Final proyect

Second delivery

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Abstract

We made some of the mathematical methods for the numerical analysis project, we created the pseudocode, also two formal codes in python (main) and MathLab, that represents some of the uses you can take in those methods.

1 Introduction

Due to the fact that in the final project we will have to present many methods and have an application development in this first delivery we will deliver several methods in implemented code and its respective pseudo-code with their tests to show an advance phase and not to be left behind in later deliveries, as we rather show what we are capable of developing from the scratch.

In this document we only show the methods with their code and tests, however you can find all the files in the following Github.

2 Tests

2.1 Python

2.1.1 Simple LU

```python
def lusimpl(A, b):
    n = A.shape[0]
    L = np.eye(n)
    U = np.zeros((n, n))
    M = A
    M = M.astype('float')
    stack = []
    etapa = 0
    # Todas las columnas de A - 1
    for i in range(0, n - 1):
        print("Etapa", etapa)
        print(M)
        print()
        etapa += 1
        # Por cada columna iterar por cada fila debajo de la diagonal
        for j in range(i+1, n):
            if M[j][i] != 0:
                # Operacion de subfila
                L[j][i] = M[j][i]/M[i][i]
                M[j][i:n+1] = M[j][i:] - (M[j][i]/M[i][i])*M[i][i:]

        U[i, i:n] = M[i, i:n]
        U[i+1, i+1:n] = M[i+1, i+1:n]

    U[n-1][n-1] = M[n-1][n-1]

    print("Etapa", etapa)
    print(M, '\n')
    stack = np.array(stack, dtype=int).reshape(-1,2)
    # Sustitucion regresiva
    #print("Despues de aplicar sustitucion regresiva\n")
    z = sustprog(np.column_stack((L, b)))
    x = sustreg(np.column_stack((U, z)))
    return x
```

Figure 1: Code of a simple LU method in python.

```
Etapa 0
[[ 4.   -1.    0.    3. ]
 [ 1.   15.5  3.    8. ]
 [ 0.   -1.3 -4.    1.1]
 [14.    5.   -2.   30. ]]

Etapa 1
[[ 4.    -1.     0.     3.  ]
 [ 0.    15.75  3.     7.25]
 [ 0.    -1.3  -4.     1.1 ]
 [ 0.     8.5  -2.    19.5 ]]

Etapa 2
[[ 4.          -1.           0.           3.        ]
 [ 0.          15.75         3.           7.25      ]
 [ 0.           0.          -3.75238095  1.6984127 ]
 [ 0.           0.          -3.61904762 15.58730159]]

Etapa 3
[[ 4.          -1.           0.           3.        ]
 [ 0.          15.75         3.           7.25      ]
 [ 0.           0.          -3.75238095  1.6984127 ]
 [ 0.           0.           0.          13.94923858]]

X:
 [ 0.52510917  0.25545852 -0.41048035 -0.28165939]
```

Figure 2: Simple LU test.

2.1.2 LU with partial pivot

```python
def lupar(A, b):
    n = A.shape[0]
    L = np.eye(n)
    U = np.zeros((n, n))
    P = np.eye(n)
    M = A
    M = M.astype('float')

    stack = []
    etapa = 0
    # Todas las columnas de A - 1
    for i in range(0, n - 1):
        print("Etapa", etapa)
        print(M)
        print()
        etapa += 1
        col = np.abs(M[i+1:, i])
        aux0 = np.max(col) # Maximo en columna
        aux1 = np.argmax(col) # Indice subcolumna

        if aux0 > abs(M[i][i]):
            aux2 = M[i+aux1+1, i:n]
            aux3 = P[i+aux1+1, :]
            M[[aux1+i+1, i], i:n] = M[[i, i+aux1+1], i:n]
            P[[i, i+aux1+1],:] = P[[i+aux1+1, i], :]
            if i>0:
                L[[i, i+aux1+1], 0:i] = L[[i+aux1+1, i], 0:i]

        # Por cada columna iterar por cada fila debajo de la diagonal
        for j in range(i+1, n):
            if M[j][i] != 0:
                # Operacion de subfila
                L[j][i] = M[j][i]/M[i][i]
                M[j][i:n+1] = M[j][i:] - (M[j][i]/M[i][i])*M[i][i:]

        U[i, i:n] = M[i, i:n]
        U[i+1, i+1:n] = M[i+1, i+1:n]

    U[n-1][n-1] = M[n-1][n-1]

    print("Etapa", etapa)
    print(M, '\n')
    stack = np.array(stack, dtype=int).reshape(-1,2)

    z = sustprog(np.column_stack((L, P.dot(b))))
    x = sustreg(np.column_stack((U, z)))
    return x
```

Figure 3: Code of a partial LU method in python.

