SCHOOL OF ELECTRICAL AND ELECTRONIC ENGINEERING

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4. RC CIRCUITS & RL CIRCUITS

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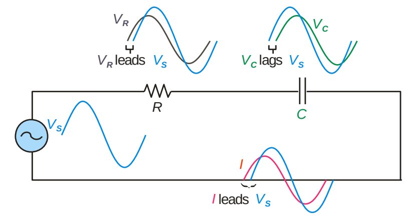
Sinusoidal response of RC circuits

When both resistance and capacitance are in a series circuit, the phase angle between the applied voltage and total current is between 0° and 90°, depending on the values of resistance and reactance.



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Impedance of series RC circuits

Impedance is a measure of opposition to sinusoidal current. In a series *RC* circuit, the total impedance is the phasor sum of *R* and X_C. Use Pythagoras' theorem for Z. Phase angle is phase difference between source voltage and total current. Since capacitor voltage lags the current (and therefore the resistor voltage) by 90°. So, we plot X_c at -90° wrt R.

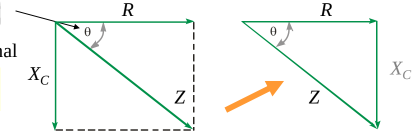
R is plotted along the positive *x*-axis.
X_C is plotted along the negative *y*-axis.

$$X_C = \frac{1}{2\pi f \cdot C}$$

$$\theta = \tan^{-1}\left(\frac{X_C}{R}\right)$$

Z is the diagonal

$$Z = \sqrt{R^2 + X_c^2}$$



It is convenient to reposition the phasors into the *impedance triangle*.

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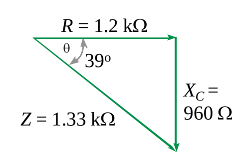
Impedance triangle of series RC circuits

Example

Sketch the impedance triangle and show the values for R = 1.2 kΩ and X_C = 960 Ω.

$$\theta = \tan^{-1}\frac{0.96\ \text{k}\Omega}{1.2\ \text{k}\Omega} = 39°$$



$$Z = \sqrt{R^2 + X_c^2}$$ Z=1.53 K

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Analysis of series RC circuits

Ohm's law is applied to series RC circuits using Z, V, and I.

$$V = IZ \qquad I = \frac{V}{Z} \qquad Z = \frac{V}{I}$$

Because I is the same everywhere in a series circuit, you can obtain the voltages across different components by multiplying the impedance of that component by the current as shown in the following example.

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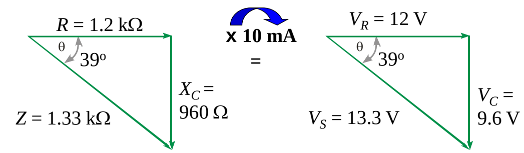
Voltage triangle of series RC circuits

Example

Assume the current in the previous example is 10 mA_{rms}. Sketch the voltage phasor diagram.

The impedance triangle from the previous example is shown for reference. The voltage phasor diagram can be found from Ohm's law.

Multiply each impedance phasor by 10 mA.



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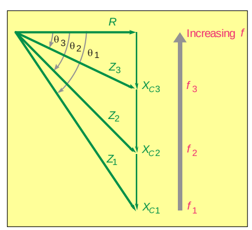
Variation of phase angle with frequency

Phasor diagrams that have reactance phasors can only be drawn for a single frequency because X is a function of frequency.

$$\theta = \tan^{-1}\left(\frac{X_C}{R}\right)$$

As frequency changes, the impedance triangle for an RC circuit changes as illustrated here because X_C decreases with increasing f.

$$G_{LP}(f) = \frac{X_C}{Z} \qquad X_C = \frac{1}{2\pi f \cdot C}$$



This determines the *frequency response* of RC circuits.

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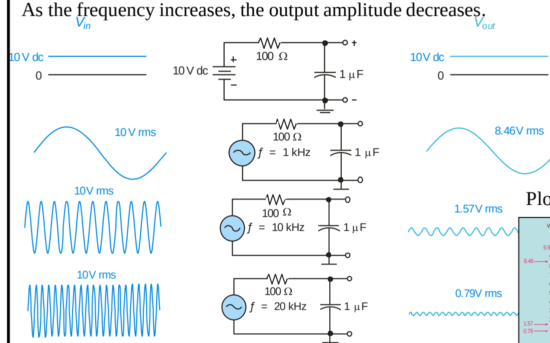
Frequency Response of RC Low pass filter

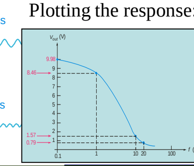
Frequency Response of RC Circuits - Low pass filter

When a signal is applied to an RC circuit, and the output is taken across the capacitor as shown, the circuit acts as a low-pass filter.

