactances as well as resistances.

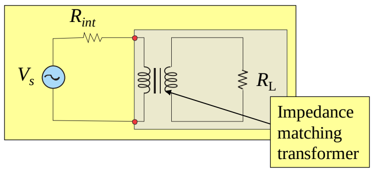


Impedance matching

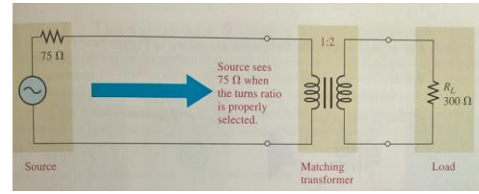
The word *impedance* is used in ac work to take into account resistance and reactance effects.

To match a load resistance to the internal source resistance **and hence transfer maximum power to the load**, a special impedance matching transformer is used.

Impedance matching transformers are designed for a wider range of frequencies than power transformers, hence tend to be not ideal.



Impedance matching



In the Figure, the source resistance is driving a 300 Ω load. The impedance-matching transformer needs to make the load resistance look like a 75 Ω resistance to the source, delivering maximum power to the load.

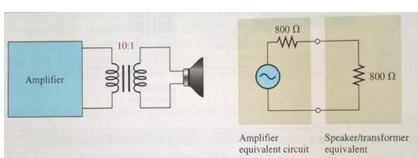
$$n = \sqrt{\frac{R_L}{R_{pri}}} = \sqrt{\frac{300}{75}} = 2$$

Impedance matching

An amplifier has an 800 Ω internal resistance. In order to provide maximum power to an 8 Ω speaker, what turns ratio must be used in the coupling transformer?

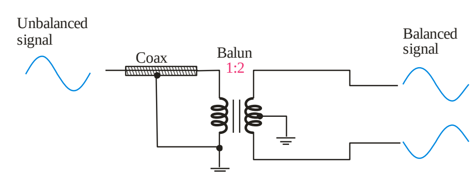
The reflected resistance must equal 800 Ω. Thus, from Equation 14–9, the turns ratio can be determined.

$$n = \sqrt{\frac{R_L}{R_{pri}}} = \sqrt{\frac{8\ \Omega}{800\ \Omega}} = \sqrt{0.01} = \mathbf{0.1}$$



Impedance matching

The balun is a specialized transformer to match a balanced line to an unbalanced line and vice-versa (hence the name *balun*).
A *balanced signal* is composed of two equal-amplitude signals that are 180º out-of-phase with each other.
An *unbalanced signal* is one that is referenced to ground.
In the illustration, an unbalanced signal is converted to a balanced signal by the balun transformer.



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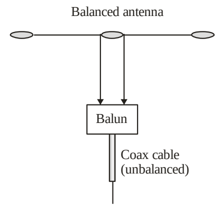
Impedance matching

One common application of a balun is in matching a balanced dipole antenna to a coax line. This is shown in the illustration.



Beside making the conversion from a balanced line to an unbalanced line, the balun can also match two different impedances. For example, a dipole antenna of 300 Ω can be matched to a 75 Ω coax using a balun.

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Non-ideal transformers

An ideal transformer has no power loss; all power applied to the primary is all delivered to the load.
Actual transformers depart from this ideal model.

Some loss mechanisms are:

1. **Winding resistance** (causing power to be dissipated in the windings.)
2. **Hysteresis loss** (due to the continuous reversal of the magnetic field.)
3. **Core losses** due to circulating current in the core (eddy currents).
4. **Flux leakage** flux from the primary that does not link to the secondary
5. **Winding capacitance** that has a bypassing effect for the windings.

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Transformer efficiency

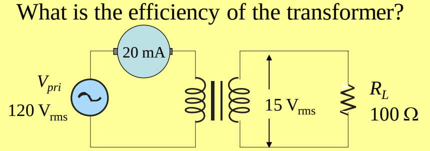
The efficiency of a transformer is the ratio of power delivered to the load (P_{out}) to the power delivered to the primary (P_{in}).

