

Statistics and Data Analysis

Unit 02 – Lecture 03: Correlation, Skewness, Kurtosis

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<https://github.com/tali7c/Statistics-and-Data-Analysis>

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Learning Outcomes

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- Explain correlation and compute Pearson correlation r
- Relate covariance and correlation, and explain why correlation is scale-free
- Identify common pitfalls: outliers, non-linearity, and correlation vs causation
- Interpret skewness (right/left skew) and kurtosis (tail heaviness)

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- Correlation standardizes covariance to a unitless number in $[-1, 1]$
- That makes correlation easier to compare across different datasets

What is Correlation?

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- $r > 0$: as x increases, y tends to increase
- $r < 0$: as x increases, y tends to decrease
- $r \approx 0$: no strong *linear* pattern (could still be non-linear)

Pearson Correlation (Formula)

For paired data (x_i, y_i) :

$$r = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^n (x_i - \bar{x})^2} \sqrt{\sum_{i=1}^n (y_i - \bar{y})^2}}$$

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- Always between -1 and 1
- Unitless (no units)
- Sensitive to outliers

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- s_{xy} : sample covariance (Lecture 02)
- s_x, s_y : sample standard deviations
- Scaling a variable (changing units) does **not** change r

Interpreting r (Rule of Thumb)

$ r $ range	Common description
0.00–0.19	very weak
0.20–0.39	weak
0.40–0.59	moderate
0.60–0.79	strong
0.80–1.00	very strong

Always confirm with a scatter plot.

Exercise 1: Pearson Correlation (Positive)

Hours studied vs Score:

Hours (x)	1	2	3	4	5
Score (y)	52	55	60	65	68

Given: $\bar{x} = 3$, $\bar{y} = 60$, $\sum(x - \bar{x})(y - \bar{y}) = 42$, $\sum(x - \bar{x})^2 = 10$, $\sum(y - \bar{y})^2 = 178$.

Task: Compute r and interpret it.

Solution 1

$$r = \frac{42}{\sqrt{10}\sqrt{178}} = \frac{42}{\sqrt{1780}} \approx 0.9955$$

Interpretation: Very strong positive linear association between hours and score.

Exercise 2: Pearson Correlation (Negative)

Price vs Demand:

Price (x)	1	2	3	4	5
Demand (y)	80	70	60	50	40

Task: Compute r . What does the sign mean?

Solution 2

Here $y = 90 - 10x$ is a perfect decreasing line.

$$r = -1$$

Interpretation: Perfect negative linear relationship.

Exercise 3: $r = 0$ Does Not Mean “No Relationship”

Consider:

$$x = [-2, -1, 0, 1, 2], \quad y = x^2 = [4, 1, 0, 1, 4]$$

Task: Compute r . Is there a relationship between x and y ?

Solution 3

$\bar{x} = 0$, $\bar{y} = 2$. The numerator $\sum(x - \bar{x})(y - \bar{y}) = 0$, so:

$$r = 0$$

Key point: $r = 0$ means no *linear* association; here the relationship is strong but non-linear.

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- A third variable can cause both (confounding)
- Sometimes correlation is accidental (spurious)
- Use domain knowledge + experiments/causal reasoning to claim causation

Exercise 4: Interpret a Correlation Claim

"Ice cream sales and drowning incidents are positively correlated."

Which statement is most correct?

- 1** Ice cream causes drowning.
- 2** Drowning causes ice cream sales.
- 3** Both may increase due to a third factor (e.g., temperature/season).

Solution 4

Correct: (3). A confounder like hot weather can increase both swimming (risk) and ice cream sales.

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- **Right-skewed (positive):** long tail to the right (few very large values)
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- Symmetric: tails are similar on both sides

Mean vs Median vs Mode (Heuristic)

- Right-skewed: mean > median > mode

Reason: the mean is pulled toward the long tail.

