Statistics and Data Analysis
Unit 03 – Lecture 04 Notes
Chi-square Tests (Goodness-of-Fit and Independence)

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Topic

Chi-square tests for counts: GOF and independence; expected counts; assumptions.

How to Use These Notes

These notes are written for students who are seeing the topic for the first time. They follow the slide order, but add the missing 'why', interpretation, and common mistakes. If you get stuck, look at the worked exercises and then run the Python demo.

Course repository (slides, demos, datasets): `https://github.com/tali7c/Statistics-and-Data-Analysis`

Time Plan (55 minutes)

- 0–10 min: Attendance + recap of previous lecture
- 10–35 min: Core concepts (this lecture's sections)
- 35–45 min: Exercises (solve 1–2 in class, rest as practice)
- 45–50 min: Mini demo + interpretation of output
- 50–55 min: Buffer / wrap-up (leave 5 minutes early)

Slide-by-slide Notes

Title Slide

State the lecture title clearly and connect it to what students already know. Tell students what they will be able to do by the end (not just what you will cover).

Quick Links / Agenda

Explain the structure of the lecture and where the exercises and demo appear.

- Overview

- Chi-square Tests
- Expected Counts
- Exercises
- Demo
- Summary

Learning Outcomes

- Explain when chi-square tests are used (counts/frequencies)
- Compute expected counts for a contingency table
- Compute chi-square statistic (basic)
- State assumptions (expected counts not too small)
- Interpret independence vs association

Why these outcomes matter. A **chi-square test** is used for **counts/frequencies** (categorical data). It compares observed counts O to expected counts E under the null hypothesis. Large deviations relative to E increase the χ^2 statistic and provide evidence against the null. In chi-square tests, **expected counts** are what you would expect to see if H_0 were true (e.g., independence). Very small expected counts can break the approximation used by the test; a common rule of thumb is that most expected counts should be at least 5.

Chi-square Tests: Key Points

- GOF: one categorical variable
- Independence: two categorical variables
- Compare observed O to expected E under H0

Explanation. The **null hypothesis** H_0 usually represents 'no effect' or a baseline value (e.g., $\mu = 60$). The **alternative** H_1 represents the effect you are looking for (e.g., $\mu \neq 60$ or $\mu > 60$). We compute a test statistic and a p-value assuming H_0 is true. A **chi-square test** is used for **counts/frequencies** (categorical data). It compares observed counts O to expected counts E under the null hypothesis. Large deviations relative to E increase the χ^2 statistic and provide evidence against the null. In chi-square tests, **expected counts** are what you would expect to see if H_0 were true (e.g., independence). Very small expected counts can break the approximation used by the test; a common rule of thumb is that most expected counts should be at least 5.

Chi-square Tests: Key Formula

$$\chi^2 = \sum \frac{(O - E)^2}{E}$$

How to read the formula. A **chi-square test** is used for **counts/frequencies** (categorical data). It compares observed counts O to expected counts E under the null hypothesis. Large deviations relative to E increase the χ^2 statistic and provide evidence against the null.

Expected Counts: Key Points

- E_rc = (row total)(col total)/N
- df = (R-1)(C-1) for independence

Explanation. **Degrees of freedom (df)** roughly represent how much independent information is available to estimate variability. For a one-sample t-test, df $= n - 1$ because one constraint is used to estimate the sample mean. df affects the critical values and the shape of the t-distribution (small df -> heavier tails). In chi-square tests, **expected counts** are what you would expect to see if H_0 were true (e.g., independence). Very small expected counts can break the approximation used by the test; a common rule of thumb is that most expected counts should be at least 5.

Exercises (with Solutions)

Attempt the exercise first, then compare with the solution. Focus on interpretation, not only arithmetic.

Exercise 1: Expected counts

Row totals 60/40, column totals 70/30, N=100. Compute E11.

Solution

- E11 = 60*70/100 = 42

Walkthrough. In chi-square tests, **expected counts** are what you would expect to see if H_0 were true (e.g., independence). Very small expected counts can break the approximation used by the test; a common rule of thumb is that most expected counts should be at least 5.

Exercise 2: Interpret reject

If you reject H0 in independence test, what do you conclude?

Solution

- Evidence of association between variables.
- Not direction; not causation.

