Statistics and Data Analysis
Unit 06 – Lecture 03 Notes
AR and MA Models

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Topic

Autoregressive (AR) and moving average (MA) models (intro).

How to Use These Notes

These notes are written for students who are seeing the topic for the first time. They follow the slide order, but add the missing 'why', interpretation, and common mistakes. If you get stuck, look at the worked exercises and then run the Python demo.

Course repository (slides, demos, datasets): `https://github.com/tali7c/Statistics-and-Data-Analysis`

Time Plan (55 minutes)

- 0–10 min: Attendance + recap of previous lecture
- 10–35 min: Core concepts (this lecture's sections)
- 35–45 min: Exercises (solve 1–2 in class, rest as practice)
- 45–50 min: Mini demo + interpretation of output
- 50–55 min: Buffer / wrap-up (leave 5 minutes early)

Slide-by-slide Notes

Title Slide

State the lecture title clearly and connect it to what students already know. Tell students what they will be able to do by the end (not just what you will cover).

Quick Links / Agenda

Explain the structure of the lecture and where the exercises and demo appear.

- Overview

- AR
- MA
- Exercises
- Demo
- Summary

Learning Outcomes

- Explain AR(p) model idea
- Explain MA(q) model idea
- Differentiate AR vs MA intuition
- Define white noise (basic)

AR: Key Points

- Current value depends on past values
- AR(1): x_t = c + phi x_t-1 + e_t
- Phi controls persistence

AR: Key Formula

$$x_t = c + \phi x_{t-1} + \epsilon_t$$

MA: Key Points

- Current value depends on past shocks
- MA(1): x_t = mu + e_t + theta e_t-1
- Captures short-term shock effects

MA: Key Formula

$$x_t = \mu + \epsilon_t + \theta \epsilon_{t-1}$$

Exercises (with Solutions)

Attempt the exercise first, then compare with the solution. Focus on interpretation, not only arithmetic.

Exercise 1: AR intuition

If phi=0.8 and last value is high (ignore noise), what happens next?

Solution

- Next value tends to be high too.

Exercise 2: MA intuition

In MA(1), what drives the series: past values or past shocks?

Solution

- Past shocks (errors).

Exercise 3: White noise

What is white noise?

Solution

- Uncorrelated errors with mean 0 and constant variance.

Mini Demo (Python)

Run from the lecture folder:

```
python demo/demo.py
```

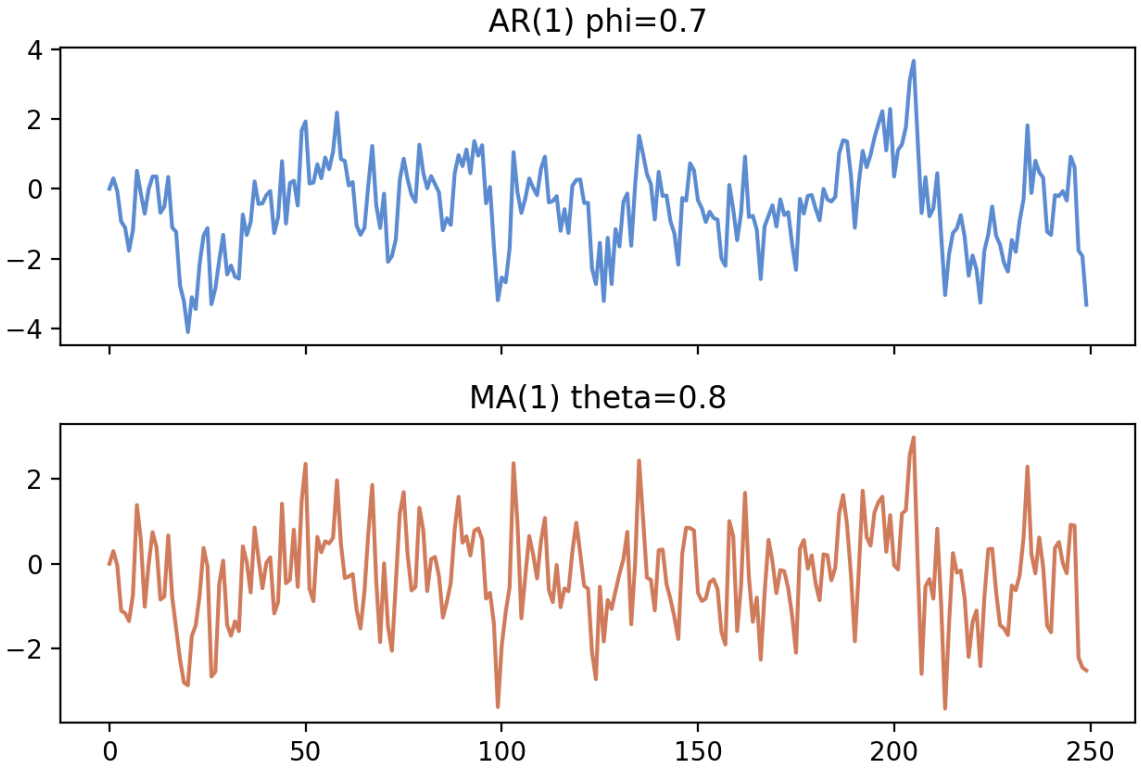
Output files:

- `images/demo.png`
- `data/results.txt`

What to show and say.

