

STA303 Assignment 2

Talia Fabregas

Kasandra Tworzynski

2025-03-28

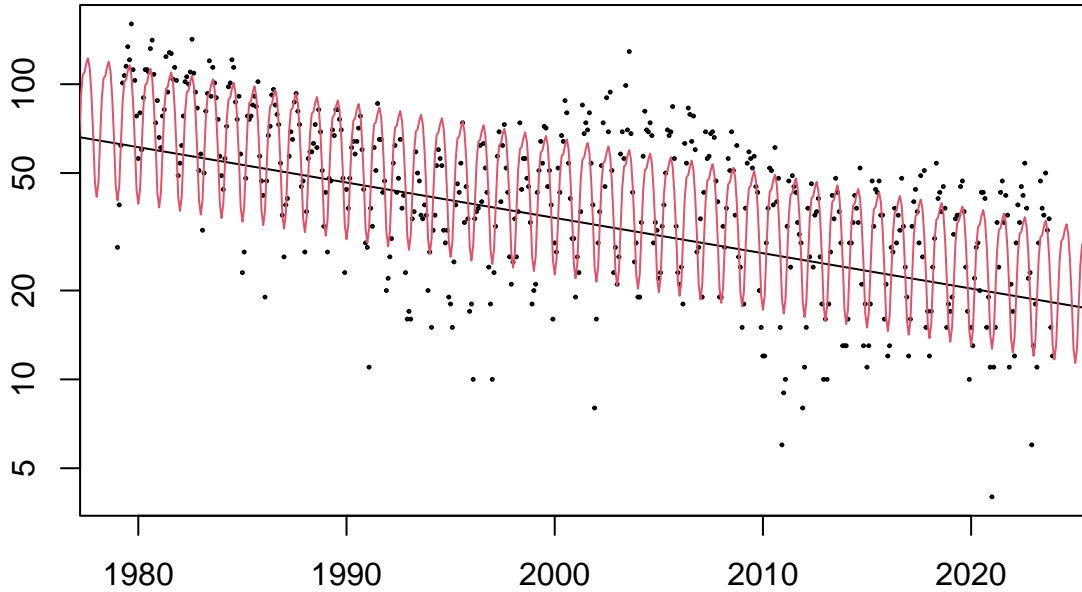


Figure 1: Provided on the assignment 2 handout

Question 1: Motorcycle Accidents

Part 1

Write down, in equations not R code, a generalized additive model suitable for this problem. Explain each of the parts of the model and give a rationale for them (i.e. “The response variable is Gamma distributed because the number of deaths must be positive”). (4 points)

A generalized additive model (GAM) suited for this problem is the Negative Binomial GAM. We chose a Negative Binomial because our response variable, Y_i (number of motorcycle deaths in month i) is a non-negative count variable. Unlike a Poisson, a Negative Binomial can account for over-dispersion (flexible enough to account for high or very low over-dispersion).

The GAM that we will use for this problem is as follows:

$$Y_i \sim \text{NegBinom}(D_i\mu_i, \tau)$$

where:

Part 2

Show R code which fits this model using the mgcv package. (2 points)

```
# fit the model
motorcycleGAM <- mgcv::gam(killed ~ month + offset(logMonthDays) + s(dateInt, k=50),
  data=x,
  family=nb,
  method='ML'
)

# show the formula (just for reference)
motorcycleGAM$formula

## killed ~ month + offset(logMonthDays) + s(dateInt, k = 50)
```

Part 3

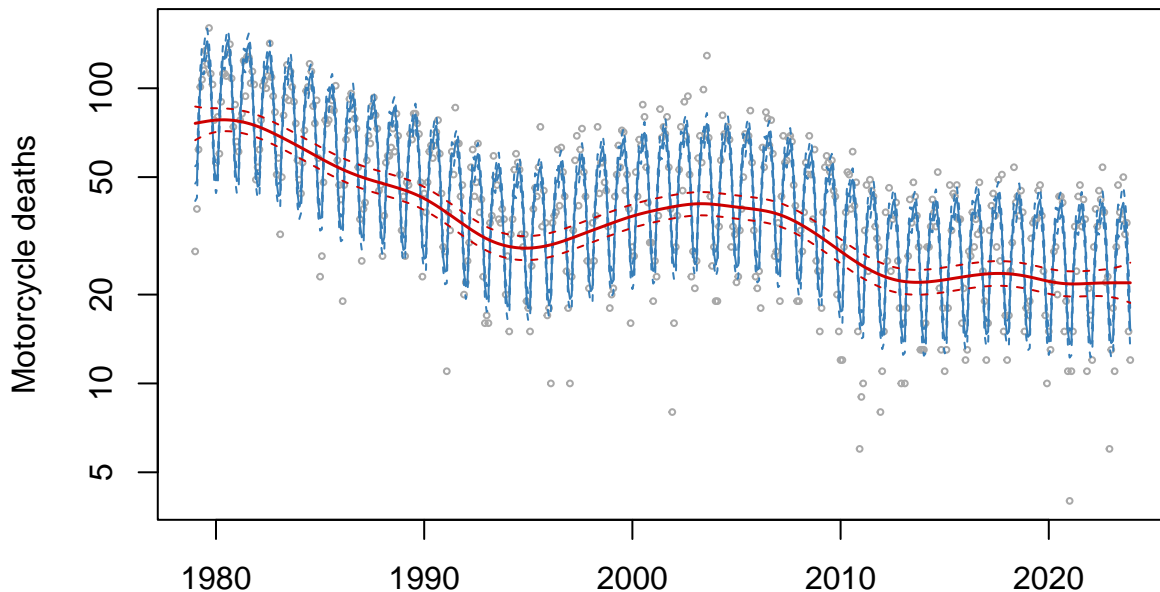


Figure 2: The figure displays motorcycle death data over time (dark gray points) with a log scale on the y-axis. The blue lines represent the seasonal effect, with the 95% prediction interval. The red lines illustrate the trend over time, with a 95% prediction interval shown by the red dotted lines. The data suggest a seasonal fluctuation in motorcycle deaths, along with a clear trend over time, as modeled by a generalized additive model (GAM)

