```
\leavevmode
\newline \\
case 1
\newline \\
$\ket{\Psi}_{0}=(x_{0}\ket{0}+y_{0}\ket{1})_{A}\otimes ((a_{0})^{00}\ket{00}+(a_
\newline \\
$\ket{\Psi}_{1}=(x_{1}\ket{0}+y_{1}\ket{1})_{A}\otimes ((a_{1})^{00}\ket{00}+(a_
\newline \\
$\ket{\Psi}_{2}=(x_{2}\ket{0}+y_{2}\ket{1})_{A}\otimes ((a_{2})^{00}\ket{00}+(a_
\newline \\
$\ket{\Psi}_{3}=(x_{3}\ket{0}+y_{3}\ket{1})_{A}\otimes((a_{3})^{00}\ket{00}+(a_{
\newline \\
$\ket{\Psi}_{4}=\ket{0}_{B}\otimes(a_{4}^{00}\ket{00}+a_{4}^{11}\ket{11})_{AC}$
```

where $|x_i|^2+|y_i|^2=1$, $i=0,1,2,3,$,

$|(a_i)^{00}|^2+|(a_i)^{01}|^2+|(a_i)^{10}|^2+|(a_i)^{11}|^2=|a|^2+|b|^2=1,$, $i=0,1,2,3,$,

suppose above 5 vectors are pairly orthogonal,then the relations between the coefficients :

$\langle\Psi_i|\,|\Psi_j\rangle=0$ where $i\neq j$, $i=0,1,2,3,j=0,1,2,3,$

$x_i=-y_j$ where $i\neq j$, $i=0,1,2,3,j=0,1,2,3,$

or

$(a_i)^{00}(a_j)^{00}+(a_i)^{01}(a_j)^{01}+(a_i)^{10}(a_j)^{10}+(a_i)^{11}(a_j)^{11}=0,$
from the orthogonal conditions we could get:

$\langle\Psi|_0\,|\Psi_i\rangle=0$ i=0,1,2,3

$$\langle\Psi|_0\,|\Psi\rangle_i=0 \quad\Rightarrow\begin{cases} x_0^*x_1=-y_0^*y_1 \\ x_0^*x_2=-y_0^*y_2 \\ \quad x_0^*x_3=-y_0^*y_3 \qquad\quad , \\ a_0^{00}a_1^{00}+a_0^{01}a_1^{01}+a_0^{10}a_1^{10}+a_0^{11}a_1^{11}=0 \\ a_0^{00}a_2^{00}+a_0^{01}a_2^{01}+a_0^{10}a_2^{10}+a_0^{11}a_2^{11}=0 \\ a_0^{00}a_3^{00}+a_0^{01}a_3^{01}+a_0^{10}a_3^{10}+a_0^{11}a_3^{11}=0 \end{cases} \tag{1}$$

similarily,

$$\begin{cases} \langle\Psi|_1 |\Psi\rangle_2 = 0, \\ \quad \langle\Psi|_1 |\Psi\rangle_3 = 0 \\ \langle\Psi|_2 |\Psi\rangle_3 = 0 \end{cases} \Rightarrow \begin{cases} x_1^* x_2 = -y_1^* y_2 \\ x_1^* x_3 = -y_1^* y_2 \\ a_1^{00} a_2^{00} + a_1^{01} a_2^{01} + a_1^{10} a_2^{10} + a_1^{11} a_2^{11} = 0 \\ a_1^{00} a_3^{00} + a_1^{01} a_3^{01} + a_1^{10} a_1^{10} + a_1^{11} a_3^{11} = 0 \\ a_2^{00} a_3^{00} + a_2^{01} a_3^{01} + a_2^{10} a_3^{10} + a_2^{11} a_3^{11} = 0 \end{cases} \tag{2}$$

$$\langle\Psi|_4 |\Psi\rangle_i = 0 \quad \Rightarrow \begin{cases} x_0^* a_0^{00} a_4^{00} + y_0^* a_0^{01} a_4^{11} = 0 \\ x_1^* a_1^{00} a_4^{00} + y_1^* a_1^{01} a_4^{11} = 0 \\ x_2^* a_2^{00} a_4^{00} + y_2^* a_2^{01} a_4^{11} = 0 \\ x_3^* a_3^{00} a_4^{00} + y_3^* a_3^{01} a_4^{11} = 0 \end{cases} \tag{3}$$

$\langle\Psi_i| |\Psi_4\rangle = 0$ where $, i = 0, 1, 2, 3,$

$|\Psi\rangle_i = x_i(a_i)^{00} |000\rangle + x_i(a_i)^{01} |001\rangle + x_i(a_i)^{10} |100\rangle + x_i(a_i)^{11} |011\rangle + y_i(a_i)^{00} |100\rangle + y_i(a_i)^{01} |101\rangle + y_i(a_i)^{10} |110\rangle + y_i(a_i)^{11} |111\rangle$

$|\Psi\rangle_4 = a|000\rangle + b|101\rangle$

for orthogonality:

$ax_i(a_i)^{00} + by_i(a_i)^{01} = 0$

Finally we get following genreal result:

$$A = \begin{cases} x_i = -y_j & i \neq j, i = 0, 1, 2, 3, j = 0, 1, 2, 3 \\ ax_i(a_i)^{00} + by_i(a_i)^{01} = 0 \end{cases} \tag{4}$$

$$B = \begin{cases} (a_i)^{00}(a_j)^{00} + (a_i)^{01}(a_j)^{01} + (a_i)^{10}(a_j)^{10} + (a_i)^{11}(a_j)^{11} = 0 & i \neq j, i = 0,1,2,3, j = 0,1,2,3 \\ ax_i(a_i)^{00} + by_i(a_i)^{01} = 0 \end{cases} \tag{5}$$

we have infinite solutions in case A of orthogonality,but we have to prove is there a vector $|\zeta\rangle$ which is let (1) ,(2) dont have any solution .

(i) : Assume we have a vector $|\zeta\rangle = (x_4 |0\rangle + y_4 |1\rangle)_A \otimes ((a_4)^{00} |00\rangle + (a_4)^{01} |01\rangle + (a_4)^{10} |10\rangle + (a_4)^{11} |11\rangle)_{BC}$, ($A|BC$cut)

we get same result :

$$C = \begin{cases} x_i = -y_j & i \neq j, i = 0,1,2,3,4. \; j = 0,1,2,3,4 \\ ax_i(a_i)^{00} + by_i(a_i)^{01} = 0 \end{cases} \tag{6}$$

$$D = \begin{cases} (a_i)^{00}(a_j)^{00} + (a_i)^{01}(a_j)^{01} + (a_i)^{10}(a_j)^{10} + (a_i)^{11}(a_j)^{11} = 0 & i \neq j, i = 0,1,2,3,4 \; ,j = 0,1,2 \\ ax_i(a_i)^{00} + by_i(a_i)^{01} = 0 \end{cases} \tag{7}$$

easy to see that (3),(4)also have infinite solutions.

(ii) :Assume we have a vector $|\zeta\rangle = (x_4 |0\rangle + y_4 |1\rangle)_B \otimes ((a_4)^{00} |00\rangle + (a_4)^{01} |01\rangle + (a_4)^{10} |10\rangle + (a_4)^{11} |11\rangle)_{AC}$, ($B|AC$ cut)

In $\langle\zeta| |\Psi\rangle_i$, $i = 0,1,2,3$.

$|\Psi\rangle_i = x_i(a_i)^{00} |000\rangle + x_i(a_i)^{01} |001\rangle + x_i(a_i)^{10} |010\rangle + x_i(a_i)^{11} |011\rangle + y_i(a_i)^{00} |100\rangle + y_i(a_i)^{01} |101\rangle + y_i(a_i)^{10} |110\rangle + y_i(a_i)^{11} |111\rangle$, i=0,1,2,3

$|\zeta\rangle = x_4(a_4)^{00} |000\rangle + x_4(a_4)^{01} |001\rangle + x_4(a_4)^{10} |100\rangle + x_4(a_4)^{11} |101\rangle + y_4(a_4)^{00} |010\rangle + y_4(a_4)^{01} |011\rangle + y_4(a_4)^{10} |110\rangle + y_4(a_4)^{11} |111\rangle$

we will get

$x_i(a_i)^{00}x_4(a_4)^{00}+x_i(a_i)^{01}x_4(a_4)^{01}+x_i(a_i)^{10}y_4(a_4)^{00}+x_i(a_i)^{11}y_4(a_4)^{01}+y_i(a_i)^{00}y_4(a_i)^{01}+y_i(a_i)^{01}x_4(a_4)^{11}+y_4(a_4)^{10}y_i(a_i)^{10}+y_i(a_i)^{11}y_4(a_4)^{11}=0$, $i=0,1,2,3$.(5)

$\langle\zeta|\,|\Psi_4\rangle$

$|\Psi\rangle_4 = a\,|000\rangle + b\,|101\rangle$

$|\zeta\rangle = x_4(a_4)^{00}\,|000\rangle+x_4(a_4)^{01}\,|001\rangle+x_4(a_4)^{10}\,|100\rangle+x_4(a_4)^{11}\,|101\rangle+y_4(a_4)^{00}\,|010\rangle+y_4(a_4)^{01}\,|011\rangle+y_4(a_4)^{10}\,|110\rangle+y_4(a_4)^{11}\,|111\rangle$

$x_4(a_4)^{00}a+bx_4(a_4)^{11}=0$, (6)

$$wehave,\begin{cases} x_i(a_i)^{00}x_4(a_4)^{00}+x_i(a_i)^{01} \\ x_4(a_4)^{01}+x_i(a_i)^{10}y_4(a_4)^{00}+x_i(a_i)^{11}y_4(a_4)^{01}+y_i(a_i)^{00}y_4(a_i)^{01} \\ +y_i(a_i)^{01}x_4(a_4)^{11}+y_4(a_4)^{10}y_i(a_i)^{10}+y_i(a_i)^{11}y_4(a_4)^{11}=0 & ,i=0,1,2,3.(5) \\ x_4(a_4)^{00}a+bx_4(a_4)^{11}=0,(6) \end{cases} \tag{8}$$

(iii) :Assume we have a vector $|\zeta\rangle = (x_5\,|0\rangle + y_5\,|1\rangle)_C \otimes ((a_5)^{00}\,|00\rangle + (a_5)^{01}\,|01\rangle + (a_5)^{10}\,|10\rangle + (a_5)^{11}\,|11\rangle)_{AB}$, ($C|AB$ cut)

