Statistics and Data Analysis
Unit 06 – Lecture 02 Notes
Smoothing (Moving Average and Exponential Smoothing)

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Topic

Smoothing techniques for time series (moving average, exponential smoothing).

How to Use These Notes

These notes are written for students who are seeing the topic for the first time. They follow the slide order, but add the missing 'why', interpretation, and common mistakes. If you get stuck, look at the worked exercises and then run the Python demo.

Course repository (slides, demos, datasets): `https://github.com/tali7c/Statistics-and-Data-Analysis`

Time Plan (55 minutes)

- 0–10 min: Attendance + recap of previous lecture
- 10–35 min: Core concepts (this lecture's sections)
- 35–45 min: Exercises (solve 1–2 in class, rest as practice)
- 45–50 min: Mini demo + interpretation of output
- 50–55 min: Buffer / wrap-up (leave 5 minutes early)

Slide-by-slide Notes

Title Slide

State the lecture title clearly and connect it to what students already know. Tell students what they will be able to do by the end (not just what you will cover).

Quick Links / Agenda

Explain the structure of the lecture and where the exercises and demo appear.

- Overview

- Moving Average
- Exponential Smoothing
- Exercises
- Demo
- Summary

Learning Outcomes

- Explain why smoothing is used (noise reduction)
- Describe moving average and its window effect
- Describe exponential smoothing and alpha effect
- Discuss responsiveness vs smoothness trade-off

Why these outcomes matter. The **significance level** α is the maximum Type I error rate you are willing to tolerate: the probability of rejecting H_0 when H_0 is actually true. Common choices are 0.05 or 0.01, but the right value depends on consequences of false alarms vs missed detections. A **moving average** smooths noise by averaging the last k points. Larger windows give smoother curves but introduce lag (the smooth curve reacts slowly to real changes). Use smoothing for visualization and as a baseline forecasting idea, not as a magic fix.

Moving Average: Key Points

- Average last k points
- Larger k -> smoother but more lag
- Good for trend visualization

Explanation. **Trend** is a long-term upward or downward movement. Trend changes the mean over time, which often creates non-stationarity. Many forecasting models handle trend by differencing or by explicitly modeling trend. A **moving average** smooths noise by averaging the last k points. Larger windows give smoother curves but introduce lag (the smooth curve reacts slowly to real changes). Use smoothing for visualization and as a baseline forecasting idea, not as a magic fix.

Exponential Smoothing: Key Points

- Weighted average with decay
- Alpha near 1 -> responsive
- Alpha near 0 -> smooth

Explanation. The **significance level** α is the maximum Type I error rate you are willing to tolerate: the probability of rejecting H_0 when H_0 is actually true. Common choices are 0.05 or 0.01, but the right value depends on consequences of false alarms vs missed detections. **Exponential smoothing** is a weighted average where recent observations get more weight. The parameter α controls the trade-off: high α reacts quickly (less smoothing), low α is smoother but slower.

Exponential Smoothing: Key Formula

$$s_t = \alpha x_t + (1 - \alpha)s_{t-1}$$

How to read the formula. The **significance level** α is the maximum Type I error rate you are willing to tolerate: the probability of rejecting H_0 when H_0 is actually true. Common choices are 0.05 or 0.01, but the right value depends on consequences of false alarms vs missed detections. **Exponential smoothing** is a weighted average where recent observations get more weight. The parameter α controls the trade-off: high α reacts quickly (less smoothing), low α is smoother but slower.

Exercises (with Solutions)

Attempt the exercise first, then compare with the solution. Focus on interpretation, not only arithmetic.

Exercise 1: Window effect

Increase window from 3 to 15: what happens?

Solution

- Smoother, more lag.

Exercise 2: Alpha

If alpha=0.9, smoothing is strong or weak?

Solution

- Weak smoothing (very responsive).

Walkthrough. The **significance level** α is the maximum Type I error rate you are willing to tolerate: the probability of rejecting H_0 when H_0 is actually true. Common choices are 0.05 or 0.01, but the right value depends on consequences of false alarms vs missed detections.

Exercise 3: Too much smoothing

Why can too much smoothing harm forecasting?

Solution

- It can hide real changes and add lag.

Mini Demo (Python)

