Statistics and Data Analysis
Unit 02 – Lecture 04 Notes
Statistical Summaries for Data

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What You Will Learn (Beginner-Friendly)

In many real problems, the dataset is too large to read row-by-row. So we ask:

1. What is a "typical" value of a feature (center)?
2. How much do values vary (spread)?
3. How can we compare groups (CSE vs ECE) or time (week 1 vs week 2)?

By the end of this lecture, you should be able to:

- interpret a common summary table (count, mean, std, quartiles, min/max),
- compute and interpret the five-number summary,
- produce and interpret grouped summaries,
- and explain what information is lost when we compress data into a few numbers.

1. Why Summaries Are Needed

If a dataset has 10,000 rows, you cannot communicate it in a report by printing the raw table. A **summary** compresses the data into a small set of numbers that still capture the important story.

1.1 "Comparison across groups or time"

Summaries let us compare:

- **groups**: e.g., mean/median final score in CSE vs ECE,
- **time periods**: e.g., average weekly sales in January vs February.

Instead of comparing 10,000 raw values, we compare a few summary values.

2. Standard Summary Table (What Each Column Means)

For a numeric variable (say `final_score`), a typical summary contains:

- **count** (n): how many values exist (after excluding missing values),
- **mean**: arithmetic average (sensitive to outliers),
- **std**: sample standard deviation (typical distance from mean),
- **min/max**: extremes,
- **25%**, **50%**, **75%**: quartiles (Q_1, median, Q_3).

Important warning. A summary table does **not** show the full distribution shape. Two datasets can have similar mean/std but look very different (skewed vs bimodal).

3. Five-Number Summary

The five-number summary is:

$$\min(x),\ Q_1,\ \text{median},\ Q_3,\ \max(x)$$

It is used to create a **boxplot** and is often more robust than mean/std.

3.1 Quartile interpretation

- Q_1 (25th percentile): about 25% of values are at or below Q_1.
- Median (50th percentile): about 50% of values are at or below the median.
- Q_3 (75th percentile): about 75% of values are at or below Q_3.

So, the middle 50% of the data lies between Q_1 and Q_3.

Exercise 1 (solution)

Dataset: 4, 5, 7, 8, 9, 10, 25
Five-number summary:

- $\min = 4,\ \max = 25$
- median $= 8$
- lower half (4, 5, 7) $\Rightarrow Q_1 = 5$
- upper half (9, 10, 25) $\Rightarrow Q_3 = 10$
- IQR $= Q_3 - Q_1 = 5$

4. Mean vs Median (Quick Skewness Clue)

Mean and median are both measures of center, but:

- mean uses all values and is pulled by outliers,
- median uses only ordering and is robust to outliers.

Rule of thumb (not a proof).

- mean $\approx$ median: might be roughly symmetric,
- mean $>$ median: often right-skewed,
- mean $<$ median: often left-skewed.

Always confirm with a plot.

Exercise 2 (solution)

Given summary:

- 25% = 65 means about 25% scored 65 or less.
- 75% = 82 means about 75% scored 82 or less.
- std = 12 means a typical score is roughly 12 points away from the mean (spread).

Exercise 3 (solution)

Group A: 60, 62, 65, 95
Group B: 70, 72, 73, 74

- Group A mean = 70.5; median = 63.5
- Group B mean = 72.25; median = 72.5

Interpretation: Group A has an outlier (95) that inflates its mean. Typical performance (median) is much higher in Group B.

5. Grouped Summaries (Stratification)

Sometimes one global summary is misleading. We compute summaries **within groups** (by program, section, gender, etc.).

5.1 Weighted mean (why it matters)

If groups have different sizes, the overall mean must weight by group size.

Exercise 4 (solution)

Section A: $n_A = 10$, mean=70; Section B: $n_B = 5$, mean=80

$$\bar{x} = \frac{70 \cdot 10 + 80 \cdot 5}{15} \approx 73.33$$

Simple average of means (75) is incorrect here.

Exercise 5 (solution)

Means:

- CSE: 72.5
- ECE: 62.5
- AIML: 82.5

Exercise 6 (solution)

When we only report mean and std, we can miss:

- outliers and skewness,
- multi-modality (two peaks),
- differences between subgroups.

6. Mini Demo (Python)

Run from the lecture folder:

```
python demo/statistical_summaries_demo.py
```

It uses `data/student_summary.csv` and prints:

- an overall summary per numeric column,
- a grouped summary of `final_score` by `program`.