$|\zeta\rangle = x_5(a_5)^{00}\,|000\rangle+x_5(a_5)^{01}\,|010\rangle+x_5(a_5)^{10}\,|100\rangle+x_5(a_5)^{11}\,|110\rangle+y_5(a_5)^{00}\,|001\rangle+y_5(a_5)^{01}\,|011\rangle+y_5(a_5)^{10}\,|101\rangle+y_5(a_5)^{11}\,|111\rangle$

$|\Psi\rangle_i = x_i(a_i)^{00}\,|000\rangle+x_i(a_i)^{01}\,|001\rangle+x_i(a_i)^{10}\,|010\rangle+x_i(a_i)^{11}\,|011\rangle+y_i(a_i)^{00}\,|100\rangle+y_i(a_i)^{01}\,|101\rangle+y_i(a_i)^{10}\,|110\rangle+y_i(a_i)^{11}\,|111\rangle$, i=0,1,2,3

In $\langle\zeta|\,|\Psi\rangle_i$, $i=0,1,2,3.$

$x_i(a_i)^{00}x_5(a_5)^{00}+x_i(a_i)^{10}x_5(a_5)^{01}+x_5(a_5)^{10}y_i(a_i)^{00}+x_5(a_5)^{11}y_i(a_i)^{10}+y_5(a_5)^{00}x_i(a_i)^{01}+y_5(a_5)^{01}x_i(a_i)^{11}+y_5(a_5)^{10}y_i(a_i)^{01}+y_5(a_5)^{11}y_i(a_i)^{11}=0$, (7)

$\langle\zeta|\,|\Psi_4\rangle$

$|\Psi\rangle_4 = a\,|000\rangle + b\,|101\rangle$

$|\zeta\rangle = x_5(a_5)^{00}|000\rangle + x_5(a_5)^{01}|010\rangle + x_5(a_5)^{10}|100\rangle + x_5(a_5)^{11}|110\rangle + y_5(a_5)^{00}|001\rangle + y_5(a_5)^{01}|011\rangle + y_5(a_5)^{10}|101\rangle + y_5(a_5)^{11}|111\rangle$

$x_5(a_5)^{00}a + by_5(a_5)^{10} = 0$, (8)

we could get

$$wehave \begin{cases} x_i(a_i)^{00}x_5(a_5)^{00} + x_i(a_i)^{10}x_5(a_5)^{01} + x_5(a_5)^{10}y_i(a_i)^{00} + \\ x_5(a_5)^{11}y_i(a_i)^{10} + y_5(a_5)^{00}x_i(a_i)^{01} + y_5(a_5)^{01}x_i(a_i)^{11} + \\ y_5(a_5)^{10}y_i(a_i)^{01} + y_5(a_5)^{11}y_i(a_i)^{11} = 0 & i = 0,1,2,3(7) \\ x_5(a_5)^{00}a + by_5(a_5)^{10} = 0 \ , (8) \end{cases} \tag{9}$$

case 2.

$|\Psi\rangle_i = (x_i|0\rangle + y_i|1\rangle)_A \otimes ((a_i)^{00}|00\rangle + (a_i)^{01}|01\rangle + (a_i)^{10}|10\rangle + (a_i)^{11}|11\rangle)_{BC}$
$i = 0,1,2,$

$|\Psi\rangle_3 = (x_3|0\rangle + y_3|1\rangle)_B \otimes ((a_3)^{00}|00\rangle + (a_3)^{01}|01\rangle + (a_3)^{10}|10\rangle + (a_3)^{11}|11\rangle)_{AC},$

$|\Psi\rangle_4 = x_4|0\rangle_B \otimes ((a_3)^{00}|00\rangle + (a_3)^{01}|01\rangle + (a_3)^{10}|10\rangle + (a_3)^{11}|11\rangle)_{AC},$
where $|x_i|^2 + |y_i|^2 = 1$, $i = 0,1,2,3,,$

$|(a_i)^{00}|^2 + |(a_i)^{01}|^2 + |(a_i)^{10}|^2 + |(a_i)^{11}|^2 = |a|^2 + |b|^2 = 1,$, $i = 0,1,2,3,,$

suppose above 5 vectors are pairly orthogonal,then the relations between the coefficients :

$\langle\Psi_i|\,|\Psi_j\rangle = 0$ where $i \neq j$, $i = 0,1,2,3, j = 0,1,2,3,$

$x_i = -y_j$ where $i \neq j$, $i = 0,1,2 j = 0,1,2,$

or

$(a_i)^{00}(a_j)^{00} + (a_i)^{01}(a_j)^{01} + (a_i)^{10}(a_j)^{10} + (a_i)^{11}(a_j)^{11} = 0,$

$\langle\Psi_i|\,|\Psi_3\rangle = 0$ where $, i = 0, 1, 2$

$|\Psi\rangle_i = x_i(a_i)^{00}\,|000\rangle + x_i(a_i)^{01}\,|001\rangle + x_i(a_i)^{10}\,|100\rangle + x_i(a_i)^{11}\,|011\rangle + y_i(a_i)^{00}\,|100\rangle + y_i(a_i)^{01}\,|101\rangle + y_i(a_i)^{10}\,|110\rangle + y_i(a_i)^{11}\,|111\rangle$

$\langle\Psi_i|\,|\Psi_4\rangle = 0$ where $, i = 0, 1, 2$

$|\Psi\rangle_3 = x_3(a_3)^{00}\,|000\rangle_{ABC} + x_3(a_3)^{01}\,|001\rangle_{ABC} + x_3(a_3)^{10}\,|100\rangle_{ABC} + x_3(a_3)^{11}\,|101\rangle_{ABC} + y_3(a_3)^{00}\,|010\rangle_{ABC} + y_3(a_3)^{01}\,|011\rangle_{ABC} + y_3(a_3)^{10}\,|110\rangle_{ABC} + y_3(a_3)^{11}\,|111\rangle_{ABC}$

from orthognality ,we could get

$x_i(a_i)^{00}x_3(a_3)^{00} + x_i(a_i)^{01}x_3(a_3)^{01} + x_i(a_i)^{10}x_3(a_3)^{10} + x_i(a_i)^{11}x_3(a_3)^{11} + y_i(a_i)^{00}y_3(a_3)^{00} + y_i(a_i)^{01}y_3(a_3)^{01} + y_i(a_i)^{10}y_3(a_3)^{10} + y_i(a_i)^{11}y_3(a_3)^{11} = 0$ where i=0,1,2

$|\Psi\rangle_i = x_i(a_i)^{00}\,|000\rangle + x_i(a_i)^{01}\,|001\rangle + x_i(a_i)^{10}\,|100\rangle + x_i(a_i)^{11}\,|011\rangle + y_i(a_i)^{00}\,|100\rangle + y_i(a_i)^{01}\,|101\rangle + y_i(a_i)^{10}\,|110\rangle + y_i(a_i)^{11}\,|111\rangle$

$|\Psi\rangle_4 = x_4(a_4)^{00}\,|000\rangle + y_4(a_4)^{01}\,|111\rangle$

for orthogonality:

$ax_i(a_i)^{00}x_4(a_4)^{00} + y_i(a_i)^{11}y_4(a_4)^{01} = 0$

$\langle\Psi_3|\,|\Psi_4\rangle = 0$

$x_3a_3^{00}x_4a_4^{00} + x_3a_3^{11}x_4a_4^{11} = 0$

Finally we get following genreal result:

$$E=\begin{cases} x_i=-y_j & i\neq j, i=0,1,2j=0,1,2 \\ ax_i(a_i)^{00}x_4(a_4)^{00}+y_i(a_i)^{11}y_4(a_4)^{01}=0 \\ x_i(a_i)^{00}x_3(a_3)^{00}+x_i(a_i)^{01}x_3(a_3)^{01}+x_i(a_i)^{10}x_3 \\ (a_3)^{10}+x_i(a_i)^{11}x_3(a_3)^{11}+y_i(a_i)^{00}y_3(a_3)^{00}+ \\ y_i(a_i)^{01}y_3(a_3)^{01}+y_i(a_i)^{10}y_3(a_3)^{10}+y_i(a_i)^{11}y_3 \\ (a_3)^{11}=0 where i=0,1,2 \\ x_3a_3^{00}x_4a_4^{00}+x_3a_3^{11}x_4a_4^{11}=0 \end{cases} \tag{10}$$

$$F=\begin{cases} (a_i)^{00}(a_j)^{00}+(a_i)^{01}(a_j)^{01}+(a_i)^{10}(a_j)^{10}+(a_i)^{11} \\ (a_j)^{11}=0 \\ ax_i(a_i)^{00}x_4(a_4)^{00}+y_i(a_i)^{11}y_4(a_4)^{01}=0 \\ x_i(a_i)^{00}x_3(a_3)^{00}+x_i(a_i)^{01}x_3(a_3)^{01}+x_i(a_i)^{10} \\ x_3(a_3)^{10}+x_i(a_i)^{11}x_3(a_3)^{11}+y_i(a_i)^{00}y_3(a_3)^{00}+ \\ y_i(a_i)^{01}y_3(a_3)^{01}+y_i(a_i)^{10}y_3(a_3)^{10}+y_i(a_i)^{11} \\ y_3(a_3)^{11}=0 where i=0,1,2 \\ x_3a_3^{00}x_4a_4^{00}+x_3a_3^{11}x_4a_4^{11}=0 \end{cases} \tag{11}$$

we have infinite solutions in case E of orthogonality,but we have to prove is there a vector $|\zeta\rangle$ which is let (7) ,(8) dont have any solution .