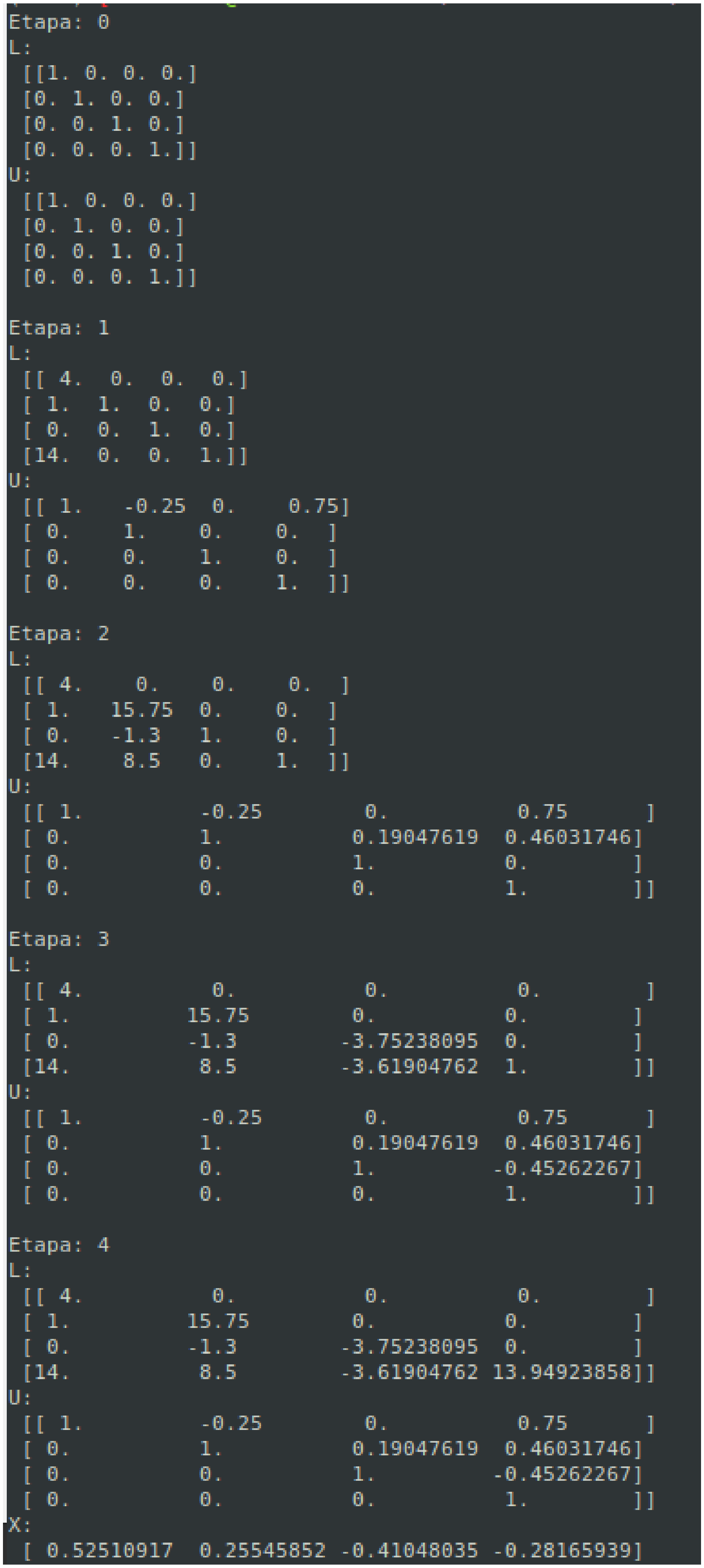
```
Etapa 0
[[ 4.   -1.    0.    3. ]
 [ 1.   15.5   3.    8. ]
 [ 0.   -1.3  -4.    1.1]
 [14.    5.   -2.   30. ]]

Etapa 1
[[14.          5.          -2.          30.        ]
 [ 0.         15.14285714   3.14285714   5.85714286]
 [ 0.         -1.3         -4.           1.1       ]
 [ 0.         -2.42857143   0.57142857  -5.57142857]]

Etapa 2
[[14.          5.          -2.          30.        ]
 [ 0.         15.14285714   3.14285714   5.85714286]
 [ 0.          0.          -3.73018868   1.60283019]
 [ 0.          0.           1.0754717   -4.63207547]]

Etapa 3
[[14.          5.          -2.          30.        ]
 [ 0.         15.14285714   3.14285714   5.85714286]
 [ 0.          0.          -3.73018868   1.60283019]
 [ 0.          0.           0.          -4.16995448]]

X:
 [ 0.52510917  0.25545852 -0.41048035 -0.28165939]
```

Figure 4: Partial LU test.

2.1.3 Crout

```python
def crout(A, b):
    n = np.array(A.shape[0])
    L = np.eye(n)
    U = np.eye(n)

    for i in range(0, n-1):

        for j in range(i, n):
            L[j][i] = A[j][i] - L[j, 0:i].dot( np.matrix.transpose(U[0:i,i]))
            print("L:\n", L)

        for j in range(i+1, n):
            U[i][j] = np.divide((A[i][j] - L[i, 0:i].dot( np.matrix.transpose(U[0:i,j]))), L[i][i])
            print("U:\n", U)

    L[n-1][n-1] = A[n-1][n-1] - L[n-1, 0:n-1].dot( np.matrix.transpose(U[0:n-1,n-1]))
    print("L:\n", L)
    z= sustprog(np.column_stack((L, b)))
    x = sustreg(np.column_stack((U, z)))

    return x
```

Figure 5: Code of crout method in python.



```
Etapa: 0
L:
 [[1. 0. 0. 0.]
 [0. 1. 0. 0.]
 [0. 0. 1. 0.]
 [0. 0. 0. 1.]]
U:
 [[1. 0. 0. 0.]
 [0. 1. 0. 0.]
 [0. 0. 1. 0.]
 [0. 0. 0. 1.]]

Etapa: 1
L:
 [[ 4.  0.  0.  0.]
 [ 1.  1.  0.  0.]
 [ 0.  0.  1.  0.]
 [14.  0.  0.  1.]]
U:
 [[ 1.   -0.25  0.    0.75]
 [ 0.    1.    0.    0.  ]
 [ 0.    0.    1.    0.  ]
 [ 0.    0.    0.    1.  ]]

Etapa: 2
L:
 [[ 4.    0.    0.    0.  ]
 [ 1.   15.75  0.    0.  ]
 [ 0.   -1.3   1.    0.  ]
 [14.    8.5   0.    1.  ]]
U:
 [[ 1.         -0.25        0.          0.75      ]
 [ 0.          1.          0.19047619  0.46031746]
 [ 0.          0.          1.          0.        ]
 [ 0.          0.          0.          1.        ]]

Etapa: 3
L:
 [[ 4.          0.          0.          0.        ]
 [ 1.         15.75        0.          0.        ]
 [ 0.         -1.3        -3.75238095  0.        ]
 [14.          8.5        -3.61904762  1.        ]]
U:
 [[ 1.         -0.25        0.          0.75      ]
 [ 0.          1.          0.19047619  0.46031746]
 [ 0.          0.          1.         -0.45262267]
 [ 0.          0.          0.          1.        ]]

Etapa: 4
L:
 [[ 4.          0.          0.          0.        ]
 [ 1.         15.75        0.          0.        ]
 [ 0.         -1.3        -3.75238095  0.        ]
 [14.          8.5        -3.61904762 13.94923858]]
U:
 [[ 1.         -0.25        0.          0.75      ]
 [ 0.          1.          0.19047619  0.46031746]
 [ 0.          0.          1.         -0.45262267]
 [ 0.          0.          0.          1.        ]]
X:
 [ 0.52510917  0.25545852 -0.41048035 -0.28165939]
```

Figure 6: Crout method test.

