Statistics and Data Analysis
Unit 03 – Lecture 05 Notes
ANOVA (One-Way) and Post-hoc Intuition

Tofik Ali

February 17, 2026

Topic

One-way ANOVA; F statistic; assumptions; post-hoc comparisons.

How to Use These Notes

These notes are written for students who are seeing the topic for the first time. They follow the slide order, but add the missing 'why', interpretation, and common mistakes. If you get stuck, look at the worked exercises and then run the Python demo.

Course repository (slides, demos, datasets): `https://github.com/tali7c/Statistics-and-Data-Analysis`

Time Plan (55 minutes)

- 0–10 min: Attendance + recap of previous lecture
- 10–35 min: Core concepts (this lecture's sections)
- 35–45 min: Exercises (solve 1–2 in class, rest as practice)
- 45–50 min: Mini demo + interpretation of output
- 50–55 min: Buffer / wrap-up (leave 5 minutes early)

Slide-by-slide Notes

Title Slide

State the lecture title clearly and connect it to what students already know. Tell students what they will be able to do by the end (not just what you will cover).

Quick Links / Agenda

Explain the structure of the lecture and where the exercises and demo appear.

- Overview

- ANOVA Concept
- Assumptions
- Exercises
- Demo
- Summary

Learning Outcomes

- Explain why ANOVA is used for comparing 3+ means
- Describe between-group vs within-group variation
- Interpret F statistic at a high level
- State main assumptions of one-way ANOVA
- Explain what a post-hoc test is

Why these outcomes matter. **ANOVA** compares means of 3+ groups using variance decomposition. The idea is: if group means are truly the same, between-group variability should look similar to within- group variability. The F-statistic is essentially a ratio of these two sources of variation. If ANOVA is significant, you usually follow with a **post-hoc** procedure to identify which specific pairs of groups differ. Post-hoc methods control the family-wise error rate so that 'looking at many pairs' does not create many false positives.

ANOVA Concept: Key Points

- One global test for equality of means
- Avoids inflating Type I error vs many t-tests
- If significant, follow with post-hoc

Explanation. A **Type I error** is a false positive: you conclude there is an effect/difference when there is none. A **Type II error** is a false negative: you miss a real effect. Lowering α reduces Type I errors but can increase Type II errors unless you also increase sample size. **ANOVA** compares means of 3+ groups using variance decomposition. The idea is: if group means are truly the same, between-group variability should look similar to within- group variability. The F-statistic is essentially a ratio of these two sources of variation. If ANOVA is significant, you usually follow with a **post-hoc** procedure to identify which specific pairs of groups differ. Post-hoc methods control the family-wise error rate so that 'looking at many pairs' does not create many false positives.

ANOVA Concept: Key Formula

$$F = \frac{\text{between-group variation}}{\text{within-group variation}}$$

How to read the formula. **ANOVA** compares means of 3+ groups using variance decomposition. The idea is: if group means are truly the same, between-group variability should look similar to within- group variability. The F-statistic is essentially a ratio of these two sources of variation.

Assumptions: Key Points

- Independent observations
- Rough normality within groups (or robust with n)
- Similar variances across groups

Explanation. Always state assumptions clearly. Common assumptions in classical tests include independence of observations, roughly normal errors (or a large-sample justification), and similar variances across groups. Violations do not automatically invalidate a result, but they change how much you should trust the p-value and confidence interval.

Exercises (with Solutions)

Attempt the exercise first, then compare with the solution. Focus on interpretation, not only arithmetic.

Exercise 1: Write H0

Compare 3 group means. What is H0?

Solution

- H0: mu1 = mu2 = mu3

Walkthrough. The **null hypothesis** H_0 usually represents 'no effect' or a baseline value (e.g., $\mu = 60$). The **alternative** H_1 represents the effect you are looking for (e.g., $\mu \neq 60$ or $\mu > 60$). We compute a test statistic and a p-value assuming H_0 is true.

Exercise 2: Within variance

If within-group variance increases, what happens to F (all else equal)?

Solution

- F tends to decrease; harder to detect differences.

Walkthrough. **Differencing** transforms a series by subtracting the previous value: $y_t - y_{t-1}$. It removes trend and can help achieve stationarity. Over-differencing can add noise, so use the smallest differencing order that works.