(i) : Assume we have a vector $|\zeta\rangle=(x_5|0\rangle+y_5|1\rangle)_A\otimes((a_5)^{00}|00\rangle+(a_5)^{01}|01\rangle+(a_5)^{10}|10\rangle+(a_5)^{11}|11\rangle)_{BC}$, ($A|BC$cut)

$$\langle\Psi_i|\,|\zeta\rangle = 0$$

$$C = \begin{cases} x_i = -y_5 (y_i = -x_5) & i \neq j, i = 0,1,2.\ j = 0,1,2 \\ \\ x_i(a_i)^{00} x_5(a_5)^{00} + x_i(a_i)^{01} x_5(a_5)^{01} + x_i(a_i)^{10} \\ x_3(a_5)^{10} + x_i(a_i)^{11} x_5(a_5)^{11} + y_i(a_i)^{00} y_5(a_5)^{00} + \\ y_i(a_i)^{01} y_5(a_5)^{01} + y_i(a_i)^{10} y_5(a_5)^{10} + y_i(a_i)^{11} \\ y_5(a_5)^{11} = 0 where i = 0,1,2 \end{cases} \tag{12}$$

$$\langle\Psi_3|\,|\zeta\rangle = 0$$

$$\begin{array}{l} x_3(a_3)^{00} x_5(a_5)^{00} + x_3(a_3)^{01} x_5(a_5)^{01} + x_3(a_3)^{10} \\ x_5(a_5)^{10} + x_3(a_3)^{11} x_5(a_5)^{11} + y_3(a_3)^{00} y_5(a_5)^{00} + \\ y_3(a_3)^{01} y_5(a_5)^{01} + y_3(a_3)^{10} y_5(a_5)^{10} + y_3(a_3)^{11} \\ y_5(a_5)^{11} = 0 \\ where i = 0,1,2. \end{array}$$

$$\langle\Psi_4|\,|\zeta\rangle = 0$$

$$x_4 a_4^{00} x_5 a_5^{00} + x_5 a_5^{01} y_4 a_4^{11} = 0$$

new ,we have to consider folowing system whether have solution or not

$$D = \begin{cases} \\ x_i = -y_5 (y_i = -x_5) \\ x_i(a_i)^{00} x_5(a_5)^{00} + x_i(a_i)^{01} x_5(a_5)^{01} + x_i(a_i)^{10} x_3(a_5)^{10} + x_i(a_i)^{11} x_5(a_5)^{11} + y_i(a_i)^{00} y_5(a_5)^{00} + \\ y_i(a_i)^{01} y_5(a_5)^{01} + y_i(a_i)^{10} y_5(a_5)^{10} + y_i(a_i)^{11} y_5(a_5)^{11} = 0 where i = 0,1,2 \\ x_3(a_3)^{00} x_5(a_5)^{00} + x_3(a_3)^{01} x_5(a_5)^{01} + x_3(a_3)^{10} x_5(a_5)^{10} + x_3(a_3)^{11} x_5(a_5)^{11} + y_3(a_3)^{00} \\ y_5(a_5)^{00} + y_3(a_3)^{01} y_5(a_5)^{01} + y_3(a_3)^{10} y_5(a_5)^{10} + y_3(a_3)^{11} y_5(a_5)^{11} = 0 \\ x_4 a_4^{00} + x_5 a_5^{11} y_4 a_4^{11} = 0 \end{cases} \tag{13}$$

(ii) : Assume we have a vector $|\zeta\rangle = (x_5 |0\rangle + y_5 |1\rangle)_B \otimes ((a_5)^{00} |00\rangle + (a_5)^{01} |01\rangle + (a_5)^{10} |10\rangle + (a_5)^{11} |11\rangle)_{AC}$, ($B|AC$cut)

$\langle\Psi_i|\,|\zeta\rangle = 0$

$x_i(a_i)^{00}x_5(a_5)^{00}+x_i(a_i)^{01}x_5(a_5)^{01}+x_i(a_i)^{10}x_3(a_5)^{10}+x_i(a_i)^{11}x_5(a_5)^{11}+y_i(a_i)^{00}y_5(a_5)^{00}+ y_i(a_i)^{01}y_5(a_5)^{01} + y_i(a_i)^{10}y_5(a_5)^{10} + y_i(a_i)^{11}y_5(a_5)^{11} = 0$

where i=0,1,2

$\langle\Psi_3|\,|\zeta\rangle = 0$

$$C = \begin{cases} x_3 = -y_5(y_3 = -x_5) \\ x_3(a_3)^{00}x_5(a_5)^{00} + x_3(a_3)^{01}x_5(a_5)^{01} + x_3(a_3)^{10}x_3(a_5)^{10} + x_3(a_3)^{11}x_5(a_5)^{11}+ \\ y_3(a_3)^{00}y_5(a_5)^{00} + y_3(a_3)^{01}y_5(a_5)^{01} + y_3(a_3)^{10}y_5(a_5)^{10} + y_3(a_3)^{11}y_5(a_5)^{11} = 0 where i = 0, 1, 2 \end{cases} \tag{14}$$

$\langle\Psi_4|\,|\zeta\rangle = 0$

$x_4a_4^{00}x_5a_5^{00} + x_5a_5^{11}y_4a_4^{11} = 0$

new ,we have to consider folowing system whether have solution or not

$$D = \begin{cases} \\ x_i(a_i)^{00}x_5(a_5)^{00} + x_i(a_i)^{01}x_5(a_5)^{01} + x_i(a_i)^{10}x_3(a_5)^{10} + x_i(a_i)^{11}x_5(a_5)^{11} + y_i(a_i)^{00}y_5(a_5)^{00} \\ +y_i(a_i)^{01}y_5(a_5)^{01} + y_i(a_i)^{10}y_5(a_5)^{10} + y_i(a_i)^{11}y_5(a_5)^{11} = 0.where, i = 0, 1, 2 \\ x_3 = -y_5(y_3 = -x_5) \\ x_3(a_3)^{00}x_5(a_5)^{00} + x_3(a_3)^{01}x_5(a_5)^{01} + x_3(a_3)^{10}x_3 \\ (a_5)^{10} + x_3(a_3)^{11}x_5(a_5)^{11} + y_3(a_3)^{00}y_5(a_5)^{00}+ \\ y_3(a_3)^{01}y_5(a_5)^{01} + y_3(a_3)^{10}y_5(a_5)^{10} + y_3(a_3)^{11}y_5(a_5)^{11} = 0 where i = 0, 1, 2 \\ x_4a_4^{00}x_5a_5^{00} + x_5a_5^{11}y_4a_4^{11} = 0 \end{cases} \tag{15}$$

(iii) : Assume we have a vector $|\zeta\rangle = (x_5\,|0\rangle + y_5\,|1\rangle)_C \otimes ((a_5)^{00}\,|00\rangle + (a_5)^{01}\,|01\rangle + (a_5)^{10}\,|10\rangle + (a_5)^{11}\,|11\rangle)_{AB}$, ($C|AB$cut)

$\langle\Psi_i|\,|\zeta\rangle = 0$

$x_i(a_i)^{00}x_5(a_5)^{00}+x_i(a_i)^{01}y5(a_5)^{00}+x_i(a_i)^{10}x_5(a_5)^{01}+x_i(a_i)^{11}y_5(a_5)^{01}+y_i(a_i)^{00}x_5(a_5)^{10}+ y_i(a_i)^{01}y_5(a_5)^{10}+y_i(a_i)^{10}x_5(a_5)^{11}+y_i(a_i)^{11}y_5(a_5)^{11}=0$

where i=0,1,2

$\langle\Psi_3|\,|\zeta\rangle = 0$

$x_3(a_3)^{00}x_5(a_5)^{00}+y_3(a_3)^{00}x_5(a_5)^{01}+x_3(a_3)^{10}x_5(a_5)^{10}+y_3(a_3)^{10}x_5(a_5)^{11}+ x_3(a_3)^{01}y_5(a_5)^{00}+x_3(a_3)^{11}y_5(a_5)^{10}+y_3a_3^{01}y_5a_5^{01}+y_3(a_3)^{11}y_5(a_5)^{11}=0$. where i=0,1,2

$\langle\Psi_4|\,|\zeta\rangle = 0$

$x_4a_4^{00}x_5a_5^{00}+x_5a_5^{11}y_4a_4^{11}=0$

new ,we have to consider folowing system whether have solution or not

$$D=\begin{cases} \\ x_i(a_i)^{00}x_5(a_5)^{00}+x_i(a_i)^{01}y5(a_5)^{00}+x_i(a_i)^{10}x_5(a_5)^{01}+ \\ x_i(a_i)^{11}y_5(a_5)^{01}+y_i(a_i)^{00}x_5(a_5)^{10}+y_i(a_i)^{01}y_5(a_5)^{10}+y_i(a_i)^{10} \\ x_5(a_5)^{11}+y_i(a_i)^{11}y_5(a_5)^{11}=0 & wherei=0,1, \\ x_3(a_3)^{00}x_5(a_5)^{00}+y_3(a_3)^{00}x_5(a_5)^{01}+x_3(a_3)^{10}x_5(a_5)^{10}+y_3(a_3)^{10}x_5(a_5)^{11}+ \\ x_3(a_3)^{01}y_5(a_5)^{00}+x_3(a_3)^{11}y_5(a_5)^{10}+y_3a_3^{01}y_5a_5^{01}+y_3(a_3)^{11}y_5(a_5)^{11}=0 \\ x_4a_4^{00}x_5a_5^{00}+x_5a_5^{11}y_4a_4^{11}=0 \end{cases} \tag{16}$$

case 3

$|\Psi\rangle_i = (x_i|0\rangle+y_i|1\rangle)_A\otimes((a_i)^{00}|00\rangle+(a_i)^{01}|01\rangle+(a_i)^{10}|10\rangle+(a_i)^{11}|11\rangle)_{BC}$
where i=0,1

$|\Psi\rangle_k = (x_k|0\rangle+y_k|1\rangle)_B\otimes((a_k)^{00}|00\rangle+(a_k)^{01}|01\rangle+(a_k)^{10}|10\rangle+(a_k)^{11}|11\rangle)_{AC}$,
where k=2,3

$|\Psi\rangle_4 = x_4|0\rangle_B\otimes((a_3)^{00}|00\rangle+(a_3)^{01}|01\rangle+(a_3)^{10}|10\rangle+(a_3)^{11}|11\rangle)_{AC}$, where $|x_i|^2 + |y_i|^2 = 1$, $i = 0, 1, 2, 3,$,

$|(a_i)^{00}|^2 + |(a_i)^{01}|^2 + |(a_i)^{10}|^2 + |(a_i)^{11}|^2 = |a|^2 + |b|^2 = 1,$, $i = 0, 1, 2, 3,$,

supposse above 5 vectors are pairly orthogonal,then the relations between the coefficients :

$\langle\Psi_0| |\Psi_1\rangle = 0$

$x_0 = -y_1 (y_0 = -x_1)$

or

$(a_0)^{00}(a_1)^{00} + (a_0)^{01}(a_1)^{01} + (a_0)^{10}(a_1)^{10} + (a_0)^{11}(a_1)^{11} = 0$