2.1.4 Doolittle

```python
def doolittle(A, b):
    n = np.array(A.shape[0])
    L = np.eye(n)
    U = np.eye(n)

    for i in range(0, n-1):
        for j in range(i, n):
            print("U:\n", U)
            U[i][j] = A[i][j] - L[i, 0:i].dot(np.matrix.transpose(U[0:i,j]))

        for j in range(i+1, n):
            print("L:\n", L)
            L[j][i] = np.divide((A[j][i] - L[j, 0:i].dot( np.matrix.transpose(U[0:i,i]))), U[i][i])

    print(L)
    print(U)
    U[n-1][n-1] = A[n-1][n-1] - L[n-1, 0:n-1].dot(np.matrix.transpose(U[0:n-1,n-1]))

    z= sustprog(np.column_stack((L, b)))
    x = sustreg(np.column_stack((U, z)))
    return x
```

Figure 7: Code of doolittle method in python.

```
Etapa: 0
L:
 [[1. 0. 0. 0.]
 [0. 1. 0. 0.]
 [0. 0. 1. 0.]
 [0. 0. 0. 1.]]
U:
 [[1. 0. 0. 0.]
 [0. 1. 0. 0.]
 [0. 0. 1. 0.]
 [0. 0. 0. 1.]]
Etapa: 1
L:
 [[1.   0.   0.   0.  ]
 [0.25 1.   0.   0.  ]
 [0.   0.   1.   0.  ]
 [3.5  0.   0.   1.  ]]
U:
 [[ 4. -1.  0.  3.]
 [ 0.  1.  0.  0.]
 [ 0.  0.  1.  0.]
 [ 0.  0.  0.  1.]]
Etapa: 2
L:
 [[ 1.          0.          0.          0.        ]
 [ 0.25        1.          0.          0.        ]
 [ 0.         -0.08253968  1.          0.        ]
 [ 3.5         0.53968254  0.          1.        ]]
U:
 [[ 4.   -1.    0.    3.  ]
 [ 0.   15.75  3.    7.25]
 [ 0.    0.    1.    0.  ]
 [ 0.    0.    0.    1.  ]]
Etapa: 3
L:
 [[ 1.          0.          0.          0.        ]
 [ 0.25        1.          0.          0.        ]
 [ 0.         -0.08253968  1.          0.        ]
 [ 3.5         0.53968254  0.96446701  1.        ]]
U:
 [[ 4.         -1.          0.          3.        ]
 [ 0.         15.75        3.          7.25      ]
 [ 0.          0.         -3.75238095  1.6984127 ]
 [ 0.          0.          0.          1.        ]]
Etapa: 4
L:
 [[ 1.          0.          0.          0.        ]
 [ 0.25        1.          0.          0.        ]
 [ 0.         -0.08253968  1.          0.        ]
 [ 3.5         0.53968254  0.96446701  1.        ]]
U:
 [[ 4.         -1.          0.          3.        ]
 [ 0.         15.75        3.          7.25      ]
 [ 0.          0.         -3.75238095  1.6984127 ]
 [ 0.          0.          0.         13.94923858]]
X:
 [ 0.52510917  0.25545852 -0.41048035 -0.28165939]
```

Figure 8: Doolittle method test.

2.1.5 Cholesky

```python
def cholesky(A, b):
    n = np.array(A.shape[0])
    L = np.eye(n, dtype=complex)
    U = np.eye(n, dtype=complex)
    etapa = 0
    print("Etapa:", etapa)
    print("L:\n", L)
    print("U:\n", U)
    etapa += 1

    for i in range(0, n-1):
        L[i][i] = cmath.sqrt(A[i][i] - L[i, 0:i].dot( np.matrix.transpose(U[0:i,i])))
        U[i][i] = L[i][i]

        for j in range(i+1, n):
            L[j][i] = np.divide((A[j][i] - L[j, 0:i].dot( np.matrix.transpose(U[0:i,i]))), U[i][i])

        for j in range(i+1, n):
            U[i][j] = np.divide((A[i][j] - L[i, 0:i].dot( np.matrix.transpose(U[0:i,j]))), L[i][i])

        print("Etapa:", etapa)
        print("L:\n", L)
        print("U:\n", U)
        etapa += 1

    L[n-1][n-1] = cmath.sqrt(A[n-1][n-1] - L[n-1, 0:n-1].dot( np.matrix.transpose(U[0:n-1,n-1])))
    U[n-1][n-1] = L[n-1][n-1]

    print("Etapa:", etapa)
    print("L:\n", L)
    print("U:\n", U)

    z= sustprog(np.column_stack((L, b)))
    x = sustreg(np.column_stack((U, z)))
    return x
```

Figure 9: Code of the Cholesky method in python.

```
Etapa: 0
L:
 [[1.+0.j 0.+0.j 0.+0.j 0.+0.j]
 [0.+0.j 1.+0.j 0.+0.j 0.+0.j]
 [0.+0.j 0.+0.j 1.+0.j 0.+0.j]
 [0.+0.j 0.+0.j 0.+0.j 1.+0.j]]
U:
 [[1.+0.j 0.+0.j 0.+0.j 0.+0.j]
 [0.+0.j 1.+0.j 0.+0.j 0.+0.j]
 [0.+0.j 0.+0.j 1.+0.j 0.+0.j]
 [0.+0.j 0.+0.j 0.+0.j 1.+0.j]]
Etapa: 1
L:
 [[2. +0.j 0. +0.j 0. +0.j 0. +0.j]
 [0.5+0.j 1. +0.j 0. +0.j 0. +0.j]
 [0. +0.j 0. +0.j 1. +0.j 0. +0.j]
 [7. +0.j 0. +0.j 0. +0.j 1. +0.j]]
U:
 [[ 2. +0.j -0.5+0.j  0. +0.j  1.5+0.j]
 [ 0. +0.j  1. +0.j  0. +0.j  0. +0.j]
 [ 0. +0.j  0. +0.j  1. +0.j  0. +0.j]
 [ 0. +0.j  0. +0.j  0. +0.j  1. +0.j]]
Etapa: 2
L:
 [[ 2.        +0.j  0.        +0.j  0.        +0.j  0.        +0.j]
 [ 0.5       +0.j  3.96862697+0.j  0.        +0.j  0.        +0.j]
 [ 0.        +0.j -0.32756921+0.j  1.        +0.j  0.        +0.j]
 [ 7.        +0.j  2.14179868+0.j  0.        +0.j  1.        +0.j]]
U:
 [[ 2.        +0.j -0.5       +0.j  0.        +0.j  1.5       +0.j]
 [ 0.        +0.j  3.96862697+0.j  0.75592895+0.j  1.82682829+0.j]
 [ 0.        +0.j  0.        +0.j  1.        +0.j  0.        +0.j]
 [ 0.        +0.j  0.        +0.j  0.        +0.j  1.        +0.j]]
Etapa: 3
L:
 [[ 2.        +0.j          0.        +0.j          0.        +0.j
   0.        +0.j        ]
 [ 0.5       +0.j          3.96862697+0.j          0.        +0.j
   0.        +0.j        ]
 [ 0.        +0.j         -0.32756921+0.j          0.        +1.93710633j
   0.        +0.j        ]
 [ 7.        +0.j          2.14179868+0.j          0.        +1.86827515j
   1.        +0.j        ]]
U:
 [[ 2.        +0.j         -0.5       +0.j          0.        +0.j
   1.5       +0.j        ]
 [ 0.        +0.j          3.96862697+0.j          0.75592895+0.j
   1.82682829+0.j        ]
 [ 0.        +0.j          0.        +0.j          0.        +1.93710633j
   0.        -0.87677825j]
 [ 0.        +0.j          0.        +0.j          0.        +0.j
   1.        +0.j        ]]
Etapa: 4
L:
 [[ 2.        +0.j          0.        +0.j          0.        +0.j
   0.        +0.j        ]
 [ 0.5       +0.j          3.96862697+0.j          0.        +0.j
   0.        +0.j        ]
 [ 0.        +0.j         -0.32756921+0.j          0.        +1.93710633j
   0.        +0.j        ]
 [ 7.        +0.j          2.14179868+0.j          0.        +1.86827515j
   3.73486795+0.j        ]]
U:
 [[ 2.        +0.j         -0.5       +0.j          0.        +0.j
   1.5       +0.j        ]
 [ 0.        +0.j          3.96862697+0.j          0.75592895+0.j
   1.82682829+0.j        ]
 [ 0.        +0.j          0.        +0.j          0.        +1.93710633j
   0.        -0.87677825j]
 [ 0.        +0.j          0.        +0.j          0.        +0.j
   3.73486795+0.j        ]]
X:
 [ 0.52510917+0.j  0.25545852+0.j -0.41048035-0.j -0.28165939+0.j]
```

