reference concepts lemma6.Let there be given a set of product vectors $B = |\Psi_x\rangle \otimes |\Psi_x\rangle_x$ from $C^m \otimes C^n$ with cardinality $|B| \geq m+n-1$. if any m-tuple of vectors $|\Psi_x\rangle$ span $C^m$and any n-tuple of vectors $|\Psi_x\rangle$ span C^n,then there is no product vector in the orthocomplement of span B.

while we care about linear indepence of the coordinates,at the same time we require the condition dim span B< d^N to hold ,that is ,the resulting GES to be nonempty.

our cnsideration In our case we consider multiple qubit Hilbert spaces, i.e.,$H_{2^3} := (C^2)^{\otimes 3}$

$$F = \left\{ |\Psi\rangle \,|\, |\Psi\rangle = (1, a + b\alpha + c\alpha^2)_A \otimes (1, A + B\alpha + C^2)_B \otimes (1, x + y\alpha + Z\alpha^2)_C \ |\alpha \in C \right\}$$

where a,b,c,A,B,C,XY,Z$\in \{-1, 0, 1\}$

case1.

	a	b	c	A	B	C	X	Y	Z	dim GES
0	√		√	√		√	√		√	0
1		√			√			√		

Table 1:

F=$\{|\Psi\rangle \,|\, |\Psi\rangle = (1, \alpha)_A \otimes (1, \alpha)_B \otimes (1, \alpha)_C \ |\alpha \in C\}$
$AB : (1, \alpha)_A \otimes (1, \alpha)_B$ =$(1, \alpha, \alpha, \alpha^2)$, number of independed linear poynomilas 3
Ac: $(1, \alpha)_A \otimes (1, \alpha)_C$ =$(1, \alpha, \alpha, \alpha^2)$, number of independed linear poynomilas 3
BC: $(1, \alpha)_B \otimes (1, \alpha)_C$ =$(1, \alpha, \alpha, \alpha^2)$, number of independed linear poynomilas 3

from lemma 6,due to construct GES ,the dimention of local part at least one part have same dimetion with local dimention d.for that reason case 1 not generate GES.

Notition: from case 1,we could conclude all the symetric partition could not generate GES.

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case2.

	a	b	c	A	B	C	X	Y	Z	dim GES
0	√		√		√		√			2
1		√		√		√		√	√	

Table 2:

F=$\{|\Psi\rangle \,|\, |\Psi\rangle = (1, \alpha)_A \otimes (1, 1 + \alpha^2)_B \otimes (1, \alpha + \alpha^2)_C \ |\alpha \in C\}$
$AB : (1, \alpha)_A \otimes (1, 1 + \alpha^2)_B$ =$(1, \alpha, 1 + \alpha^2, \alpha + \alpha^3)$, number of independed linear poynomilas 4

AC: $(1,\alpha)_A \otimes (1,\alpha+\alpha^2)_C$ =$(1,\alpha,\alpha+\alpha^2,\alpha^2+\alpha^3)$, number of independed linear poynomilas 4
BC: $(1,1+\alpha^2)_B \otimes (1,\alpha+\alpha^2)_C$ =$(1,1+\alpha^2,\alpha+\alpha^2,\alpha+\alpha^2+\alpha^3+\alpha^4)$, number of independed linear poynomilas 4

$|\Psi\rangle = (1,\alpha,1+\alpha^2,\alpha+\alpha^3)\otimes(1,\alpha+\alpha^2) = (1,1+\alpha^2,\alpha,\alpha^2+\alpha^3,\alpha+\alpha^2,\alpha^2+\alpha^3,\alpha^2+\alpha^3+\alpha^4+\alpha^5,\alpha+\alpha^3,\alpha^2+\alpha^3+\alpha^4+\alpha^6)_{ABC}$, dim 6

case 2 could generate dimention of 2 GES,which means there is have two vectors orthogonal to all the vectors inB.

$$(0,-1,1,1,0,0,0,0)$$
$$(0,1,0,-1,0,0,0,0)$$

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case3.

	a	b	c	A	B	C	X	Y	Z	dim GES
0	$\surd$	$\surd$								2
1			$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	

Table 3:

F=$\{|\Psi\rangle \mid |\Psi\rangle = (1,\alpha)_A \otimes (1,1+\alpha+\alpha^2)_B \otimes (1,1+\alpha+\alpha^2)_C \; |\alpha\in C\}$
$AB: (1,\alpha)_A \otimes (1,1+\alpha+\alpha^2)_B$ =$(1,\alpha,1+\alpha+\alpha^2,\alpha+\alpha^2+\alpha^3)$, number of independed linear poynomilas 4
Ac: $(1,\alpha)_A \otimes (1,1+\alpha+\alpha^2)_C$ =$(1,\alpha,1+\alpha+\alpha^2,\alpha+\alpha^2+\alpha^3)$, number of independed linear poynomilas 4
BC: $(1,1+\alpha+\alpha^2)_B \otimes (1,1+\alpha+\alpha^2)_C$ =$(1,1+\alpha+\alpha^2,1+\alpha+\alpha^2,(1+\alpha+\alpha^2)^2)$, number of independed linear poynomilas 3

