

# POWER FREQUENCY CONTROL

General requirements for operation of a power network:

**Frequency** and **voltage** be maintained within limits

- Frequency
  - system-wide parameter in the steady state
  - $\pm 0.4\%$  ( $\pm 0.2\text{Hz}$  in a  $50\text{Hz}$  system)  $49.8 \rightarrow 50.2\text{Hz}$
- Voltage
  - varies considerably within power network
  - depends on the loading
  - $\pm 6\%$  (depends on voltage level)

Control of voltage and frequency

- Frequency by system wide control (AGC)
- Voltage by local control (OLTC, SVC, etc.)

# Frequency and Active Power

Strong correlation between

- load or rotor angle (frequency) and active power  $f \Leftrightarrow P$
- voltage and reactive power  $V \Leftrightarrow Q$

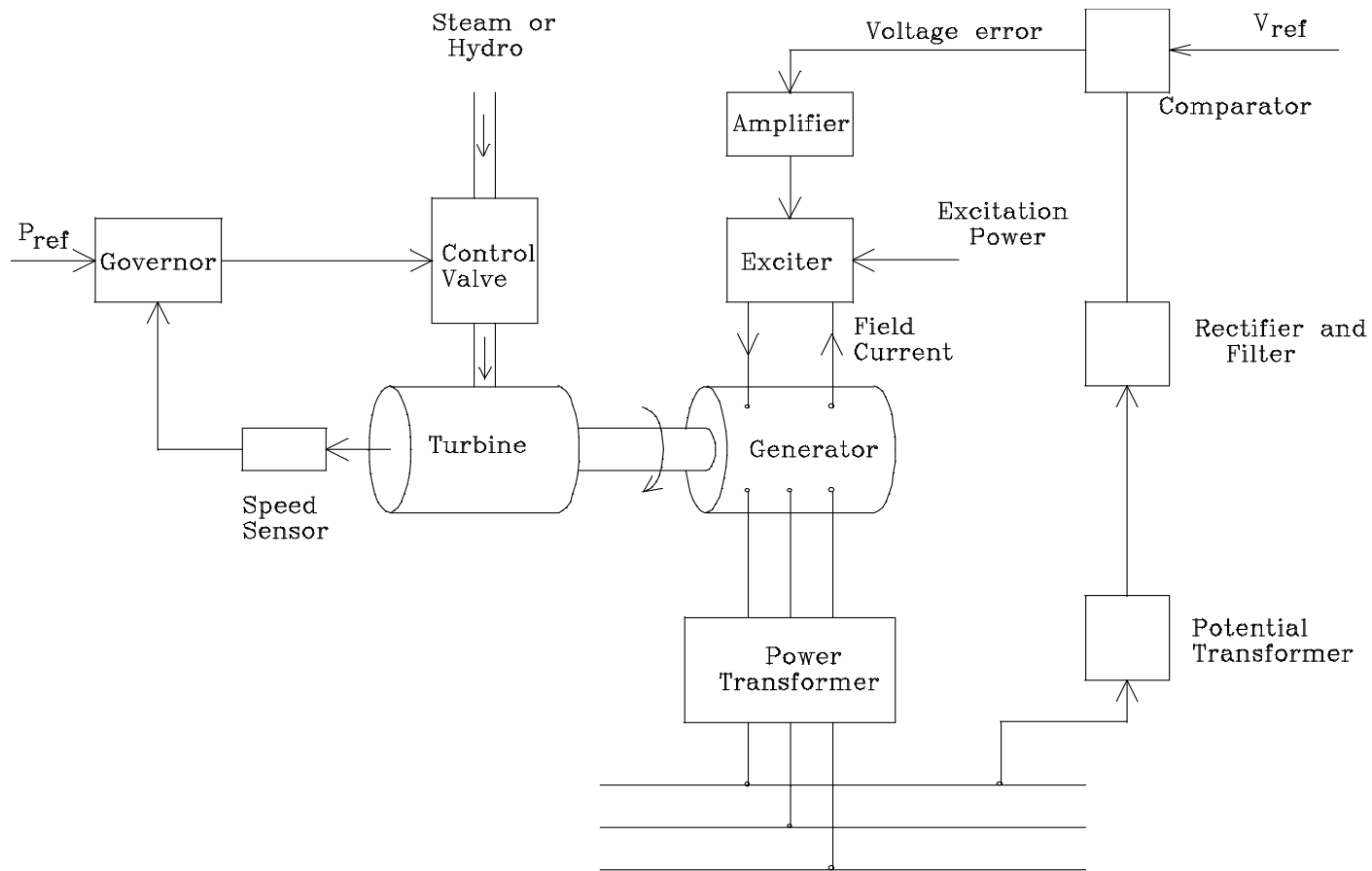
This section deals with

- Control of frequency (speed of generators)
- Effect of interconnection to other systems
- Voltage control is considered separately

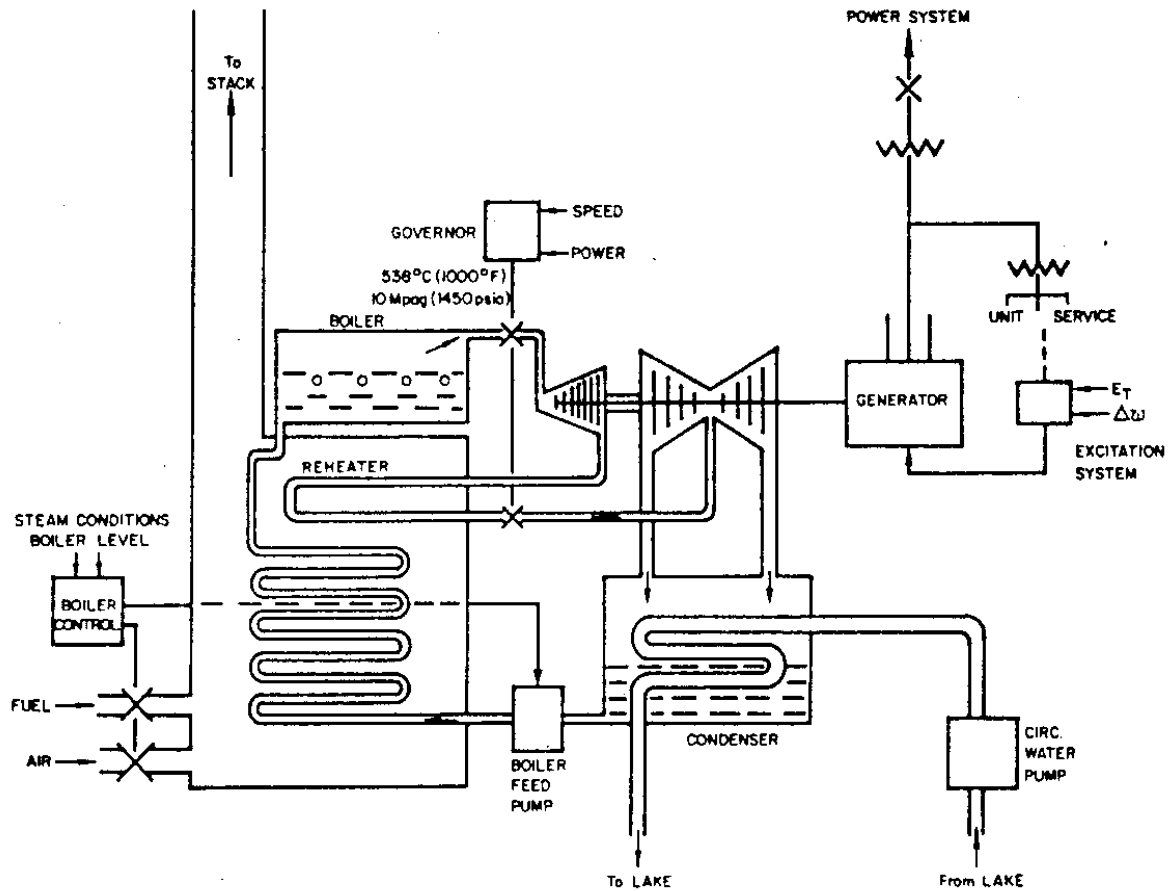
Two control loops:

- Automatic load frequency control loop (ALFC)
- Automatic voltage control loop (AVR)
- Little interaction from the ALFC loop to the AVR loop
- AVR loop does affect the ALFC loop to a certain extent
- AVR loop tends to be much faster than the ALFC loop
- Time constants of much less than one second