Figure 10: Cholesky method test (Can not handle negative roots yet).

2.1.6 Jacobi

```python
def jacobi(A, b, x0, tol, Nmax):
    det = np.linalg.det(A) # Determinant of matrix
    if det == 0:
        print("Determinant of matrix is 0")
        return
    D = np.diag(np.diag(A))
    L = -np.tril(A)+D
    U = -np.triu(A)+D

    T = np.linalg.inv(D).dot(L+U)
    C = np.linalg.inv(D).dot(b)

    xant = x0
    E = 1000
    cont = 0

    while E>tol and cont<Nmax:
        np.set_printoptions(precision=8, suppress=True)
        print("{i:2d} | X{i} = {xant} | E = {Err:.1E}".format(i=cont, xant=xant, Err=E))
        xact = T.dot(xant) + C
        E = np.linalg.norm(xant-xact)
        xant = xact
        cont += 1

    if E>tol or cont == Nmax:
        print("{i} | X{i} = {xant} | E = {Err:.1E}".format(i=cont, xant=xant, Err=E))
        print("This method can not get solution to the matrix")
    else:
        print("{i} | X{i} = {xant} | E = {Err:.1E}".format(i=cont, xant=xant, Err=E))
```

Figure 11: Code of the Jacobi method in python.

```
0 | X0 = [0 0 0 0] | E = 1.0E+03
1 | X1 = [ 0.25        0.06451613 -0.25        0.03333333] | E = 3.6E-01
2 | X2 = [ 0.24112903  0.07956989 -0.26180108 -0.11075269] | E = 1.5E-01
3 | X3 = [ 0.35295699  0.15679327 -0.3063172  -0.1099086 ] | E = 1.4E-01
4 | X4 = [ 0.37162977  0.15775893 -0.33118268 -0.17793329] | E = 7.5E-02
5 | X5 = [ 0.4228897   0.19647642 -0.35020331 -0.18846589] | E = 6.8E-02
6 | X6 = [ 0.44046853  0.20228693 -0.36568296 -0.22010815] | E = 4.0E-02
7 | X7 = [ 0.46565284  0.22048036 -0.37627299 -0.230312  ] | E = 3.4E-02
8 | X8 = [ 0.47785409  0.22617175 -0.38499192 -0.24580292] | E = 2.2E-02
9 | X9 = [ 0.49089513  0.23506742 -0.39110162 -0.25302666] | E = 1.8E-02
10 | X10 = [ 0.49853685  0.23913697 -0.39597924 -0.2610024 ] | E = 1.3E-02
11 | X11 = [ 0.50553605  0.24370452 -0.39949518 -0.26557197] | E = 1.0E-02
12 | X12 = [ 0.51010511  0.24629195 -0.40223626 -0.26983392] | E = 7.3E-03
13 | X13 = [ 0.51394843  0.24872742 -0.40424921 -0.27258013] | E = 5.7E-03
14 | X14 = [ 0.51661695  0.25028647 -0.40579595 -0.27491378] | E = 4.2E-03
15 | X15 = [ 0.51875696  0.25161814 -0.40694439 -0.27652205] | E = 3.2E-03
16 | X16 = [ 0.52029607  0.25253243 -0.40781946 -0.27781923] | E = 2.4E-03
17 | X17 = [ 0.52149753  0.25327201 -0.40847333 -0.2787482 ] | E = 1.8E-03
18 | X18 = [ 0.52237915  0.25380052 -0.40896916 -0.27947574] | E = 1.4E-03
19 | X19 = [ 0.52305693  0.25421511 -0.409341   -0.2800083 ] | E = 1.0E-03
20 | X20 = [ 0.52356     0.25451822 -0.40962219 -0.28041849] | E = 7.7E-04
21 | X21 = [ 0.52394342  0.2547519  -0.40983351 -0.28072252] | E = 5.8E-04
22 | X22 = [ 0.52422986  0.25492498 -0.40999306 -0.28095448] | E = 4.4E-04
23 | X23 = [ 0.52444711  0.25505711 -0.4101131  -0.28112764] | E = 3.3E-04
24 | X24 = [ 0.52461001  0.2551557  -0.41020366 -0.28125904] | E = 2.5E-04
25 | X25 = [ 0.52473321  0.25523054 -0.41027184 -0.28135753] | E = 1.9E-04
26 | X26 = [ 0.52482578  0.25528662 -0.41032324 -0.28143204] | E = 1.4E-04
27 | X27 = [ 0.52489568  0.25532905 -0.41036196 -0.28148802] | E = 1.1E-04
28 | X28 = [ 0.52494828  0.25536092 -0.41039115 -0.28153029] | E = 8.0E-05
29 | X29 = [ 0.52498795  0.255385   -0.41041313 -0.28156209] | E = 6.0E-05
30 | X30 = [ 0.52501782  0.25540311 -0.4104297  -0.28158609] | E = 4.6E-05
31 | X31 = [ 0.52504034  0.25541677 -0.41044218 -0.28160415] | E = 3.4E-05
32 | X32 = [ 0.5250573   0.25542706 -0.41045159 -0.28161777] | E = 2.6E-05
33 | X33 = [ 0.52507009  0.25543481 -0.41045868 -0.28162802] | E = 1.9E-05
34 | X34 = [ 0.52507972  0.25544065 -0.41046402 -0.28163576] | E = 1.5E-05
35 | X35 = [ 0.52508698  0.25544506 -0.41046805 -0.28164158] | E = 1.1E-05
36 | X36 = [ 0.52509245  0.25544837 -0.41047108 -0.28164597] | E = 8.3E-06
37 | X37 = [ 0.52509657  0.25545087 -0.41047336 -0.28164928] | E = 6.3E-06
38 | X38 = [ 0.52509968  0.25545276 -0.41047509 -0.28165177] | E = 4.7E-06
39 | X39 = [ 0.52510202  0.25545418 -0.41047638 -0.28165365] | E = 3.6E-06
40 | X40 = [ 0.52510378  0.25545525 -0.41047736 -0.28165506] | E = 2.7E-06
41 | X41 = [ 0.52510511  0.25545605 -0.4104781  -0.28165613] | E = 2.0E-06
42 | X42 = [ 0.52510611  0.25545666 -0.41047865 -0.28165693] | E = 1.5E-06
43 | X43 = [ 0.52510686  0.25545712 -0.41047907 -0.28165754] | E = 1.1E-06
44 | X44 = [ 0.52510743  0.25545746 -0.41047939 -0.28165799] | E = 8.7E-07
45 | X45 = [ 0.52510786  0.25545772 -0.41047962 -0.28165834] | E = 6.5E-07
46 | X46 = [ 0.52510818  0.25545792 -0.4104798  -0.2816586 ] | E = 4.9E-07
47 | X47 = [ 0.52510843  0.25545806 -0.41047994 -0.28165879] | E = 3.7E-07
48 | X48 = [ 0.52510861  0.25545818 -0.41048004 -0.28165894] | E = 2.8E-07
49 | X49 = [ 0.52510875  0.25545826 -0.41048012 -0.28165905] | E = 2.1E-07
50 | X50 = [ 0.52510885  0.25545832 -0.41048017 -0.28165913] | E = 1.6E-07
51 | X51 = [ 0.52510893  0.25545837 -0.41048022 -0.2816592 ] | E = 1.2E-07
52 | X52 = [ 0.52510899  0.25545841 -0.41048025 -0.28165924] | E = 9.0E-08
```

