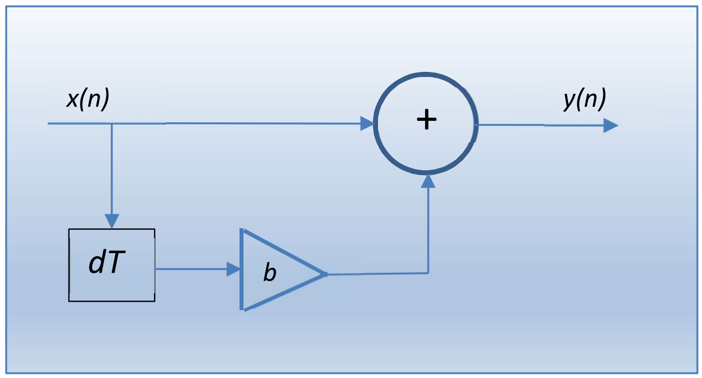
Audio Filters

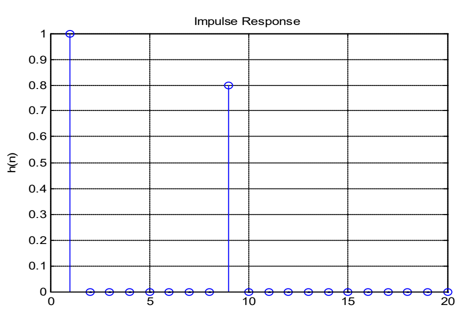
Basic echo filter (1st order filter FIR)

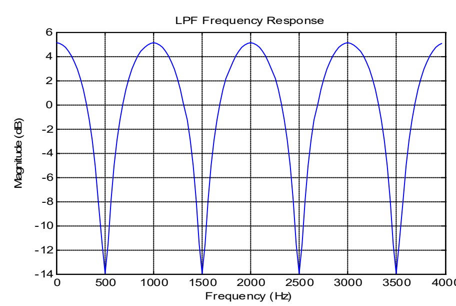
The filter below is a simple echo filter. It adds a copy of the input to itself *dT* seconds later, where *d* is the number of samples in the delay used.

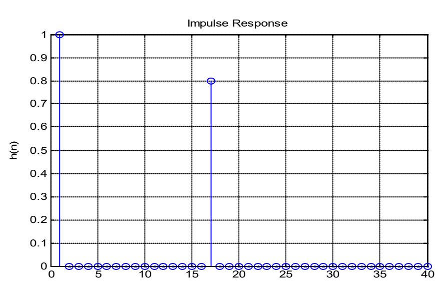


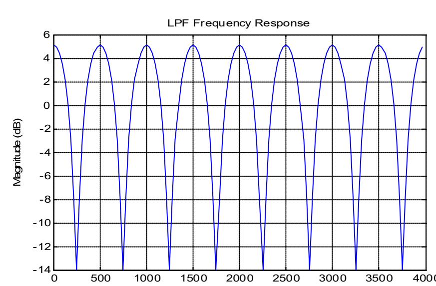
$$y(n) = x(n) + bx(n-d) \Rightarrow Y(z) = X(z) + bX(z)z^{-d}$$
$$\Rightarrow H(z) = \frac{Y(z)}{X(z)} = 1 + bz^{-d}$$

The impulse response and the frequency response of the echo filter are shown below for delays of 8 samples and 16 samples respectively.





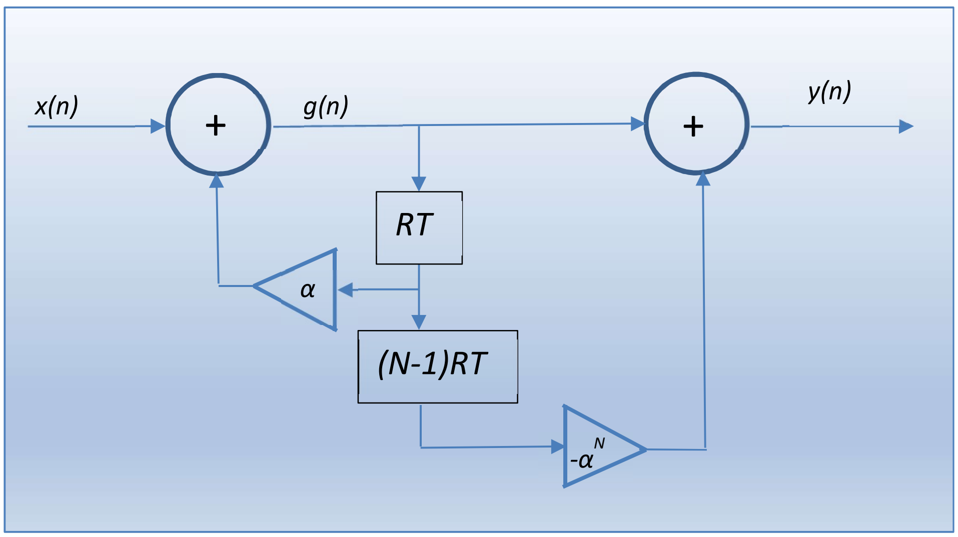




This filter called a *comb* filter because its frequency response resembles a comb.