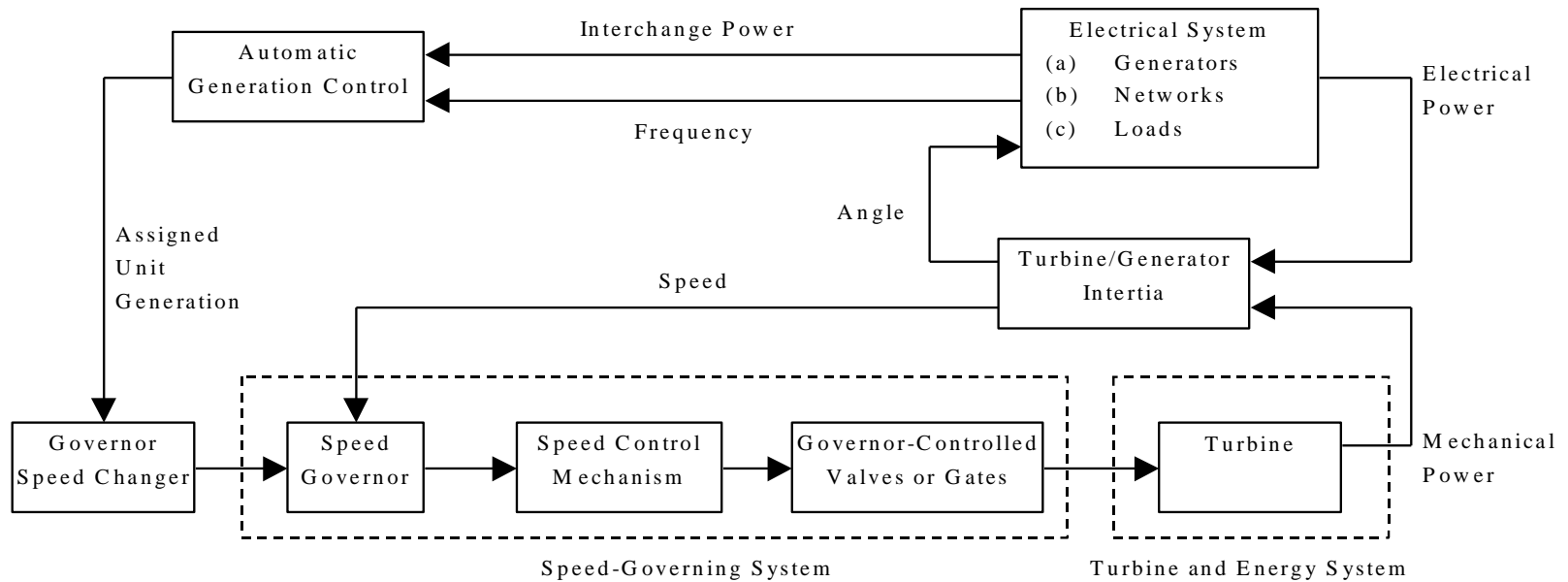
# AVR and Turbine Control Loops



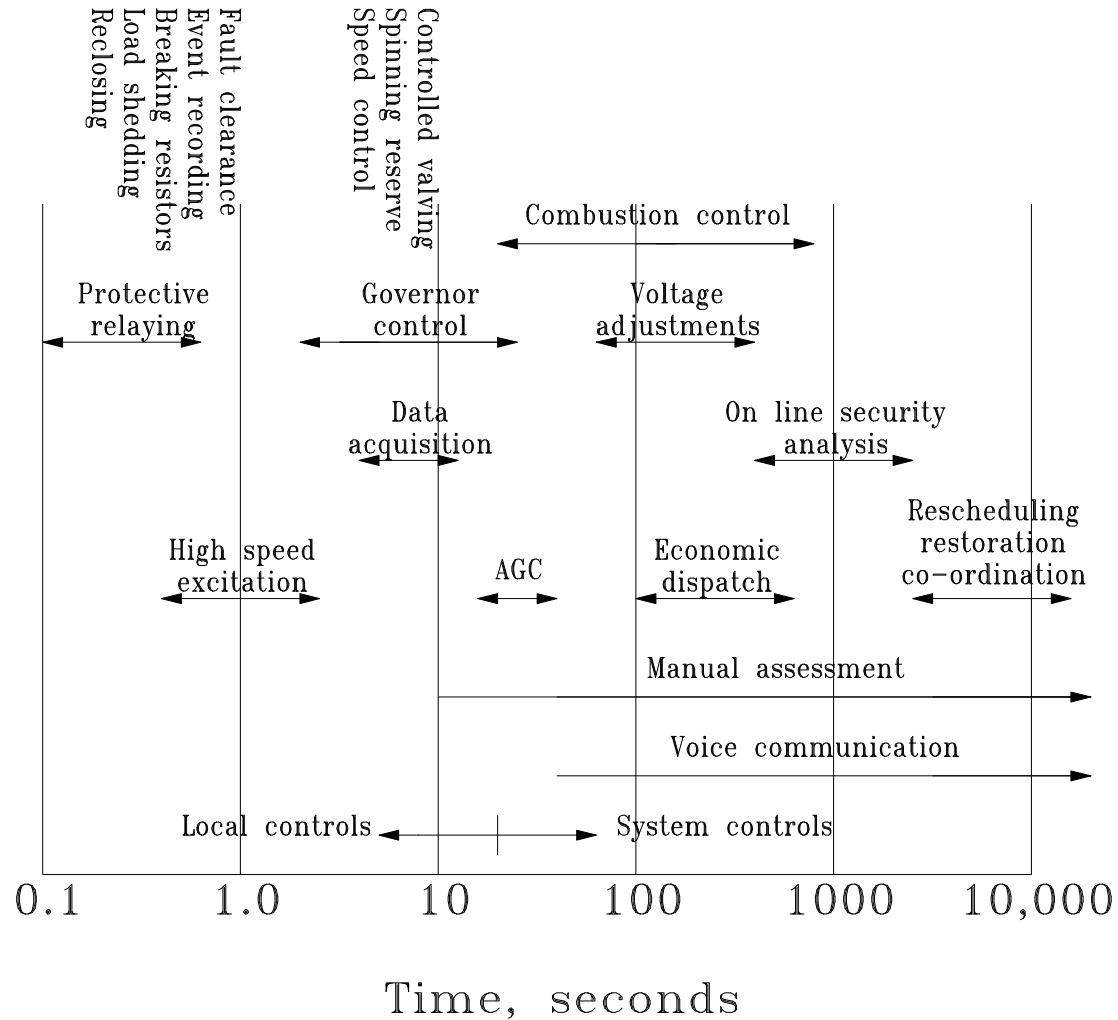
# Boiler/Turbine/Generator Schematic Diagram



# Speed-Governing System and Turbine



# Spectrum of responses in power networks



# Small-signal Analysis of Power Systems

Analysis of power networks divided into:

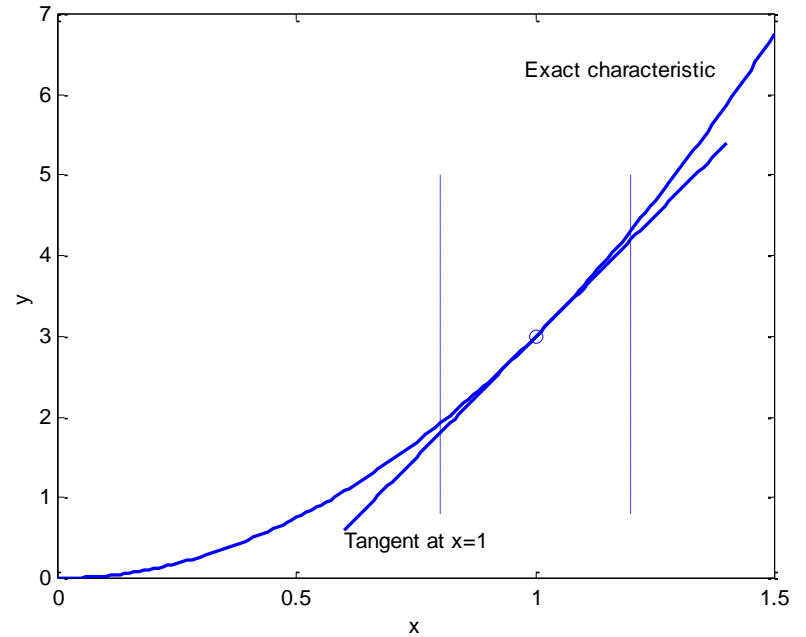
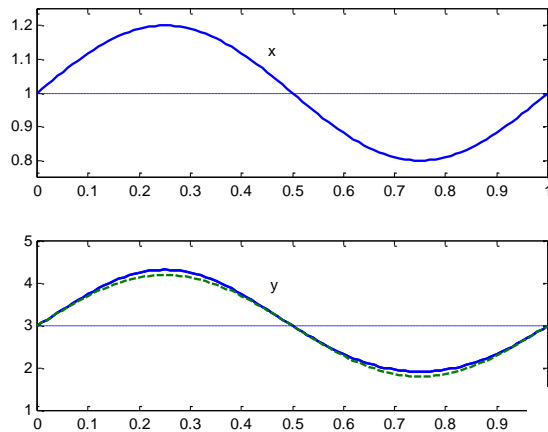
- Large-signal analysis:
  - Time-domain analysis of events
  - Major faults, voltages can change by 100%
- Small-signal analysis
  - Small-changes:
  - Response considered linear
  - Laplace transform can be used
  - Frequency domain response
  - Power frequency control in power systems is usually investigated using linear or small-signal methods

# Small-Signal Analysis – An Example

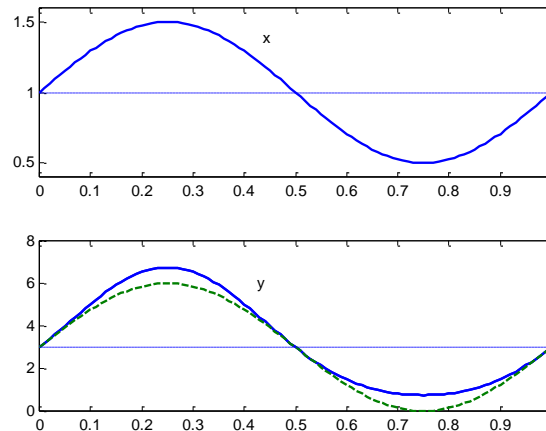
Non-linear relationship:  $y = 3x^2$

Let  $x$  vary about 1, sinusoidal function

Try:  $x = 1 + 0.2 \sin \omega t$



and then:  $x = 1 + 0.5 \sin \omega t$





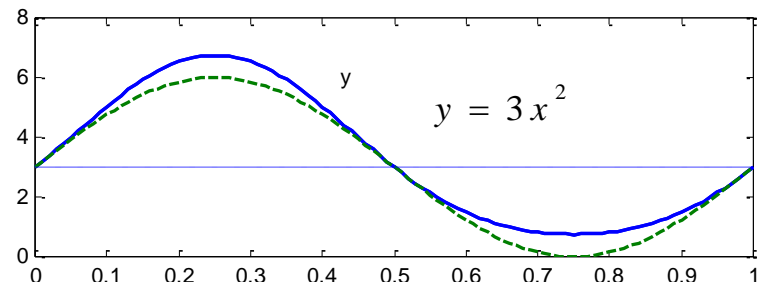
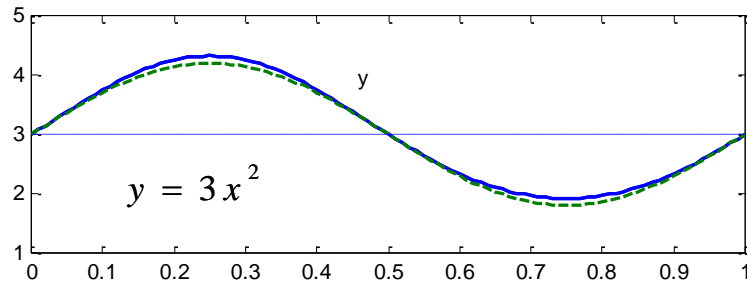
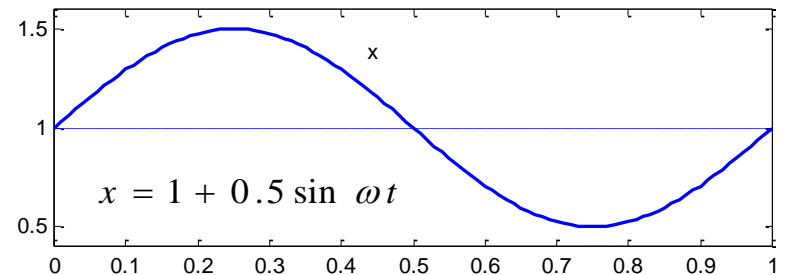
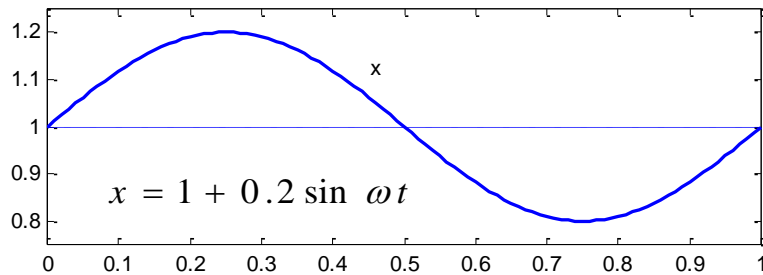
# Small-Signal Analysis – An Example