It also saves:

- `data/overall_summary.csv`
- `data/summary_by_program.csv`
- `images/mean_final_by_program.png` (if matplotlib is installed)

References

- Montgomery, D. C., & Runger, G. C. *Applied Statistics and Probability for Engineers*, Wiley, 7th ed., 2020.
- McKinney, W. *Python for Data Analysis*, O'Reilly, 2022.
- Freedman, D., Pisani, R., & Purves, R. *Statistics*, W. W. Norton, 4th ed., 2007.

Appendix: Slide Deck Content (Reference)

The material below is a reference copy of the slide deck content. Exercise solutions are explained in the main notes where applicable.

Title Slide

https://github.com/tali7c/Statistics-and-Data-Analysis

Quick Links

Overview Summary Tables Grouped Summary Demo Summary

Agenda

- Overview
- Summary Tables
- Grouped Summary
- Demo
- Summary

Learning Outcomes

- Explain why we summarize data (communication and comparison)
- Interpret a standard summary table (count, mean, std, quartiles, min/max)
- Compute and interpret a five-number summary (min, Q1, median, Q3, max)
- Produce grouped summaries (mean/median by category)
- Explain what is lost when we compress data into a few numbers

Why Summaries?

A summary answers: "What does the dataset look like in one page?"

- We cannot read thousands of rows one-by-one
- We need quick **comparison** across groups (CSE vs ECE) or time (week 1 vs week 2)
- Summaries are used in reports, dashboards, and as a first step in analysis

A Standard Summary Table (Common Columns)

For a numeric feature (example: `final_score`), a typical summary includes:

- **count** (n): number of non-missing values
- **mean**, **std** (sample standard deviation)
- **min**, **max**
- **25% (Q1)**, **50% (median)**, **75% (Q3)**

Idea: these numbers quickly describe center + spread + typical range.

Five-Number Summary (Very Important)

For a dataset x (sorted), the five-number summary is:

$$\min(x),\ Q_1,\ \text{median},\ Q_3,\ \max(x)$$

- It is the foundation of the boxplot
- It is more robust than mean/std when outliers exist

Exercise 1: Five-Number Summary

Dataset:

4 5 7 8 9 10 25

Task: Compute min, Q_1, median, Q_3, max and IQR.

Solution 1

Sorted: 4, 5, 7, 8, 9, 10, 25 (n=7)

- $\min = 4,\ \max = 25$
- median = 8
- lower half: 4, 5, 7 $\Rightarrow Q_1 = 5$
- upper half: 9, 10, 25 $\Rightarrow Q_3 = 10$
- IQR $= Q_3 - Q_1 = 10 - 5 = 5$

Reading Quartiles (Interpretation)

Quartiles are percentiles:

- Q_1 (25%): 25% of values are *at or below* Q_1
- median (50%): 50% of values are at or below the median
- Q_3 (75%): 75% of values are at or below Q_3

Checkpoint: the middle 50% of values lie between Q_1 and Q_3.

Exercise 2: Interpret a Summary Row

Suppose a feature has summary:

	count	mean	std	min	25%	50%	75%	max
final_score	24	71.0	12.0	40	65	74	82	92

Task: What do 25% and 75% mean? What does std tell us?

Solution 2

- 25% (Q1)=65: about 25% of students scored 65 or less.
- 75% (Q3)=82: about 75% of students scored 82 or less.
- std=12: a typical score is about 12 points away from the mean (rough idea of spread).

Important: summaries do not show the full distribution shape.

Mean vs Median in Summaries

- If mean $\approx$ median, distribution may be roughly symmetric
- If mean $>$ median, data is often right-skewed (high outliers pull mean up)
- If mean $<$ median, data is often left-skewed (low outliers pull mean down)

Rule of thumb: always confirm with a plot (histogram/boxplot).

Exercise 3: Group Comparison (Outlier Effect)

Two groups:
Group A
60, 62, 65, 95
Group B
70, 72, 73, 74

Task: Compute mean and median for both groups. Which group is "better"?

Solution 3

- Group A mean $= (60 + 62 + 65 + 95)/4 = 70.5$; median $= (62 + 65)/2 = 63.5$
- Group B mean $= (70 + 72 + 73 + 74)/4 = 72.25$; median $= (72 + 73)/2 = 72.5$

Interpretation: Group A has an outlier (95) that inflates its mean. Typical performance (median) is much lower in Group A.

Grouped Summaries (Stratification)

Instead of one summary for the entire dataset, we summarize **by group**:

- mean/median `final_score` by `program`
- mean attendance by section or batch
- revenue by category, etc.