$\langle\Psi_i| |\Psi_k\rangle = 0$ where , $i = 0, 1, k = 2, 3$

$|\Psi\rangle_i = x_i(a_i)^{00} |000\rangle + x_i(a_i)^{01} |001\rangle + x_i(a_i)^{10} |100\rangle + x_i(a_i)^{11} |011\rangle + y_i(a_i)^{00} |100\rangle + y_i(a_i)^{01} |101\rangle + y_i(a_i)^{10} |110\rangle + y_i(a_i)^{11} |111\rangle$

$|\Psi\rangle_k = x_k(a_k)^{00} |000\rangle_{ABC} + x_k(a_k)^{01} |001\rangle_{ABC} + x_k(a_k)^{10} |100\rangle_{ABC} + x_k(a_k)^{11} |101\rangle_{ABC} + y_k(a_k)^{00} |010\rangle_{ABC} + y_k(a_k)^{01} |011\rangle_{ABC} + y_k(a_k)^{10} |110\rangle_{ABC} + y_k(a_k)^{11} |111\rangle_{ABC}$

from orthognality ,we could get

$x_i(a_i)^{00}x_k(a_k)^{00} + x_i(a_i)^{01}x_k(a_k)^{01} + x_i(a_i)^{10}x_k(a_k)^{10} + x_i(a_i)^{11}x_k(a_k)^{11} + y_i(a_i)^{00}y_k(a_k)^{00} + y_i(a_i)^{01}y_k(a_k)^{01} + y_i(a_i)^{10}y_k(a_k)^{10} + y_i(a_i)^{11}y_k(a_k)^{11} = 0$ where i=0,1,2

$\langle\Psi_i| |\Psi_4\rangle = 0$ where , $i = 0, 1, 2$

$|\Psi\rangle_i = x_i(a_i)^{00} |000\rangle + x_i(a_i)^{01} |001\rangle + x_i(a_i)^{10} |100\rangle + x_i(a_i)^{11} |011\rangle + y_i(a_i)^{00} |100\rangle + y_i(a_i)^{01} |101\rangle + y_i(a_i)^{10} |110\rangle + y_i(a_i)^{11} |111\rangle$

$|\Psi\rangle_4 = x_4(a_4)^{00} |000\rangle + y_4(a_4)^{01} |111\rangle$

for orthogonality:

$ax_i(a_i)^{00}x_4(a_4)^{00} + y_i(a_i)^{11}y_4(a_4)^{01} = 0$

$\langle\Psi_k|\,|\Psi_4\rangle = 0$ where k=2,3

$x_k a_k^{00} x_4 a_4^{00} + x_k a_k^{11} x_4 a_4^{11} = 0$

Finally we get following genreal result:

$$E = \begin{cases} x_0 = -y_1(y_0 = -x_1) \\ \\ x_i(a_i)^{00}x_k(a_k)^{00} + x_i(a_i)^{01}x_k(a_k)^{01} + x_i(a_i)^{10} \\ x_k(a_k)^{10} + x_i(a_i)^{11}x_k(a_k)^{11} + y_i(a_i)^{00}y_k(a_k)^{00} + \\ y_i(a_i)^{01}y_k(a_k)^{01} + y_i(a_i)^{10}y_k(a_k)^{10} + y_i(a_i)^{11}y_k(a_k)^{11} = 0, wherei = 0,1, k = 2,3 \\ ax_i(a_i)^{00}x_4(a_4)^{00} + y_i(a_i)^{11}y_4(a_4)^{01} = 0 \\ x_k a_k^{00} x_4 a_4^{00} + x_k a_k^{11} x_4 a_4^{11} = 0 \end{cases} \tag{17}$$

$$F = \begin{cases} (a_0)^{00}(a_1)^{00} + (a_0)^{01}(a_1)^{01} + (a_0)^{10}(a_1)^{10} + (a_0)^{11}(a_1)^{11} = 0 \\ x_i(a_i)^{00}x_k(a_k)^{00} + x_i(a_i)^{01}x_k(a_k)^{01} + x_i(a_i)^{10} \\ x_k(a_k)^{10} + x_i(a_i)^{11}x_k(a_k)^{11} + y_i(a_i)^{00}y_k(a_k)^{00} + \\ y_i(a_i)^{01}y_k(a_k)^{01} + y_i(a_i)^{10}y_k(a_k)^{10} + y_i(a_i)^{11}y_k(a_k)^{11} = 0, wherei = 0,1, k = 2,3 \\ ax_i(a_i)^{00}x_4(a_4)^{00} + y_i(a_i)^{11}y_4(a_4)^{01} = 0 \\ x_k a_k^{00} x_4 a_4^{00} + x_k a_k^{11} x_4 a_4^{11} = 0 \end{cases} \tag{18}$$

we have infinite solutions in case E of orthogonality,but we have to prove is there a vector $|\zeta\rangle$ which is let (14) ,(15) dont have any solution .

(i) : Assume we have a vector $|\zeta\rangle = (x_5\,|0\rangle + y_5\,|1\rangle)_A \otimes ((a_5)^{00}\,|00\rangle + (a_5)^{01}\,|01\rangle + (a_5)^{10}\,|10\rangle + (a_5)^{11}\,|11\rangle)_{BC}$, ($A|BC$cut)

$\langle \Psi_i | |\zeta\rangle = 0$

$$C = \begin{cases} x_i = -y_5(y_i = -x_5) & , i = 0, 1 \\ \\ x_i(a_i)^{00} x_5(a_5)^{00} + x_i(a_i)^{01} x_5(a_5)^{01} + x_i(a_i)^{10} \\ x_3(a_5)^{10} + x_i(a_i)^{11} x_5(a_5)^{11} + y_i(a_i)^{00} y_5(a_5)^{00} + \\ y_i(a_i)^{01} y_5(a_5)^{01} + y_i(a_i)^{10} y_5(a_5)^{10} + y_i(a_i)^{11} \\ y_5(a_5)^{11} = 0, i = 0, 1 \end{cases} \tag{19}$$

$\langle \Psi_k | |\zeta\rangle = 0$

$x_k(a_k)^{00} x_5(a_5)^{00} + x_k(a_k)^{01} x_5(a_5)^{01} + x_k(a_k)^{10}$
$x_5(a_5)^{10} + x_k(a_k)^{11} x_5(a_5)^{11} + y_k(a_k)^{00} y_5(a_5)^{00} + y_k(a_k)^{01} y_5(a_5)^{01} + y_k(a_k)^{10} y_5(a_5)^{10} +$
$y_k(a_k)^{11} y_5(a_5)^{11} = 0, k = 2, 3$

$\langle \Psi_4 | |\zeta\rangle = 0$

$x_4 a_4^{00} x_5 a_5^{00} + x_5 a_5^{01} y_4 a_4^{11} = 0$

new ,we have to consider folowing system whether have solution or not

$$D = \begin{cases} \\ x_i = -y_5(y_i = -x_5), i = 0, 1 \\ x_k(a_k)^{00} x_5(a_5)^{00} + x_k(a_k)^{01} x_5(a_5)^{01} + x_k(a_k)^{10} \\ x_5(a_5)^{10} + x_k(a_k)^{11} x_5(a_5)^{11} + y_k(a_k)^{00} y_5(a_5)^{00} \\ + y_k(a_k)^{01} y_5(a_5)^{01} + y_k(a_k)^{10} y_5(a_5)^{10} + y_k(a_k)^{11} y_5(a_5)^{11} = 0, k = 2, 3 \\ x_4 a_4^{00} x_5 a_5^{00} + x_5 a_5^{01} y_4 a_4^{11} = 0 \end{cases} \tag{20}$$

$$E=\begin{cases} \\ x_i(a_i)^{00}x_5(a_5)^{00}+x_i(a_i)^{01}x_5(a_5)^{01}+x_i(a_i)^{10} \\ x_3(a_5)^{10}+x_i(a_i)^{11}x_5(a_5)^{11}+y_i(a_i)^{00}y_5(a_5)^{00}+ \\ y_i(a_i)^{01}y_5(a_5)^{01}+y_i(a_i)^{10}y_5(a_5)^{10}+y_i(a_i)^{11} \\ y_5(a_5)^{11}=0, i=0,1 \\ x_k(a_k)^{00}x_5(a_5)^{00}+x_k(a_k)^{01}x_5(a_5)^{01}+x_k(a_k)^{10} \\ x_5(a_5)^{10}+x_k(a_k)^{11}x_5(a_5)^{11}+y_k(a_k)^{00}y_5(a_5)^{00} \\ +y_k(a_k)^{01}y_5(a_5)^{01}+y_k(a_k)^{10}y_5(a_5)^{10}+y_k(a_k)^{11}y_5(a_5)^{11}=0, k=2,3 \\ x_4a_4^{00}x_5a_5^{00}+x_5a_5^{01}y_4a_4^{11}=0 \end{cases} \tag{21}$$

(ii) : Assume we have a vector $|\zeta\rangle = (x_5|0\rangle + y_5|1\rangle)_B \otimes ((a_5)^{00}|00\rangle + (a_5)^{01}|01\rangle + (a_5)^{10}|10\rangle + (a_5)^{11}|11\rangle)_{AC}$, ($B|AC$cut)

$$\langle\Psi_i|\,|\zeta\rangle = 0$$

$x_i(a_i)^{00}x_5(a_5)^{00}+x_i(a_i)^{01}x_5(a_5)^{01}+x_i(a_i)^{10}x_3(a_5)^{10}+x_i(a_i)^{11}x_5(a_5)^{11}+y_i(a_i)^{00}y_5(a_5)^{00}+y_i(a_i)^{01}y_5(a_5)^{01}+y_i(a_i)^{10}y_5(a_5)^{10}+y_i(a_i)^{11}y_5(a_5)^{11}=0$

where i=0,1

$\langle\Psi_k|\,|\zeta\rangle = 0$ where k=2,3

$$C=\begin{cases} x_k=-y_5(y_k=-x_5) & k=2,3 \\ x_k(a_k)^{00}x_5(a_5)^{00}+x_k(a_k)^{01}x_5(a_5)^{01}+x_k(a_k)^{10} \\ x_5(a_5)^{10}+x_k(a_k)^{11}x_5(a_5)^{11}+y_k(a_k)^{00}y_5(a_5)^{00} \\ +y_k(a_k)^{01}y_5(a_5)^{01}+y_k(a_k)^{10}y_5(a_5)^{10}+y_k(a_k)^{11}y_5(a_5)^{11}=0, k=2,3 \end{cases} \tag{22}$$

