SCHOOL OF ELECTRICAL AND ELECTRONIC ENGINEERING

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5. RLC CIRCUITS & RESONANCE

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Series RLC Circuits

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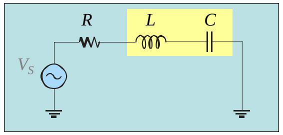
Series RLC circuits

When a circuit contains an inductor and capacitor in series, the reactance of each tend to cancel.

The total reactance is given by $X_{tot} = |X_L - X_C|$

The total impedance is given by $Z_{tot} = \sqrt{R^2 + X_{tot}^2}$

The phase angle is given by $\theta = \tan^{-1}\left(\frac{X_{tot}}{R}\right)$



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Problem

1. In the circuit below, R= 10Ω, L= 5mH, C=0.047µF. Determine the impedance and phase angle.
What is the total reactance?

1. $X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(5\text{ kHz})(0.047\ \mu\text{F})} = 677\ \Omega$

$X_L = 2\pi fL = 2\pi(5\text{ kHz})(5\text{ mH}) = 157\ \Omega$

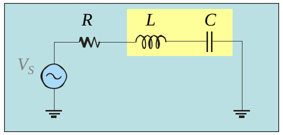
$Z = \sqrt{R^2 + (X_C - X_L)^2}$

$= \sqrt{(10\ \Omega)^2 + (677\ \Omega - 157\ \Omega)^2} = \sqrt{(10\ \Omega)^2 + (520\ \Omega)^2} = \mathbf{520\ \Omega}$

$\theta = \tan^{-1}\left(\frac{X_C - X_L}{R}\right) = \tan^{-1}\left(\frac{520\ \Omega}{10\ \Omega}\right) = \mathbf{88.9°}$ (V_s lagging I)

$X_{tot} = X_C - X_L = \mathbf{520\ \Omega}$ **Capacitive**

$X_C > X_L$ …capacitive



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Variation of X_L and X_C with frequency

In a series *RLC* circuit, the circuit can be capacitive or inductive, depending on the frequency.

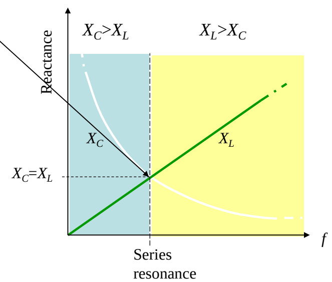
$X_C = \frac{1}{2\pi f C}$

$X_L = 2\pi f L =$

At the frequency where $X_C = X_L$, the circuit is at series resonance.

Below the resonant frequency, the circuit is predominantly capacitive.

Above the resonant frequency, the circuit is predominantly inductive.



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Impedance of series RLC circuits

Example-1

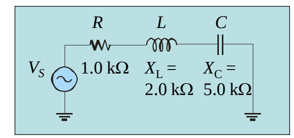
What is the total impedance and phase angle of the series *RLC* circuit if R= 1.0 kΩ, X_L = 2.0 kΩ, and X_C = 5.0 kΩ?

The total reactance is $X_{tot} = |X_L - X_C| = |2.0\text{ k}\Omega - 5.0\text{ k}\Omega| = 3.0\text{ k}\Omega$

The total impedance is $Z_{tot} = \sqrt{R^2 + X_{tot}^2} = \sqrt{1.0\text{ k}\Omega^2 + 3.0\text{ k}\Omega^2} = 3.16\text{ k}\Omega$

The phase angle is $\theta = \tan^{-1}\left(\frac{X_{tot}}{R}\right) = \tan^{-1}\left(\frac{3.0\text{ k}\Omega}{1.0\text{ k}\Omega}\right) = 71.6^o$

The circuit is capacitive, so I leads V by 71.6°.