Why? A single global average can hide important group differences.

Exercise 4: Weighted Mean (Correct Overall Average)

Suppose:

- Section A: $n_A = 10$, mean score = 70
- Section B: $n_B = 5$, mean score = 80

Task: Compute the overall mean score (all 15 students together).

Solution 4

Overall mean is a **weighted mean**:

$$\bar{x} = \frac{70 \cdot 10 + 80 \cdot 5}{10 + 5} = \frac{700 + 400}{15} = \frac{1100}{15} \approx 73.33$$

Note: $(70 + 80)/2 = 75$ is wrong because group sizes are different.

Exercise 5: Mean by Program (Small Table)

Program	final_score values
CSE	70, 75
ECE	60, 65
AIML	80, 85

Task: Compute mean final_score for each program.

Solution 5

- CSE mean = $(70 + 75)/2 = 72.5$
- ECE mean = $(60 + 65)/2 = 62.5$
- AIML mean = $(80 + 85)/2 = 82.5$

Interpretation: group summaries let us compare programs directly.

Exercise 6: What Is Lost in a Summary Table?

Question: If we only report mean and std for a dataset, what could we miss?

- Think about outliers, skewness, and multi-modal distributions.

Solution 6

A small set of numbers can hide:

- outliers (one extreme value can distort mean/std)
- skewness (mean $\neq$ median) and long tails
- multi-modality (two peaks) where "average" is not typical
- subgroup differences (one group high, one group low)

Takeaway: summaries are useful, but always validate with plots.

Mini Demo (Python)

Run from the lecture folder:

```
python demo/statistical_summaries_demo.py
```

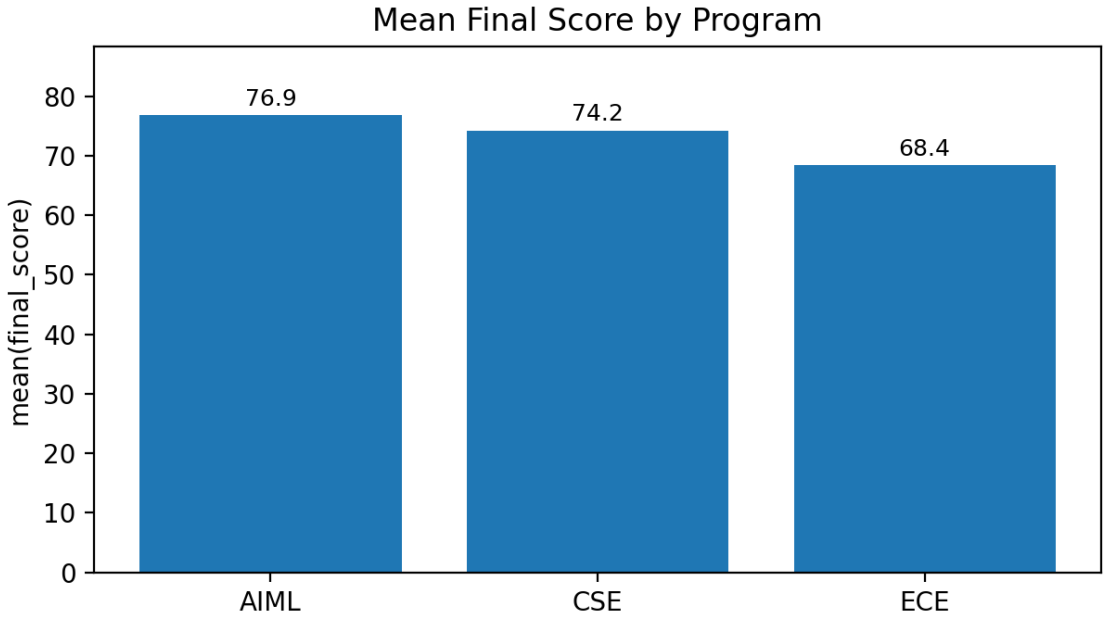
Uses:

- `data/student_summary.csv`

Outputs:

- `data/overall_summary.csv`
- `data/summary_by_program.csv`
- `images/mean_final_by_program.png` (if matplotlib is installed)

Demo Output (Example Plot)



Summary

- Summary tables compress data into center + spread + typical range (quartiles)
- Five-number summary is robust and supports boxplot thinking
- Grouped summaries (stratification) reveal differences hidden by global averages
- Summaries can hide distribution shape and outliers ⇒ use plots too

Exit question: Why is a weighted mean needed when groups have different sizes?