$|\Psi\rangle = (1,\alpha)_A \otimes (1,1+\alpha+\alpha^2)_B \otimes (1,1+\alpha+\alpha^2)_C = (1,\alpha,1+\alpha+\alpha^2,\alpha+\alpha^2+\alpha^3,\alpha+\alpha^2+\alpha^3,(1+\alpha+\alpha^2)^2,\alpha+2\alpha^2+3\alpha^4+2\alpha^3+\alpha^5,1+\alpha+\alpha^2)_{ABC}$, dim 6

case 3 could generate dimention of 2 GES,which means there is have two vectors orthogonal to all the vectors inB.

$$(0,0,1,,0,0,0,-1)$$
$$(0,0,0,1,-1,0,0,0)$$

Notion:if in the three parties any have diffrent coordinates or any two of parites symeteric ,it could generate GES dimention of 2.
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case 4

	a	b	c	A	B	C	X	Y	Z	dim GES
0			$\surd$			$\surd$			$\surd$	0
-1	$\surd$	$\surd$		$\surd$	$\surd$		$\surd$	$\surd$		

Table 4:

$|\Psi\rangle = (1, -1-\alpha)_A \otimes (1, -1-\alpha)_B \otimes (1, -1-\alpha)_C$
AB:$(1, -1-\alpha, -1-\alpha, 1+2\alpha+\alpha^2)$,dim 3
AC:$(1, -1-\alpha, -1-\alpha, 1+2\alpha+\alpha^2)$,dim 3
BC:$(1, -1-\alpha, -1-\alpha, 1+2\alpha+\alpha^2)$,dim 3
case 4 not generate GES.

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case5

	a	b	c	A	B	C	X	Y	Z	dim GES
0			$\surd$	$\surd$			$\surd$			2
-1	$\surd$	$\surd$			$\surd$	$\surd$		$\surd$	$\surd$	

Table 5:

$|\Psi\rangle = (1, -1-\alpha)_A \otimes (1, -\alpha-\alpha^2)_B \otimes (1, -\alpha-\alpha^2)_C$
AB:$(1, -1-\alpha, -\alpha-\alpha^2, \alpha+2\alpha+\alpha^2)$,dim 4
AC:$(1, -1-\alpha, -\alpha-\alpha^2, \alpha+2\alpha+\alpha^2)$,dim 4
BC:$(1, -\alpha-\alpha^2, -\alpha-\alpha^2, \alpha^2+2\alpha^3+\alpha^4)$,dim 4

$|\Psi\rangle = (1, -1-\alpha, -\alpha-\alpha^2, \alpha+2\alpha+\alpha^2, -\alpha-\alpha^2, \alpha+2\alpha^2+\alpha^3, \alpha^2+2\alpha^3+\alpha^4, -\alpha^2-3\alpha^3-3\alpha^4-\alpha^5)$,dim 6
then ,it could generate dimention of 2 GES.

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case6

	a	b	c	A	B	C	X	Y	Z	dim GES
0				$\surd$	$\surd$			$\surd$		2
-1	$\surd$	$\surd$	$\surd$			$\surd$	$\surd$		$\surd$	

Table 6:

$|\Psi\rangle = (1, -1-\alpha-\alpha^2)_A \otimes (1, -\alpha^2)_B \otimes (1, -1-\alpha^2)_C$
AB:$(1, -1-\alpha-\alpha^2, -\alpha^2, \alpha^2+\alpha^3+\alpha^4)$,dim 4
AC:$(1, -1-\alpha-\alpha^2, -1-\alpha^2, 1+2\alpha^2+\alpha+\alpha^3+\alpha^4)$,dim 4
BC:$(1, -\alpha^2, -1-\alpha^2, \alpha^2+\alpha^4)$,dim 4

$|\Psi\rangle = (1, -1-\alpha-\alpha^2, -\alpha^2, \alpha^2+\alpha^3+\alpha^4, -1-\alpha^2, 1+\alpha+2\alpha^2+\alpha^3+\alpha^4, \alpha^2+\alpha^4, -\alpha^2-\alpha^3-2\alpha^4-\alpha^5-\alpha^6)$,dim 6
it could generate dimention of 2 GES.

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case 7
$|\Psi\rangle = (1, -1-\alpha^2)_A \otimes (1, -1+\alpha^2)_B \otimes (1, -1+\alpha^2)_C$

	a	b	c	A	B	C	X	Y	Z	dim GES
0		$\surd$			$\surd$			$\surd$		2
1						$\surd$			$\surd$	
-1	$\surd$		$\surd$	$\surd$			$\surd$			

Table 7:

AB:$(1, -1-\alpha^2, -1+\alpha^2, 1+\alpha^4)$,dim 4
AC$(1, -1-\alpha^2, -1+\alpha^2, 1+\alpha^4)$,dim 4
BC:$(1, -1+\alpha^2, -1+\alpha^2, 1+2\alpha^2+\alpha^4)$,dim 3
$|\Psi\rangle = (1, -1-\alpha^2)_A \otimes (1, -1+\alpha^2)_B \otimes (1, -1+\alpha^2)_C = (1, -1-\alpha^2, -1+\alpha^2, 1+\alpha^4, -1+\alpha^2, 1+\alpha^4, 1+2\alpha^2+\alpha^4, -1-\alpha^4+\alpha^2+\alpha^4)$,dim 6
it could generate dimention of 2 GES.

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conclution,in A,B,C partition if all the party have same coordinater then the B not generate GES.if any of two part have same coordinates or all the partites have different coordinates ,then B could genreate dimetion of 2 GES.