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Impedance of series RLC circuits

Example-2

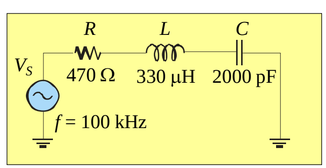
What is the magnitude of the impedance for the circuit?

$$X_L = 2\pi fL = 2\pi(100\text{ kHz})(330\ \mu\text{H}) = 207\ \Omega$$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(100\text{ kHz})(2000\text{ pF})} = 796\ \Omega$$

$$X_{tot} = |X_L - X_C| = |207\ \Omega - 796\ \Omega| = 589\ \Omega$$

$$Z = \sqrt{(470\ \Omega)^2 + (589\ \Omega)^2} = 753\ \Omega$$



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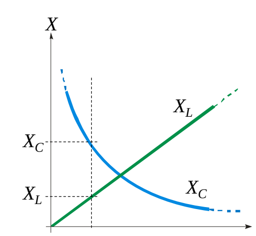
Impedance of series RLC circuits

Depending on the frequency, the circuit can appear to be capacitive or inductive.

The circuit in Example-2 was capacitive because $X_C > X_L$.



Impedance of series RLC circuits

Example-3

What is the total impedance for the circuit when the frequency is increased to 400 Hz?

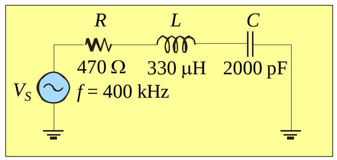
$$X_L = 2\pi fL = 2\pi(400\text{ kHz})(330\ \mu\text{H}) = 829\ \Omega$$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(400\text{ kHz})(2000\text{ pF})} = 199\ \Omega$$

$$X_{tot} = |X_L - X_C| = |829\ \Omega - 199\ \Omega| = 630\ \Omega$$

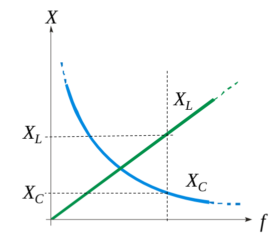
$$Z = \sqrt{(470\ \Omega)^2 + (630\ \Omega)^2} = 786\ \Omega$$

The circuit is now inductive.



Impedance of series RLC circuits

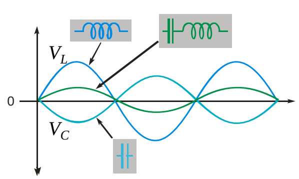
By changing the frequency, the circuit in Example-3 is now inductive because $X_L > X_C$



Voltages in a series RLC circuits

The voltages across the *RLC* components must add to the source voltage in accordance with KVL.
Because of the opposite phase shift due to *L* and *C*, V_L and V_C effectively subtract.

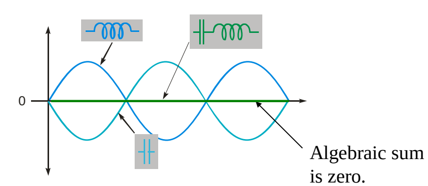
Notice that V_C is out of phase with V_L. When they are algebraically added, the result is….



This example is inductive.

Series resonance

At series resonance, X_C and X_L cancel. V_C and V_L also cancel because the voltages are equal and opposite.
The circuit is purely resistive at resonance.



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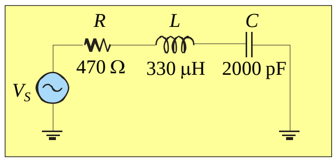
Resonance Frequency

The formula for resonance can be found by setting $X_C = X_L$. The result is

$$\boxed{f_r = \frac{1}{2\pi\sqrt{LC}}}$$

Example What is the resonant frequency for the circuit?

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$
$$= \frac{1}{2\pi\sqrt{(330\ \mu\text{H})(2000\ \text{pF})}}$$
$$= 196\ \text{kHz}$$



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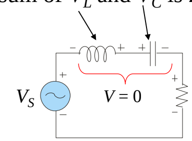
Series resonance

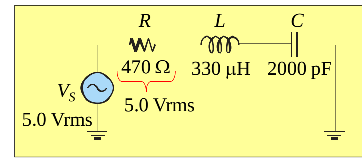
Ideally, at resonance the sum of V_L and V_C is zero.



By KVL, $V_R = V_S$

Example What is V_R at resonance?

