Statistics and Data Analysis
Unit 06 – Lecture 07 Notes
ACF and PACF Interpretation

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Topic

ACF/PACF interpretation to guide AR/MA orders (rough).

How to Use These Notes

These notes are written for students who are seeing the topic for the first time. They follow the slide order, but add the missing 'why', interpretation, and common mistakes. If you get stuck, look at the worked exercises and then run the Python demo.

Course repository (slides, demos, datasets): `https://github.com/tali7c/Statistics-and-Data-Analysis`

Time Plan (55 minutes)

- 0–10 min: Attendance + recap of previous lecture
- 10–35 min: Core concepts (this lecture's sections)
- 35–45 min: Exercises (solve 1–2 in class, rest as practice)
- 45–50 min: Mini demo + interpretation of output
- 50–55 min: Buffer / wrap-up (leave 5 minutes early)

Slide-by-slide Notes

Title Slide

State the lecture title clearly and connect it to what students already know. Tell students what they will be able to do by the end (not just what you will cover).

Quick Links / Agenda

Explain the structure of the lecture and where the exercises and demo appear.

- Overview

- ACF
- PACF
- Exercises
- Demo
- Summary

Learning Outcomes

- Define ACF and PACF (intuition)
- Use ACF/PACF patterns to guess AR and MA orders (rough)
- Recognize slow ACF decay as non-stationarity hint
- Explain why validation/diagnostics are still needed

Why these outcomes matter. **Stationarity** (intuition) means the series behavior is stable over time: roughly constant mean/variance and correlation structure. AR/MA/ARIMA models assume stationarity (after differencing). If the process changes over time, parameters learned from the past may not hold. **ACF** shows correlation of the series with its lagged versions. It helps identify MA-like behavior and seasonality. **PACF** shows the correlation at a lag after removing shorter-lag effects and helps identify AR-like behavior.

ACF: Key Points

- Corr(x_t, x_t-k) vs lag k
- Slow decay can suggest non-stationarity
- ACF cutoff after q lags can suggest MA(q) (rough)

Explanation. **Stationarity** (intuition) means the series behavior is stable over time: roughly constant mean/variance and correlation structure. AR/MA/ARIMA models assume stationarity (after differencing). If the process changes over time, parameters learned from the past may not hold. **ACF** shows correlation of the series with its lagged versions. It helps identify MA- like behavior and seasonality. **PACF** shows the correlation at a lag after removing shorter-lag effects and helps identify AR-like behavior.

PACF: Key Points

- Partial correlation after removing lower lags
- PACF cutoff after p lags can suggest AR(p) (rough)
- Use with caution and validate

Explanation. **Correlation** measures the strength of a linear association between two variables. It is symmetric (no X/Y direction) and does not imply causation. Outliers can inflate or hide correlation, so always look at the scatter plot. **ACF** shows correlation of the series with its lagged versions. It helps identify MA- like behavior and seasonality. **PACF** shows the correlation at a lag after removing shorter-lag effects and helps identify AR-like behavior.

Exercises (with Solutions)

Attempt the exercise first, then compare with the solution. Focus on interpretation, not only arithmetic.

Exercise 1: ACF pattern

If ACF stays significant for many lags, what might it suggest?

Solution

- Non-stationary; try differencing.

Walkthrough. **Differencing** transforms a series by subtracting the previous value: $y_t - y_{t-1}$. It removes trend and can help achieve stationarity. Over-differencing can add noise, so use the smallest differencing order that works. **Stationarity** (intuition) means the series behavior is stable over time: roughly constant mean/variance and correlation structure. AR/MA/ARIMA models assume stationarity (after differencing). If the process changes over time, parameters learned from the past may not hold.

Exercise 2: PACF AR hint

PACF cuts off after lag 2: what AR order to try?

Solution

- Try AR(2) as starting point.

Walkthrough. **ACF** shows correlation of the series with its lagged versions. It helps identify MA- like behavior and seasonality. **PACF** shows the correlation at a lag after removing shorter-lag effects and helps identify AR-like behavior.

Exercise 3: Rules not perfect

Why ACF/PACF rules are not perfect?

Solution

- Finite sample noise, seasonality, mixed ARMA.

Walkthrough. **Seasonality** is a repeating pattern with a fixed period (weekly, monthly, yearly). You must account for it; otherwise forecasts systematically miss repeating rises/falls. Seasonal differencing and SARIMA are common tools. **ACF** shows correlation of the series with its lagged versions. It helps identify MA- like behavior and seasonality. **PACF** shows the correlation at a lag after removing shorter-lag effects and helps identify AR-like behavior.