Than is $$\boxed{\eta = \left(\frac{P_{out}}{P_{in}}\right)100\%}$$

$$= \left(\frac{V_L^2/R_L}{(V_{pri})(I_{pri})}\right)100\% = \left(\frac{15\ \text{V}^2/100\ \Omega}{(120\ \text{V})(0.020\ \text{A})}\right)100\% = 94\%$$

Example

What is the efficiency of the transformer? 94%



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Problem

- A transformer has a primary resistance of 16Ω. Calculate the turns ratio needed to deliver maximum power to a 4 Ω speaker. If the voltage applied to the primary is 25V, what is the maximum power delivered to the speaker.

$$R_{pri} = \left(\frac{1}{n}\right)^2 R_L$$

$$\left(\frac{1}{n}\right)^2 = \frac{R_{pri}}{R_L}$$

$$\frac{1}{n} = \sqrt{\frac{R_{pri}}{R_L}} = \sqrt{\frac{16\ \Omega}{4\ \Omega}} = \sqrt{4} = 2$$

$$n = \frac{1}{2} = \mathbf{0.5}$$

$$R_{pri} = \left(\frac{1}{n}\right)^2 R_{\text{speaker}} = \left(\frac{1}{0.25}\right)4\ \Omega = 16\ \Omega$$

$$I_{pri} = \frac{25\ \text{V}}{16\ \Omega} = 1.56\ \text{A}$$

$$I_{sec} = \left(\frac{1}{n}\right)I_{pri} = 2(1.56\ \text{A}) = 3.12\ \text{A}$$

$$P_{\text{speaker}} = I_{sec}^2 R_{\text{speaker}} = (3.12\ \text{A})^2(4\ \Omega) = \mathbf{38.9\ W}$$

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Problem

A transformer is rated at 1kVA. It operates on 60Hz, 120V ac. The secondary voltage is 600V.

- (a) what is the maximum load current?
- (b) What is the smallest R_L you can drive?
- (c) What is the largest capacitor you can connect as a load?

30. (a) $I_{L(max)} = \frac{P_a}{V_{sec}} = \frac{1\text{ kVA}}{600\text{ V}} = \mathbf{1.67\text{ A}}$

(b) $R_{L(min)} = \frac{V_{sec}}{I_{L(max)}} = \frac{600\text{ V}}{1.67\text{ A}} = \mathbf{359\ \Omega}$

(c) $X_C = \frac{V_{sec}}{I_L} = \mathbf{359\ \Omega}$

$$C_{max} = \frac{1}{2\pi f X_C} = \frac{1}{2\pi(60\text{ Hz})(359\ \Omega)} = \mathbf{7.4\ \mu F}$$

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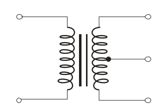
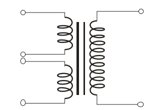
Tapped and multiple-winding transformers

Frequently, it is useful to tap a transformer to allow for a different reference or to achieve different voltage ratings, either on the primary side or the secondary side.

Multiple windings can be on either the primary or secondary side. One application for multiple windings is to connect to either 120 V or 240 V operation.



Secondary with center-tap



Primary with multiple-windings

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Typical exam Question

- A transformer used on a nuclear submarine subsystem has 2000 turns in the primary winding and 1000 turns in the secondary winding. A sine wave of amplitude 30 V_{rms} is applied to the primary coil. The load resistance is 300 Ω.

a) Calculate the turns ratio. [3 marks]
b) Calculate the voltage appearing across the secondary coil. [3 marks]
c) Calculate the reflected resistance, viewed from the primary side. [3 marks]
d) Calculate the current in the primary coil. [3 marks]
e) Calculate the current in the secondary coil. [3 marks]
f) Calculate the power dissipated in the primary coil. [3 marks]
g) Calculate the power dissipated in the secondary coil. [3 marks]
h) Calculate the mutual inductance of two coils with inductances of $L_1 = 1.4\ \mu H$, $L_2 = 3.5\ \mu H$. The coupling coefficient, k = 0.8. [3 marks]
i) Briefly discuss the origin of **Winding loss** and **Core loss** in a transformer. [6 marks]