Question 2: Heat

The formulas for `res1`, `res2`, `res3` :

```
## Max.Temp ~ s(dateInt, pc = as.integer(as.Date("1990/7/1")), k = 100) +
##   s(yearFac, bs = "re") + sinpi(dateInt/182.625) + cospi(dateInt/182.625) +
##   sinpi(dateInt/91.3125) + cospi(dateInt/91.3125)

## Max.Temp ~ s(dateInt, pc = as.integer(as.Date("1990/7/1")), k = 4) +
##   s(yearFac, bs = "re") + sinpi(dateInt/182.625) + cospi(dateInt/182.625) +
##   sinpi(dateInt/91.3125) + cospi(dateInt/91.3125)
```

```
## Max.Temp ~ s(dateInt, pc = as.integer(as.Date("1990/7/1")), k = 100) +
##      sinpi(dateInt/182.625) + cospi(dateInt/182.625) + sinpi(dateInt/91.3125) +
##      cospi(dateInt/91.3125)
```

Part 1

Equations for `res1`

$$Y_i \sim \mathcal{N}(\mu_i, \sigma^2) \mu_i = \beta_0 + \beta_1 \cdot \cos\left(\frac{2\pi \text{dateInt}_i}{365.25}\right) + \beta_2 \cdot \sin\left(\frac{2\pi \text{dateInt}_i}{365.25}\right) + \beta_3 \cdot \cos\left(\frac{2\pi \text{dateInt}_i}{182.625}\right) + \beta_4 \cdot \sin\left(\frac{2\pi \text{dateInt}_i}{182.625}\right) + f_1(\text{dateInt}_i) + f_2(\text{yearFac}_i)$$

where:

- Y_i is the Max Temp recorded in month i
- β_1, β_2 are the coefficients of the cosine and sine terms of the 12-month cycle
- β_3 and β_4 are the coefficients of the cosine and sine terms of the 6-month cycle
- f_1 is the smooth trend over time (`dateInt`), modeled using a spline with $k=100$ knots
- f_2 is the year random effect which accounts for variability between years. The use of `bs = "re"` in the code tells `mgcv::gam` to treat it as a random effect, and not a smoothing term.

Equations for `res2`

$$Y_i \sim \mathcal{N}(\mu_i, \sigma^2)$$

$$\mu_i = \beta_0 + \beta_1 \cdot \cos\left(\frac{2\pi \text{dateInt}_i}{365.25}\right) + \beta_2 \cdot \sin\left(\frac{2\pi \text{dateInt}_i}{365.25}\right) + \beta_3 \cdot \cos\left(\frac{2\pi \text{dateInt}_i}{182.625}\right) + \beta_4 \cdot \sin\left(\frac{2\pi \text{dateInt}_i}{182.625}\right) + f_1(\text{dateInt}_i) + f_2(\text{yearFac}_i)$$

where:

- Y_i is the Max Temp recorded in month i
- β_1, β_2 are the coefficients of the cosine and sine terms of the 12-month cycle
- β_3 and β_4 are the coefficients of the cosine and sine terms of the 6-month cycle
- `f_1` is the smooth trend over time (`dateInt`), modeled using a spline with $k=4$ knots. The number of knots, k is the key difference between ``res1`` and ``res2``
- f_2 is the year random effect which accounts for variability between years. The use of `bs = "re"` in the code tells `mgcv::gam` to treat it as a random effect, and not a smoothing term.

Equations for `res3`

$$Y_i \sim \mathcal{N}(\mu_i, \sigma^2)$$

$$\mu_i = \beta_0 + \beta_1 \cdot \cos\left(\frac{2\pi \text{dateInt}_i}{365.25}\right) + \beta_2 \cdot \sin\left(\frac{2\pi \text{dateInt}_i}{365.25}\right) + \beta_3 \cdot \cos\left(\frac{2\pi \text{dateInt}_i}{182.625}\right) + \beta_4 \cdot \sin\left(\frac{2\pi \text{dateInt}_i}{182.625}\right) + f(\text{dateInt}_i)$$

where:

- Y_i is the Max Temp recorded in month i
- β_1, β_2 are the coefficients of the cosine and sine terms of the 12-month cycle
- β_3 and β_4 are the coefficients of the cosine and sine terms of the 6-month cycle
- f is the smooth trend over time (`dateInt`), modeled using a spline with $k=100$ knots.

Differences between the three models The models defined in `res1`, `res2`, and `res3` have a couple of key differences. Firstly, the models in `res1` and `res2` have one key difference: the number of knots, k . The `res1` model uses $k = 100$ knots which allows for a more detailed and flexible smoothing function, whereas the `res2` model uses $k = 4$ knots which allows for a “smoother”, less detailed smoothing function. Larger k is generally better because it fits the data better; the model in `res2` with $k = 4$ probably has too few knots to fit the data well or capture details. The model in `res3` has $k = 100$ knots just like the model in `res1`, but it omits the `yearFac` random effect and only includes a smooth term for `dateInt`. The `res3` model is the simplest but it does not capture unobserved temperature variability between years.

Part 2

Part 3

Part 4