Walkthrough. The **null hypothesis** H_0 usually represents 'no effect' or a baseline value (e.g., $\mu = 60$). The **alternative** H_1 represents the effect you are looking for (e.g., $\mu \neq 60$ or $\mu > 60$). We compute a test statistic and a p-value assuming H_0 is true.

Exercise 3: Assumption

Why do we worry about small expected counts?

Solution

- Chi-square approximation can break when E is very small.

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Mini Demo (Python)

Run from the lecture folder:

```
python demo/demo.py
```

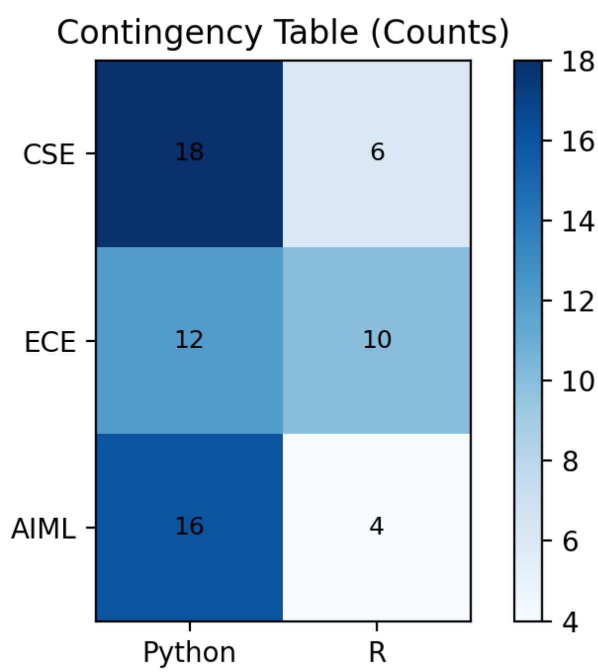
Output files:

- `images/demo.png`
- `data/results.txt`

What to show and say.

- Builds a small contingency table (counts) and runs a chi-square independence test.
- Visualizes counts as a heatmap to make 'observed vs expected' intuitive.
- Use the output to discuss df and why expected counts should not be too small.

Demo Output (Example)



Summary

- Key definitions and the main formula.
- How to interpret results in context.
- How the demo connects to the theory.

Exit Question

Why do chi-square tests use expected counts instead of only raw percentages?

Suggested answer (for revision). Expected counts incorporate the row/column totals and sample size; they define what 'no association' predicts so we can measure deviations fairly.

References

- Montgomery, D. C., & Runger, G. C. *Applied Statistics and Probability for Engineers*, Wiley.
- Devore, J. L. *Probability and Statistics for Engineering and the Sciences*, Cengage.
- McKinney, W. *Python for Data Analysis*, O'Reilly.

Appendix: Slide Deck Content (Reference)

The material below is a reference copy of the slide deck content. Exercise solutions are explained in the main notes where applicable.

Title Slide

https://github.com/tali7c/Statistics-and-Data-Analysis

Quick Links

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Solution 1

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Exercise 2: Interpret reject

If you reject H0 in independence test, what do you conclude?

Solution 2

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- Not direction; not causation.

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Solution 3

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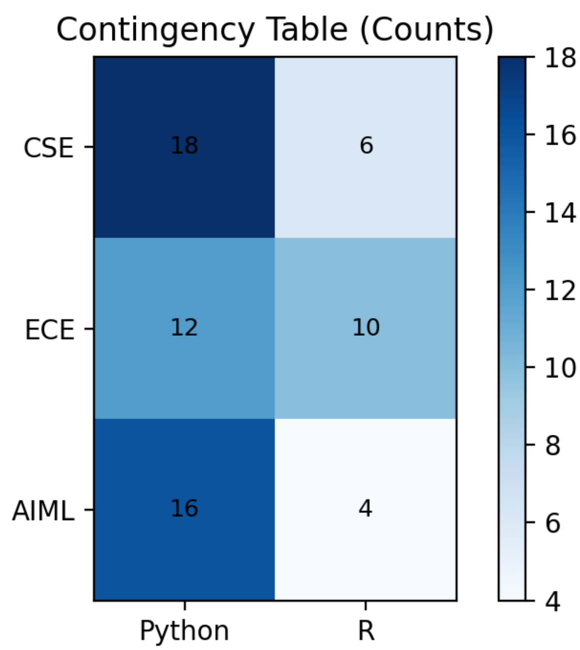

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