- Simulates an AR(1) and an MA(1) process and plots the series.
- Use it to explain 'memory of past values' (AR) vs 'memory of shocks' (MA).
- Connect to ACF/PACF intuition for order selection.

Demo Output (Example)



Summary

- Key definitions and the main formula.
- How to interpret results in context.
- How the demo connects to the theory.

Exit Question

How are AR and MA models different in what they remember?

Suggested answer (for revision). AR remembers past values; MA remembers past shocks (errors).

References

- Montgomery, D. C., & Runger, G. C. *Applied Statistics and Probability for Engineers*, Wiley.
- Devore, J. L. *Probability and Statistics for Engineering and the Sciences*, Cengage.
- McKinney, W. *Python for Data Analysis*, O'Reilly.

Appendix: Slide Deck Content (Reference)

The material below is a reference copy of the slide deck content. Exercise solutions are explained in the main notes where applicable.

Title Slide

https://github.com/tali7c/Statistics-and-Data-Analysis

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Exercise 1: AR intuition

If phi=0.8 and last value is high (ignore noise), what happens next?

Solution 1

- Next value tends to be high too.

Exercise 2: MA intuition

In MA(1), what drives the series: past values or past shocks?

Solution 2

- Past shocks (errors).

Exercise 3: White noise

What is white noise?

Solution 3

- Uncorrelated errors with mean 0 and constant variance.

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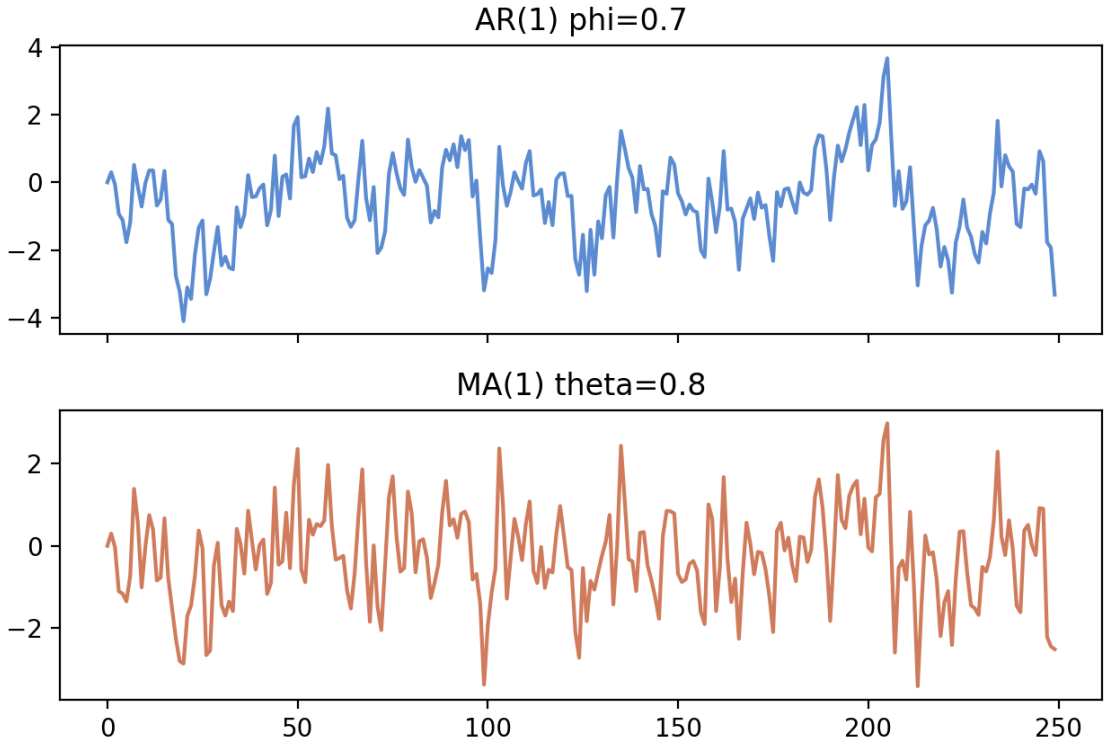

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