$\langle\Psi_4|\,|\zeta\rangle = 0$

$x_4a_4^{00}x_5a_5^{00}+x_5a_5^{11}y_4a_4^{11}=0$

new ,we have to consider folowing system whether have solution or not

$$D=\begin{cases} \\ x_k=-y_5(y_k=-x_5),k=2,3 \\ x_4a_4^{00}x_5a_5^{00}+x_5a_5^{11}y_4a_4^{11}=0 \\ x_i(a_i)^{00}x_5(a_5)^{00}+x_i(a_i)^{01}x_5(a_5)^{01}+ \\ x_i(a_i)^{10}x_3(a_5)^{10}+x_i(a_i)^{11}x_5(a_5)^{11}+y_i(a_i)^{00}y_5(a_5)^{00}+ \\ y_i(a_i)^{01}y_5(a_5)^{01}+y_i(a_i)^{10}y_5(a_5)^{10}+y_i(a_i)^{11}y_5(a_5)^{11}=0 \\ x_4a_4^{00}x_5a_5^{00}+x_5a_5^{11}y_4a_4^{11}=0 \end{cases} \tag{23}$$

$$E=\begin{cases} \\ x_k(a_k)^{00}x_5(a_5)^{00}+x_k(a_k)^{01}x_5(a_5)^{01}+x_k(a_k)^{10} \\ x_5(a_5)^{10}+x_k(a_k)^{11}x_5(a_5)^{11}+y_k(a_k)^{00}y_5(a_5)^{00} \\ +y_k(a_k)^{01}y_5(a_5)^{01}+y_k(a_k)^{10}y_5(a_5)^{10}+y_k(a_k)^{11}y_5(a_5)^{11}=0,k=2,3 \\ x_4a_4^{00}x_5a_5^{00}+x_5a_5^{11}y_4a_4^{11}=0 \\ x_i(a_i)^{00}x_5(a_5)^{00}+x_i(a_i)^{01}x_5(a_5)^{01}+ \\ x_i(a_i)^{10}x_3(a_5)^{10}+x_i(a_i)^{11}x_5(a_5)^{11}+y_i(a_i)^{00}y_5(a_5)^{00}+ \\ y_i(a_i)^{01}y_5(a_5)^{01}+y_i(a_i)^{10}y_5(a_5)^{10}+y_i(a_i)^{11}y_5(a_5)^{11}=0 \\ x_4a_4^{00}x_5a_5^{00}+x_5a_5^{11}y_4a_4^{11}=0 \end{cases} \tag{24}$$

(iii) : Assume we have a vector $|\zeta\rangle = (x_5|0\rangle + y_5|1\rangle)_C \otimes ((a_5)^{00}|00\rangle + (a_5)^{01}|01\rangle + (a_5)^{10}|10\rangle + (a_5)^{11}|11\rangle)_{AB}$, ($C|AB$cut)

$\langle\Psi_i|\,|\zeta\rangle = 0$, i=0,1

$x_i(a_i)^{00}x_5(a_5)^{00}+x_i(a_i)^{10}x5(a_5)^{01}+y_i(a_i)^{00}x_5(a_5)^{10}+y_i(a_i)^{10}x_5(a_5)^{11}+x_i(a_i)^{01}y_5(a_5)^{00}+x_i(a_i)^{11}y_5(a_5)^{01}+y_i(a_i)^{10}y_5(a_5)^{10}+y_i(a_i)^{11}y_5(a_5)^{11}=0$

where i=0,1,2

$\langle \Psi_k | \, |\zeta\rangle = 0$,k=2,3

$x_k(a_k)^{00}x_5(a_5)^{00} + x_k(a_k)^{01}y_5(a_5)^{00} + x_k(a_k)^{10}x_5(a_5)^{10} + x_k(a_k)^{11}y_5(a_5)^{10} + y_k(a_k)^{00}x_5(a_5)^{01} + y_k(a_k)^{01}y_5(a_5)^{01} + y_k a_k^{10}x_5 a_5^{11} + y_k(a_k)^{11}y_5(a_5)^{11} = 0.$ where k=2,3

$\langle \Psi_4 | \, |\zeta\rangle = 0$

$x_4 a_4^{00} x_5 a_5^{00} + x_5 a_5^{11} y_4 a_4^{11} = 0$

new ,we have to consider folowing system whether have solution or not

$$D = \begin{cases} \\ x_i(a_i)^{00}x_5(a_5)^{00} + x_i(a_i)^{10}x5(a_5)^{01} + y_i(a_i)^{00}x_5 \\ (a_5)^{10} + y_i(a_i)^{10}x_5(a_5)^{11} + x_i(a_i)^{01}y_5(a_5)^{00} + \\ x_i(a_i)^{11}y_5(a_5)^{01} + y_i(a_i)^{10}y_5(a_5)^{10} + y_i(a_i)^{11}y_5(a_5)^{11} = 0, i = 0,1 \\ x_k(a_k)^{00}x_5(a_5)^{00} + x_k(a_k)^{01}y_5(a_5)^{00} + x_k(a_k)^{10}x_5(a_5)^{10} + x_k(a_k)^{11}y_5(a_5)^{10} + \\ y_k(a_k)^{00}x_5(a_5)^{01} + y_k(a_k)^{01}y_5(a_5)^{01} + y_k a_k^{10}x_5 a_5^{11} + y_k(a_k)^{11}y_5(a_5)^{11} = 0, k = 2,3 \\ x_4 a_4^{00} x_5 a_5^{00} + x_5 a_5^{11} y_4 a_4^{11} = 0 \end{cases} \tag{25}$$

case 5;

$|\Psi\rangle_0 = (x_0 |0\rangle)_B \otimes ((a_0)^{00} |00\rangle + (a_i)^{11} |11\rangle)_{AC}$

$|\Psi\rangle_1 = (x_1 |0\rangle + y_1 |1\rangle)_C \otimes ((a_1)^{00} |00\rangle + (a_1)^{01} |01\rangle + (a_1)^{10} |10\rangle + (a_1)^{11} |11\rangle)_{AB}$

$|\Psi\rangle_i = (x_i |0\rangle + y_i |1\rangle)_B \otimes ((a_i)^{00} |00\rangle + (a_i)^{01} |01\rangle + (a_i)^{10} |10\rangle + (a_i)^{11} |11\rangle)_{AC}$
where i=2,3

$|\Psi\rangle_4 = x_4 |0\rangle_A \otimes ((a_4)^{00} |00\rangle + (a_4)^{01} |01\rangle + (a_4)^{10} |10\rangle + (a_4)^{11} |11\rangle)_{BC},$

$|(a_i)^{00}|^2 + |(a_i)^{01}|^2 + |(a_i)^{10}|^2 + |(a_i)^{11}|^2 = 1, \, , i = 0,1,2,3,,$

suppose above 5 vectors are pairly orthogonal,then the relations between the coefficients :

$\langle \Psi_0 | \, |\Psi_1\rangle = 0$

$x_0(a_0)^{00}x_1(a_1)^{00} + x_0(a_0)^{01}y_1(a_1)^{00} + x_0(a_0)^{10}x_1(a_1)^{10} + x_0(a_0)^{11}y_1(a_1)^{10} = 0$

$\langle\Psi_i|\,|\Psi_0\rangle = 0$ where , $i = 2, 3$

$x_0(a_0)^{00}x_i(a_i)^{00} + x_0(a_0)^{01}x_i(a_i)^{01} + x_0(a_0)^{10}x_i(a_i)^{10} + x_0(a_0)^{11}x_i(a_i)^{11} = 0$

$\langle\Psi_i|\,|\Psi_1\rangle = 0$

where , $i = 2, 3$. from orthognality ,we could get

$x_1(a_1)^{00}x_i(a_i)^{00}+x_1(a_1)^{01}y_i(a_i)^{00}+x_1(a_1)^{10}x_i(a_i)^{10}+x_1(a_1)^{11}y_i(a_i)^{10}+y_1(a_1)^{00}x_i(a_i)^{01}+$
$y_1(a_1)^{01}y_i(a_i)^{01} + y_1(a_1)^{10}x_i(a_i)^{11} + y_1(a_1)^{11}y_i(a_i)^{11} = 0$ where i=2,3

$\langle\Psi_i|\,|\Psi_4\rangle = 0$ where , $i = 2, 3$

$x_i(a_i)^{00}x_4(a_4)^{00}+x_i(a_i)^{01}x_4(a_4)^{01}+x_i(a_i)^{10}y_4(a_4)^{00}+x_i(a_i)^{11}y_4(a_4^{01}+y_i(a_i)^{00}x_4(a_4)^{10}+$
$y_i(a_i)^{01}x_4(a_4)^{11} + y_i(a_i)^{10}y_4(a_4)^{10} + y_i(a_i)^{11}y_4(a_4)^{11} = 0$ where i=2,3

$\langle\Psi_0|\,|\Psi_4\rangle = 0$
$x_0(a_0)^{00}x_4(a_4)^{00} + x_0(a_0)^{01}x_4(a_4)^{01} + x_0(a_0)^{10}y_4(a_4)^{00} + x_0(a_0)^{11}y_4(a_4)^{01} = 0$

$\langle\Psi_1|\,|\Psi_4\rangle = 0$
$x_1(a_1)^{00}x_4(a_4)^{00}+x_1(a_1)^{01}x_4(a_4)^{10}+x_1(a_1)^{10}y_4(a_4)^{00}+x_1(a_1)^{11}y_4(a_4^{10}+y_1(a_1)^{00}x_4(a_4)^{11}+$
$y_1(a_1)^{01}x_4(a_4)^{11} + y_1(a_1)^{10}y_4(a_4)^{01} + y_1(a_1)^{11}y_4(a_4)^{11} = 0$ where i=2,3