Exercise 3: Next step

ANOVA p-value is 0.01 at alpha=0.05. What next?

Solution

- Reject H0.
- Run post-hoc to find which pairs differ.

Walkthrough. The **null hypothesis** H_0 usually represents 'no effect' or a baseline value (e.g., $\mu = 60$). The **alternative** H_1 represents the effect you are looking for (e.g., $\mu \neq 60$ or $\mu > 60$). We compute a test statistic and a p-value assuming H_0 is true. A **p-value** is computed assuming the null hypothesis H_0 is true. It measures how surprising the observed data (or something more extreme) would be under H_0. A small p-value suggests the data is hard to explain by H_0 alone, but it does not tell you how large the effect is or whether it is practically important.

Mini Demo (Python)

Run from the lecture folder:

```
python demo/demo.py
```

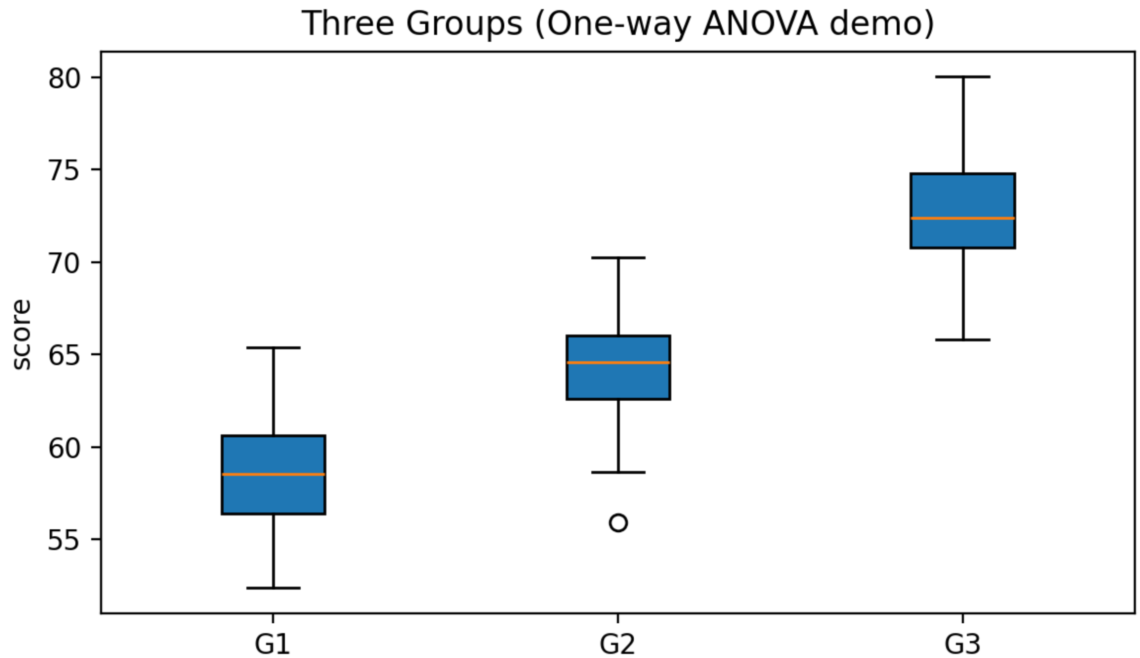
Output files:

- `images/demo.png`
- `data/results.txt`

What to show and say.

- Generates 3 groups with different means and runs one-way ANOVA (F-test).
- Shows a boxplot of the groups to connect F to 'between vs within' variation.
- Use the p-value to explain what ANOVA can/cannot tell (needs post-hoc for pairs).

Demo Output (Example)



Summary

- Key definitions and the main formula.
- How to interpret results in context.
- How the demo connects to the theory.

Exit Question

Why are several pairwise t-tests not equivalent to one ANOVA?

Suggested answer (for revision). Many pairwise t-tests inflate the family-wise Type I error rate; ANOVA performs one global test to control false positives before post-hoc comparisons.

References

- Montgomery, D. C., & Runger, G. C. *Applied Statistics and Probability for Engineers*, Wiley.
- Devore, J. L. *Probability and Statistics for Engineering and the Sciences*, Cengage.
- McKinney, W. *Python for Data Analysis*, O'Reilly.