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- Symmetric: mean \approx median \approx mode

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Moment Skewness (One Common Formula)

Let m_k be the k th central moment (divide by n):

$$m_k = \frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^k$$

Moment skewness:

$$g_1 = \frac{m_3}{m_2^{3/2}}$$

- $g_1 > 0$ right-skewed, $g_1 < 0$ left-skewed

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- $g_1 > 0$ right-skewed, $g_1 < 0$ left-skewed
- Different software may use small-sample corrections

Exercise 5: Identify Skewness Direction

Dataset A (Income, INR thousands):

20 22 23 24 25 26 27 28 60

Dataset B (Scores):

50 80 85 88 90 92 93 94 95 96

Task: For each dataset, decide if it is right-skewed or left-skewed.
Predict whether $\text{mean} > \text{median}$ or $\text{mean} < \text{median}$.

Solution 5

- Dataset A: one large value (60) creates a right tail \Rightarrow right-skewed, $\text{mean} > \text{median}$
- Dataset B: one small value (50) creates a left tail \Rightarrow left-skewed, $\text{mean} < \text{median}$

Exercise 6: Compute Moment Skewness

For Dataset A (income), suppose:

$$\bar{x} = 28.33, \quad m_2 = 130.89, \quad m_3 = 3404.07$$

Task: Compute $g_1 = \frac{m_3}{m_2^{3/2}}$ and interpret the sign.

Solution 6

$$g_1 = \frac{3404.07}{(130.89)^{3/2}} \approx 2.27$$

Interpretation: Positive and large \Rightarrow strongly right-skewed distribution.

Kurtosis (Tail Heaviness)

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- Often reported as **excess kurtosis** = kurtosis – 3
- Normal distribution has excess kurtosis 0
- Positive excess: heavier tails; negative excess: lighter tails

Moment Kurtosis (One Common Formula)

Moment kurtosis:

$$g_2 = \frac{m_4}{m_2^2}$$

Excess kurtosis:

$$\text{Excess} = g_2 - 3$$

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- If excess > 0 , more extreme values than normal (heavy tails)
- If excess < 0 , fewer extremes than normal (light tails)

Exercise 7: Excess Kurtosis (Small Symmetric Data)

Dataset: 1, 2, 3, 4, 5.

Mean = 3, deviations: $[-2, -1, 0, 1, 2]$.

Task: Compute m_2 , m_4 , then g_2 and excess kurtosis.

Solution 7

$$m_2 = \frac{4 + 1 + 0 + 1 + 4}{5} = 2$$

$$m_4 = \frac{16 + 1 + 0 + 1 + 16}{5} = \frac{34}{5} = 6.8$$

$$g_2 = \frac{6.8}{2^2} = 1.7, \quad \text{excess} = 1.7 - 3 = -1.3$$

Interpretation: Negative excess \Rightarrow lighter tails than normal (platykurtic).

Exercise 8: Excess Kurtosis (Income Example)

For the income dataset (Exercise 6), suppose:

$$m_2 = 130.89, \quad m_4 = 112590.30$$

Task: Compute $g_2 = \frac{m_4}{m_2^2}$ and excess kurtosis. Interpret.

Solution 8

$$g_2 \approx \frac{112590.30}{(130.89)^2} \approx 6.57, \quad \text{excess} \approx 3.57$$

Interpretation: Large positive excess \Rightarrow heavy tails / extreme values (outliers).

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- Different formulas exist (bias corrections), so values may differ across tools
- Always verify with plots (histogram/boxplot) and context

Mini Demo (Python)

Run:

```
python demo/correlation_skew_kurt_demo.py
```

What it does:

- Computes Pearson correlation for three paired datasets
- Prints correlation matrix for data/student_metrics.csv
- Computes moment skewness and excess kurtosis for example distributions
- (Optional) Saves plots to images/ if matplotlib is installed

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Exit question: Give one real-life example where correlation might be misleading and explain why.

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- Correlation standardizes covariance to $[-1, 1]$ and measures linear association
- $r = 0$ does not mean independence; it only indicates no linear relation
- Skewness describes tail direction; mean is pulled toward the tail
- Excess kurtosis relates to tail heaviness and outliers

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