Digital convolution: Use of Z-transform

Use of Tables & z-Transform h(n)=[4,3,2], x(n)=[1,3,7,9]

First we obtain the Z-transform of both functions. Then multiplication of these functions is equivalent to convolution in the time domain.

$$x(n) = [1\,3\,7\,9] \Rightarrow X(z) = 1 + 3z^{-1} + 7z^{-2} + 9z^{-3}$$
$$h(n) = [4\,3\,2\,] \Rightarrow H(z) = 4 + 3z^{-1} + 2z^{-2}$$

$$\begin{array}{rl}
X(z)X(z) = & 1 + 3z^{-1} + 7z^{-2} + 9z^{-3} \\
 & \times 4 + 3z^{-1} + 2z^{-2} \\
 & \text{-----------------} \\
= & 4 + 12z^{-1} + 28z^{-2} + 36z^{-3} \\
+ & \quad 3z^{-1} + 9z^{-2} + 21z^{-3} + 27z^{-4} \\
+ & \quad\quad 2z^{-2} + 6z^{-3} + 14z^{-4} + 18z^{-5} \\
= & \text{---------------------------} \\
= & 4 + 15z^{-1} + 39z^{-2} + 63z^{-3} + 41z^{-4} + 18z^{-5}
\end{array}$$

$$h(n) = [4 \quad 15 \quad 39 \quad 63 \quad 41 \quad 18\,]$$

Consider another example:

TABLE 3.1: Convolution in table format

$y(n)$	$x(n)h(n)$			$\sum xh$
$y(1)$	$h_0 x_0 = 1 \times 1$			1
$y(2)$	$h_0 x_1 = 1 \times 0.5$	$h_1 x_0 = 2 \times 1$		2.5
$y(3)$	$h_0 x_2 = 1 \times 0.25$	$h_1 x_1 = 2 \times 0.5$	$h_2 x_0 = 3 \times 1$	4.25
$y(4)$		$h_1 x_2 = 2 \times 0.25$	$h_2 x_1 = 3 \times 0.5$	2
$y(5)$			$h_2 x_2 = 3 \times 0.25$	0.75

If we obtain the z-transform of the input signal and the impulse response we can obtain the system output by simply multiplying the 2 z-transforms. For example:

$$X(z) = 1 + 0.5z^{-1} + 0.25z^{-2}$$
$$H(z) = 1 + 2z^{-1} + 3z^{-2}$$

$$\begin{aligned}
&= 1 + 0.5z^{-1} + 0.25z^{-2} \\
&\qquad\; 2z^{-1} \quad + 1z^{-2} \quad + 0.5z^{-3} \\
&\qquad\qquad\qquad + 3z^{-2} \quad + 1.5z^{-3} + 0.75z^{-4}
\end{aligned}$$

$$Y(z) = \mathbf{1 + 2.5z^{-1} + 4.25z^{-2} + 2z^{-3} + 0.75z^{-4}}$$