As the frequency increases, the output amplitude decreases.



Plotting the response:

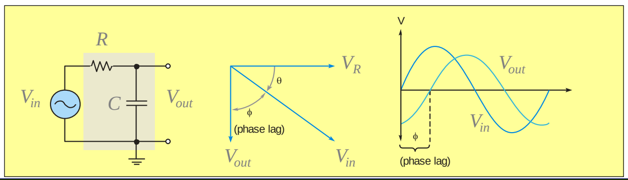


Applications – Low pass RC filter

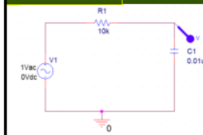
For a given frequency, a series *RC* circuit can be used to produce a phase lag by a specific amount between an input voltage and an output by taking the output across the capacitor.

This circuit is also a basic *low-pass filter*, a circuit that passes low frequencies and rejects high frequencies. This filter passes low frequencies up to a frequency f_c , called the cutoff frequency, the 3dB frequency or the break frequency.

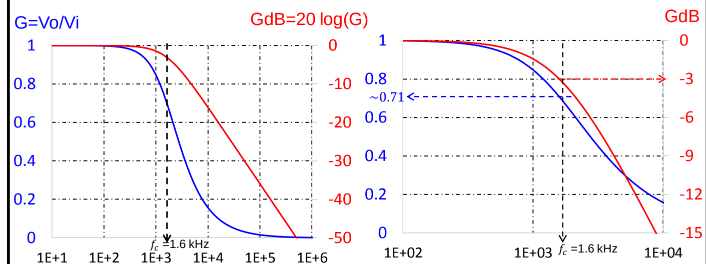
At the cut-off frequency, the i/p signal has been attenuated by 3dB. In other words, the gain is 0.707. $f_c = \frac{1}{2\pi RC}$



Low pass RC filter – cutoff frequency



$$f_c = \frac{1}{2\pi RC} = \frac{1}{2\pi 10^4 10^{-8}} = 1.6 \text{ kHz}$$



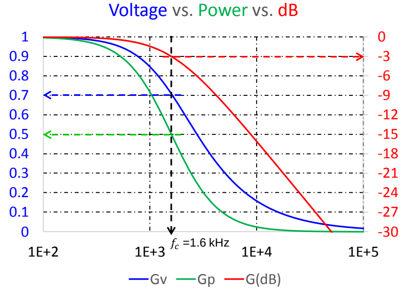
The 3dB frequency

- A -3 dB drop occurs, relative to some maximum, when the signal voltage drops to 0.707 of that maximum. In other words, 0.707 X voltage is -3dB.

  $0.707 = \sqrt{\frac{1}{2}}$

  $P=\frac{V^2}{R}$ so Power is proportional to the square of the voltage.

Voltage vs. Power vs. dB



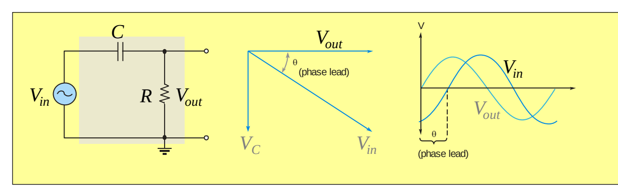
If the voltage drops to $\sqrt{\frac{1}{2}}$ multiplied by its original value, then since $P=\frac{V^2}{R}$ the Power has reduced by ½.

High Pass RC Filter

Reversing the components in the previous circuit produces a circuit that is a basic lead network. This circuit is also a basic *high-pass filter*, a circuit that passes high frequencies and rejects all others. This filter passes high frequencies down to a frequency called the *cutoff* frequency.

The formula for the cutoff frequency is the same!

$$f_c = \frac{1}{2\pi RC}$$



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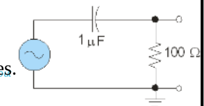
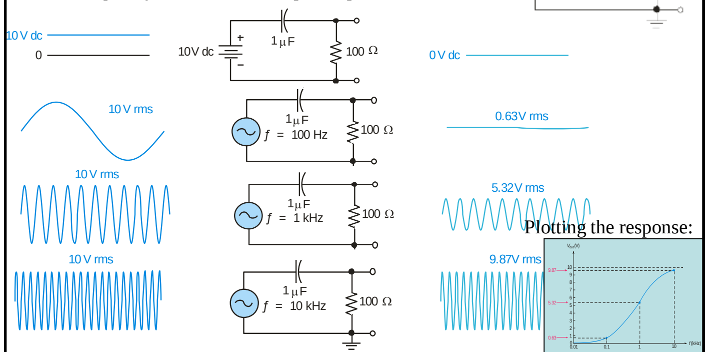
Frequency Response of RC High pass filter

Reversing the components, and taking the output across the resistor as shown, the circuit acts as a high-pass filter.

