

## QMM Assignment #5

```
library(Benchmarking)

## Loading required package: lpSolveAPI

## Loading required package: ucminf

## Loading required package: quadprog

library(lpSolveAPI)
library(ucminf)
library(quadprog)
```

#1) Formulate and perform DEA analysis under all DEA assumptions of FDH, CRS, VRS, IRS, DRS, and FRH.

*# Using Benchmarking Libraries for DEA*

```
library(Benchmarking)
library(lpSolveAPI)
library(ucminf)
library(quadprog)
X <- matrix(c(150,400,320,520,350,320,0.2,0.7,1.2,2.0,1.2,.7), ncol=2)
Y <-
matrix(c(14000,14000,42000,28000,19000,14000,3500,21000,10500,42000,25000,15000), ncol=2)
```

```
colnames(X) <- c("TP", "RP")
colnames(Y) <- c("Staffing Labor Per Hour", "Cost of Supplies per day")
X
```

```
##      TP  RP
## [1,] 150 0.2
## [2,] 400 0.7
## [3,] 320 1.2
## [4,] 520 2.0
## [5,] 350 1.2
## [6,] 320 0.7
```

Y

```
##      Staffing Labor Per Hour Cost of Supplies per day
## [1,]          14000          3500
## [2,]          14000          21000
## [3,]          42000          10500
## [4,]          28000          42000
## [5,]          19000          25000
## [6,]          14000          15000
```

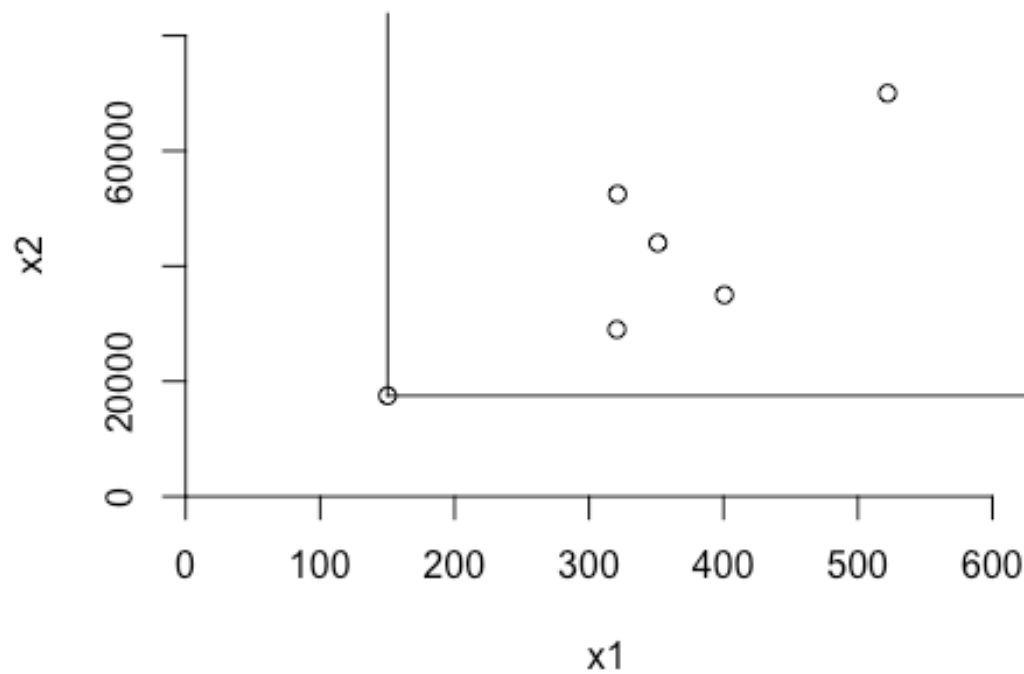
### #DEA Analysis

```
e <- dea(X,Y,RTS = "crs")           # provide the input and output
e
## [1] 1.0000 1.0000 1.0000 1.0000 0.9775 0.8675

peers(e)                             # identify the peers
##      peer1 peer2 peer3
## [1,]     1    NA    NA
## [2,]     2    NA    NA
## [3,]     3    NA    NA
## [4,]     4    NA    NA
## [5,]     1     2     4
## [6,]     1     2     4

lambda(e)                            # identify the relative weights given to
the peers
##           L1           L2 L3           L4
## [1,] 1.0000000 0.0000000 0 0.0000000
## [2,] 0.0000000 1.0000000 0 0.0000000
## [3,] 0.0000000 0.0000000 1 0.0000000
## [4,] 0.0000000 0.0000000 0 1.0000000
## [5,] 0.2000000 0.08048142 0 0.5383307
## [6,] 0.3428571 0.39499264 0 0.1310751

dea.plot.isoquant(X,Y,RTS="crs")     # plot the results
```