Original function:  $y = 3x^2$

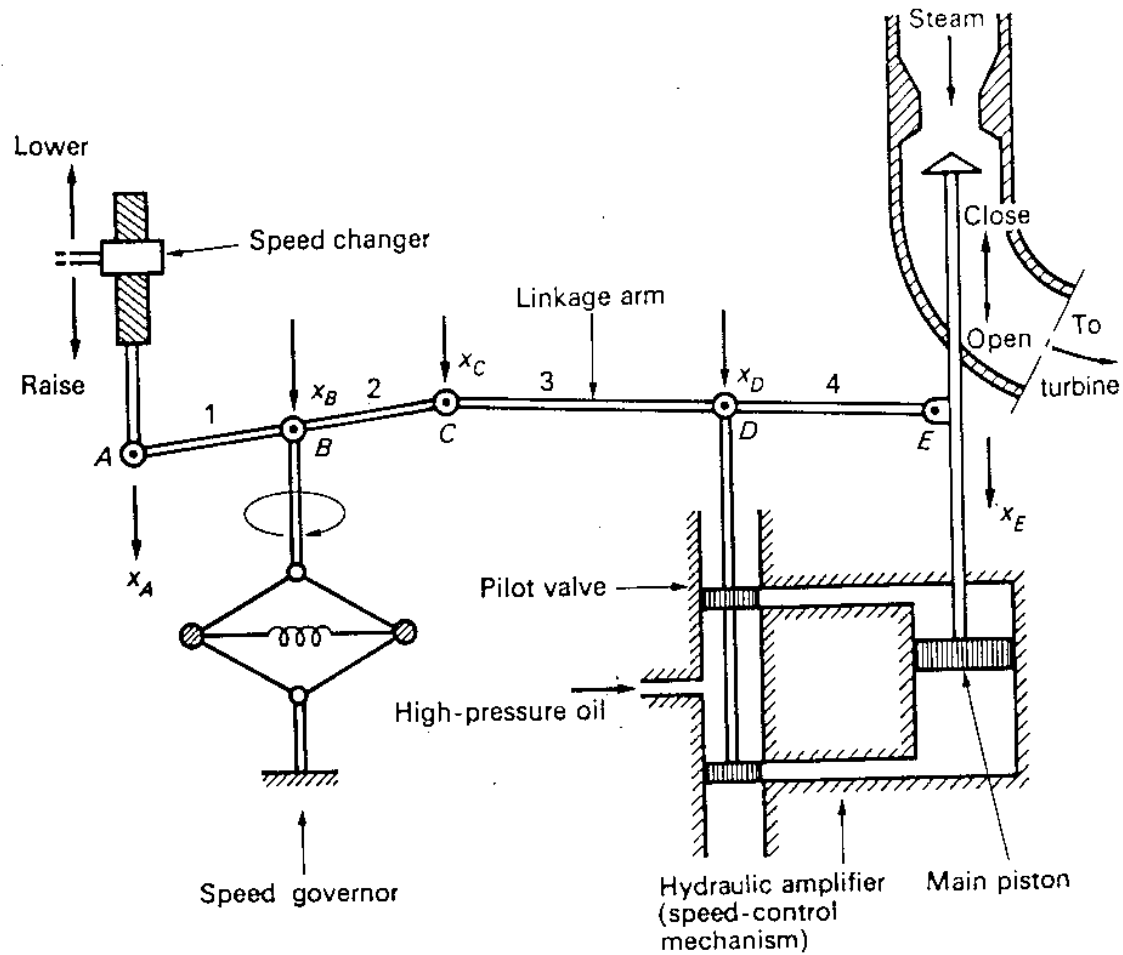
Linear approximation about  $x_0=1$ :

$$y = y_0 + \left( \frac{dy}{dx} \bigg|_{x=x_0} \right) \Delta x = 3 + 6 \Delta x$$

Works well for small  $\Delta x$ , not so well as  $\Delta x$  increases



# Power Control Mechanism



# Analysis of Power Control Mechanism

- Downward movement is +ve

$$\Delta x_c = k_1 \Delta f - k_2 \Delta P_{ref}$$

$$\Delta x_d = k_3 \Delta x_c + k_4 \Delta x_e$$

- k's determined by linkage arms, etc
- Pressure on value
  - determines position of main piston
  - proportional to fluid into amplifier

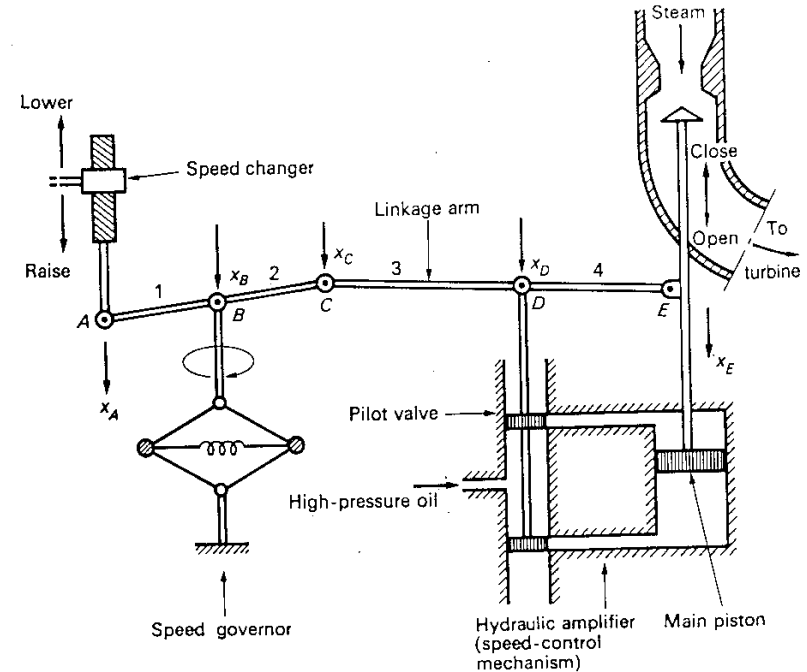
$$\Delta x_e = -k_s \int \Delta x_d dt$$

- Take Laplace Transform

$$\Delta x_e(s) = -\frac{k_s}{s} \Delta x_d(s)$$

$$\Delta x_d = k_3 k_1 \Delta f - k_2 k_3 \Delta P_{ref} + k_4 \Delta x_e$$

$$\Delta x_e = \frac{-k_1 k_3 \Delta F(s) + k_2 k_3 \Delta P_{ref}}{k_3 + s/k_s} = \frac{K_G}{1 + sT_G} \left( \Delta P_{ref} - \frac{1}{R} \Delta F(s) \right)$$



# Small-Signal Representation of Speed Control (1)

$$\Delta x_e = \frac{K_G}{1 + sT_G} \left( \Delta P_{ref} - \frac{1}{R} \Delta F(s) \right)$$

- R (Hz/MW): Regulation or droop  
 $\Delta P_C(s)$ ,  $\Delta P_{ref}(s)$ : LT of change in reference power  
 $\Delta F(s)$ : Change in frequency

Transfer function for turbine:

Relating the power in to the mechanical power out

$$G_T(s) = \frac{\Delta P_T(s)}{\Delta x_e(s)} = \frac{K_T}{1 + sT_T} \quad \text{Steam turbine, no reheat}$$

$$G_T(s) = \frac{K_T}{1 + sT_T} \left[ \frac{1 + s\alpha T_{RH}}{1 + sT_{RH}} \right] \quad \text{Steam turbine, with reheat}$$

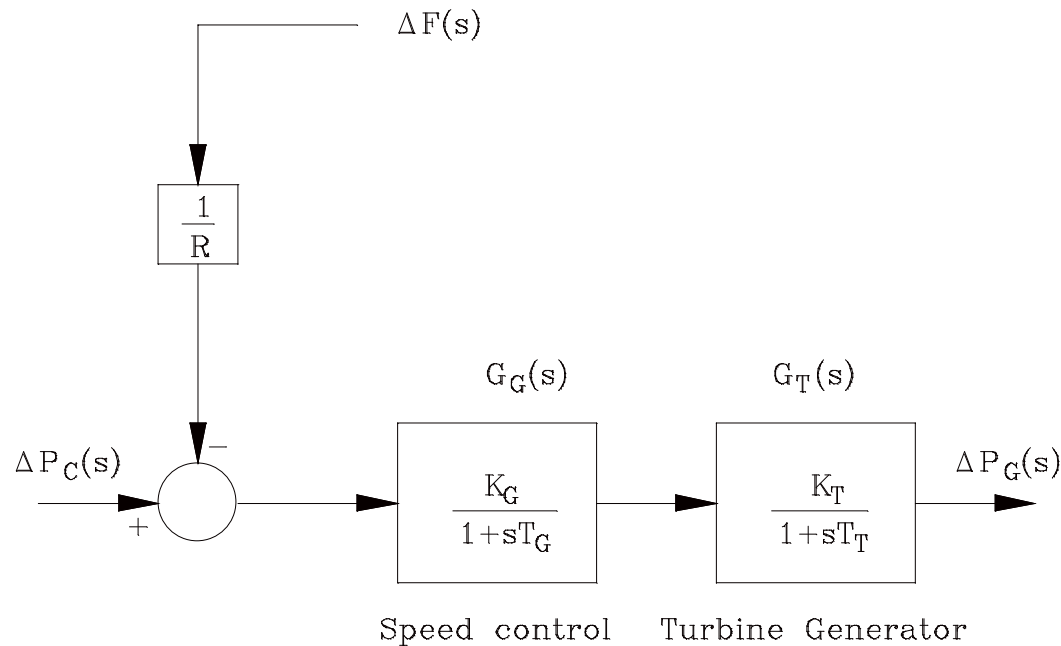
$$G_T(s) = K_T \frac{1 - 2sT_W}{1 + sT_W} \quad \text{Hydro}$$

# Small-Signal Representation of Speed Control (2)

Incorporate generator representation

$\Delta P_T(s)$ : Laplace transform of electrical power output

$K_G K_T = 1$  Change in input power is same as change at output



# Static Performance of Speed Control (1)

Steady state response (Final Value theorem)

$$\Delta P_G = \Delta P_c - \frac{1}{R} \Delta f$$

If  $\Delta f = 0$ , generator connected to an infinite bus:

