Statistics and Data Analysis
Unit 06 – Lecture 05 Notes
Stationarity and Non-stationarity

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February 17, 2026

Topic

Stationarity concept; why it matters; fixes like differencing.

How to Use These Notes

These notes are written for students who are seeing the topic for the first time. They follow the slide order, but add the missing 'why', interpretation, and common mistakes. If you get stuck, look at the worked exercises and then run the Python demo.

Course repository (slides, demos, datasets): `https://github.com/tali7c/Statistics-and-Data-Analysis`

Time Plan (55 minutes)

- 0–10 min: Attendance + recap of previous lecture
- 10–35 min: Core concepts (this lecture's sections)
- 35–45 min: Exercises (solve 1–2 in class, rest as practice)
- 45–50 min: Mini demo + interpretation of output
- 50–55 min: Buffer / wrap-up (leave 5 minutes early)

Slide-by-slide Notes

Title Slide

State the lecture title clearly and connect it to what students already know. Tell students what they will be able to do by the end (not just what you will cover).

Quick Links / Agenda

Explain the structure of the lecture and where the exercises and demo appear.

- Overview

- Stationarity
- Fixes
- Exercises
- Demo
- Summary

Learning Outcomes

- Define stationarity (intuition)
- Recognize non-stationary patterns (trend/seasonality)
- Explain why stationarity matters for ARIMA-type models
- List basic fixes (differencing, transforms)

Why these outcomes matter. **Trend** is a long-term upward or downward movement. Trend changes the mean over time, which often creates non-stationarity. Many forecasting models handle trend by differencing or by explicitly modeling trend. **Seasonality** is a repeating pattern with a fixed period (weekly, monthly, yearly). You must account for it; otherwise forecasts systematically miss repeating rises/falls. Seasonal differencing and SARIMA are common tools.

Stationarity: Key Points

- Mean/variance roughly constant
- Autocorrelation depends on lag only
- Trend/seasonality often implies non-stationarity

Explanation. **Correlation** measures the strength of a linear association between two variables. It is symmetric (no X/Y direction) and does not imply causation. Outliers can inflate or hide correlation, so always look at the scatter plot. **Trend** is a long-term upward or downward movement. Trend changes the mean over time, which often creates non-stationarity. Many forecasting models handle trend by differencing or by explicitly modeling trend. **Seasonality** is a repeating pattern with a fixed period (weekly, monthly, yearly). You must account for it; otherwise forecasts systematically miss repeating rises/falls. Seasonal differencing and SARIMA are common tools.

Fixes: Key Points

- Differencing removes trend
- Seasonal differencing removes seasonality
- Log transform can stabilize variance

Explanation. **Trend** is a long-term upward or downward movement. Trend changes the mean over time, which often creates non-stationarity. Many forecasting models handle trend by differencing or by explicitly modeling trend. **Seasonality** is a repeating pattern with a fixed period (weekly, monthly, yearly). You must account for it; otherwise forecasts systematically miss repeating rises/falls. Seasonal differencing and SARIMA are common tools. **Differencing** transforms a series by subtracting the previous value: $y_t - y_{t-1}$. It removes trend and can help achieve stationarity. Over-differencing can add noise, so use the smallest differencing order that works.

Exercises (with Solutions)

Attempt the exercise first, then compare with the solution. Focus on interpretation, not only arithmetic.

Exercise 1: Trend

Is a strong upward trend likely stationary?

Solution

- No; mean changes over time.

Walkthrough. **Trend** is a long-term upward or downward movement. Trend changes the mean over time, which often creates non-stationarity. Many forecasting models handle trend by differencing or by explicitly modeling trend. **Stationarity** (intuition) means the series behavior is stable over time: roughly constant mean/variance and correlation structure. AR/MA/ARIMA models assume stationarity (after differencing). If the process changes over time, parameters learned from the past may not hold.

Exercise 2: Variance change

If fluctuations grow over time, is variance constant?

Solution

- No; non-stationary variance.