### #FDH Assumptions

```
FDH <- dea (X,Y, RTS = "fdh")
FDH
```

```
## [1] 1 1 1 1 1 1
```

```
peers(FDH)
```

```
##      peer1
```

```
## [1,]      1
```

```
## [2,]      2
```

```
## [3,]      3
```

```
## [4,]      4
```

```
## [5,]      5
```

```
## [6,]      6
```

```
lambda(FDH)
```

```
##      L1 L2 L3 L4 L5 L6
```

```
## [1,]  1  0  0  0  0  0
```

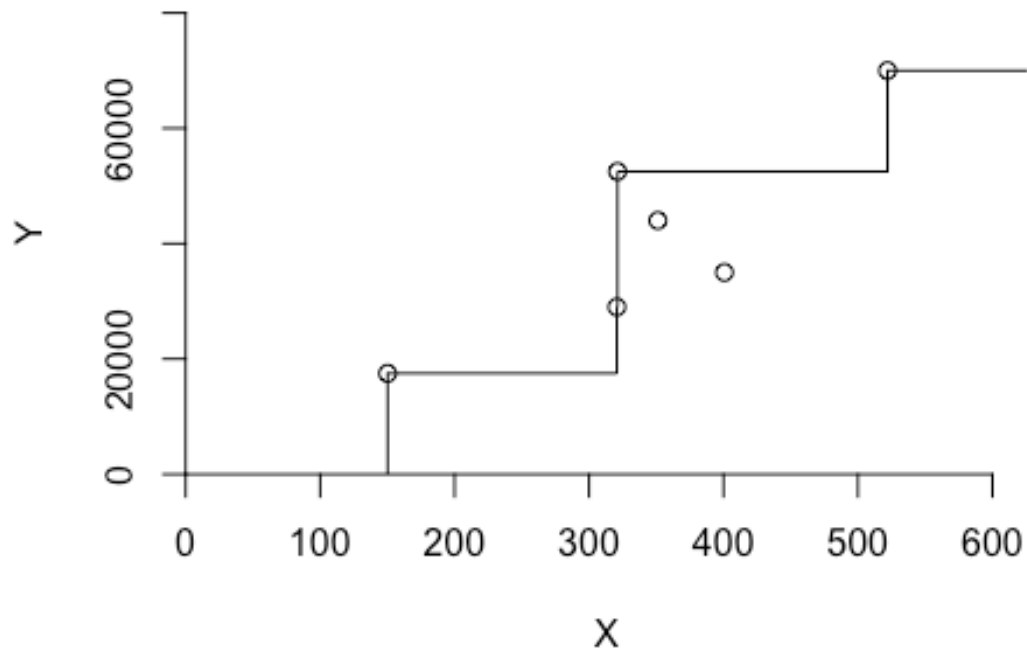
```
## [2,]  0  1  0  0  0  0
```

```
## [3,]  0  0  1  0  0  0
```

```
## [4,]  0  0  0  1  0  0
```

```
## [5,] 0 0 0 0 1 0
## [6,] 0 0 0 0 0 1

dea.plot.frontier(X,Y,RTS="FDH")
```



```
summary(FDH, digits=4)

## Summary of efficiencies
## FDH technology and input orientated efficiency
## Number of firms with efficiency==1 are 6 out of 6
## Mean efficiency: 1
## ---
##   Eff range      #    %
##       E ==1      6 100
##   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##       1       1       1       1       1       1

#VRS Assumptions

VRS <- dea (X,Y, RTS = "vrs")
VRS

## [1] 1.0000 1.0000 1.0000 1.0000 1.0000 0.8963
```