$$\Delta P_G = \Delta P_c$$

Changing reference power = change in power being delivered to infinite bus

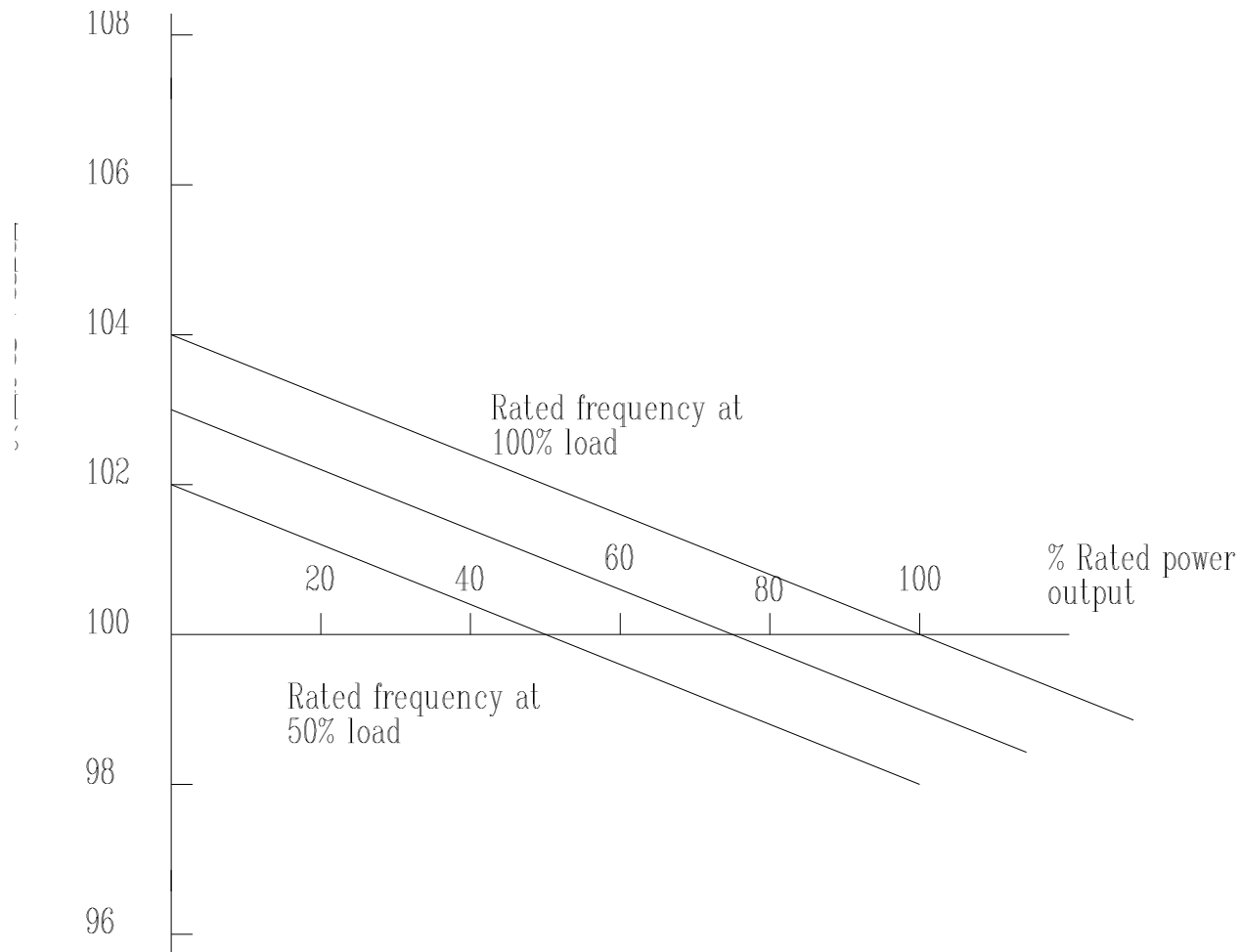
Isolated generator, without changing reference:

$$\Delta P_G = -\frac{1}{R} \Delta f$$

- Increase in output power means drop in frequency
- Determined by the droop characteristic R, measure of how responsive the speed controller is to changes in frequency
- Hz/MW, Hz/pu MW, %Hz/pu MW

4% droop means 2Hz/pu MW for a 50Hz system.

## Static Performance of Speed Control (2)



# Static Performance of Speed Control - Example

Two generators, 50MW and 500MW, connected to common bus

- Each at half loading, Change in the load of 110MW
- Frequency drop to 49.6Hz

$$R_1 = \frac{0.4}{10} = 0.04 \text{ Hz} / \text{MW} \quad (\text{Small unit})$$

$$R_2 = \frac{0.4}{100} = 0.004 \text{ Hz} / \text{MW} \quad (\text{Large unit})$$

Correct distribution of 1:10 between the machines

$$R_1 = 0.04 \text{ Hz} / \text{MW} = 0.04 \frac{50}{50} \text{ pu Hz/pu MW} = 4.0 \text{ \%Hz/pu MW}$$

$$R_2 = 0.004 \text{ Hz} / \text{MW} = 0.004 \frac{500}{50} \text{ pu Hz/pu MW} = 4.0 \text{ \%Hz/pu MW}$$

when expressed as %Hz/pu MW



# The Power System Model (1)

To close the ALFC loop, require model for the power system

- Representation of dynamic relationship between system frequency, demand power and generator output
- System load generally decreases as frequency decreases

Pre-disturbance operating condition:

$f^0$  - initial frequency

$$\Delta P_G = 0$$

$W_{kin}^0$  - initial kinetic energy

$$\Delta P_D = 0$$

Changes in system frequency lead to changes in kinetic energy of rotating machinery:

$$W_{kin} = W_{kin}^0 \left( \frac{f}{f^0} \right)^2$$

Change in power:

$$\Delta P_G - \Delta P_D = \text{change in generator output} - \text{change in customer demand}$$

$$= \frac{d}{dt}(W_{kin}) + D \Delta f$$

## The Power System Model (2)

$$D = \frac{\partial P_D}{\partial f} \quad (\text{MW/Hz})$$

- describes change in system demand due to change in frequency (+ve)

Frequency:  $f = f^0 + \Delta f$

$$W_{\text{kin}} = W_{\text{kin}}^0 \left[ \frac{f^0 + \Delta f}{f^0} \right]^2 = W_{\text{kin}}^0 \left[ 1 + 2 \frac{\Delta f}{f^0} + \left( \frac{\Delta f}{f^0} \right)^2 \right]$$

$$\approx W_{\text{kin}}^0 \left[ 1 + 2 \frac{\Delta f}{f^0} \right]$$

$$\Delta P_G - \Delta P_D = \frac{2 W_{\text{kin}}^0}{f^0} \frac{d}{dt} [\Delta f] + D \Delta f$$

Kinetic energy and inertia constant H:

$$H = \frac{\text{kinetic energy}}{\text{rated MW}} = \frac{\text{MW} \times \text{s}}{\text{MW}} (= \text{seconds})$$

# The Power System Model (3)

In per unit:

$$\Delta P_G - \Delta P_D = \frac{2 W_{kin}^0}{f^0} \frac{d}{dt} [\Delta f] + D \Delta f$$

Laplace transform:

$$\Delta P_G(s) - \Delta P_D(s) = \frac{2 H}{f^0} s \Delta F(s) + D \Delta F(s)$$

$$\Delta F(s) = G_P(s) [\Delta P_G(s) - \Delta P_D(s)]$$

or

$$G_P(s) = \frac{K_P}{1 + s T_P}$$

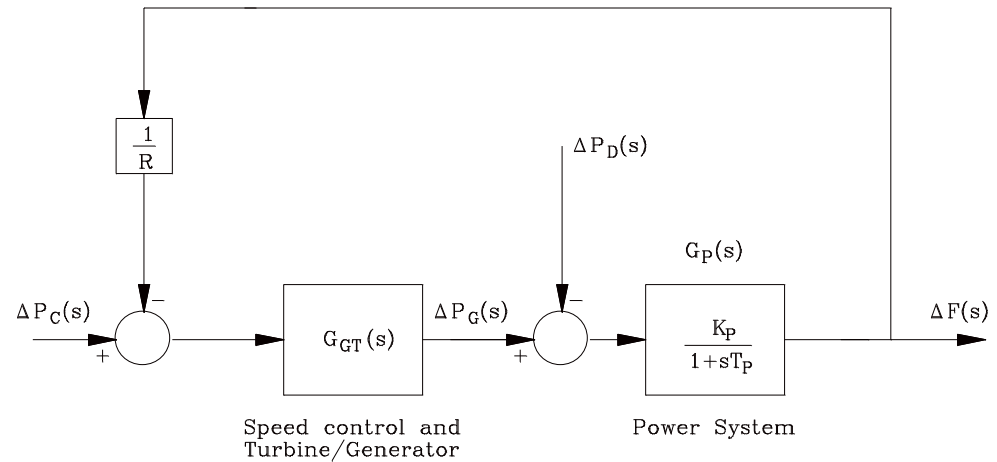
$$T_P = \frac{2 H}{f^0 D}$$

$$K_P = \frac{1}{D}$$

$K_P$  relates change in power to  
change in frequency

Static performance of the  
power system:

$$\Delta f = K_P [\Delta P_G - \Delta P_D]$$



# The Power System Model (4)

Overall transfer function:

$$\Delta F = \frac{G_P G_G G_T}{1 + \frac{1}{R} G_P G_G G_T} \Delta P_C - \frac{G_P}{1 + \frac{1}{R} G_P G_G G_T} \Delta P_D$$

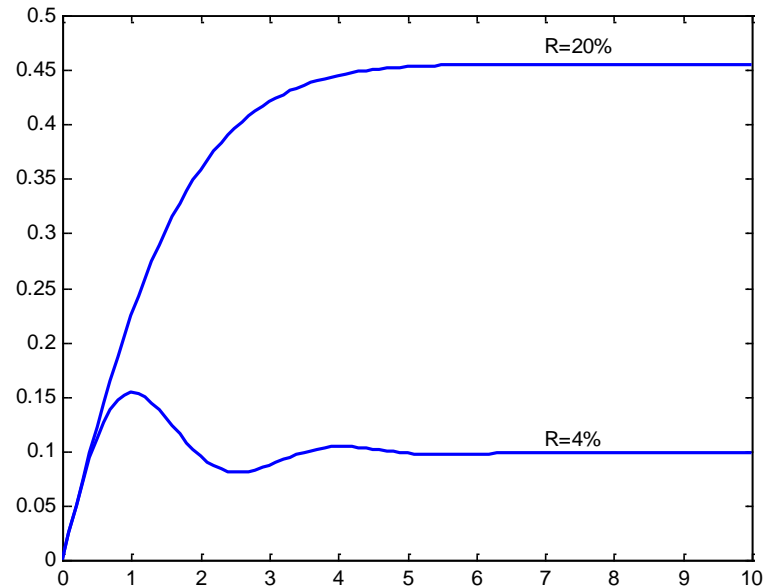
If  $\Delta P_C=0$ , and step change occurs in the load demand:

$$\Delta P_D(s) = \frac{M}{s}$$

Change in frequency is given by:

$$\Delta F = - \frac{G_P}{1 + \frac{1}{R} G_P G_G G_T} \frac{M}{s}$$

$$\Delta P_D = -0.05 \text{ p.u.}$$



# The Power System Model (5)

Steady-state frequency change: 
$$\Delta f = \lim_{t \rightarrow \infty} s \Delta F(s) = \frac{-K_p}{1 + K_p / R} M$$
$$= \frac{-M}{D + 1 / R} = \frac{-M}{\beta}$$

Area Frequency Response Characteristic (AFRC):  $\beta = D + 1 / R$

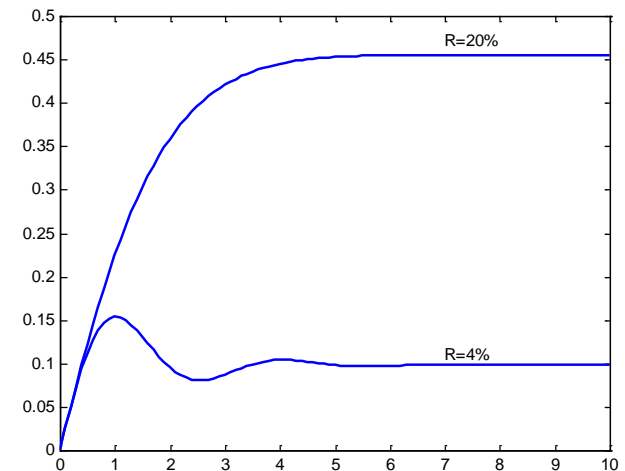
For example, if  $\Delta P_p = -0.05$  pu and  $D = 0.01$  pu MW/Hz:

$R = 4\%$ ,  $\beta = 0.01 + 0.5 = 0.51$ ,  $\Delta f = 0.098$  Hz

$R = 20\%$ ,  $\beta = 0.01 + 0.1 = 0.11$ ,  $\Delta f = 0.454$  Hz

$R = \infty$ ,  $\beta = 0.01$ , change in frequency is 5 Hz.