Walkthrough. **Stationarity** (intuition) means the series behavior is stable over time: roughly constant mean/variance and correlation structure. AR/MA/ARIMA models assume stationarity (after differencing). If the process changes over time, parameters learned from the past may not hold.

Exercise 3: Fix choice

Name one fix for non-stationary mean.

Solution

- Differencing.

Walkthrough. **Differencing** transforms a series by subtracting the previous value: $y_t - y_{t-1}$. It removes trend and can help achieve stationarity. Over-differencing can add noise, so use the smallest differencing order that works. **Stationarity** (intuition) means the series behavior is stable over time: roughly constant mean/variance and correlation structure. AR/MA/ARIMA models assume stationarity (after differencing). If the process changes over time, parameters learned from the past may not hold.

Mini Demo (Python)

Run from the lecture folder:

```
python demo/demo.py
```

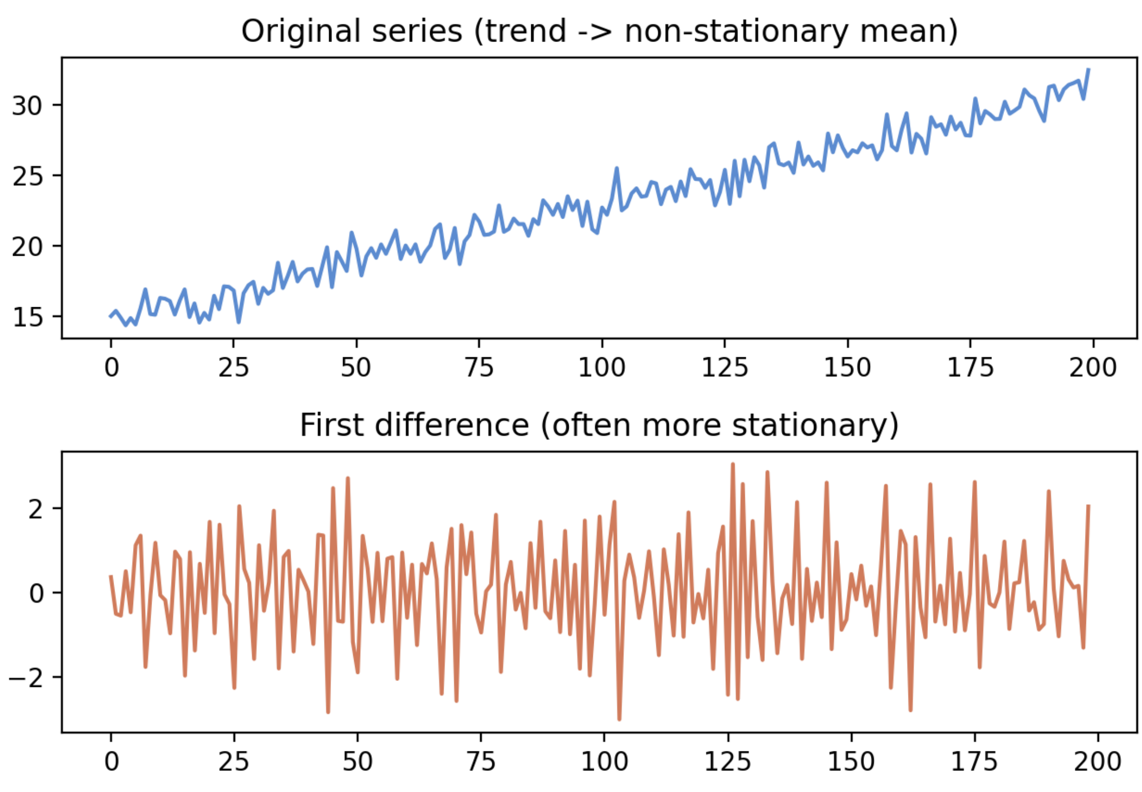
Output files:

- `images/demo.png`
- `data/results.txt`

What to show and say.