Finally we get following genreal result:

$$E = \begin{cases} \\ x_0(a_0)^{00}x_i(a_i)^{00} + x_0(a_0)^{01}x_i(a_i)^{01} + x_0(a_0)^{10}x_i(a_i)^{10} + x_0(a_0)^{11}x_i(a_i)^{11} = 0 \\ x_1(a_1)^{00}x_i(a_i)^{00} + x_1(a_1)^{01}y_i(a_i)^{00} + \\ x_1(a_1)^{10}x_i(a_i)^{10} + x_1(a_1)^{11}y_i(a_i)^{10} + \\ y_1(a_1)^{00}x_i(a_i)^{01} + y_1(a_1)^{01}y_i(a_i)^{01} + y_1(a_1)^{10}x_i(a_i)^{11} + y_1(a_1)^{11}y_i(a_i)^{11} = 0 \\ x_i(a_i)^{00}x_4(a_4)^{00} + x_i(a_i)^{01}x_4(a_4)^{01} + \\ x_i(a_i)^{10}y_4(a_4)^{00} + x_i(a_i)^{11}y_4(a_4^{01} + y_i(a_i)^{00}x_4(a_4)^{10} + y_i(a_i)^{01}x_4(a_4)^{11} + \\ y_i(a_i)^{10}y_4(a_4)^{10} + y_i(a_i)^{11}y_4(a_4)^{11} = 0 \\ x_0(a_0)^{00}x_4(a_4)^{00} + x_0(a_0)^{01}x_4(a_4)^{01} + x_0(a_0)^{10}y_4(a_4)^{00} + x_0(a_0)^{11}y_4(a_4)^{01} = 0 \\ x_0(a_0)^{00}x_4(a_4)^{00} + x_0(a_0)^{01}x_4(a_4)^{01} + x_0(a_0)^{10}y_4(a_4)^{00} + x_0(a_0)^{11}y_4(a_4)^{01} = 0 \\ x_1(a_1)^{00}x_4(a_4)^{00} + x_1(a_1)^{01}x_4(a_4)^{10} + \\ x_1(a_1)^{10}y_4(a_4)^{00} + x_1(a_1)^{11}y_4(a_4^{10} + y_1(a_1)^{00}x_4(a_4)^{11} + y_1(a_1)^{01}x_4(a_4)^{11} + \\ y_1(a_1)^{10}y_4(a_4)^{01} + y_1(a_1)^{11}y_4(a_4)^{11} = 00 \end{cases} \tag{26}$$

we have infinite solutions in case E of orthogonality,but we have to prove is there a vector $|\zeta\rangle$ which is let (23) dont have any solution .

(i) : Assume we have a vector $|\zeta\rangle = (x_5\,|0\rangle + y_5\,|1\rangle)_A \otimes ((a_5)^{00}\,|00\rangle + (a_5)^{01}\,|01\rangle + (a_5)^{10}\,|10\rangle + (a_5)^{11}\,|11\rangle)_{BC}$, ($A|BC$cut)

$$\langle \Psi_i|\,|\zeta\rangle = 0$$

$$C = \begin{cases} x_i = -y_5 (y_i = -x_5) & , i = 0,1 \\ \\ x_i(a_i)^{00}x_5(a_5)^{00} + x_i(a_i)^{01}x_5(a_5)^{01} + x_i(a_i)^{10} \\ x_3(a_5)^{10} + x_i(a_i)^{11}x_5(a_5)^{11} + y_i(a_i)^{00}y_5(a_5)^{00} + \\ y_i(a_i)^{01}y_5(a_5)^{01} + y_i(a_i)^{10}y_5(a_5)^{10} + y_i(a_i)^{11} \\ y_5(a_5)^{11} = 0, i = 0,1 \end{cases} \tag{27}$$

$\langle\Psi_k|\,|\zeta\rangle = 0$

$x_k(a_k)^{00}x_5(a_5)^{00} + x_k(a_k)^{01}x_5(a_5)^{01} + x_k(a_k)^{10}$
$x_5(a_5)^{10}+x_k(a_k)^{11}x_5(a_5)^{11}+y_k(a_k)^{00}y_5(a_5)^{00}+y_k(a_k)^{01}y_5(a_5)^{01}+y_k(a_k)^{10}y_5(a_5)^{10}+$
$y_k(a_k)^{11}y_5(a_5)^{11} = 0, k = 2, 3$

$\langle\Psi_4|\,|\zeta\rangle = 0$

$x_4a_4^{00}x_5a_5^{00} + x_5a_5^{01}y_4a_4^{11} = 0$

new ,we have to consider folowing system whether have solution or not

$$D = \begin{cases} \\ x_i = -y_5(y_i = -x_5), i = 0, 1 \\ x_k(a_k)^{00}x_5(a_5)^{00} + x_k(a_k)^{01}x_5(a_5)^{01} + x_k(a_k)^{10} \\ x_5(a_5)^{10} + x_k(a_k)^{11}x_5(a_5)^{11} + y_k(a_k)^{00}y_5(a_5)^{00} \\ +y_k(a_k)^{01}y_5(a_5)^{01} + y_k(a_k)^{10}y_5(a_5)^{10} + y_k(a_k)^{11}y_5(a_5)^{11} = 0, k = 2, 3 \\ x_4a_4^{00}x_5a_5^{00} + x_5a_5^{01}y_4a_4^{11} = 0 \end{cases} \tag{28}$$

$$E = \begin{cases} \\ x_i(a_i)^{00}x_5(a_5)^{00} + x_i(a_i)^{01}x_5(a_5)^{01} + x_i(a_i)^{10} \\ x_3(a_5)^{10} + x_i(a_i)^{11}x_5(a_5)^{11} + y_i(a_i)^{00}y_5(a_5)^{00}+ \\ y_i(a_i)^{01}y_5(a_5)^{01} + y_i(a_i)^{10}y_5(a_5)^{10} + y_i(a_i)^{11} \\ y_5(a_5)^{11} = 0, i = 0, 1 \\ x_k(a_k)^{00}x_5(a_5)^{00} + x_k(a_k)^{01}x_5(a_5)^{01} + x_k(a_k)^{10} \\ x_5(a_5)^{10} + x_k(a_k)^{11}x_5(a_5)^{11} + y_k(a_k)^{00}y_5(a_5)^{00} \\ +y_k(a_k)^{01}y_5(a_5)^{01} + y_k(a_k)^{10}y_5(a_5)^{10} + y_k(a_k)^{11}y_5(a_5)^{11} = 0, k = 2, 3 \\ x_4a_4^{00}x_5a_5^{00} + x_5a_5^{01}y_4a_4^{11} = 0 \end{cases} \tag{29}$$

(ii) : Assume we have a vector $|\zeta\rangle = (x_5\,|0\rangle + y_5\,|1\rangle)_B \otimes ((a_5)^{00}\,|00\rangle + (a_5)^{01}\,|01\rangle + (a_5)^{10}\,|10\rangle + (a_5)^{11}\,|11\rangle)_{AC}$, ($B|AC$cut)

$\langle\Psi_i|\,|\zeta\rangle = 0$

$x_i(a_i)^{00}x_5(a_5)^{00}+x_i(a_i)^{01}x_5(a_5)^{01}+x_i(a_i)^{10}x_3(a_5)^{10}+x_i(a_i)^{11}x_5(a_5)^{11}+y_i(a_i)^{00}y_5(a_5)^{00}+ y_i(a_i)^{01}y_5(a_5)^{01}+y_i(a_i)^{10}y_5(a_5)^{10}+y_i(a_i)^{11}y_5(a_5)^{11}=0$

where i=0,1

$\langle\Psi_k|\,|\zeta\rangle=0$ where k=2,3

$$C=\begin{cases} x_k=-y_5(y_k=-x_5) & k=2,3 \\ x_k(a_k)^{00}x_5(a_5)^{00}+x_k(a_k)^{01}x_5(a_5)^{01}+x_k(a_k)^{10} \\ x_5(a_5)^{10}+x_k(a_k)^{11}x_5(a_5)^{11}+y_k(a_k)^{00}y_5(a_5)^{00} \\ +y_k(a_k)^{01}y_5(a_5)^{01}+y_k(a_k)^{10}y_5(a_5)^{10}+y_k(a_k)^{11}y_5(a_5)^{11}=0, k=2,3 \end{cases} \tag{30}$$

$\langle\Psi_4|\,|\zeta\rangle=0$

$x_4a_4^{00}x_5a_5^{00}+x_5a_5^{11}y_4a_4^{11}=0$

new ,we have to consider folowing system whether have solution or not

$$D=\begin{cases} \\ x_k=-y_5(y_k=-x_5), k=2,3 \\ x_4a_4^{00}x_5a_5^{00}+x_5a_5^{11}y_4a_4^{11}=0 \\ x_i(a_i)^{00}x_5(a_5)^{00}+x_i(a_i)^{01}x_5(a_5)^{01}+ \\ x_i(a_i)^{10}x_3(a_5)^{10}+x_i(a_i)^{11}x_5(a_5)^{11}+y_i(a_i)^{00}y_5(a_5)^{00}+ \\ y_i(a_i)^{01}y_5(a_5)^{01}+y_i(a_i)^{10}y_5(a_5)^{10}+y_i(a_i)^{11}y_5(a_5)^{11}=0 \\ x_4a_4^{00}x_5a_5^{00}+x_5a_5^{11}y_4a_4^{11}=0 \end{cases} \tag{31}$$

$$E = \begin{cases} \\ x_k(a_k)^{00}x_5(a_5)^{00} + x_k(a_k)^{01}x_5(a_5)^{01} + x_k(a_k)^{10} \\ x_5(a_5)^{10} + x_k(a_k)^{11}x_5(a_5)^{11} + y_k(a_k)^{00}y_5(a_5)^{00} \\ +y_k(a_k)^{01}y_5(a_5)^{01} + y_k(a_k)^{10}y_5(a_5)^{10} + y_k(a_k)^{11}y_5(a_5)^{11} = 0, k = 2, 3 \\ x_4a_4^{00}x_5a_5^{00} + x_5a_5^{11}y_4a_4^{11} = 0 \\ x_i(a_i)^{00}x_5(a_5)^{00} + x_i(a_i)^{01}x_5(a_5)^{01}+ \\ x_i(a_i)^{10}x_3(a_5)^{10} + x_i(a_i)^{11}x_5(a_5)^{11} + y_i(a_i)^{00}y_5(a_5)^{00}+ \\ y_i(a_i)^{01}y_5(a_5)^{01} + y_i(a_i)^{10}y_5(a_5)^{10} + y_i(a_i)^{11}y_5(a_5)^{11} = 0 \\ x_4a_4^{00}x_5a_5^{00} + x_5a_5^{11}y_4a_4^{11} = 0 \end{cases} \tag{32}$$