$$H(\theta)=1+be^{-j\theta d}=1+b\{\cos(\theta d)-j\sin(\theta d)\}$$
$$|H(\theta)|^2=\{1+b\cos(\theta d)\}^2+\{b\sin(\theta d)\}^2=1+2b\cos(\theta d)+b^2\{\cos^2(\theta d)+\sin^2(\theta d)\}$$
$$|H(\theta)|^2=1+2b\cos(\theta d)+b^2$$
$$|H(\theta)|^2_{\max}=1+2b+b^2=(1+b)^2\Rightarrow|H(\theta)|_{\max}=(1+b)$$
$$|H(\theta)|^2_{\min}=1-2b+b^2=(1-b)^2\Rightarrow|H(\theta)|_{\min}=(1-b)$$
$$|H(\theta)|_{\max}\ @\ \ \cos(\theta d)=1\ \ \Rightarrow\ \ \theta d=2n\pi\Rightarrow\theta=\frac{2n\pi}{d}$$
$$If\ d=8,\theta=\frac{2n\pi}{8}=\frac{n\pi}{4}=n\times1\,kHz\ for\ f_s=8\,kHz$$

Multiple echo filter



The multiple echo filter adds N-1 delays reducing in amplitude for each delay (α<1). The objective is to realise the following transfer function:

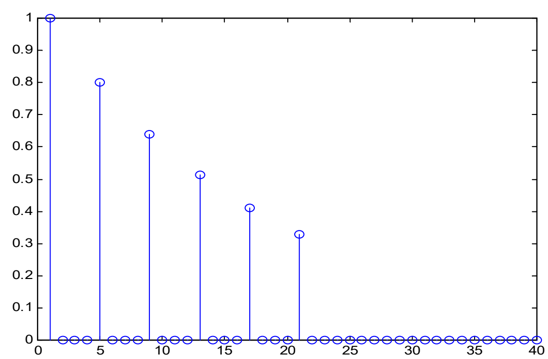
$$H(z)=\frac{Y(z)}{X(z)}=1+\alpha z^{-R}+\alpha^2z^{-2R}+\alpha^3z^{-3R}+.........+\alpha^{N-1}z^{-(N-1)R}$$

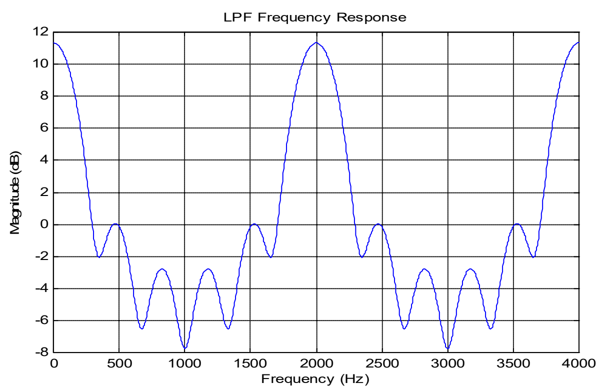
We can however re-write this into a more compact form.

$$H(z)=\frac{Y(z)}{X(z)}=1+\alpha z^{-R}+\alpha^2z^{-2R}+\alpha^3z^{-3R}+.........+\alpha^{N-1}z^{-(N-1)R}$$
$$\alpha z^{-R}H(z)=\alpha z^{-R}+\alpha^2z^{-2R}+\alpha^3z^{-3R}+.........+\alpha^Nz^{-NR}$$
$$H(z)-\alpha z^{-R}H(z)=1+\alpha z^{-R}+\alpha^2z^{-2R}+\alpha^3z^{-3R}+.........+\alpha^{N-1}z^{-(N-1)R}-\{\alpha z^{-R}+\alpha^2z^{-2R}+\alpha^3z^{-3R}+.........+\alpha^Nz^{-NR}\}$$
$$H(z)(1-\alpha z^{-R})=1-\alpha^Nz^{-NR}$$
$$H(z)=\frac{1-\alpha^Nz^{-NR}}{1-\alpha z^{-R}}$$