- Generates non-stationary vs stationary examples and plots them.
- Demonstrates differencing / log transform as simple fixes.
- Use it to explain why ARIMA-type models assume stationarity after transforms.

Demo Output (Example)



Summary

- Key definitions and the main formula.
- How to interpret results in context.
- How the demo connects to the theory.

Exit Question

Why does non-stationarity make forecasting harder?

Suggested answer (for revision). If mean/variance change over time, patterns learned from the past may not hold; stationarity makes modeling and forecasting more reliable.

References

- Montgomery, D. C., & Runger, G. C. *Applied Statistics and Probability for Engineers*, Wiley.
- Devore, J. L. *Probability and Statistics for Engineering and the Sciences*, Cengage.
- McKinney, W. *Python for Data Analysis*, O'Reilly.

Appendix: Slide Deck Content (Reference)

The material below is a reference copy of the slide deck content. Exercise solutions are explained in the main notes where applicable.

Title Slide

https://github.com/tali7c/Statistics-and-Data-Analysis

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Exercise 1: Trend

Is a strong upward trend likely stationary?

Solution 1

- No; mean changes over time.

Exercise 2: Variance change

If fluctuations grow over time, is variance constant?

Solution 2

- No; non-stationary variance.

Exercise 3: Fix choice

Name one fix for non-stationary mean.

Solution 3

- Differencing.

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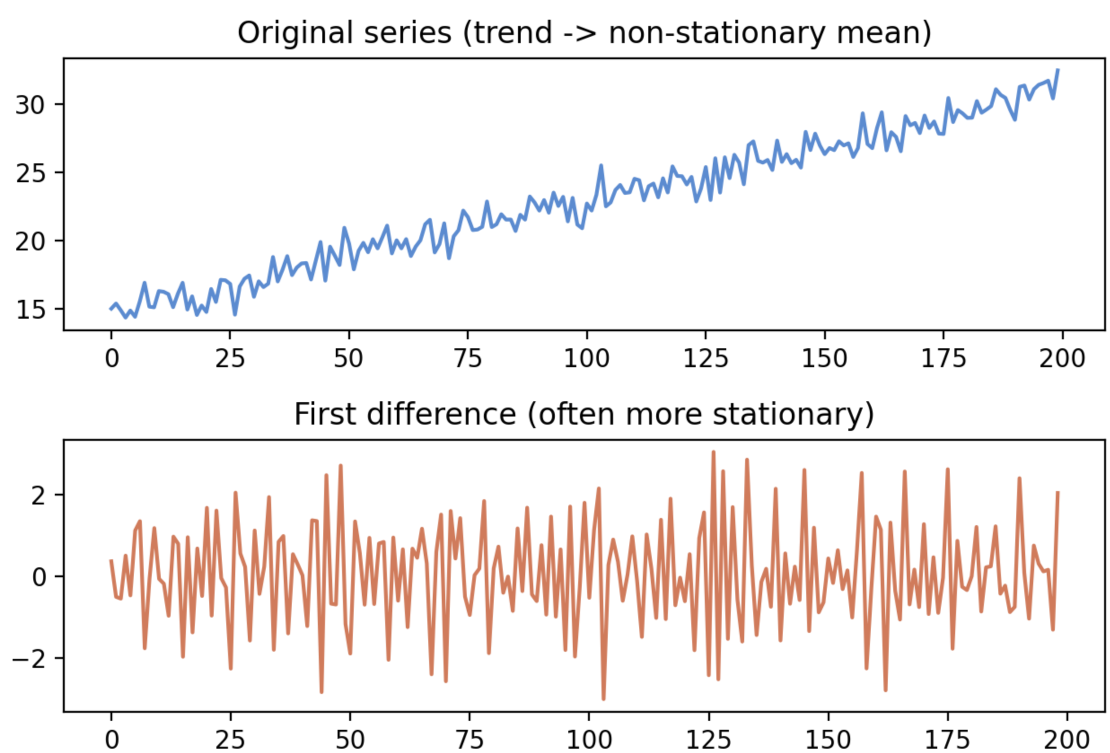

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