As the frequency increases, the output amplitude also increases.





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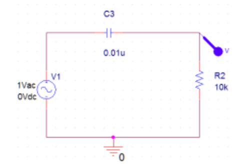
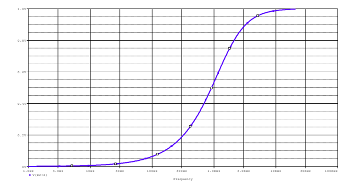
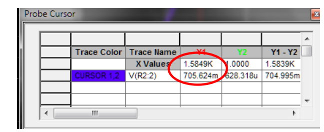
Applications – High pass RC filter







$f_C = \dfrac{1}{2\pi RC}$ = 1.6 kHz

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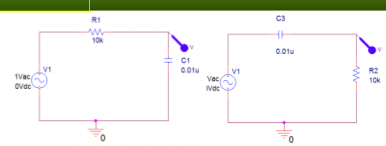
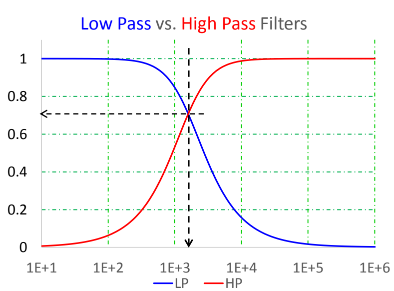
Low pass vs High pass filter



$f_C = \dfrac{1}{2\pi \cdot RC} = 1.6\,kHz$



$$X_C = \frac{1}{2\pi f \cdot C}$$

$$Z = \sqrt{R^2 + X_C^2}$$

$$G_{LP} = \frac{X_C}{Z} \qquad G_{HP} = \frac{R}{Z}$$

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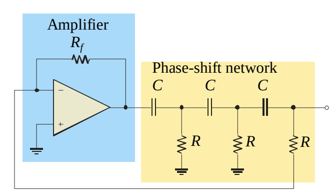
Applications

An application showing how the phase-shift network is useful is the phase-shift oscillator, which uses a combination of *RC* networks to produce the required 180º phase shift for the oscillator.



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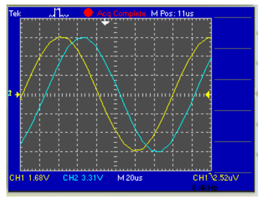
Measuring Phase Angle

An oscilloscope is commonly used to measure phase angle in reactive circuits. The easiest way to measure phase angle is to set up the two signals to have the same apparent amplitude and measure the period. An example of a Multisim simulation is shown, but the technique is the same in lab.

Set up the oscilloscope so that two waves *appear* to have the same amplitude as shown.

Determine the period. For the wave shown, the period is

$$T = 8.0\ \text{div}\left(\frac{20\ \mu\text{s}}{\text{div}}\right) = 160\ \mu\text{s}$$



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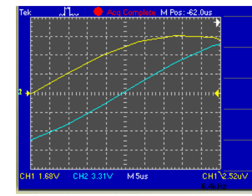
Measuring Phase Angle

Next, spread the waves out using the SEC/DIV control in order to make an accurate measurement of the time difference between the waves. In the case illustrated, the time difference is

$$\Delta t = 4.9\ \text{div}\left(\frac{5\ \mu\text{s}}{\text{div}}\right) = 24.5\ \mu\text{s}$$

The phase shift is calculated from

$$\theta = \left(\frac{\Delta t}{T}\right)360° = \left(\frac{24.5\ \mu\text{s}}{160\ \mu\text{s}}\right)360° = 55°$$



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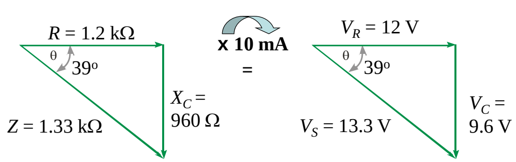
The voltage triangle

Recall that in a series *RC* circuit, you could multiply the impedance phasors by the current to obtain the voltage phasors.