Appendix: Slide Deck Content (Reference)

The material below is a reference copy of the slide deck content. Exercise solutions are explained in the main notes where applicable.

Title Slide

https://github.com/tali7c/Statistics-and-Data-Analysis

Quick Links

Overview ANOVA Concept Assumptions Exercises Demo Summary

Agenda

- Overview
- ANOVA Concept
- Assumptions
- Exercises
- Demo
- Summary

Learning Outcomes

- Explain why ANOVA is used for comparing 3+ means
- Describe between-group vs within-group variation
- Interpret F statistic at a high level
- State main assumptions of one-way ANOVA
- Explain what a post-hoc test is

ANOVA Concept: Key Points

- One global test for equality of means
- Avoids inflating Type I error vs many t-tests
- If significant, follow with post-hoc

ANOVA Concept: Key Formula

$$F = \frac{\text{between-group variation}}{\text{within-group variation}}$$

Assumptions: Key Points

- Independent observations
- Rough normality within groups (or robust with n)
- Similar variances across groups

Exercise 1: Write H0

Compare 3 group means. What is H0?

Solution 1

- H0: mu1 = mu2 = mu3

Exercise 2: Within variance

If within-group variance increases, what happens to F (all else equal)?

Solution 2

- F tends to decrease; harder to detect differences.

Exercise 3: Next step

ANOVA p-value is 0.01 at alpha=0.05. What next?

Solution 3

- Reject H0.
- Run post-hoc to find which pairs differ.

Mini Demo (Python)

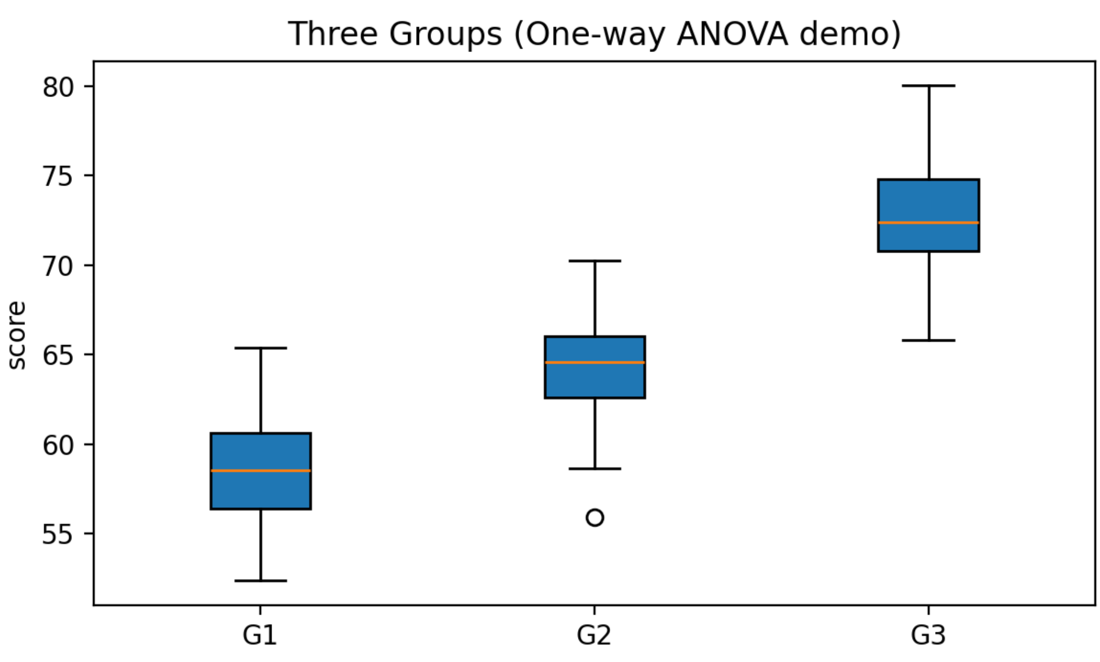
Run from the lecture folder:

```
python demo/demo.py
```

Outputs:

- `images/demo.png`
- `data/results.txt`

Demo Output (Example)



Summary

- Key definitions and the main formula.
- How to interpret results in context.
- How the demo connects to the theory.

Exit Question

Why are several pairwise t-tests not equivalent to one ANOVA?