Original change in demand completely balanced  
by equal and opposite drop in system demand



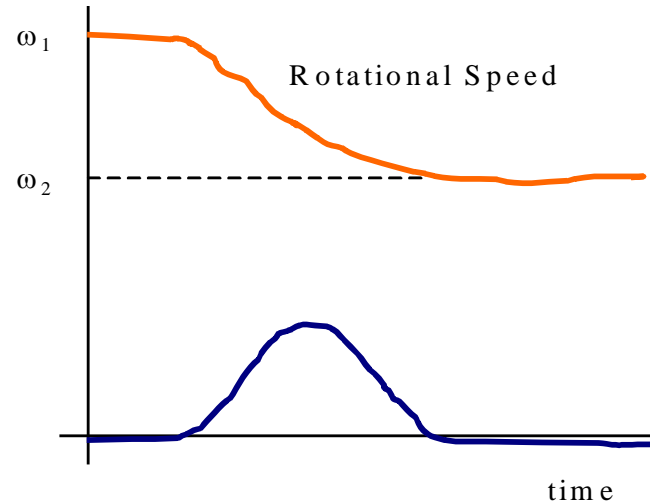
# The Power System Model (6)

3 sources of power in system to meet increase in demand:

- kinetic energy of the rotating machinery (short term)
- increased generation because of the speed control loop.
- reduction in load due to a decrease in frequency.

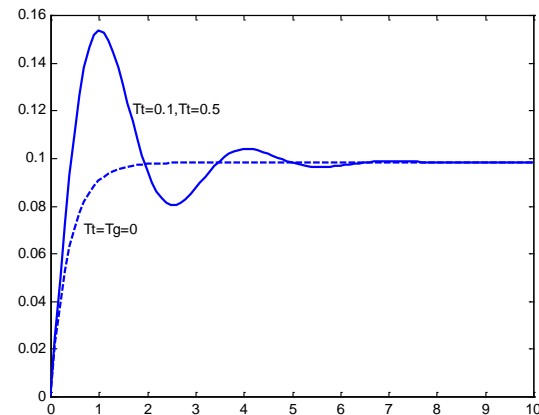
$$\frac{\Delta f}{R}$$

$$\frac{\Delta f}{K_P} = D \Delta f$$



Often, time constants  $T_G$  and  $T_T$  are set to zero

- Simplify the analysis, reduces system to a first order model
- Steady-state values remain unchanged
- Dynamic behaviour changes greatly



# The Reset Loop (1)

Speed governor action only:

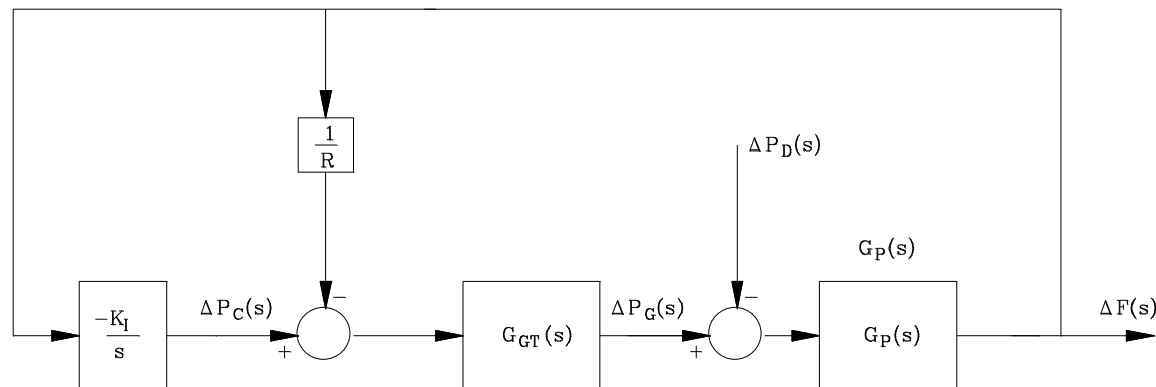
finite error in frequency in steady-state

$$\Delta f = \frac{-M}{\beta}$$

Integral control action added

$$\Delta P_C = -K_I \int \Delta f dt$$

- Changes reference power level
- Returns the system frequency error to zero
- Reference power continues to change until frequency error = 0
- Then  $\Delta P_C = \Delta P_D$



## The Reset Loop (2)

If  $T_G = T_T = 0$ , then:

$$\begin{aligned} \frac{\Delta F(s)}{\Delta P_D(s)} &= \frac{\frac{-K_p}{1 + sT_p}}{1 + \left[ \frac{K_p}{1 + sT_p} \right] \left[ \frac{1}{R} + \frac{K_I}{s} \right]} = \frac{-K_p R s}{s^2 T_p R + s(R + K_p) + K_p K_I R} \\ &= \frac{-K_p s / T_p}{s^2 + s \left( 1 + \frac{K_p}{R} \right) / T_p + K_p K_I / T_p} \end{aligned}$$

Characteristic equation:  $\left( s + \frac{1 + K_p / R}{2 T_p} \right)^2 + \frac{K_p K_I}{T_p} - \left( \frac{1 + K_p / R}{2 T_p} \right)^2 = 0$

System is oscillatory if:

$$K_I > \frac{1}{4 T_p K_p} (1 + K_p / R)^2$$



# Example

$T_p=20$  sec.,  $D=0.01$  puMW/Hz,  $R=2$ Hz/puMW

System becomes oscillatory at  $K_I=0.325$

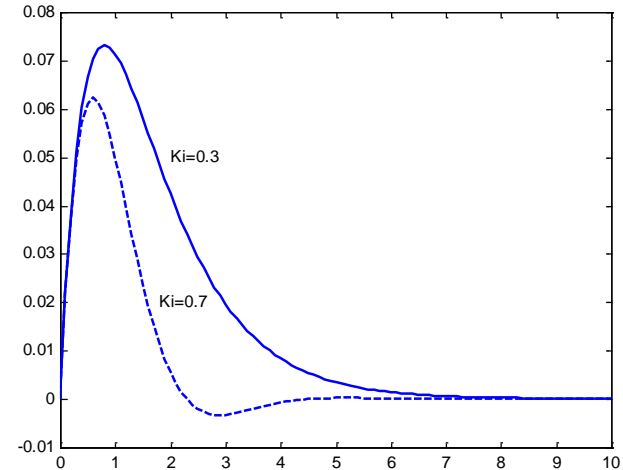
Higher gain in reset:

- faster response
- also cause oscillations or instability

Clocks controlled by system frequency:

$$t_e = \frac{1}{f} \int_0^T \Delta f dt$$

Change in frequency (even transient) cause time errors



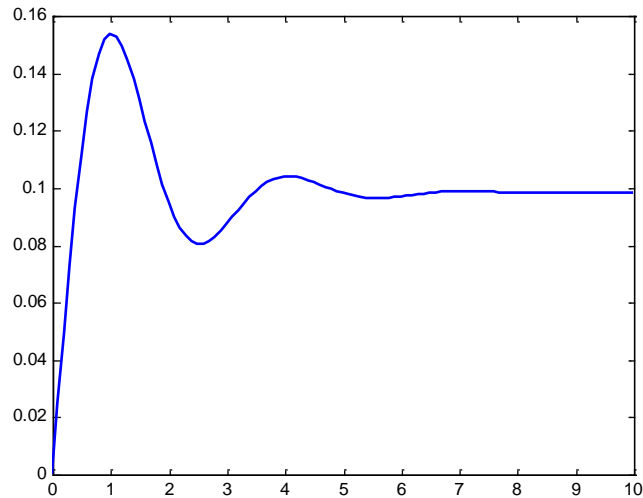
## Problem:

By applying the Final Value Theorem, calculate the time error due to a change in demand of 0.05pu in the system shown above if the integral control gain is 0.7. Suggest how this error might be subsequently set to zero.

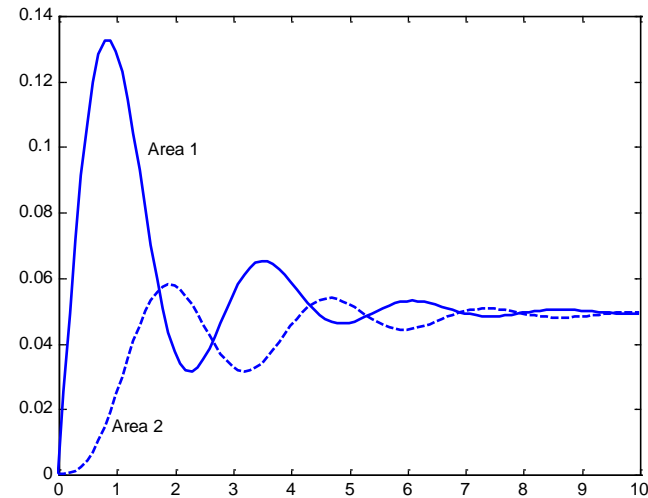
# Pool Operation (1)

Advantages to connecting power systems by interties or interconnectors:

- Interconnected systems provide support in the event of sudden large load or fault condition
- Lead to more economic operation
- Impact of load change on frequency will be reduced



One area



Two areas

# Time Error

<https://www.theverge.com/2018/3/8/17095440/europe-clocks-running-slow-electricity-frequency-kosovo-serbia>

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## Clocks are running slow across Europe because of an argument over who pays the electricity bill

12

*A dispute between Kosovo and Serbia means clocks are running up to six minutes slow*

By James Vincent | @jjvincent | Mar 8, 2018, 8:30am EST

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# Time Error

Over the past couple of months, Europeans have noticed time slipping away from them. It's not just their imaginations: all across the continent, clocks built into home appliances like ovens, microwaves, and coffee makers have been running up to six minutes slow. The unlikely cause? A dispute between Kosovo and Serbia over who pays the electricity bill.

To make sense of all this, you need to know that the clocks in many household devices use the frequency of electricity to keep time. Electric power is delivered to our homes in the form of an alternating current, where the direction of the flow of electricity switches back and forth many times a second. (How this system came to be established is complex, but the advantage is that it allows electricity to be transmitted efficiently.) In Europe, this frequency is 50 Hertz — meaning a current alternating of 50 times a second. In America, it's 60 Hz.

Since the 1930s, manufacturers have taken advantage of this feature to keep time. Each clock needs a metronome — something with a consistent rhythm that helps space out each second — and an alternating current provides one, saving the cost of extra components. Customers simply set the time on their oven or microwave once, and the frequency keeps it precise.