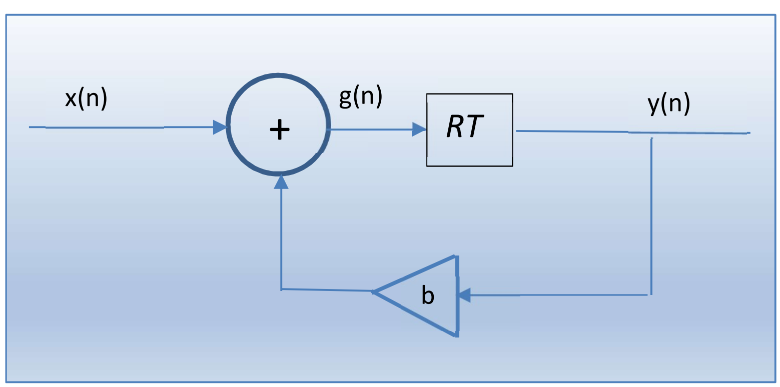
Now we need to verify that the above block diagram does indeed produce the compact transfer function derived above. Let the output from the 1st summer be defined as *g(n).* Then:

$$G(z) = X(z) + \alpha G(z) z^{-R}$$
$$G(z) - \alpha G(z) z^{-R} = X(z)$$
$$X(z) = G(z)\left\{1 - \alpha z^{-R}\right\}$$
$$Y(z) = G(z) - \alpha^N G(z) z^{-NR} = G(z)\left\{1 - \alpha^N z^{-NR}\right\}$$
$$Y(z) = \frac{X(z)}{\left\{1 - \alpha z^{-R}\right\}}\left\{1 - \alpha^N z^{-NR}\right\}$$
$$H(z) = \frac{Y(z)}{X(z)} = \alpha z^{-R} + \alpha^2 z^{-2R} + \alpha^3 z^{-3R} + \ldots\ldots\ldots + \alpha^N z^{-NR} = \frac{1 - \alpha^N z^{-NR}}{1 - \alpha z^{-R}}$$

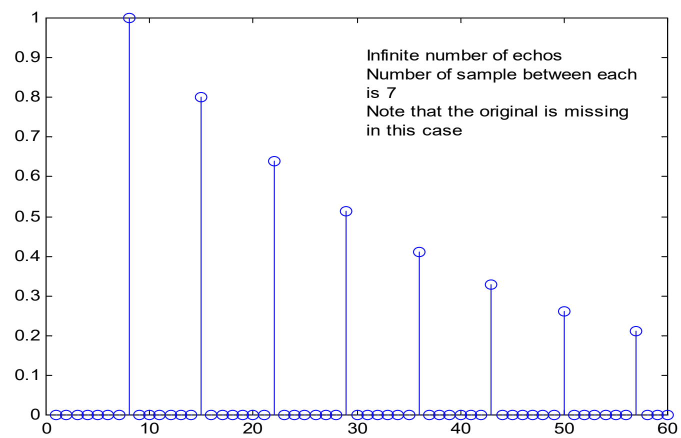


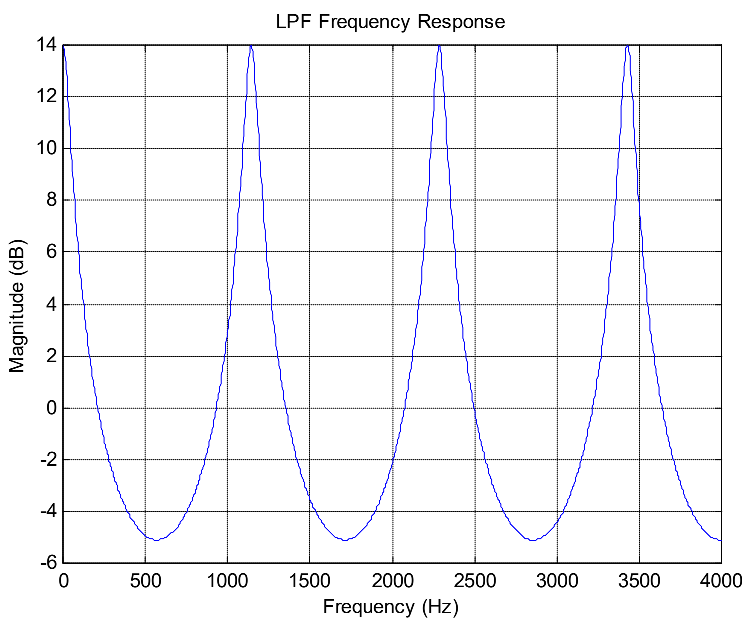


Infinite echo filter



$$g(n) = x(n) + by(n)$$
$$y(n) = g(n-R) = x(n-R) + by(n-R)$$
$$Y(z) = G(z)z^{-R} = X(z)z^{-R} + bY(z)z^{-R}$$
$$Y(z)\left(1 - bz^{-R}\right) = X(z)z^{-R}$$
$$H(z) = \frac{Y(z)}{X(z)} = \frac{z^{-R}}{1 - bz^{-R}}$$





It should be reasonably clear that this filter makes an infinite number of copies of itself scaled by '*b*'.