

Statistics and Data Analysis

Unit 02 – Lecture 04 Notes

Statistical Summaries for Data

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What You Will Learn (Beginner-Friendly)

In many real problems, the dataset is too large to read row-by-row. So we ask:

1. What is a “typical” value of a feature (center)?
2. How much do values vary (spread)?
3. How can we compare groups (CSE vs ECE) or time (week 1 vs week 2)?

By the end of this lecture, you should be able to:

- interpret a common summary table (count, mean, std, quartiles, min/max),
- compute and interpret the five-number summary,
- produce and interpret grouped summaries,
- and explain what information is lost when we compress data into a few numbers.

1. Why Summaries Are Needed

If a dataset has 10,000 rows, you cannot communicate it in a report by printing the raw table. A **summary** compresses the data into a small set of numbers that still capture the important story.

1.1 “Comparison across groups or time”

Summaries let us compare:

- **groups**: e.g., mean/median final score in CSE vs ECE,
- **time periods**: e.g., average weekly sales in January vs February.

Instead of comparing 10,000 raw values, we compare a few summary values.

2. Standard Summary Table (What Each Column Means)

For a numeric variable (say `final_score`), a typical summary contains:

- **count** (n): how many values exist (after excluding missing values),
- **mean**: arithmetic average (sensitive to outliers),
- **std**: sample standard deviation (typical distance from mean),
- **min/max**: extremes,
- **25%, 50%, 75%**: quartiles (Q_1 , median, Q_3).

Important warning. A summary table does **not** show the full distribution shape. Two datasets can have similar mean/std but look very different (skewed vs bimodal).

3. Five-Number Summary

The five-number summary is:

$$\min(x), Q_1, \text{median}, Q_3, \max(x)$$

It is used to create a **boxplot** and is often more robust than mean/std.

3.1 Quartile interpretation

- Q_1 (25th percentile): about 25% of values are at or below Q_1 .
- Median (50th percentile): about 50% of values are at or below the median.
- Q_3 (75th percentile): about 75% of values are at or below Q_3 .

So, the middle 50% of the data lies between Q_1 and Q_3 .

Exercise 1 (solution)

Dataset: 4, 5, 7, 8, 9, 10, 25

Five-number summary:

- $\min = 4, \max = 25$
- $\text{median} = 8$
- lower half (4, 5, 7) $\Rightarrow Q_1 = 5$
- upper half (9, 10, 25) $\Rightarrow Q_3 = 10$
- $\text{IQR} = Q_3 - Q_1 = 5$

4. Mean vs Median (Quick Skewness Clue)

Mean and median are both measures of center, but:

- mean uses all values and is pulled by outliers,
- median uses only ordering and is robust to outliers.

Rule of thumb (not a proof).

- mean \approx median: might be roughly symmetric,
- mean $>$ median: often right-skewed,
- mean $<$ median: often left-skewed.

Always confirm with a plot.

Exercise 2 (solution)

Given summary:

- 25% = 65 means about 25% scored 65 or less.
- 75% = 82 means about 75% scored 82 or less.
- std = 12 means a typical score is roughly 12 points away from the mean (spread).

Exercise 3 (solution)

Group A: 60, 62, 65, 95

Group B: 70, 72, 73, 74

- Group A mean = 70.5; median = 63.5
- Group B mean = 72.25; median = 72.5

Interpretation: Group A has an outlier (95) that inflates its mean. Typical performance (median) is much higher in Group B.

5. Grouped Summaries (Stratification)

Sometimes one global summary is misleading. We compute summaries **within groups** (by program, section, gender, etc.).

5.1 Weighted mean (why it matters)

If groups have different sizes, the overall mean must weight by group size.

Exercise 4 (solution)

Section A: $n_A = 10$, mean=70; Section B: $n_B = 5$, mean=80

$$\bar{x} = \frac{70 \cdot 10 + 80 \cdot 5}{15} \approx 73.33$$

Simple average of means (75) is incorrect here.

Exercise 5 (solution)

Means:

- CSE: 72.5
- ECE: 62.5
- AIML: 82.5

Exercise 6 (solution)

When we only report mean and std, we can miss:

- outliers and skewness,
- multi-modality (two peaks),
- differences between subgroups.

6. Mini Demo (Python)

Run from the lecture folder:

```
python demo/statistical_summaries_demo.py
```

It uses `data/student_summary.csv` and prints:

- an overall summary per numeric column,
- a grouped summary of `final_score` by `program`.

It also saves:

- `data/overall_summary.csv`
- `data/summary_by_program.csv`
- `images/mean_final_by_program.png` (if matplotlib is installed)

References

- Montgomery, D. C., & Runger, G. C. *Applied Statistics and Probability for Engineers*, Wiley, 7th ed., 2020.
- McKinney, W. *Python for Data Analysis*, O'Reilly, 2022.
- Freedman, D., Pisani, R., & Purves, R. *Statistics*, W. W. Norton, 4th ed., 2007.