The earlier example is shown for review:



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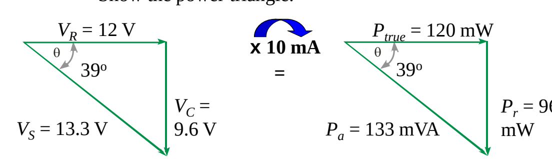
The power triangle

Multiplying the voltage phasors by I_{rms} gives the power triangle (equivalent to multiplying the impedance phasors by I^2). Apparent power is the product of the magnitude of the current and magnitude of the voltage and is plotted along the hypotenuse of the power triangle.

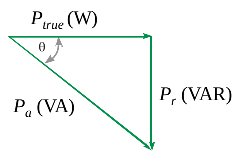
Example The rms current in the earlier example was 10 mA. Show the power triangle.



Apparent power

Apparent power consists of two components; a true power component, that does the work, and a reactive power component, that is simply power shuttled back and forth between source and load.

Some components such as transformers, motors, and generators are rated in VA rather than watts.



Power factor

The power factor is the relationship between the apparent power in volt-amperes and true power in watts. Volt-amperes multiplied by the power factor equals true power.

Power factor is defined mathematically as

$$PF = \cos\theta$$

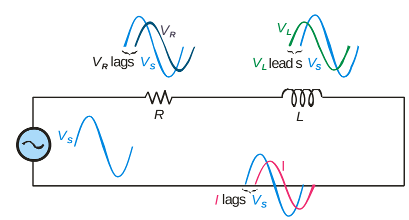
The power factor can vary from 0 for a purely reactive circuit to 1 for a purely resistive circuit.

RL Circuits

Electronics Fundamentals 8th edition
Floyd/Buchla

Sinusoidal response of RL circuits

When both resistance and inductance are in a series circuit, the phase angle between the applied voltage and total current is between 0° and 90°, depending on the values of resistance and reactance.



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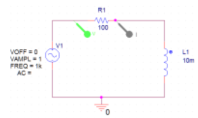
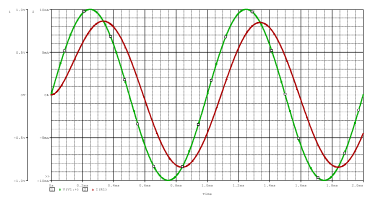
Sinusoidal response of RL circuits

Note I lags V_s





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Impedance of series RL circuits

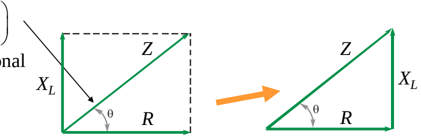
In a series *RL* circuit, the total impedance is the phasor sum of *R* and X_L.

R is plotted along the positive *x*-axis.

X_L is plotted along the positive *y*-axis.

$$\theta = \tan^{-1}\left(\frac{X_L}{R}\right)$$

Z is the diagonal



It is convenient to reposition the phasors into the *impedance triangle*.

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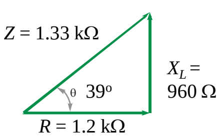
Impedance of series RL circuits

Example

Sketch the impedance triangle and show the values for R = 1.2 kΩ and X_L = 960 Ω.

$$Z = \sqrt{(1.2\ \text{k}\Omega)^2 + (0.96\ \text{k}\Omega)^2}$$
$$= 1.33\ \text{k}\Omega$$
$$\theta = \tan^{-1}\frac{0.96\ \text{k}\Omega}{1.2\ \text{k}\Omega}$$
$$= 39^\circ$$



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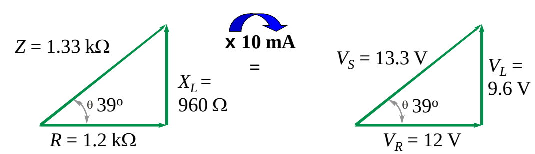
Analysis of series RL circuits

Example

Assume the current in the previous example is 10 mA_{rms}. Sketch the voltage phasors.

The impedance triangle from the previous example is shown for reference.

The voltage phasors can be found from Ohm's law. Multiply each impedance phasor by 10 mA.



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Variation of phase angle with frequency

Phasor diagrams that have reactance phasors can only be drawn for a single frequency because X is a function of frequency.

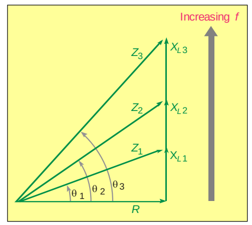
$$\theta(f) = tan^{-1}\left(\frac{X_L}{Z}\right)$$

As frequency changes, the impedance triangle for an RL circuit changes as illustrated here because X_L increases with increasing f.

$$X_L = 2\pi f L$$

This determines the *frequency response* of RL circuits.