5.0 Vrms



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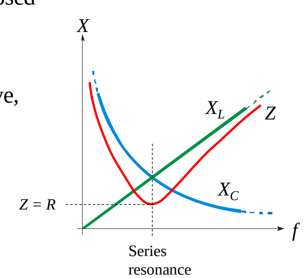
Impedance of series RLC circuits

The general shape of the impedance versus frequency for a series *RLC* circuit is superimposed on the curves for X_L and X_C.

Notice that at the resonant frequency, the circuit is resistive, and $Z = R$.



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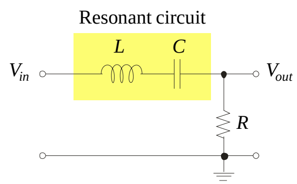
Series resonance

Summary of important concepts for series resonance:

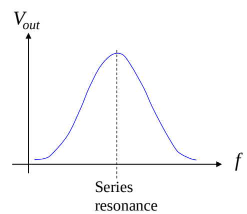
- Capacitive and inductive reactances are equal.
- Total impedance is a minimum and is resistive.
- The current is maximum.
- The phase angle between V_S and I_S is zero.
- f_r is given by $f_r = \dfrac{1}{2\pi\sqrt{LC}}$

Series resonant filters (band-pass)

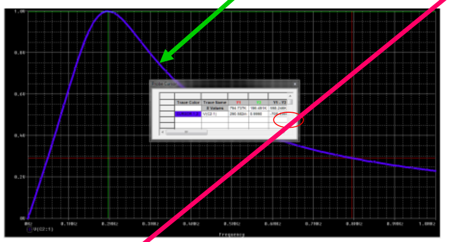
An application of series resonant circuits is in filters.
A band-pass filter allows signals within a range of frequencies to pass.



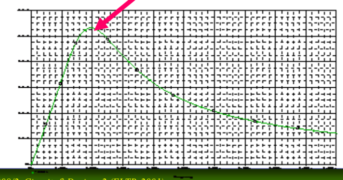
Circuit response:

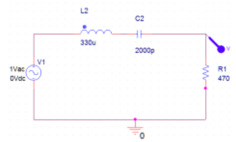


RLC Series resonant filters PSPICE – Voltage o/p and Current

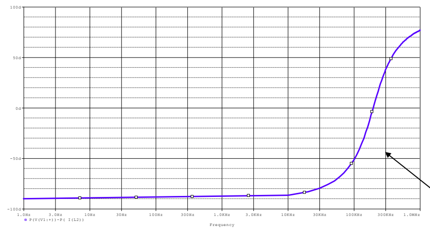


$$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(330\ \mu\text{H})(2000\ \text{pF})}} = 196\ \text{kHz}$$



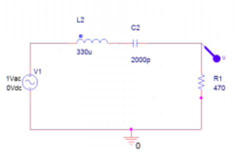


RLC Series resonant filters PSPICE – Phase difference between Voltage source and Current



$$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(330\ \mu\text{H})(2000\ \text{pF})}} = 196\ \text{kHz}$$

Phase difference = 0 at resonance

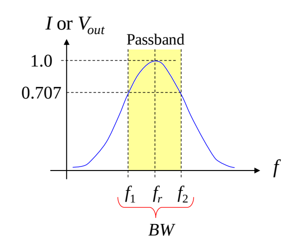


Series resonant filters (band width)

The response has a peak because at the *series* resonant frequency, the current is maximum at resonance and falls off before and after resonance. This develops the maximum voltage across the resistor at resonance.

The bandwidth (*BW*) of the filter is the range of frequencies for which the output is equal to or greater than 70.7% of the maximum value.

f_1 and f_2 are commonly referred to as the *critical frequencies*, *cutoff frequencies* or *half-power frequencies*.