Mini Demo (Python)

Run from the lecture folder:

```
python demo/demo.py
```

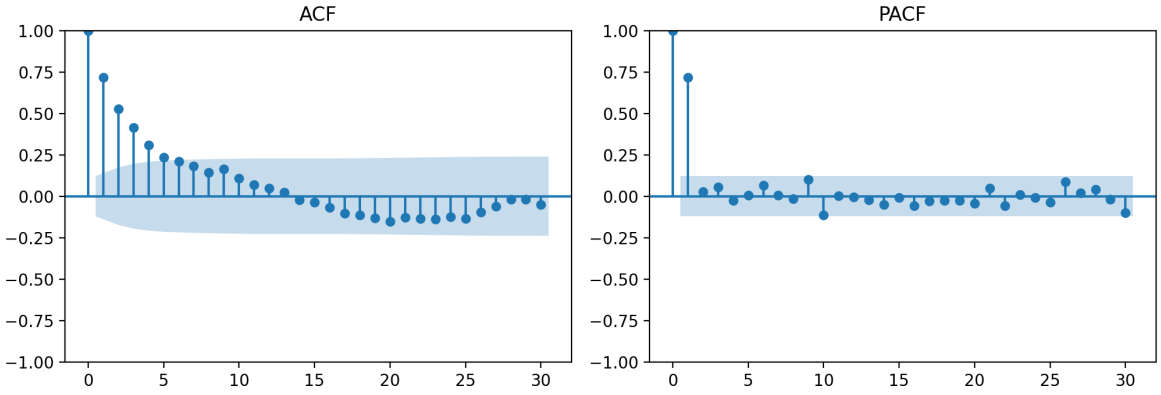
Output files:

- `images/demo.png`
- `data/results.txt`

What to show and say.

- Computes and plots ACF/PACF-like behavior for a simulated process.
- Use it to practice reading cutoff/decay patterns (rough guidance).
- Emphasize validation and diagnostics because rules are noisy in finite samples.

Demo Output (Example)



Summary

- Key definitions and the main formula.
- How to interpret results in context.
- How the demo connects to the theory.

Exit Question

How do ACF and PACF help choose ARIMA orders p and q?

Suggested answer (for revision). ACF/PACF patterns give rough hints: PACF cutoff AR(p), ACF cutoff MA(q); use diagnostics/validation because real data is noisy.

References

- Montgomery, D. C., & Runger, G. C. *Applied Statistics and Probability for Engineers*, Wiley.
- Devore, J. L. *Probability and Statistics for Engineering and the Sciences*, Cengage.
- McKinney, W. *Python for Data Analysis*, O'Reilly.

Appendix: Slide Deck Content (Reference)

The material below is a reference copy of the slide deck content. Exercise solutions are explained in the main notes where applicable.

Title Slide

https://github.com/tali7c/Statistics-and-Data-Analysis

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Exercise 1: ACF pattern

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Solution 1

- Non-stationary; try differencing.

Exercise 2: PACF AR hint

PACF cuts off after lag 2: what AR order to try?

Solution 2

- Try AR(2) as starting point.

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- Finite sample noise, seasonality, mixed ARMA.

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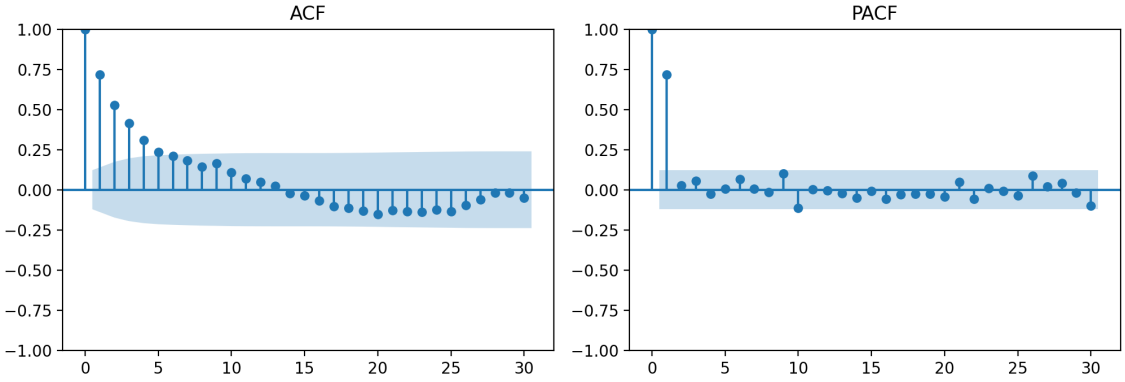

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