Figure 12: Jacobi method test.

2.1.7 Gauss-Seidel

```python
def gseidel(A, b, x0, tol, Nmax):
    det = np.linalg.det(A) # Determinant of matrix
    if det == 0:
        print("Determinant of matrix is 0")
        return
    D = np.diag(np.diag(A))
    L = -np.tril(A)+D
    U = -np.triu(A)+D


    T = np.linalg.inv(D-L).dot(U)
    C = np.linalg.inv(D-L).dot(b)

    xant = x0
    E = 1000
    cont = 0

    while E>tol and cont<Nmax:
        np.set_printoptions(precision=8, suppress=True)
        print("{i:2d} | X{i} = {xant} | E = {Err:.1E}".format(i=cont, xant=xant, Err=E))
        xact = T.dot(xant) + C
        E = np.linalg.norm(xant-xact)
        xant = xact
        cont += 1

    if E>tol or cont == Nmax:
        print("{i} | X{i} = {xant} | E = {Err:.1E}".format(i=cont, xant=xant, Err=E))
        print("This method can not get solution to the matrix")
    else:
        print("{i} | X{i} = {xant} | E = {Err:.1E}".format(i=cont, xant=xant, Err=E))
```

Figure 13: Code of the Gauss-Seidel method in python.

```
 0 | X0 = [0 0 0 0] | E = 1.0E+03
 1 | X1 = [ 0.25        0.0483871  -0.26572581 -0.1091129 ] | E = 3.8E-01
 2 | X2 = [ 0.34393145  0.15007414 -0.32878014 -0.17409904] | E = 1.7E-01
 3 | X3 = [ 0.41809282  0.19103484 -0.35996356 -0.21761336] | E = 1.0E-01
 4 | X4 = [ 0.46096873  0.21676315 -0.3802917  -0.24326538] | E = 6.0E-02
 5 | X5 = [ 0.48663982  0.23228118 -0.39238936 -0.25863807] | E = 3.6E-02
 6 | X6 = [ 0.50204885  0.24156283 -0.39963339 -0.26785883] | E = 2.1E-02
 7 | X7 = [ 0.51128483  0.24712813 -0.40397782 -0.27338613] | E = 1.3E-02
 8 | X8 = [ 0.51682163  0.25046457 -0.40658217 -0.27669967] | E = 7.7E-03
 9 | X9 = [ 0.52014089  0.25246471 -0.40814344 -0.2786861 ] | E = 4.6E-03
10 | X10 = [ 0.52213075  0.25366376 -0.4090794  -0.27987694] | E = 2.8E-03
11 | X11 = [ 0.52332364  0.25438258 -0.4096405  -0.28059083] | E = 1.7E-03
12 | X12 = [ 0.52403877  0.25481351 -0.40997687 -0.2810188 ] | E = 1.0E-03
13 | X13 = [ 0.52446748  0.25507184 -0.41017852 -0.28127536] | E = 6.0E-04
14 | X14 = [ 0.52472448  0.25522671 -0.41029941 -0.28142917] | E = 3.6E-04
15 | X15 = [ 0.52487856  0.25531955 -0.41037188 -0.28152138] | E = 2.1E-04
16 | X16 = [ 0.52497092  0.25537521 -0.41041532 -0.28157665] | E = 1.3E-04
17 | X17 = [ 0.52502629  0.25540857 -0.41044137 -0.28160979] | E = 7.7E-05
18 | X18 = [ 0.52505948  0.25542858 -0.41045698 -0.28162965] | E = 4.6E-05
19 | X19 = [ 0.52507938  0.25544057 -0.41046634 -0.28164156] | E = 2.8E-05
20 | X20 = [ 0.52509131  0.25544776 -0.41047195 -0.2816487 ] | E = 1.7E-05
21 | X21 = [ 0.52509847  0.25545206 -0.41047531 -0.28165298] | E = 1.0E-05
22 | X22 = [ 0.52510275  0.25545465 -0.41047733 -0.28165555] | E = 6.0E-06
23 | X23 = [ 0.52510532  0.2554562  -0.41047854 -0.28165709] | E = 3.6E-06
24 | X24 = [ 0.52510686  0.25545713 -0.41047926 -0.28165801] | E = 2.1E-06
25 | X25 = [ 0.52510779  0.25545768 -0.4104797  -0.28165856] | E = 1.3E-06
26 | X26 = [ 0.52510834  0.25545802 -0.41047996 -0.28165889] | E = 7.7E-07
27 | X27 = [ 0.52510867  0.25545822 -0.41048012 -0.28165909] | E = 4.6E-07
28 | X28 = [ 0.52510887  0.25545834 -0.41048021 -0.28165921] | E = 2.8E-07
29 | X29 = [ 0.52510899  0.25545841 -0.41048027 -0.28165928] | E = 1.7E-07
30 | X30 = [ 0.52510906  0.25545845 -0.4104803  -0.28165932] | E = 1.0E-07
```

Figure 14: Gauss-Seidel method test.