(iii) : Assume we have a vector $|\zeta\rangle = (x_5 |0\rangle + y_5 |1\rangle)_C \otimes ((a_5)^{00} |00\rangle + (a_5)^{01} |01\rangle + (a_5)^{10} |10\rangle + (a_5)^{11} |11\rangle)_{AB}$, ($C|AB$cut)

$\langle\Psi_i| |\zeta\rangle = 0$, i=0,1

$x_i(a_i)^{00}x_5(a_5)^{00}+x_i(a_i)^{10}x5(a_5)^{01}+y_i(a_i)^{00}x_5(a_5)^{10}+y_i(a_i)^{10}x_5(a_5)^{11}+x_i(a_i)^{01}y_5(a_5)^{00}+ x_i(a_i)^{11}y_5(a_5)^{01} + y_i(a_i)^{10}y_5(a_5)^{10} + y_i(a_i)^{11}y_5(a_5)^{11} = 0$

where i=0,1,2

$\langle\Psi_k| |\zeta\rangle = 0$,k=2,3

$x_k(a_k)^{00}x_5(a_5)^{00} + x_k(a_k)^{01}y_5(a_5)^{00} + x_k(a_k)^{10}x_5(a_5)^{10} + x_k(a_k)^{11}y_5(a_5)^{10} + y_k(a_k)^{00}x_5(a_5)^{01} + y_k(a_k)^{01}y_5(a_5)^{01} + y_ka_k^{10}x_5a_5^{11} + y_k(a_k)^{11}y_5(a_5)^{11} = 0$. where k=2,3

$\langle\Psi_4| |\zeta\rangle = 0$

$x_4a_4^{00}x_5a_5^{00} + x_5a_5^{11}y_4a_4^{11} = 0$

new ,we have to consider folowing system whether have solution or not

$$D = \begin{cases} \\ x_i(a_i)^{00}x_5(a_5)^{00} + x_i(a_i)^{10}x5(a_5)^{01} + y_i(a_i)^{00}x_5 \\ (a_5)^{10} + y_i(a_i)^{10}x_5(a_5)^{11} + x_i(a_i)^{01}y_5(a_5)^{00} + \\ x_i(a_i)^{11}y_5(a_5)^{01} + y_i(a_i)^{10}y_5(a_5)^{10} + y_i(a_i)^{11}y_5(a_5)^{11} = 0, i = 0, 1 \\ x_k(a_k)^{00}x_5(a_5)^{00} + x_k(a_k)^{01}y_5(a_5)^{00} + x_k(a_k)^{10}x_5(a_5)^{10} + x_k(a_k)^{11}y_5(a_5)^{10} + \\ y_k(a_k)^{00}x_5(a_5)^{01} + y_k(a_k)^{01}y_5(a_5)^{01} + y_k a_k^{10} x_5 a_5^{11} + y_k(a_k)^{11}y_5(a_5)^{11} = 0, k = 2, 3 \\ x_4 a_4^{00} x_5 a_5^{00} + x_5 a_5^{11} y_4 a_4^{11} = 0 \end{cases} \quad (33)$$

case 6

case 3

$|\Psi\rangle_i = (x_i |0\rangle + y_i |1\rangle)_A \otimes ((a_i)^{00} |00\rangle + (a_i)^{01} |01\rangle + (a_i)^{10} |10\rangle + (a_i)^{11} |11\rangle)_{BC}$
where i=0,1

$|\Psi\rangle_k = (x_k |0\rangle + y_k |1\rangle)_B \otimes ((a_k)^{00} |00\rangle + (a_k)^{01} |01\rangle + (a_k)^{10} |10\rangle + (a_k)^{11} |11\rangle)_{AC}$,
where k=2,3

$|\Psi\rangle_4 = x_4 |0\rangle_B \otimes ((a_3)^{00} |00\rangle + (a_3)^{01} |01\rangle + (a_3)^{10} |10\rangle + (a_3)^{11} |11\rangle)_{AC}$,
where $|x_i|^2 + |y_i|^2 = 1$, $i = 0, 1, 2, 3,,$

$|(a_i)^{00}|^2 + |(a_i)^{01}|^2 + |(a_i)^{10}|^2 + |(a_i)^{11}|^2 = |a|^2 + |b|^2 = 1, \ , i = 0, 1, 2, 3,,$

suppose above 5 vectors are pairly orthogonal,then the relations between the coefficients :

$\langle \Psi_0 | |\Psi_1\rangle = 0$

$x_0 = -y_1 (y_0 = -x_1)$

or

$(a_0)^{00}(a_1)^{00} + (a_0)^{01}(a_1)^{01} + (a_0)^{10}(a_1)^{10} + (a_0)^{11}(a_1)^{11} = 0$

$\langle\Psi_i|\,|\Psi_k\rangle = 0$ where $, i = 0, 1, k = 2, 3$

$|\Psi\rangle_i = x_i(a_i)^{00}|000\rangle + x_i(a_i)^{01}|001\rangle + x_i(a_i)^{10}|100\rangle + x_i(a_i)^{11}|011\rangle + y_i(a_i)^{00}|100\rangle + y_i(a_i)^{01}|101\rangle + y_i(a_i)^{10}|110\rangle + y_i(a_i)^{11}|111\rangle$

$|\Psi\rangle_k = x_k(a_k)^{00}|000\rangle_{ABC} + x_k(a_k)^{01}|001\rangle_{ABC} + x_k(a_k)^{10}|100\rangle_{ABC} + x_k(a_k)^{11}|101\rangle_{ABC} + y_k(a_k)^{00}|010\rangle_{ABC} + y_k(a_k)^{01}|011\rangle_{ABC} + y_k(a_k)^{10}|110\rangle_{ABC} + y_k(a_k)^{11}|111\rangle_{ABC}$

from orthognality ,we could get

$x_i(a_i)^{00}x_k(a_k)^{00} + x_i(a_i)^{01}x_k(a_k)^{01} + x_i(a_i)^{10}x_k(a_k)^{10} + x_i(a_i)^{11}x_k(a_k)^{11} + y_i(a_i)^{00}y_k(a_k)^{00} + y_i(a_i)^{01}y_k(a_k)^{01} + y_i(a_i)^{10}y_k(a_k)^{10} + y_i(a_i)^{11}y_k(a_k)^{11} = 0$ where i=0,1,2

$\langle\Psi_i|\,|\Psi_4\rangle = 0$ where $, i = 0, 1, 2$

$|\Psi\rangle_i = x_i(a_i)^{00}|000\rangle + x_i(a_i)^{01}|001\rangle + x_i(a_i)^{10}|100\rangle + x_i(a_i)^{11}|011\rangle + y_i(a_i)^{00}|100\rangle + y_i(a_i)^{01}|101\rangle + y_i(a_i)^{10}|110\rangle + y_i(a_i)^{11}|111\rangle$

$|\Psi\rangle_4 = x_4(a_4)^{00}|000\rangle + y_4(a_4)^{01}|111\rangle$

for orthogonality:

$ax_i(a_i)^{00}x_4(a_4)^{00} + y_i(a_i)^{11}y_4(a_4)^{01} = 0$

$\langle\Psi_k|\,|\Psi_4\rangle = 0$ where k=2,3

$x_k a_k^{00} x_4 a_4^{00} + x_k a_k^{11} x_4 a_4^{11} = 0$

Finally we get following genreal result:

$$E = \begin{cases} x_0 = -y_1(y_0 = -x_1) \\ \\ x_i(a_i)^{00}x_k(a_k)^{00} + x_i(a_i)^{01}x_k(a_k)^{01} + x_i(a_i)^{10} \\ x_k(a_k)^{10} + x_i(a_i)^{11}x_k(a_k)^{11} + y_i(a_i)^{00}y_k(a_k)^{00} + \\ y_i(a_i)^{01}y_k(a_k)^{01} + y_i(a_i)^{10}y_k(a_k)^{10} + y_i(a_i)^{11}y_k(a_k)^{11} = 0, where i = 0, 1, k = 2, 3 \\ ax_i(a_i)^{00}x_4(a_4)^{00} + y_i(a_i)^{11}y_4(a_4)^{01} = 0 \\ x_k a_k^{00} x_4 a_4^{00} + x_k a_k^{11} x_4 a_4^{11} = 0 \end{cases} \tag{34}$$

$$F=\begin{cases}(a_0)^{00}(a_1)^{00}+(a_0)^{01}(a_1)^{01}+(a_0)^{10}(a_1)^{10}+(a_0)^{11}(a_1)^{11}=0\\ x_i(a_i)^{00}x_k(a_k)^{00}+x_i(a_i)^{01}x_k(a_k)^{01}+x_i(a_i)^{10}\\ x_k(a_k)^{10}+x_i(a_i)^{11}x_k(a_k)^{11}+y_i(a_i)^{00}y_k(a_k)^{00}+\\ y_i(a_i)^{01}y_k(a_k)^{01}+y_i(a_i)^{10}y_k(a_k)^{10}+y_i(a_i)^{11}y_k(a_k)^{11}=0, wherei=0,1,k=2,3\\ ax_i(a_i)^{00}x_4(a_4)^{00}+y_i(a_i)^{11}y_4(a_4)^{01}=0\\ x_ka_k^{00}x_4a_4^{00}+x_ka_k^{11}x_4a_4^{11}=0\end{cases} \tag{35}$$

we have infinite solutions in case E of orthogonality,but we have to prove is there a vector $|\zeta\rangle$ which is let (14) ,(15) dont have any solution .