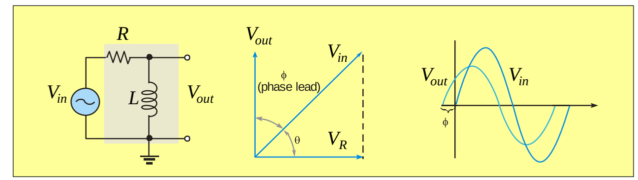
$$G_L(f) = \frac{X_L}{Z}$$



Application - Phase shift

For a given frequency, a series RL circuit can be used to produce a phase lead by a specific amount between an input voltage and an output by taking the output across the inductor.

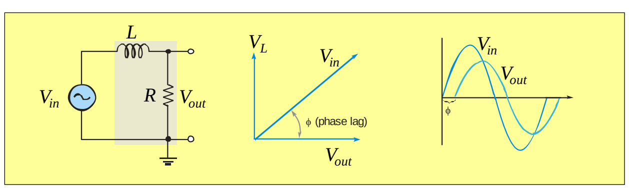
This circuit is also a basic high-pass filter, a circuit that passes high frequencies and rejects all others.



Application: Phase shift

Reversing the components in the previous circuit produces a circuit that is a basic lag network.

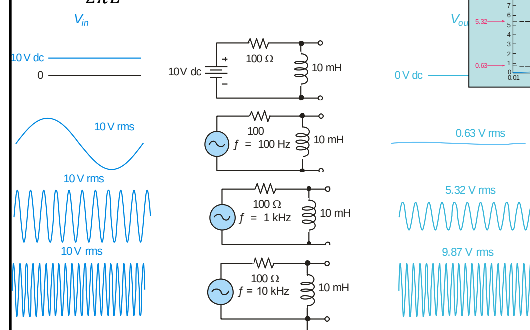
This circuit is also a basic low-pass filter, a circuit that passes low frequencies and rejects all others.

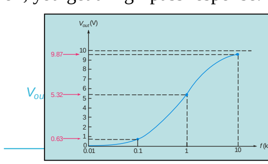


Application - High Pass RL filter

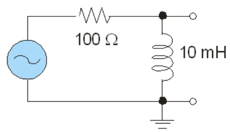
When the R and L components are in this configuration, you get a high-pass response. The output is taken across the inductor.

$$f_c = \frac{R}{2\pi L} = \frac{100}{2\pi 10x10^{-3}} = 1.6\ kHz$$

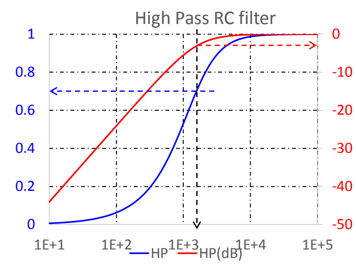




High Pass RL Filter PSPICE



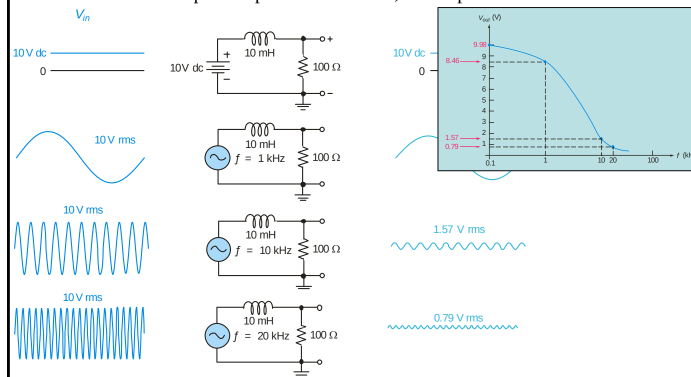
$$f_c = \frac{R}{2\pi L} = \frac{100}{2\pi 10x10^{-3}} = 1.6\ kHz$$



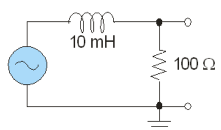
RL Low Pass filter

Series *RL* circuits have a frequency response similar to series *RC* circuits.

In the case of the low-pass response shown here, the output is taken across the resistor.



RL Low Pass filter dB plot



$$f_c = \frac{R}{2\pi L} = \frac{100}{2\pi 10x10^{-3}} = 1.6\ kHz$$