2.1.8 SOR

```python
def sor(A, b, x0, w, tol, Nmax):
    det = np.linalg.det(A) # Determinant of matrix
    if det == 0:
        print("Determinant of matrix is 0")
        return
    D = np.diag(np.diag(A))
    L = -np.tril(A)+D
    U = -np.triu(A)+D


    T = np.linalg.inv(D -(w*L)).dot((1-w)*D + w*U)
    C = w*np.linalg.inv(D- (w*L)).dot(b)

    xant = x0
    E = 1000
    cont = 0

    while E>tol and cont<Nmax:
        np.set_printoptions(precision=8, suppress=True)
        print("{i:2d} | X{i} = {xant} | E = {Err:.1E}".format(i=cont, xant=xant, Err=E))
        xact = T.dot(xant) + C
        E = np.linalg.norm(xant-xact)
        xant = xact
        cont += 1

    if E>tol or cont == Nmax:
        print("{i} | X{i} = {xant} | E = {Err:.1E}".format(i=cont, xant=xant, Err=E))
        print("This method can not get solution to the matrix")
    else:
        print("{i} | X{i} = {xant} | E = {Err:.1E}".format(i=cont, xant=xant, Err=E))
```

Figure 15: Code of the SOR method in python.

```
 0 | X0 = [0 0 0 0] | E = 1.0E+03
 1 | X1 = [ 0.375       0.06048387 -0.40448589 -0.26806956] | E = 6.2E-01
 2 | X2 = [ 0.5117597   0.34197623 -0.45004916 -0.30469599] | E = 3.2E-01
 3 | X3 = [ 0.59014422  0.23522845 -0.39033638 -0.30859371] | E = 1.5E-01
 4 | X4 = [ 0.51530648  0.28152633 -0.4443708  -0.27123634] | E = 1.1E-01
 5 | X5 = [ 0.52806002  0.24390875 -0.38360511 -0.28336154] | E = 7.4E-02
 6 | X6 = [ 0.52121751  0.25512531 -0.42445767 -0.27939858] | E = 4.3E-02
 7 | X7 = [ 0.52438664  0.25800267 -0.40379938 -0.28225196] | E = 2.1E-02
 8 | X8 = [ 0.52709114  0.25251377 -0.41262971 -0.28222923] | E = 1.1E-02
 9 | X9 = [ 0.52365497  0.25813679 -0.41094639 -0.2810727 ] | E = 6.9E-03
10 | X10 = [ 0.5261806   0.25369679 -0.40914648 -0.28212891] | E = 5.5E-03
11 | X11 = [ 0.52444102  0.25638029 -0.41179033 -0.28131836] | E = 4.2E-03
12 | X12 = [ 0.52540526  0.25508527 -0.40950273 -0.28184609] | E = 2.8E-03
13 | X13 = [ 0.5250312   0.2555134  -0.41107293 -0.28158444] | E = 1.7E-03
14 | X14 = [ 0.52508442  0.25554748 -0.41019651 -0.2816734 ] | E = 8.8E-04
15 | X15 = [ 0.52517067  0.25533652 -0.41056857 -0.28167376] | E = 4.4E-04
16 | X16 = [ 0.52504884  0.25556209 -0.41049266 -0.2816371 ] | E = 2.7E-04
17 | X17 = [ 0.5251531   0.25538879 -0.41043101 -0.28167892] | E = 2.2E-04
18 | X18 = [ 0.52508303  0.2554967  -0.41053169 -0.28164601] | E = 1.7E-04
19 | X19 = [ 0.52512151  0.25544277 -0.41044149 -0.28166689] | E = 1.1E-04
20 | X20 = [ 0.52510554  0.25546126 -0.41050422 -0.28165617] | E = 6.8E-05
21 | X21 = [ 0.52510839  0.25546165 -0.41046862 -0.28166007] | E = 3.6E-05
22 | X22 = [ 0.5251115   0.25545384 -0.41048422 -0.2816599 ] | E = 1.8E-05
23 | X23 = [ 0.52510682  0.2554626  -0.41048062 -0.28165854] | E = 1.1E-05
24 | X24 = [ 0.52511092  0.25545573 -0.41047851 -0.28166015] | E = 8.4E-06
25 | X25 = [ 0.52510811  0.25546007 -0.41048235 -0.28165885] | E = 6.6E-06
26 | X26 = [ 0.52510968  0.25545785 -0.41047881 -0.28165969] | E = 4.5E-06
27 | X27 = [ 0.52510901  0.25545865 -0.41048131 -0.28165925] | E = 2.7E-06
28 | X28 = [ 0.52510915  0.25545862 -0.41047987 -0.28165942] | E = 1.5E-06
29 | X29 = [ 0.52510926  0.25545834 -0.41048052 -0.28165941] | E = 7.2E-07
30 | X30 = [ 0.52510908  0.25545868 -0.41048035 -0.28165936] | E = 4.2E-07
31 | X31 = [ 0.52510924  0.2554584  -0.41048028 -0.28165942] | E = 3.3E-07
32 | X32 = [ 0.52510913  0.25545858 -0.41048043 -0.28165937] | E = 2.6E-07
33 | X33 = [ 0.52510919  0.25545849 -0.41048029 -0.2816594 ] | E = 1.8E-07
34 | X34 = [ 0.52510916  0.25545852 -0.41048039 -0.28165938] | E = 1.1E-07
35 | X35 = [ 0.52510917  0.25545852 -0.41048033 -0.28165939] | E = 5.9E-08
```

Figure 16: SOR method test

2.1.9 Vandermonde

```python
def vandermonde(X, Y):
    n = X.size
    A = np.zeros((n, n)) # (shape)
    X = np.matrix.transpose(X)

    for i in range(n):
        A[:,i] = np.matrix.transpose(np.power(X, (n-i-1)))

    print("A:\n", A)
    coef = gausstot(A, np.matrix.transpose(Y))
    print("Coef:\n", coef)
    return coef
```