(i) : Assume we have a vector $|\zeta\rangle=(x_5\,|0\rangle+y_5\,|1\rangle)_A\otimes((a_5)^{00}\,|00\rangle+(a_5)^{01}\,|01\rangle+(a_5)^{10}\,|10\rangle+(a_5)^{11}\,|11\rangle)_{BC}$, ($A|BC$cut)

$$\langle\Psi_i|\,|\zeta\rangle=0$$

$$C=\begin{cases}x_i=-y_5(y_i=-x_5) & ,i=0,1\\ \\ x_i(a_i)^{00}x_5(a_5)^{00}+x_i(a_i)^{01}x_5(a_5)^{01}+x_i(a_i)^{10}\\ x_3(a_5)^{10}+x_i(a_i)^{11}x_5(a_5)^{11}+y_i(a_i)^{00}y_5(a_5)^{00}+\\ y_i(a_i)^{01}y_5(a_5)^{01}+y_i(a_i)^{10}y_5(a_5)^{10}+y_i(a_i)^{11}\\ y_5(a_5)^{11}=0,i=0,1\end{cases} \tag{36}$$

$$\langle\Psi_k|\,|\zeta\rangle=0$$

$x_k(a_k)^{00}x_5(a_5)^{00}+x_k(a_k)^{01}x_5(a_5)^{01}+x_k(a_k)^{10}$
$x_5(a_5)^{10}+x_k(a_k)^{11}x_5(a_5)^{11}+y_k(a_k)^{00}y_5(a_5)^{00}+y_k(a_k)^{01}y_5(a_5)^{01}+y_k(a_k)^{10}y_5(a_5)^{10}+$
$y_k(a_k)^{11}y_5(a_5)^{11}=0,k=2,3$

$$\langle\Psi_4|\,|\zeta\rangle=0$$

$x_4a_4^{00}x_5a_5^{00} + x_5a_5^{01}y_4a_4^{11} = 0$

new ,we have to consider folowing system whether have solution or not

$$D = \begin{cases} x_i = -y_5(y_i = -x_5), i = 0,1 \\ x_k(a_k)^{00}x_5(a_5)^{00} + x_k(a_k)^{01}x_5(a_5)^{01} + x_k(a_k)^{10} \\ x_5(a_5)^{10} + x_k(a_k)^{11}x_5(a_5)^{11} + y_k(a_k)^{00}y_5(a_5)^{00} \\ +y_k(a_k)^{01}y_5(a_5)^{01} + y_k(a_k)^{10}y_5(a_5)^{10} + y_k(a_k)^{11}y_5(a_5)^{11} = 0, k = 2,3 \\ x_4a_4^{00}x_5a_5^{00} + x_5a_5^{01}y_4a_4^{11} = 0 \end{cases} \tag{37}$$

$$E = \begin{cases} x_i(a_i)^{00}x_5(a_5)^{00} + x_i(a_i)^{01}x_5(a_5)^{01} + x_i(a_i)^{10} \\ x_3(a_5)^{10} + x_i(a_i)^{11}x_5(a_5)^{11} + y_i(a_i)^{00}y_5(a_5)^{00} + \\ y_i(a_i)^{01}y_5(a_5)^{01} + y_i(a_i)^{10}y_5(a_5)^{10} + y_i(a_i)^{11} \\ y_5(a_5)^{11} = 0, i = 0,1 \\ x_k(a_k)^{00}x_5(a_5)^{00} + x_k(a_k)^{01}x_5(a_5)^{01} + x_k(a_k)^{10} \\ x_5(a_5)^{10} + x_k(a_k)^{11}x_5(a_5)^{11} + y_k(a_k)^{00}y_5(a_5)^{00} \\ +y_k(a_k)^{01}y_5(a_5)^{01} + y_k(a_k)^{10}y_5(a_5)^{10} + y_k(a_k)^{11}y_5(a_5)^{11} = 0, k = 2,3 \\ x_4a_4^{00}x_5a_5^{00} + x_5a_5^{01}y_4a_4^{11} = 0 \end{cases} \tag{38}$$

(ii) : Assume we have a vector $|\zeta\rangle = (x_5|0\rangle + y_5|1\rangle)_B \otimes ((a_5)^{00}|00\rangle + (a_5)^{01}|01\rangle + (a_5)^{10}|10\rangle + (a_5)^{11}|11\rangle)_{AC}$, ($B|AC$cut)

$\langle\Psi_i|\,|\zeta\rangle = 0$

$x_i(a_i)^{00}x_5(a_5)^{00}+x_i(a_i)^{01}x_5(a_5)^{01}+x_i(a_i)^{10}x_3(a_5)^{10}+x_i(a_i)^{11}x_5(a_5)^{11}+y_i(a_i)^{00}y_5(a_5)^{00}+ y_i(a_i)^{01}y_5(a_5)^{01} + y_i(a_i)^{10}y_5(a_5)^{10} + y_i(a_i)^{11}y_5(a_5)^{11} = 0$

where i=0,1

$\langle\Psi_k|\,|\zeta\rangle = 0$ where k=2,3

$$C = \begin{cases} x_k = -y_5 (y_k = -x_5) & k = 2,3 \\ x_k(a_k)^{00} x_5(a_5)^{00} + x_k(a_k)^{01} x_5(a_5)^{01} + x_k(a_k)^{10} \\ x_5(a_5)^{10} + x_k(a_k)^{11} x_5(a_5)^{11} + y_k(a_k)^{00} y_5(a_5)^{00} \\ +y_k(a_k)^{01} y_5(a_5)^{01} + y_k(a_k)^{10} y_5(a_5)^{10} + y_k(a_k)^{11} y_5(a_5)^{11} = 0, k = 2,3 \end{cases} \tag{39}$$

$\langle \Psi_4 | \, | \zeta \rangle = 0$

$x_4 a_4^{00} x_5 a_5^{00} + x_5 a_5^{11} y_4 a_4^{11} = 0$

new ,we have to consider folowing system whether have solution or not

$$D = \begin{cases} \\ x_k = -y_5 (y_k = -x_5), k = 2,3 \\ x_4 a_4^{00} x_5 a_5^{00} + x_5 a_5^{11} y_4 a_4^{11} = 0 \\ x_i(a_i)^{00} x_5(a_5)^{00} + x_i(a_i)^{01} x_5(a_5)^{01} + \\ x_i(a_i)^{10} x_3(a_5)^{10} + x_i(a_i)^{11} x_5(a_5)^{11} + y_i(a_i)^{00} y_5(a_5)^{00} + \\ y_i(a_i)^{01} y_5(a_5)^{01} + y_i(a_i)^{10} y_5(a_5)^{10} + y_i(a_i)^{11} y_5(a_5)^{11} = 0 \\ x_4 a_4^{00} x_5 a_5^{00} + x_5 a_5^{11} y_4 a_4^{11} = 0 \end{cases} \tag{40}$$

$$E = \begin{cases} \\ x_k(a_k)^{00} x_5(a_5)^{00} + x_k(a_k)^{01} x_5(a_5)^{01} + x_k(a_k)^{10} \\ x_5(a_5)^{10} + x_k(a_k)^{11} x_5(a_5)^{11} + y_k(a_k)^{00} y_5(a_5)^{00} \\ +y_k(a_k)^{01} y_5(a_5)^{01} + y_k(a_k)^{10} y_5(a_5)^{10} + y_k(a_k)^{11} y_5(a_5)^{11} = 0, k = 2,3 \\ x_4 a_4^{00} x_5 a_5^{00} + x_5 a_5^{11} y_4 a_4^{11} = 0 \\ x_i(a_i)^{00} x_5(a_5)^{00} + x_i(a_i)^{01} x_5(a_5)^{01} + \\ x_i(a_i)^{10} x_3(a_5)^{10} + x_i(a_i)^{11} x_5(a_5)^{11} + y_i(a_i)^{00} y_5(a_5)^{00} + \\ y_i(a_i)^{01} y_5(a_5)^{01} + y_i(a_i)^{10} y_5(a_5)^{10} + y_i(a_i)^{11} y_5(a_5)^{11} = 0 \\ x_4 a_4^{00} x_5 a_5^{00} + x_5 a_5^{11} y_4 a_4^{11} = 0 \end{cases} \tag{41}$$

(iii) : Assume we have a vector $|\zeta\rangle = (x_5|0\rangle + y_5|1\rangle)_C \otimes ((a_5)^{00}|00\rangle + (a_5)^{01}|01\rangle + (a_5)^{10}|10\rangle + (a_5)^{11}|11\rangle)_{AB}$, ($C|AB$cut)

$\langle\Psi_i|\,|\zeta\rangle = 0$, i=0,1

$x_i(a_i)^{00}x_5(a_5)^{00}+x_i(a_i)^{10}x5(a_5)^{01}+y_i(a_i)^{00}x_5(a_5)^{10}+y_i(a_i)^{10}x_5(a_5)^{11}+x_i(a_i)^{01}y_5(a_5)^{00}+ x_i(a_i)^{11}y_5(a_5)^{01} + y_i(a_i)^{10}y_5(a_5)^{10} + y_i(a_i)^{11}y_5(a_5)^{11} = 0$

where i=0,1,2

$\langle\Psi_k|\,|\zeta\rangle = 0$,k=2,3

$x_k(a_k)^{00}x_5(a_5)^{00} + x_k(a_k)^{01}y_5(a_5)^{00} + x_k(a_k)^{10}x_5(a_5)^{10} + x_k(a_k)^{11}y_5(a_5)^{10} + y_k(a_k)^{00}x_5(a_5)^{01} + y_k(a_k)^{01}y_5(a_5)^{01} + y_k a_k^{10} x_5 a_5^{11} + y_k(a_k)^{11}y_5(a_5)^{11} = 0.$ where k=2,3

$\langle\Psi_4|\,|\zeta\rangle = 0$

$x_4 a_4^{00} x_5 a_5^{00} + x_5 a_5^{11} y_4 a_4^{11} = 0$

new ,we have to consider folowing system whether have solution or not

$$D = \begin{cases} \\ x_i(a_i)^{00}x_5(a_5)^{00} + x_i(a_i)^{10}x5(a_5)^{01} + y_i(a_i)^{00}x_5 \\ (a_5)^{10} + y_i(a_i)^{10}x_5(a_5)^{11} + x_i(a_i)^{01}y_5(a_5)^{00} + \\ x_i(a_i)^{11}y_5(a_5)^{01} + y_i(a_i)^{10}y_5(a_5)^{10} + y_i(a_i)^{11}y_5(a_5)^{11} = 0, i = 0,1 \\ x_k(a_k)^{00}x_5(a_5)^{00} + x_k(a_k)^{01}y_5(a_5)^{00} + x_k(a_k)^{10}x_5(a_5)^{10} + x_k(a_k)^{11}y_5(a_5)^{10} + \\ y_k(a_k)^{00}x_5(a_5)^{01} + y_k(a_k)^{01}y_5(a_5)^{01} + y_k a_k^{10} x_5 a_5^{11} + y_k(a_k)^{11}y_5(a_5)^{11} = 0, k = 2,3 \\ x_4 a_4^{00} x_5 a_5^{00} + x_5 a_5^{11} y_4 a_4^{11} = 0 \end{cases} \tag{42}$$