At least, that's the theory. But because this timekeeping method is reliant on electrical frequency, when the frequency changes, so do the clocks. That is what has been happening

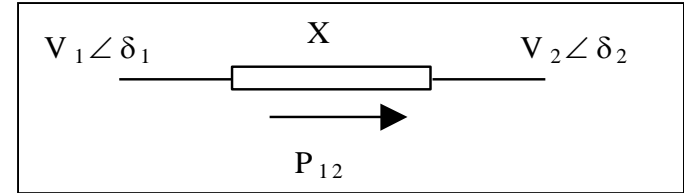
# Pool Operation (1)

Power transfer between two areas:

Initial point:

$$P_{12} = \frac{|V_1||V_2|}{X} \sin(\delta_1 - \delta_2)$$

$$P_{12}^0 = \frac{|V_1^0||V_2^0|}{X} \sin(\delta_1^0 - \delta_2^0)$$



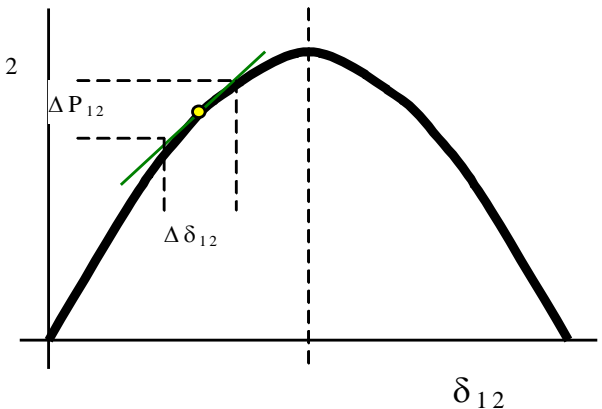
Small deviations from this point:

$$\Delta P_{12} = \frac{|V_1^0||V_2^0|}{X} \cos(\delta_1^0 - \delta_2^0) (\Delta \delta_1 - \Delta \delta_2) = T_{12}^0 (\Delta \delta_1 - \Delta \delta_2)$$

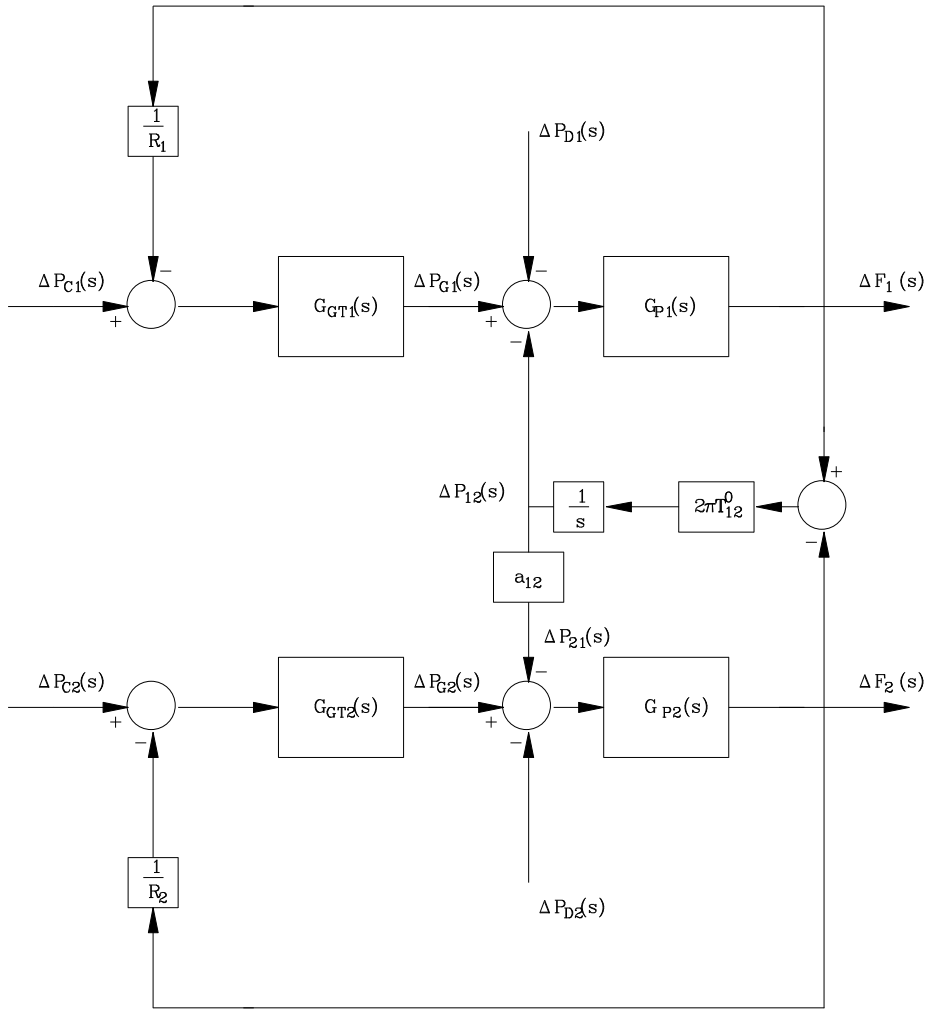
$$T_{12}^0 = \frac{|V_1^0||V_2^0|}{X} \cos(\delta_1^0 - \delta_2^0) \quad \text{synchronising coefficient}$$

$$\Delta f = \frac{1}{2\pi} \left[ \frac{d}{dt} (\delta^0 + \Delta \delta) \right] = \frac{1}{2\pi} \frac{d(\Delta \delta)}{dt} \quad \Delta \delta = 2\pi \int_0^t \Delta f dt$$

$$\Delta P_{12} = 2\pi T_{12}^0 \left( \int_0^t \Delta f_1 dt - \int_0^t \Delta f_2 dt \right) = \frac{2\pi T_{12}^0}{s} (\Delta F_1(s) - \Delta F_2(s))$$



## Pool Operation (2)



$$\Delta F_1(s) = \frac{-G_{P1}(s)}{1 + G_{P1}(s)/R_1} (\Delta P_{D1} + \Delta P_{12})$$

$$\Delta F_2(s) = \frac{-G_{P2}(s)}{1 + G_{P2}(s)/R_2} (\Delta P_{D2} + a_{12} \Delta P_{12})$$

$$\Delta P_{12} = \frac{2\pi T_{12}^0}{s} (\Delta F_1(s) - \Delta F_2(s))$$

$$a_{12} = - \frac{\text{Power Base Area 1}}{\text{Power Base Area 2}}$$

## Pool Operation (3)

$$\Delta F_1(s) = \frac{-G_{P1}(s)}{1 + G_{P1}(s)/R_1} (\Delta P_{D1} + \Delta P_{12})$$

$$\Delta P_{12} = \frac{2\pi T_{12}^0}{s} (\Delta F_1(s) - \Delta F_2(s))$$

$$\Delta F_2(s) = \frac{-G_{P2}(s)}{1 + G_{P2}(s)/R_2} (\Delta P_{D2} + a_{12} \Delta P_{12})$$

Change in load of  $M_1$  in area 1 and  $M_2$  in area 2: 
$$\Delta P_{12} = \frac{\beta_1 M_2 - \beta_2 M_1}{\beta_1 + \beta_2}$$

where: 
$$\beta_1 = D_1 + 1/R_1; \quad \beta_2 = D_2 + 1/R_2$$

Frequencies: 
$$\Delta f_1 = -\left[ \frac{M_1 + M_2}{\beta_1 + \beta_2} \right]; \quad \Delta f_2 = -\left[ \frac{M_1 + M_2}{\beta_1 + \beta_2} \right]$$

### Problem:

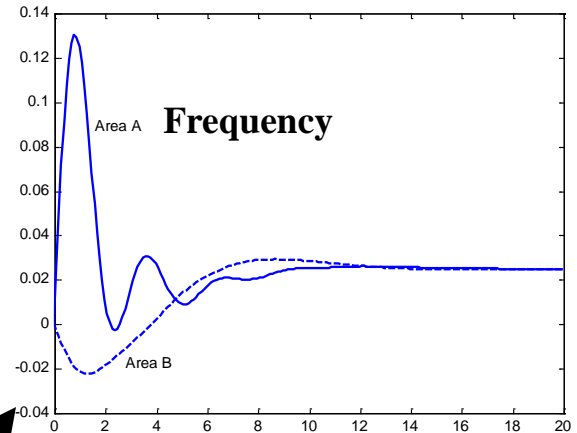
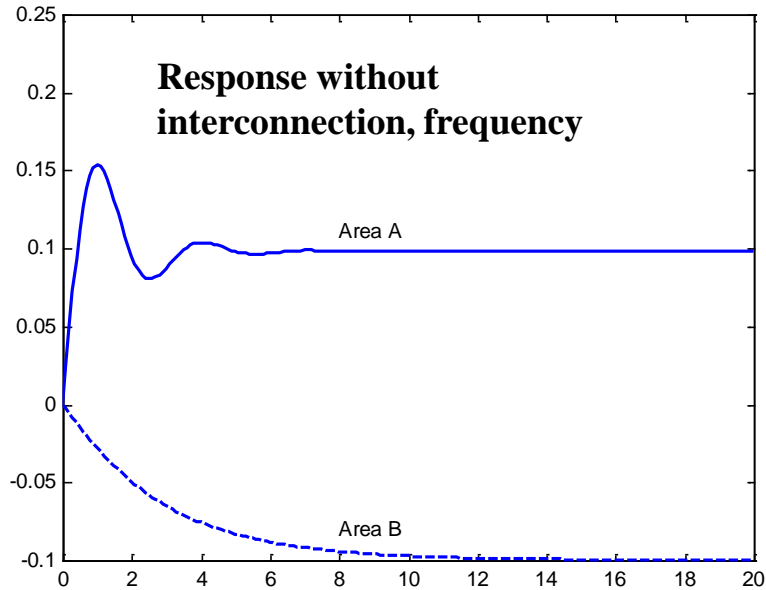
Two areas connected by a transmission line and have the following values:

Area 1:  $R=4\%$ ,  $T_P=20\text{sec}$ ,  $D=0.01$ ,  $\Delta P_D=-0.05$

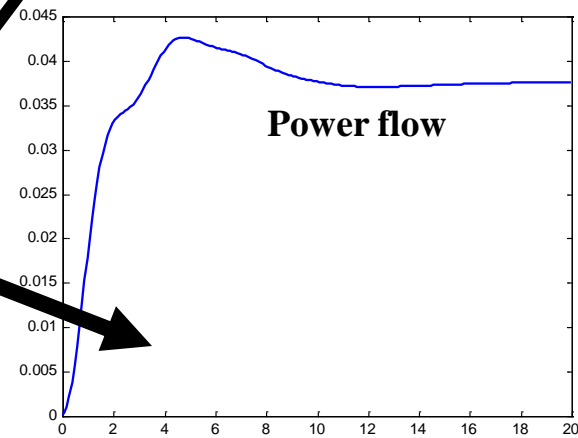
Area 2:  $R=8\%$ ,  $T_P=20\text{sec}$ ,  $D=0.05$ ,  $\Delta P_D=0.03$

Calculate eventual steady-state power flow between the areas and the frequency deviation.