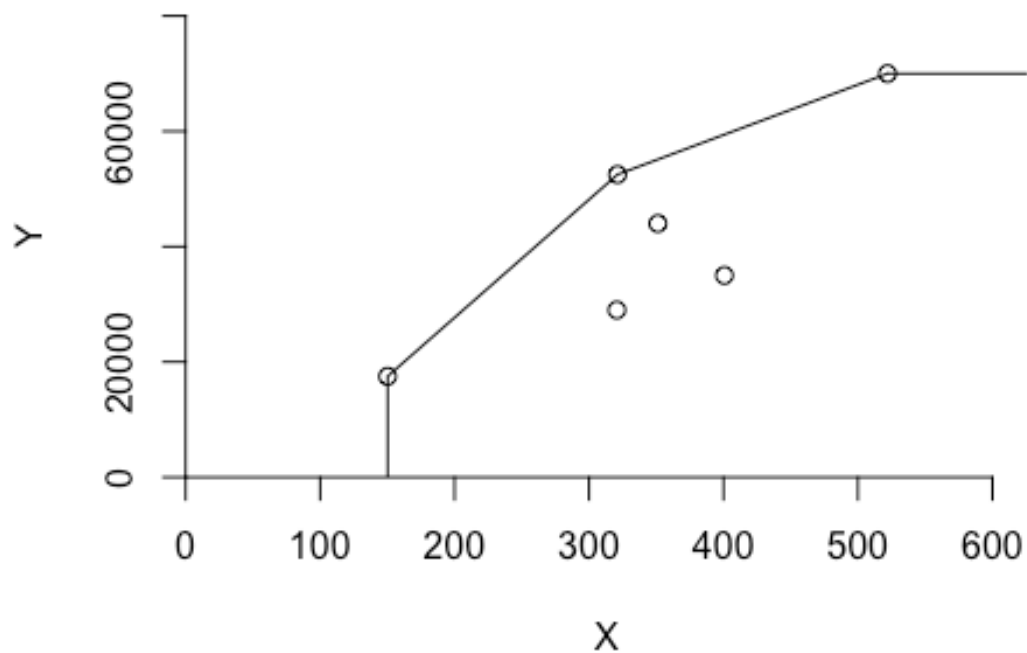
```
peers(VRS)
```

```
##      peer1 peer2 peer3
## [1,]      1    NA    NA
## [2,]      2    NA    NA
## [3,]      3    NA    NA
## [4,]      4    NA    NA
## [5,]      5    NA    NA
## [6,]      1     2     5
```

```
lambda(VRS)
```

```
##      L1      L2 L3 L4      L5
## [1,] 1.0000000 0.0000000 0 0 0.0000000
## [2,] 0.0000000 1.0000000 0 0 0.0000000
## [3,] 0.0000000 0.0000000 1 0 0.0000000
## [4,] 0.0000000 0.0000000 0 1 0.0000000
## [5,] 0.0000000 0.0000000 0 0 1.0000000
## [6,] 0.4014399 0.3422606 0 0 0.2562995
```

```
dea.plot.frontier(X,Y,RTS="VRS")
```



```
summary(VRS, digits=4)
```

```
## Summary of efficiencies
## VRS technology and input orientated efficiency
## Number of firms with efficiency==1 are 5 out of 6
## Mean efficiency: 0.983
## ---
##   Eff range      #   %
##   0.8<= E <0.9    1 17
##   0.9<= E <1      0  0
##           E ==1    5 83
##   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
## 0.8963  1.0000  1.0000  0.9827  1.0000  1.0000
```

### *#IRS Assumptions*

```
IRS <- dea (X,Y, RTS = "irs")
IRS
```

```
## [1] 1.0000 1.0000 1.0000 1.0000 1.0000 0.8963
```