Figure 17: Code of vandermonde method in python.

```
A:
 [[-1.  1. -1.  1.]
 [ 0.  0.  0.  1.]
 [27.  9.  3.  1.]
 [64. 16.  4.  1.]]
Etapa 0
[[-1.   1.  -1.   1.  15.5]
 [ 0.   0.   0.   1.   3. ]
 [27.   9.   3.   1.   8. ]
 [64.  16.   4.   1.   1. ]]

Cambio de fila
[[64.  16.   4.   1.   1. ]
 [ 0.   0.   0.   1.   3. ]
 [27.   9.   3.   1.   8. ]
 [-1.   1.  -1.   1.  15.5]]

Etapa 1
[[64.       16.        4.        1.        1.      ]
 [ 0.        0.        0.        1.        3.      ]
 [ 0.        2.25      1.3125    0.578125  7.578125]
 [ 0.        1.25     -0.9375    1.015625 15.515625]]

Cambio de fila
[[64.       16.        4.        1.        1.      ]
 [ 0.        2.25      1.3125    0.578125  7.578125]
 [ 0.        0.        0.        1.        3.      ]
 [ 0.        1.25     -0.9375    1.015625 15.515625]]

Etapa 2
[[64.         16.          4.          1.          1.        ]
 [ 0.          2.25        1.3125      0.578125    7.578125  ]
 [ 0.          0.          0.          1.          3.        ]
 [ 0.          0.         -1.66666667  0.69444444 11.30555556]]

Cambio de fila
[[64.         16.          4.          1.          1.        ]
 [ 0.          2.25        1.3125      0.578125    7.578125  ]
 [ 0.          0.         -1.66666667  0.69444444 11.30555556]
 [ 0.          0.          0.          1.          3.        ]]

Etapa 3
[[64.         16.          4.          1.          1.        ]
 [ 0.          2.25        1.3125      0.578125    7.578125  ]
 [ 0.          0.         -1.66666667  0.69444444 11.30555556]
 [ 0.          0.          0.          1.          3.        ]]

Despues de aplicar sustitucion regresiva

X antes del cambio de columnas:
[-1.14166667  5.825      -5.53333333  3.        ]
X despues del cambio de columnas:
[-1.14166667  5.825      -5.53333333  3.        ]
Coef:
 [-1.14166667  5.825      -5.53333333  3.        ]
```

Figure 18: Vandermonde method test.

2.1.10 Newton

```python
def difdivididas(X, Y):
    n = X.size
    D = np.zeros((n, n)) # (shape)
    X = np.matrix.transpose(X)

    D[:,0] = np.matrix.transpose(Y)
    for i in range(1, n):
        aux0 = D[i-1:n, i-1]
        aux = np.diff(aux0)
        aux2 = X[i:n] - X[0:n-i]
        D[i:n, i] = np.divide(aux, aux2)

    coef = np.diag(D)
    return coef
```

Figure 19: Code of the Newton method in python.

```
Etapa: 0
[[0. 0. 0. 0.]
 [0. 0. 0. 0.]
 [0. 0. 0. 0.]
 [0. 0. 0. 0.]]

Etapa: 1
[[15.5  0.   0.   0. ]
 [ 3.   0.   0.   0. ]
 [ 8.   0.   0.   0. ]
 [ 1.   0.   0.   0. ]]

Etapa: 2
[[ 15.5         0.          0.          0.        ]
 [  3.        -12.5        0.          0.        ]
 [  8.          1.66666667  0.          0.        ]
 [  1.         -7.         0.          0.        ]]

Etapa: 3
[[ 15.5         0.          0.          0.        ]
 [  3.        -12.5        0.          0.        ]
 [  8.          1.66666667  3.54166667  0.        ]
 [  1.         -7.        -2.16666667  0.        ]]

Etapa: 4
[[ 15.5         0.          0.          0.        ]
 [  3.        -12.5        0.          0.        ]
 [  8.          1.66666667  3.54166667  0.        ]
 [  1.         -7.        -2.16666667 -1.14166667]]

Coef:
 [ 15.5       -12.5         3.54166667 -1.14166667]
```

Figure 20: Newton method test.

2.1.11 Lagrange

```python
def lagrange(X, Y):
    n = X.size
    L = np.zeros((n, n)) # (shape)

    for i in range(n):
        aux0 = np.setdiff1d(X, X[i])
        aux = [1, -aux0[0]]
        for j in range(1, n-1):
            aux = np.convolve(aux, [1, -aux0[j]])
        L[i,:] = aux / np.polyval(aux, X[i])

    coef = Y.dot(L)
    return L, coef
```

Figure 21: Code of the lagrange method in python.

```
Etapa: 0
L:
 [[0. 0. 0. 0.]
 [0. 0. 0. 0.]
 [0. 0. 0. 0.]
 [0. 0. 0. 0.]]

Etapa: 1
L:
 [[-0.05  0.35 -0.6  -0.  ]
 [ 0.    0.    0.    0.  ]
 [ 0.    0.    0.    0.  ]
 [ 0.    0.    0.    0.  ]]

Etapa: 2
L:
 [[-0.05        0.35       -0.6        -0.        ]
 [ 0.08333333 -0.5         0.41666667  1.        ]
 [ 0.          0.          0.          0.        ]
 [ 0.          0.          0.          0.        ]]

Etapa: 3
L:
 [[-0.05        0.35       -0.6        -0.        ]
 [ 0.08333333 -0.5         0.41666667  1.        ]
 [-0.08333333  0.25        0.33333333 -0.        ]
 [ 0.          0.          0.          0.        ]]

Etapa: 4
L:
 [[-0.05        0.35       -0.6        -0.        ]
 [ 0.08333333 -0.5         0.41666667  1.        ]
 [-0.08333333  0.25        0.33333333 -0.        ]
 [ 0.05       -0.1        -0.15        0.        ]]

Etapa: 5
L:
 [[-0.05        0.35       -0.6        -0.        ]
 [ 0.08333333 -0.5         0.41666667  1.        ]
 [-0.08333333  0.25        0.33333333 -0.        ]
 [ 0.05       -0.1        -0.15        0.        ]]
Coef:
 [-1.14166667  5.825      -5.53333333  3.        ]
```

Figure 22: Lagrange method test.

2.1.12 Line plotter

```python
def trazlin(X, Y):
    n = X.size
    m = 2*(n-1)
    A = np.zeros((m, m)) # (shape)
    b = np.zeros(m)
    coef = np.zeros((n-1, 2))

    for i in range(n - 1):
        A[i+1, [2*i, 2*i+1]] = np.array([X[i+1], 1])
        b[i+1] = Y[i+1]

    A[0, [0, 1]] = np.array([X[0], 1])
    b[0] = Y[0]
    print(A)
    print(b)

    for i in range(1, n-1):
        A[n+i-1, 2*i-2:2*i+2] = np.array([X[i], 1, -X[i], -1])
        b[n+i-1] = 0

    Saux = gausstot(A, b)

    for i in range(n-1):
        coef[i,:] = Saux[[2*i, 2*i+1]]
    return coef
```