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Decibels

Filter responses are often given in terms of decibels, which is defined as

$$\mathrm{dB} = 10\log\left(\frac{P_{out}}{P_{in}}\right)$$

Because it is a ratio, the decibel is dimensionless. One of the most important decibel ratios occurs when the power ratio is 1:2. This is called the *–3 dB frequency*, because

$$\mathrm{dB} = 10\log\left(\frac{1}{2}\right) = -3\ \mathrm{dB}$$

Another useful definition for the decibel, when measuring voltages across the same impedance is

$$\mathrm{dB} = 20\log\left(\frac{V_{out}}{V_{in}}\right)$$

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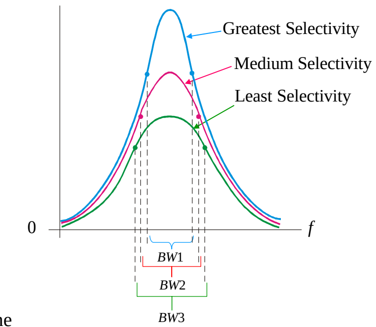
Selectivity

Selectivity describes the basic frequency response of a resonant circuit. (The –3 dB frequencies are marked by the dots.)

The bandwidth is inversely proportional to Quality Factor Q in accordance with the formula, $BW = \frac{f_r}{Q}$



Question

Which curve represents the highest Q? The one with the greatest selectivity.

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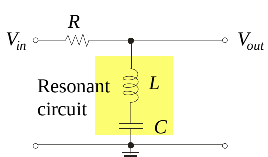
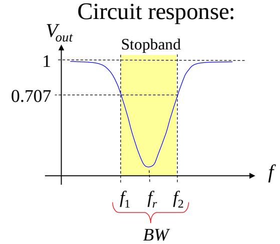
Series resonant filters (band-stop)

By taking the output across the resonant circuit, a band-stop (or notch) filter is produced.

Series resonant filters



Circuit response:



f_2

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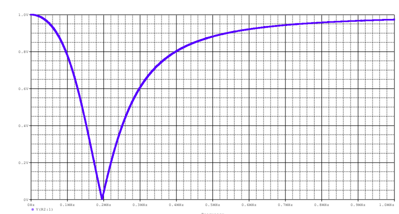
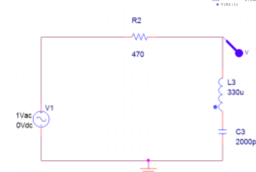
Notch filter or band reject filter





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Parallel RLC Circuits

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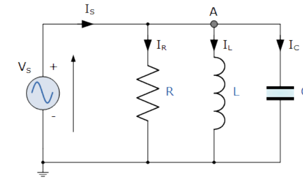
Parallel RLC Circuit

- The Parallel RLC Circuit is the exact opposite to the series circuit we looked at in the previous, although some of the previous concepts and equations still apply.

In the parallel RLC circuit, the supply voltage, V_S is common to all three components, whilst the supply current I_S consists of three parts.
The current flowing through the resistor, I_R, the current flowing through the inductor, I_L and the current through the capacitor, I_C.



Like the series RLC circuit, we can solve this circuit using the phasor or vector method but this time the vector diagram will have the voltage as its reference with the three current vectors plotted with respect to the voltage.
The phasor diagram for a parallel RLC circuit is produced by combining together the three individual phasors for each component and adding the currents vectorially.

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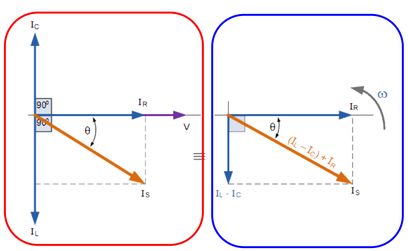
Phasor Diagram for Parallel RLC Circuit

Since the voltage across the circuit is common to all three circuit elements we can use this as the reference vector with the three current vectors drawn relative to this at their corresponding angles.
The resulting vector I_S is obtained by adding together two of the vectors, I_L and I_C and then adding this sum to the remaining vector I_R.
The resulting angle obtained between V and I_S will be the circuits phase angle as shown below.