Run from the lecture folder:

```
python demo/demo.py
```

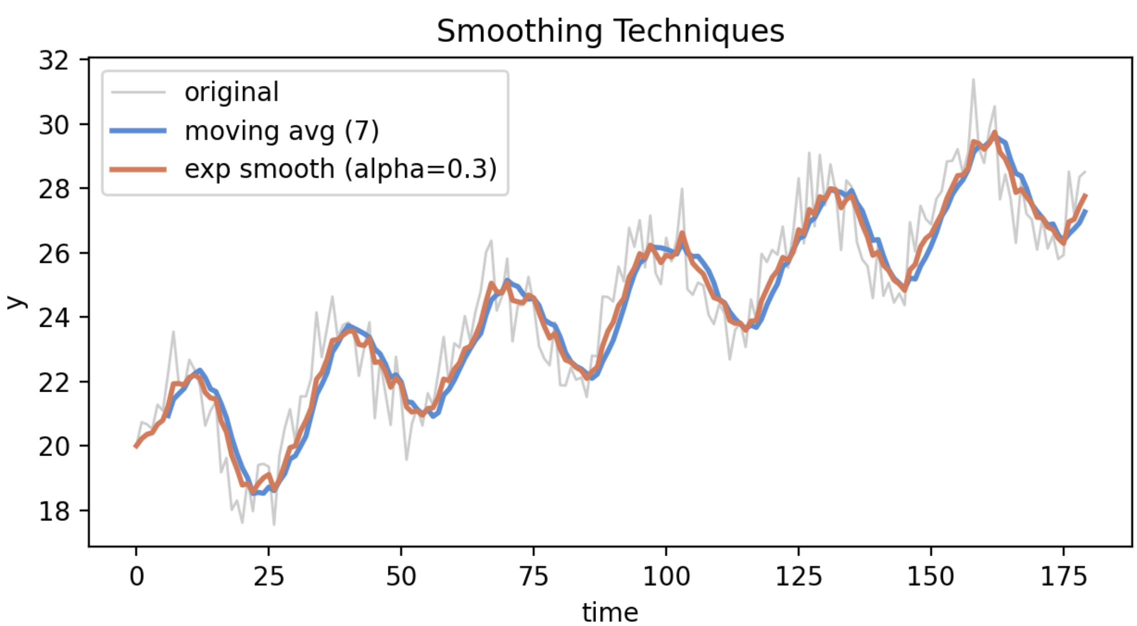
Output files:

- `images/demo.png`
- `data/results.txt`

What to show and say.

- Applies moving average and exponential smoothing to a noisy series.
- Shows the lag vs smoothness trade-off for different window/alpha values.
- Use it as a baseline for forecasting and for visual trend extraction.

Demo Output (Example)



Summary

- Key definitions and the main formula.
- How to interpret results in context.
- How the demo connects to the theory.

Exit Question

What is one sign that your smoothing window is too large?

Suggested answer (for revision). If the smoothed curve reacts too slowly and misses real changes, your window is too large (too much lag).

References

- Montgomery, D. C., & Runger, G. C. *Applied Statistics and Probability for Engineers*, Wiley.
- Devore, J. L. *Probability and Statistics for Engineering and the Sciences*, Cengage.
- McKinney, W. *Python for Data Analysis*, O'Reilly.

Appendix: Slide Deck Content (Reference)

The material below is a reference copy of the slide deck content. Exercise solutions are explained in the main notes where applicable.

Title Slide

https://github.com/tali7c/Statistics-and-Data-Analysis

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Solution 1

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If alpha=0.9, smoothing is strong or weak?

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Why can too much smoothing harm forecasting?

Solution 3

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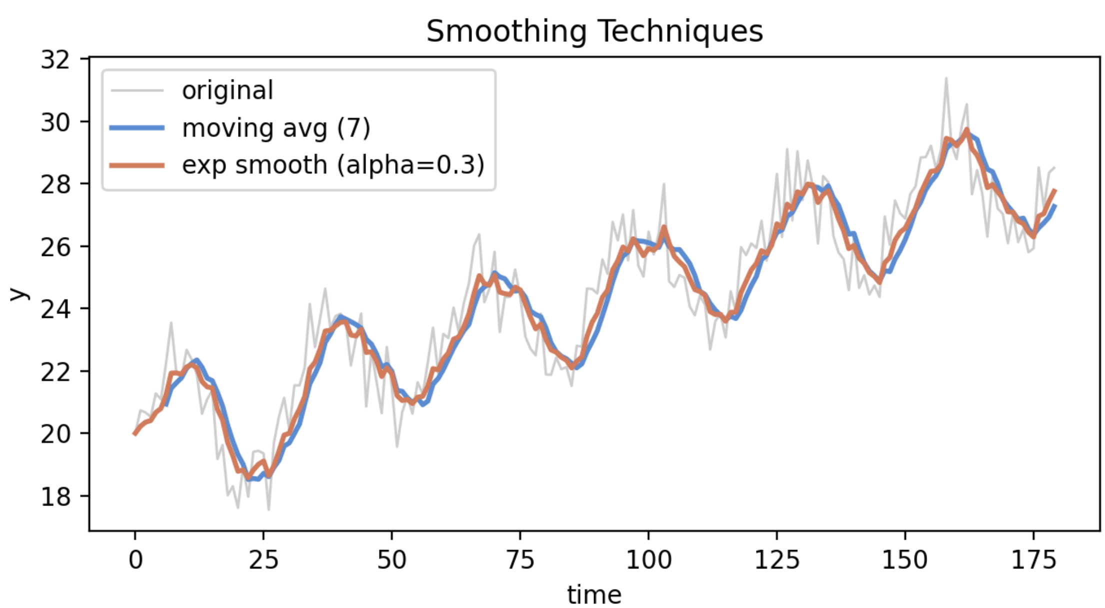

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