## Pool Operation (4)



**Response with interconnection**





# Pool Operation (5)

Area control error (ACE) consists of components of power error and the frequency error:

$$ACE_1 = \Delta P_{12} + B_1 \Delta f_1$$

$$ACE_2 = \Delta P_{21} + B_2 \Delta f_2$$

Laplace Transform of this:

$$\Delta P_{C1}(s) = -\frac{K_{11}}{s}(\Delta P_{12}(s) + B_1 \Delta F_1(s))$$

$$\Delta P_{C2}(s) = -\frac{K_{12}}{s}(\Delta P_{21}(s) + B_2 \Delta F_2(s))$$

Static response is given by:

$$ACE_1 = \Delta P_{12} + B_1 \Delta f_1 = \Delta P_{12} + B_1 \Delta f = 0$$

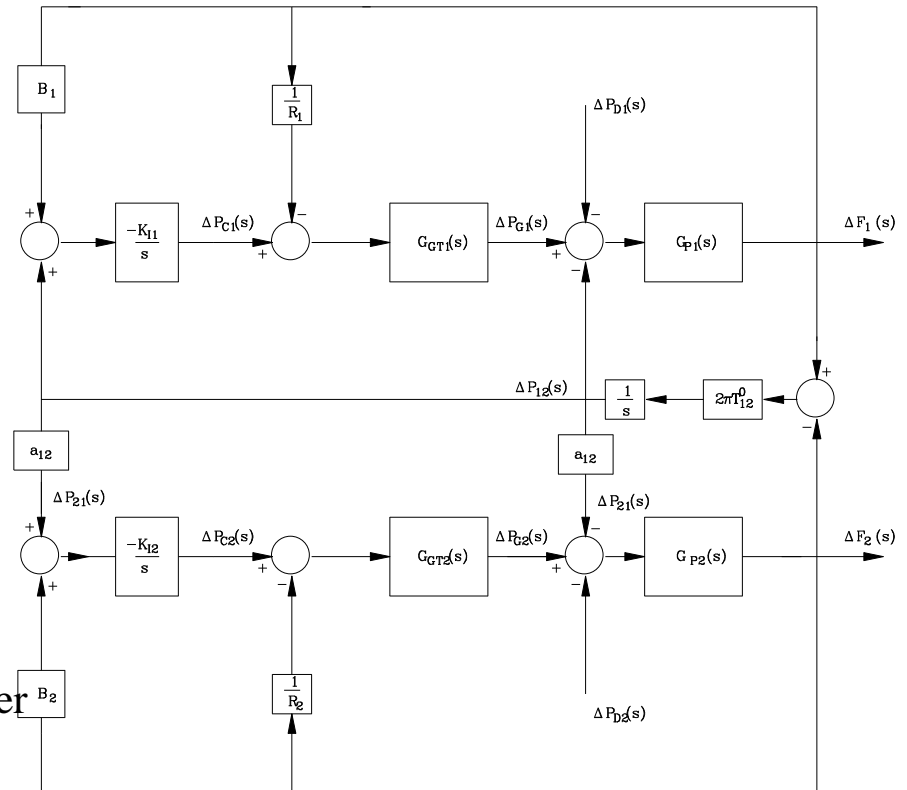
$$ACE_2 = \Delta P_{21} + B_2 \Delta f_2 = \Delta P_{21} + B_2 \Delta f = 0$$

Condition met for  $\Delta f=0$  and  $\Delta P_{12}=\Delta P_{21}=0$

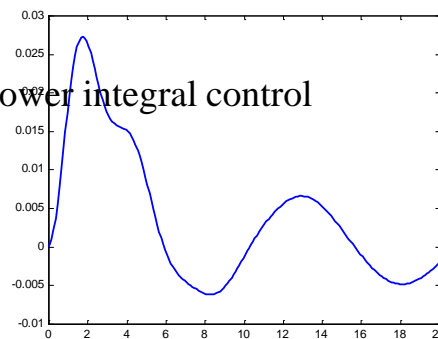
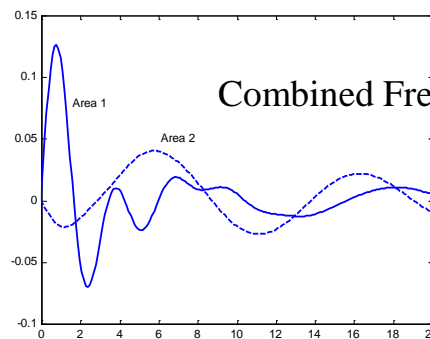
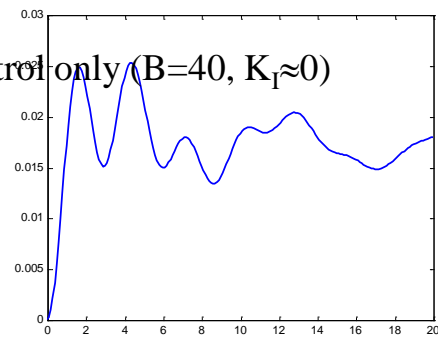
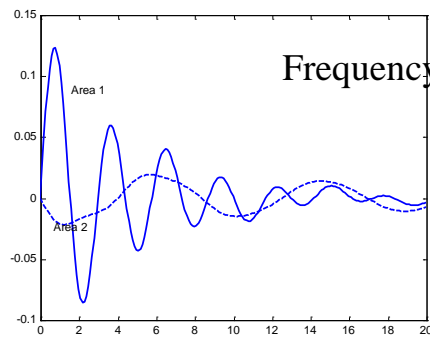
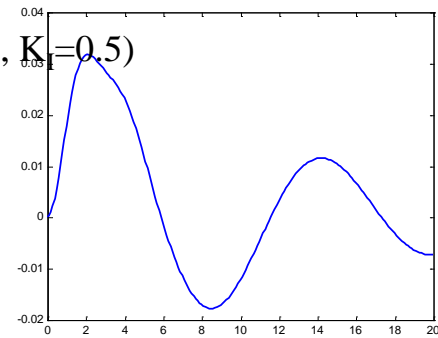
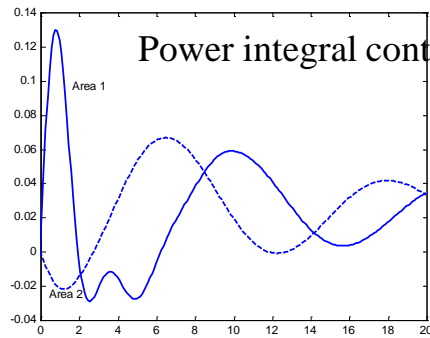
Relative values of B and  $K_I$  selected to control how quickly the frequency and power are driven to zero after a disturbance

$$\Delta P_{C1} = -K_{11} \int (\Delta P_{12} + B_1 \Delta f_1) dt$$

$$\Delta P_{C2} = -K_{12} \int (\Delta P_{21} + B_2 \Delta f_2) dt$$



# Pool Operation (6)



# State Space Representation of Two-area System (1)

Individual states are identified as follows:

$$x_1(s) = \Delta F_1(s) = \frac{K_{P1}}{1 + sT_{P1}} [\Delta P_{G1} - \Delta P_{D1} - \Delta P_{12}]$$

$$x_2(s) = \Delta F_2(s) = \frac{K_{P2}}{1 + sT_{P2}} [\Delta P_{G2} - \Delta P_{D2} - a_{12} \Delta P_{12}]$$

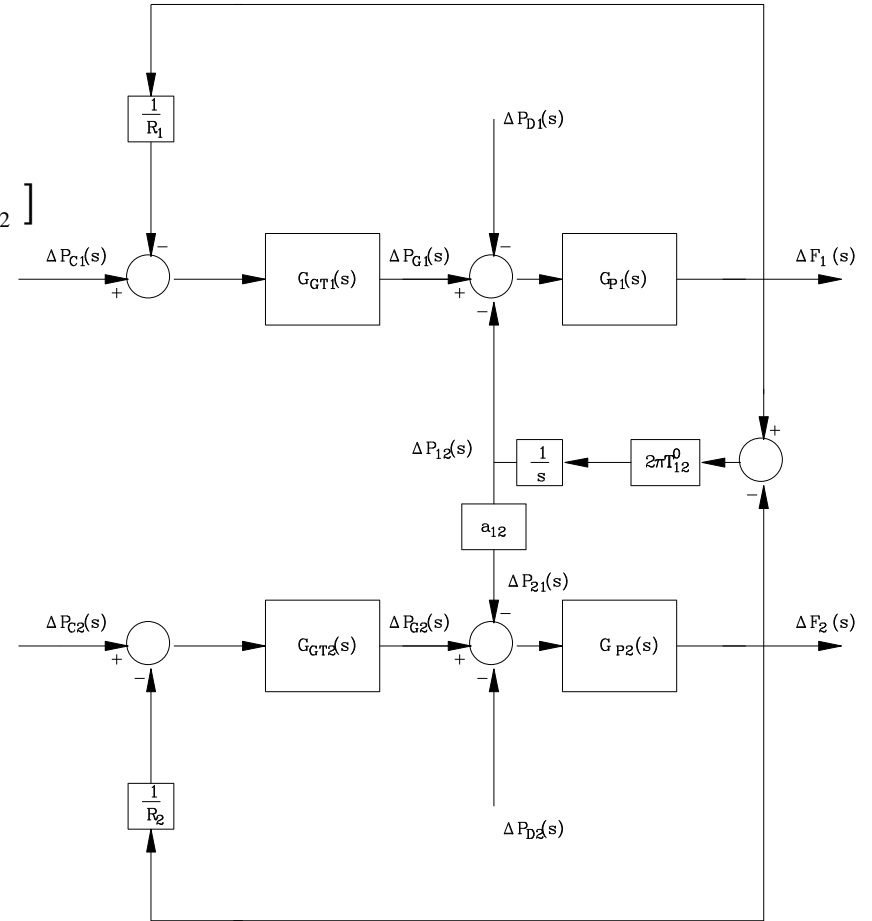
$$x_3(s) = \frac{1}{1 + sT_{G1}} \left[ \Delta P_{C1} - \frac{1}{R_1} \Delta F_1 \right]$$

$$x_4(s) = \frac{1}{1 + sT_{G2}} \left[ \Delta P_{C2} - \frac{1}{R_2} \Delta F_2 \right]$$

$$x_5(s) = \Delta P_{G1} = \frac{1}{1 + sT_{T1}} x_3(s)$$

$$x_6(s) = \Delta P_{G2} = \frac{1}{1 + sT_{T2}} x_4(s)$$

$$x_7(s) = \frac{2\pi T_{12}^0}{s} [\Delta F_1(s) - \Delta F_2(s)] = \Delta P_{12}$$