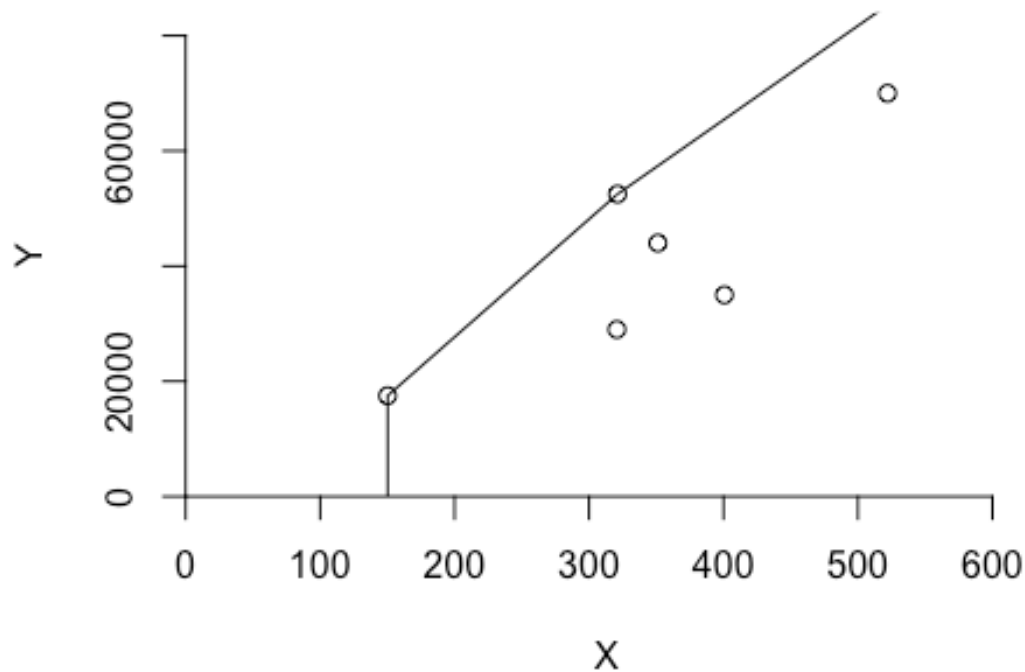
```
peers(IRS)
```

```
##      peer1 peer2 peer3
## [1,]      1    NA    NA
## [2,]      2    NA    NA
## [3,]      3    NA    NA
## [4,]      4    NA    NA
## [5,]      5    NA    NA
## [6,]      1     2     5
```

```
lambda(IRS)
```

```
##           L1           L2 L3 L4           L5
## [1,] 1.0000000 0.0000000  0  0 0.0000000
## [2,] 0.0000000 1.0000000  0  0 0.0000000
## [3,] 0.0000000 0.0000000  1  0 0.0000000
## [4,] 0.0000000 0.0000000  0  1 0.0000000
## [5,] 0.0000000 0.0000000  0  0 1.0000000
## [6,] 0.4014399 0.3422606  0  0 0.2562995
```

```
dea.plot.frontier(X,Y,RTS="IRS")
```



```
summary(IRS, digits=4)
```

```
## Summary of efficiencies
## IRS technology and input orientated efficiency
## Number of firms with efficiency==1 are 5 out of 6
## Mean efficiency: 0.983
## ---
```

```
##   Eff range      # %
##   0.8<= E <0.9   1 17
##   0.9<= E <1     0  0
##           E ==1   5 83
##   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##  0.8963  1.0000  1.0000  0.9827  1.0000  1.0000
```

```
#DRS Assumptions
```

```
DRS <- dea (X,Y, RTS = "drs")
VRS
```

```
## [1] 1.0000 1.0000 1.0000 1.0000 1.0000 0.8963
```