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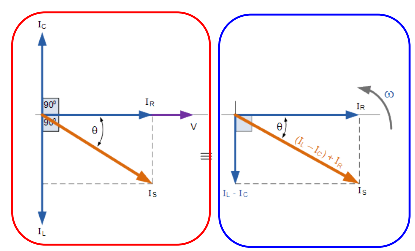


Current Triangle for a Parallel RLC

- .

$$I_S^2 = I_R^2 + (I_L - I_C)^2$$

$$I_S = \sqrt{I_R^2 + (I_L - I_C)^2}$$

$$\therefore I_S = \sqrt{\left(\frac{V}{R}\right)^2 + \left(\frac{V}{X_L} - \frac{V}{X_C}\right)^2} = \frac{V}{Z}$$

$$\text{where:} \quad I_R = \frac{V}{R}, \quad I_L = \frac{V}{X_L}, \quad I_C = \frac{V}{X_C}$$

Impedance of a Parallel RLC Circuit

- the final equation for a parallel RLC circuit produces complex impedance's for each parallel branch as each element becomes the reciprocal of impedance, (1/Z) with the reciprocal of impedance being called Admittance.

In parallel AC circuits it is more convenient to use admittance, symbol (Y) to solve complex branch impedance's especially when two or more parallel branch impedance's are involved (helps with the math's).

The total admittance of the circuit can simply be found by the addition of the parallel admittances.

Then the total impedance, ZT of the circuit will therefore be 1/YT Siemens as shown.

$$R = \frac{V}{I_R} \quad X_L = \frac{V}{I_L} \quad X_C = \frac{V}{I_C}$$

$$Z = \frac{1}{\sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2}}$$

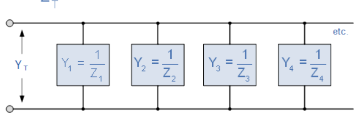
$$\therefore \frac{1}{Z} = \sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2}$$

Admittance of a Parallel RLC Circuit

- The new unit for admittance is the Siemens, abbreviated as S, (old unit mho's ℧, ohm's in reverse).
- Admittances are added together in parallel branches, whereas impedance's are added together in series branches.

A reciprocal of impedance also a reciprocal of resistance and reactance as impedance consists of two components, R and X.
Then the reciprocal of resistance is called Conductance and the reciprocal of reactance is called Susceptance.

$$\frac{1}{Z_T} = Y_T = Y_1 + Y_2 + Y_3 + Y_4 + \text{......etc}$$



Conductance, Admittance & Susceptance

- The units used for conductance, admittance and susceptance are all the same namely Siemens (S), which can also be thought of as the reciprocal of Ohms or ohm-1, but the symbol used for each element is different and in a pure component this is given as:

Admittance (Y) :
Admittance is the reciprocal of impedance, Z and is given the symbol Y.

$$Y = \frac{1}{Z} \; [S]$$

Conductance (G) :
Conductance is the reciprocal of resistance, R and is given the symbol G.

$$G = \frac{1}{R} \; [S]$$

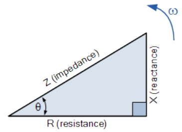
Susceptance (B) :
Susceptance is the reciprocal of a pure reactance, X and is given the symbol B.

$$B_L = \frac{1}{X_L} \; [S]$$

$$B_C = \frac{1}{X_C} \; [S]$$

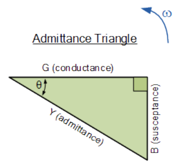
Admittance Triangle for a Parallel RLC

- Now that we have an admittance triangle, we can use Pythagoras to calculate the magnitudes of all three sides as well as the phase angle as shown.



Impedance Triangle

Admittance Triangle



- from Pythagoras

$$Y = \sqrt{G^2 + \left(B_L - B_C\right)^2}$$

where: $Y = \dfrac{1}{Z}$ $\quad G = \dfrac{1}{R}$

$B_L = \dfrac{1}{\omega L}$ $\quad B_C = \omega C$

Admittance & Impedance

Admittance: $Y = \sqrt{\left(\dfrac{1}{R}\right)^2 + \left(\dfrac{1}{(\omega L)} - (\omega C)\right)^2}$

Impedance: $Z = \dfrac{1}{Y} = \dfrac{1}{\sqrt{\left(\dfrac{1}{R}\right)^2 + \left(\dfrac{1}{(\omega L)} - (\omega C)\right)^2}}$