# State Space Representation of Two-area System (2)

Differential equations are given by:

$$\dot{x}_1 = -\frac{1}{T_{P1}}x_1 + \frac{K_{P1}}{T_{P1}}x_5 - \frac{K_{P1}}{T_{P1}}x_7 - \frac{K_{P1}}{T_{P1}}\Delta P_{D1}$$

$$\dot{x}_2 = -\frac{1}{T_{P2}}x_2 + \frac{K_{P2}}{T_{P2}}x_6 - a_{12}\frac{K_{P2}}{T_{P2}}x_7 - \frac{K_{P2}}{T_{P2}}\Delta P_{D2}$$

$$\dot{x}_3 = -\frac{1}{T_{G1}}x_3 - \frac{1}{R_1T_{G1}}x_1 + \frac{1}{T_{G1}}\Delta P_{C1}$$

$$\dot{x}_4 = -\frac{1}{T_{G2}}x_4 - \frac{1}{R_2T_{G2}}x_2 + \frac{1}{T_{G2}}\Delta P_{C2}$$

$$\dot{x}_5 = \frac{1}{T_{T1}}x_3 - \frac{1}{T_{T1}}x_5$$

$$\dot{x}_6 = \frac{1}{T_{T2}}x_4 - \frac{1}{T_{T2}}x_6$$

$$\dot{x}_7 = 2\pi T_{12}^0 [x_1 - x_2]$$

*Example :*

$$x_3(s) = \frac{1}{1 + sT_{G1}} \left[ \Delta P_{C1} - \frac{x_1(s)}{R_1} \right]$$

$$(1 + sT_{G1})x_3(s) = \Delta P_{C1} - \frac{1}{R_1}\Delta F$$

$$sT_{G1}x_3(s) = -x_3(s) + \Delta P_{C1} - \frac{1}{R_1}x_1(s)$$

$$sx_3(s) = -\frac{1}{T_{G1}}x_3(s) + \frac{1}{T_{G1}}\Delta P_{C1} - \frac{1}{T_{G1}R_1}x_1(s)$$

$$\dot{x}_3 = -\frac{1}{T_{G1}}x_3 + \frac{1}{T_{G1}}\Delta P_{C1} - \frac{1}{T_{G1}R_1}x_1$$

# State Space Representation of Two-area System (3)

Conventional state-space form:

$$\dot{x} = Ax + Bu + \Gamma p$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix}$$

$$u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} \Delta P_{C1} \\ \Delta P_{C2} \end{bmatrix}$$

$$p = \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} \Delta P_{D1} \\ \Delta P_{D2} \end{bmatrix}$$

$$A = \begin{bmatrix} -\frac{1}{T_{P1}} & 0 & 0 & 0 & \frac{K_{P1}}{T_{P1}} & 0 & -\frac{K_{P1}}{T_{P1}} \\ 0 & -\frac{1}{T_{P2}} & 0 & 0 & 0 & \frac{K_{P2}}{T_{P2}} & -a_{12} \frac{K_{P2}}{T_{P2}} \\ -\frac{1}{R_1 T_{G1}} & 0 & -\frac{1}{T_{G1}} & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{R_2 T_{G2}} & 0 & -\frac{1}{T_{G2}} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{T_{T1}} & 0 & -\frac{1}{T_{T1}} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{T_{T2}} & 0 & -\frac{1}{T_{T2}} & 0 \\ 2\pi T_{12}^0 & -2\pi T_{12}^0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{1}{T_{G1}} & 0 \\ 0 & \frac{1}{T_{G2}} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Gamma = \begin{bmatrix} -\frac{K_{P1}}{T_{G1}} & 0 \\ 0 & -\frac{K_{P2}}{T_{G2}} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

# State Space Representation of Two-area System (4)

If ACE control is implemented:

$$x_8(s) = -\frac{K_{I1}}{s} [B_1 \Delta F_1 + \Delta P_{12}] = \Delta P_{C1}$$

$$x_9(s) = -\frac{K_{I2}}{s} [B_2 \Delta F_2 + a_{12} \Delta P_{12}] = \Delta P_{C2}$$

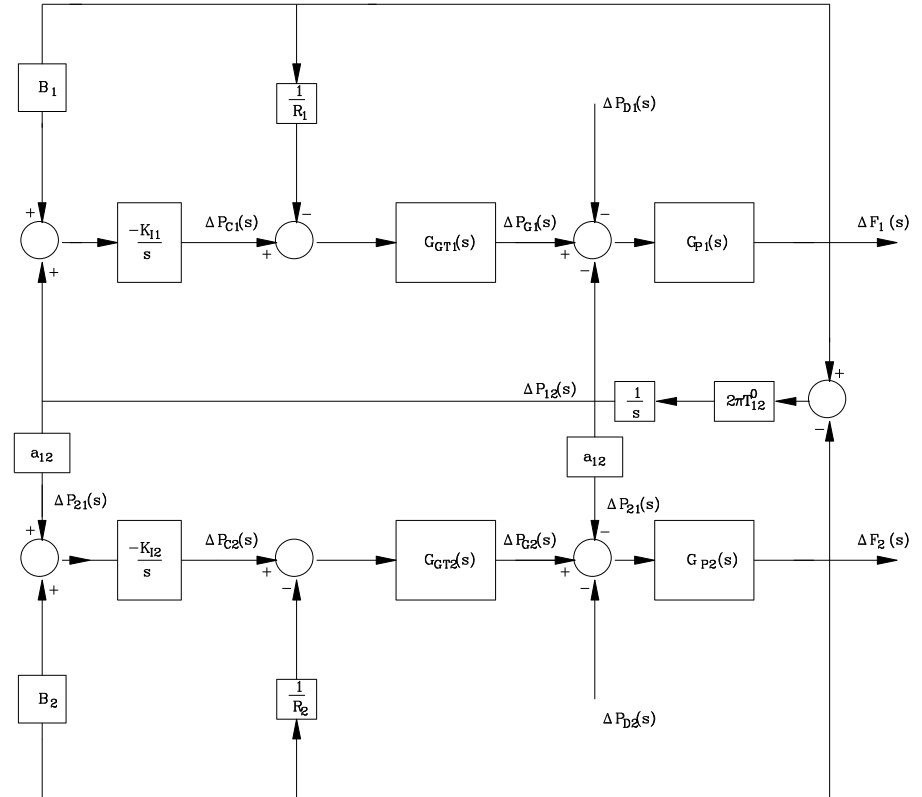
and these changes are made:

$$\dot{x}_3 = -\frac{1}{T_{G1}} x_3 - \frac{1}{R_1 T_{G1}} x_1 + \frac{1}{T_{G1}} x_8$$

$$\dot{x}_4 = -\frac{1}{T_{G2}} x_4 - \frac{1}{R_2 T_{G2}} x_2 + \frac{1}{T_{G2}} x_9$$

$$\dot{x}_8 = -K_{I1} B_1 x_1 - K_{I1} x_7$$

$$\dot{x}_9 = -K_{I2} B_2 x_2 - a_{12} K_{I2} x_7$$



# State Space Representation of Two-area System (5)

State-space representation:

$$\dot{x} = Ax + \Gamma p$$

$$A = \begin{bmatrix} -\frac{1}{T_{P1}} & 0 & 0 & 0 & \frac{K_{P1}}{T_{P1}} & 0 & -\frac{K_{P1}}{T_{P1}} & 0 & 0 \\ 0 & -\frac{1}{T_{P2}} & 0 & 0 & 0 & \frac{K_{P2}}{T_{P2}} & -a_{12}\frac{K_{P2}}{T_{P2}} & 0 & 0 \\ -\frac{1}{R_1 T_{G1}} & 0 & -\frac{1}{T_{G1}} & 0 & 0 & 0 & 0 & \frac{1}{T_{G1}} & 0 \\ 0 & -\frac{1}{R_2 T_{G2}} & 0 & -\frac{1}{T_{G2}} & 0 & 0 & 0 & 0 & \frac{1}{T_{G2}} \\ 0 & 0 & \frac{1}{T_{T1}} & 0 & -\frac{1}{T_{T1}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{T_{T2}} & 0 & -\frac{1}{T_{T2}} & 0 & 0 & 0 \\ 2\pi T_{12}^0 & -2\pi T_{12}^0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -K_{I1} B_1 & 0 & 0 & 0 & 0 & 0 & -K_{I1} & 0 & 0 \\ 0 & -K_{I2} B_2 & 0 & 0 & 0 & 0 & -a_{12} K_{I2} & 0 & 0 \end{bmatrix}$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \end{bmatrix} \quad \Gamma = \begin{bmatrix} -\frac{K_{P1}}{T_{G1}} & 0 \\ 0 & -\frac{K_{P2}}{T_{G2}} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$p = \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} \Delta P_{D1} \\ \Delta P_{D2} \end{bmatrix}$$

## Further Issues

- Reserve generation required to control frequency
- Reserve generation must be fast-acting to respond to changes
- Level of reserve depends on size of system and required speed of response
- Reserve forms one of the ancillary services in a power system
- In Ireland:

Reserve class	Response time	Capability % of capacity
Primary	5-15 sec	5%
Secondary	15-90 sec	5%
Tertiary 1	90 sec-5 min	8%
Tertiary 2	5-20 min	10%
Replacement	>20 min	

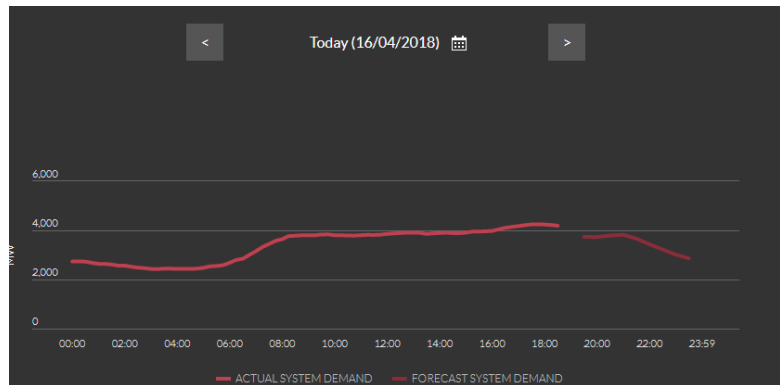
- For primary reserve, 5% of 5000MW=250MW
- Spinning reserve, interruptible load

CER, Operating Reserves in a Centralised Market, March 2003

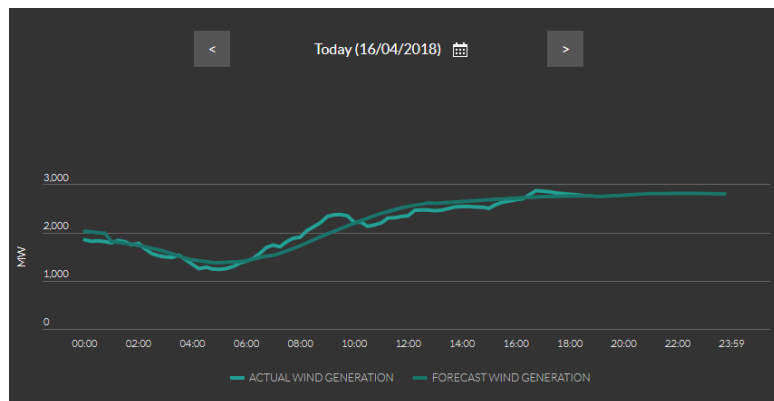


# System Inertia and Wind Energy

- System Demand, 16 April 2018



- Wind Generation, 16 April 2018



# System Inertia and Wind Energy

- High levels of wind penetration required to reach GHG emission targets and targets for electricity from renewable sources (RES-E)
- Objective of 40% of electricity from renewable sources by 2020 (effectively 37% from wind)
- Little or no inertia provided by wind generation
- Measured as System Non-Synchronous Penetration (SNSP)
- CRU recently approved levels up to 65%
- Continuous monitoring of system inertia