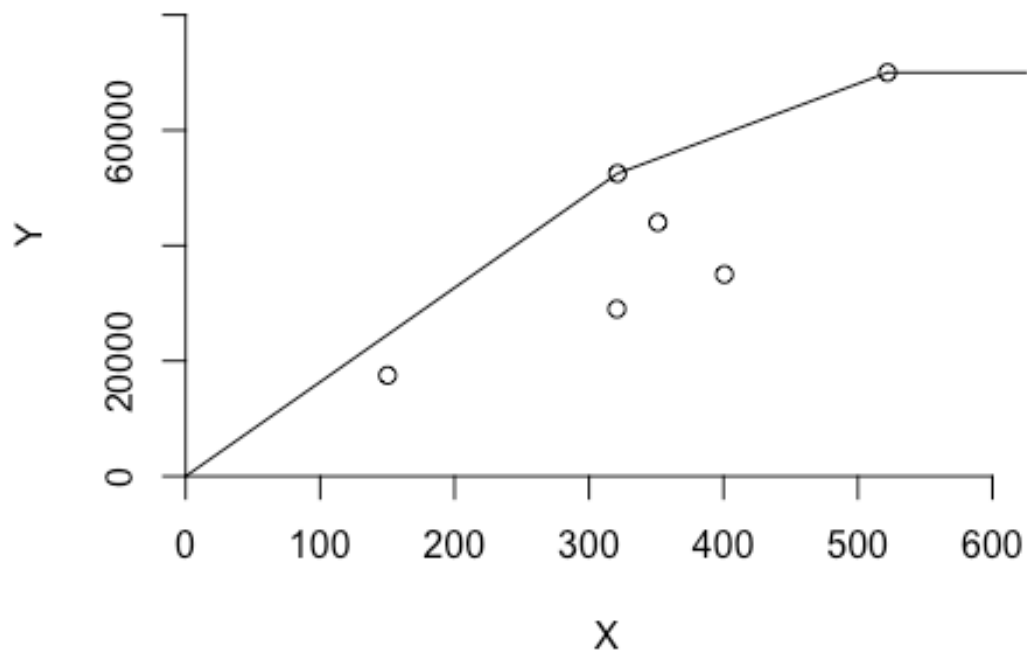
```
peers(DRS)
```

```
##      peer1 peer2 peer3
## [1,]     1    NA    NA
## [2,]     2    NA    NA
## [3,]     3    NA    NA
## [4,]     4    NA    NA
## [5,]     1     2     4
## [6,]     1     2     4
```

```
lambda(DRS)
```

```
##      L1      L2 L3      L4
## [1,] 1.0000000 0.0000000 0 0.0000000
## [2,] 0.0000000 1.0000000 0 0.0000000
## [3,] 0.0000000 0.0000000 1 0.0000000
## [4,] 0.0000000 0.0000000 0 1.0000000
## [5,] 0.2000000 0.08048142 0 0.5383307
## [6,] 0.3428571 0.39499264 0 0.1310751
```

```
dea.plot.frontier(X,Y,RTS="DRS")
```



```
summary(DRS, digits=4)
```

```
## Summary of efficiencies
## DRS technology and input orientated efficiency
```



```
## Number of firms with efficiency==1 are 4 out of 6
## Mean efficiency: 0.974
## ---
##   Eff range      #   %
##   0.8<= E <0.9   1  17
##   0.9<= E <1     1  17
##           E ==1   4  67
##   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##  0.8675  0.9831  1.0000   0.9742  1.0000   1.0000
```

### *#CRS Assumptions*

```
CRS <- dea (X,Y, RTS = "crs")
CRS
```

```
## [1] 1.0000 1.0000 1.0000 1.0000 0.9775 0.8675
```