Figure 23: Code of Line plotter method in python.

```
Etapa: 0
A:
 [[0. 0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0. 0.]]
b:
 [0. 0. 0. 0. 0. 0.]

Etapa: 1
A:
 [[-1.  1.  0.  0.  0.  0.]
 [ 0.  1.  0.  0.  0.  0.]
 [ 0.  0.  3.  1.  0.  0.]
 [ 0.  0.  0.  0.  4.  1.]
 [ 0.  0.  0.  0.  0.  0.]
 [ 0.  0.  0.  0.  0.  0.]]
b:
 [15.5  3.   8.   1.   0.   0. ]

Etapa: 2
A:
 [[-1.  1.  0.  0.  0.  0.]
 [ 0.  1.  0.  0.  0.  0.]
 [ 0.  0.  3.  1.  0.  0.]
 [ 0.  0.  0.  0.  4.  1.]
 [ 0.  1.  0. -1.  0.  0.]
 [ 0.  0.  3.  1. -3. -1.]]
b:
 [15.5  3.   8.   1.   0.   0. ]

INICIO - GAUSSTOT
```

Figure 24: First image of the line plotter method test.

```
X despues del cambio de columnas:
[-12.5          3.           1.66666667   3.          -7.
  29.        ]
TERMINA - GAUSSTOT
Coef:
 [[-12.5          3.        ]
 [  1.66666667   3.        ]
 [ -7.          29.        ]]
```

Figure 25: Second image of the line plotter method test.

2.1.13 Cuadratic plotter

```python
def trazcuad(X, Y):
    n = X.size
    m = 3*(n-1)
    A = np.zeros((m, m)) # (shape)
    b = np.zeros(m)
    coef = np.zeros((n-1, 3))

    # Condiciones de interpolacion
    for i in range(n - 1):
        A[i+1, 3*i:3*i+3] = np.array([X[i+1]**2, X[i+1], 1])
        b[i+1] = Y[i+1]

    A[0, 0:3] = np.array([X[0]**2, X[0], 1])
    b[0] = Y[0]

    # Condiciones de continuidad
    for i in range(1, n-1):
        A[n+i-1, 3*i-3:3*i+3] = np.array([X[i]**2, X[i], 1, -X[i]**2, -X[i], -1])
        b[n+i-1] = 0

    # Condiciones de suavidad
    for i in range(1, n-1):
        A[2*n+i-3, 3*i-3:3*i+3] = np.array([2*X[i], 1, 0, -2*X[i], -1, 0])
        b[2*n+i-3] = 0

    A[m-1, 0] = 2
    b[m-1] = 0

    Saux = gausstot(A, b)

    for i in range(n-1):
        coef[i,:] = Saux[3*i:3*i+3]
    return coef
```

Figure 26: Code of a cuadratic plotter method in python.

```
Etapa: 0
A:
 [[0. 0. 0. 0. 0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0. 0. 0. 0. 0.]]
b:
 [0. 0. 0. 0. 0. 0. 0. 0. 0.]

Etapa: 1
A:
 [[ 1. -1.  1.  0.  0.  0.  0.  0.  0.]
 [ 0.  0.  1.  0.  0.  0.  0.  0.  0.]
 [ 0.  0.  0.  9.  3.  1.  0.  0.  0.]
 [ 0.  0.  0.  0.  0.  0. 16.  4.  1.]
 [ 0.  0.  0.  0.  0.  0.  0.  0.  0.]
 [ 0.  0.  0.  0.  0.  0.  0.  0.  0.]
 [ 0.  0.  0.  0.  0.  0.  0.  0.  0.]
 [ 0.  0.  0.  0.  0.  0.  0.  0.  0.]
 [ 0.  0.  0.  0.  0.  0.  0.  0.  0.]]
b:
 [15.5  3.   8.   1.   0.   0.   0.   0.   0. ]

Etapa: 2
A:
 [[ 1. -1.  1.  0.  0.  0.  0.  0.  0.]
 [ 0.  0.  1.  0.  0.  0.  0.  0.  0.]
 [ 0.  0.  0.  9.  3.  1.  0.  0.  0.]
 [ 0.  0.  0.  0.  0.  0. 16.  4.  1.]
 [ 0.  0.  1.  0.  0. -1.  0.  0.  0.]
 [ 0.  0.  0.  9.  3.  1. -9. -3. -1.]
 [ 0.  0.  0.  0.  0.  0.  0.  0.  0.]
 [ 0.  0.  0.  0.  0.  0.  0.  0.  0.]
 [ 0.  0.  0.  0.  0.  0.  0.  0.  0.]]
b:
 [15.5  3.   8.   1.   0.   0.   0.   0.   0. ]

Etapa: 3
A:
 [[ 1. -1.  1.  0.  0.  0.  0.  0.  0.]
 [ 0.  0.  1.  0.  0.  0.  0.  0.  0.]
 [ 0.  0.  0.  9.  3.  1.  0.  0.  0.]
 [ 0.  0.  0.  0.  0.  0. 16.  4.  1.]
 [ 0.  0.  1.  0.  0. -1.  0.  0.  0.]
 [ 0.  0.  0.  9.  3.  1. -9. -3. -1.]
 [ 0.  1.  0.  0. -1.  0.  0.  0.  0.]
 [ 0.  0.  0.  6.  1.  0. -6. -1.  0.]
 [ 2.  0.  0.  0.  0.  0.  0.  0.  0.]]
b:
 [15.5  3.   8.   1.   0.   0.   0.   0.   0. ]

INICIO - GAUSSTOT
```

Figure 27: First image of a cuadratic plotter method test.

```
    12.5          3.          152.83333333 -245.        ]
X despues del cambio de columnas:
[   0.          -12.5           3.            4.72222222  -12.5
    3.          -22.83333333  152.83333333 -245.        ]
TERMINA - GAUSSTOT
Etapa: 4
A:
 [[ 1. -1.  1.  0.  0.  0.  0.  0.  0.]
 [ 0.  0.  1.  0.  0.  0.  0.  0.  0.]
 [ 0.  0.  0.  9.  3.  1.  0.  0.  0.]
 [ 0.  0.  0.  0.  0.  0. 16.  4.  1.]
 [ 0.  0.  1.  0.  0. -1.  0.  0.  0.]
 [ 0.  0.  0.  9.  3.  1. -9. -3. -1.]
 [ 0.  1.  0.  0. -1.  0.  0.  0.  0.]
 [ 0.  0.  0.  6.  1.  0. -6. -1.  0.]
 [ 2.  0.  0.  0.  0.  0.  0.  0.  0.]]
b:
 [15.5  3.   8.   1.   0.   0.   0.   0.   0. ]

Coef:
 [[   0.          -12.5           3.        ]
 [   4.72222222  -12.5           3.        ]
 [ -22.83333333  152.83333333 -245.        ]]
```

Figure 28: Second image of a cuadratic plotter method test.