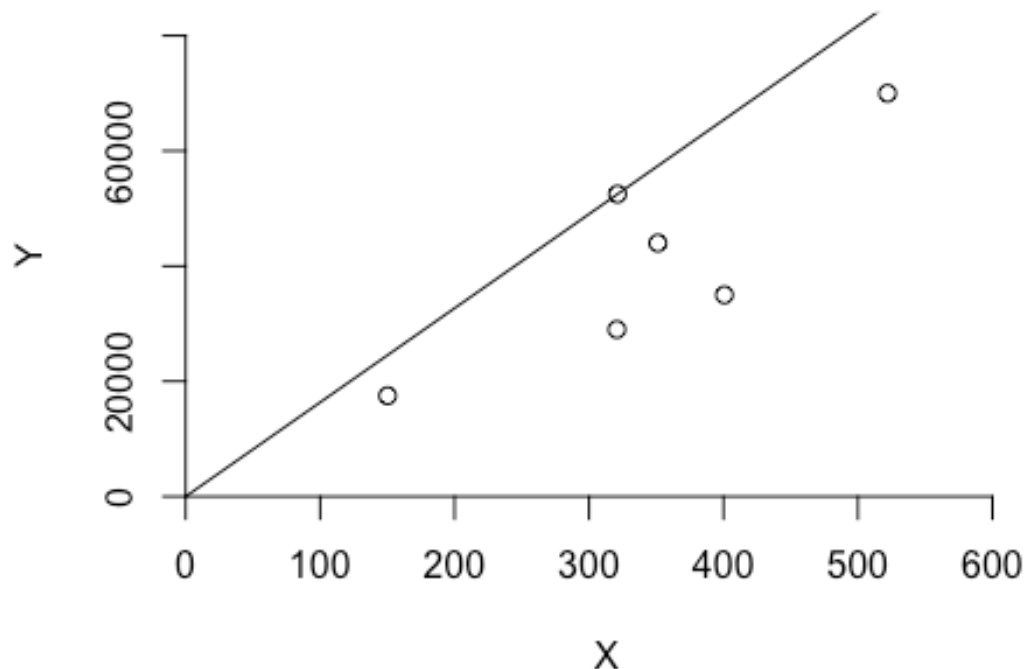
```
peers(CRS)
```

```
##      peer1 peer2 peer3
## [1,]     1    NA    NA
## [2,]     2    NA    NA
## [3,]     3    NA    NA
## [4,]     4    NA    NA
## [5,]     1     2     4
## [6,]     1     2     4
```

```
lambda(CRS)
```

```
##           L1           L2 L3           L4
## [1,] 1.0000000 0.0000000  0 0.0000000
## [2,] 0.0000000 1.0000000  0 0.0000000
## [3,] 0.0000000 0.0000000  1 0.0000000
## [4,] 0.0000000 0.0000000  0 1.0000000
## [5,] 0.2000000 0.08048142  0 0.5383307
## [6,] 0.3428571 0.39499264  0 0.1310751
```

```
dea.plot.frontier(X,Y,RTS="CRS")
```



```
summary(CRS, digits=4)

## Summary of efficiencies
## CRS technology and input orientated efficiency
## Number of firms with efficiency==1 are 4 out of 6
## Mean efficiency: 0.974
## ---
##   Eff range      #   %
##   0.8<= E <0.9    1  17
##   0.9<= E <1      1  17
##           E ==1    4  67
##   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##  0.8675  0.9831  1.0000  0.9742  1.0000  1.0000
```

#Express management's objective function in terms of  $x_1, x_2, x_3, y_1^+, y_1^-, y_2^+$  and  $y_2^-$ .  
Formulate and solve the linear programming model. What are your findings?

```
library(lpSolve)
library(lpSolveAPI)
lprec <- read.lp("QMM Assignment #5.lp")
solve(lprec)

## [1] 0
```

```

get.objective(lprec)
## [1] 225
get.variables(lprec)
## [1] 0 0 15 25 0 0 0
get.constraints(lprec)
## [1] 50 75
X <- "QMM Assignment #5.lp"

```

#Solving the lp, it can be found that 225 million is the maximum profit.

```

library(lpSolve)
library(lpSolveAPI)
lprec <- read.lp("QMMnumber2.lp")
solve(lprec)

## [1] 0
get.objective(lprec)
## [1] 208.3333
get.variables(lprec)
## [1] 0.000000 8.333333 3.333333 0.000000 0.000000 0.000000 0.000000
get.constraints(lprec)
## [1] 50 75
Y <- "QMMnumber2.lp"

```

**The optimal solution is 208.333 for maximum profit. I that suggest the EMAX produces 0 from product 1, 8 from product 2, 3 from product 3.**