

MIDDLE EAST TECHNICAL UNIVERSITY

EE568 - SELECTED TOPICS ON ELECTRICAL MACHINES

ELECTRICAL ELECTRONICS ENGINEERING

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# Torque in a Variable Reluctance Machine

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Project 1

<http://keysan.me/ee568/>



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## Introduction

Reluctance, inductance and torque equations are derived for variable reluctance motor in this report. Finite element analysis (FEA) is carried out for given parameters and results are compared with analytical results. FEMM is used for FEA. Bonus questions are also tried to be handled.

### Question 1

a)

Since core permeability ( $\mu_r$ ) is assumed to be infinite, reluctance of core is zero, as it can be easily calculated in Eq-1 where  $l$  is length,  $\mu$  is permeability and  $A$  is cross section area. Therefore, total reluctance of flux path comes into existence only because of air-gaps. There are two air-gaps around rotor. In this report, each length of air-gap is called 'g' and the depth of core is called 'h'. To calculate reluctance of air gap, cross section area of flux path needs to be evaluated. The area is function of angle to stator which rotor has.

$$\mathcal{R} = \frac{l}{\mu_r \mu_0 A} \quad (1)$$

Flux flows through over more permeable path. Flux chooses the path which has 0.5 mm air-gap until certain angle that calculated as  $\arcsin(7.5/12.5)$ . Because of rotor geometry, air-gap is 2.5 mm from  $\arcsin(7.5/12.5)$  to  $\arccos(7.5/12.5)$ . The certain angle is obtained with help of Fig-1. Accordingly, reluctance needs to be defined as partial function.

For this system,  $l$  is  $2g$  and  $A$  can be obtained with Eq-2 and generalized function of reluctance is given in Eq-3.  $h$  is already given as 20 mm. When  $\alpha$  is smaller than  $\arcsin(7.5/12.5)$ ,  $r$  is 12 mm and  $g$  is 0.5 mm, but when  $\alpha$  is between  $\arcsin(7.5/12.5)$  and  $\arccos(7.5/12.5)$ ,  $r$  is 10 mm and  $g$  is 2.5 mm.

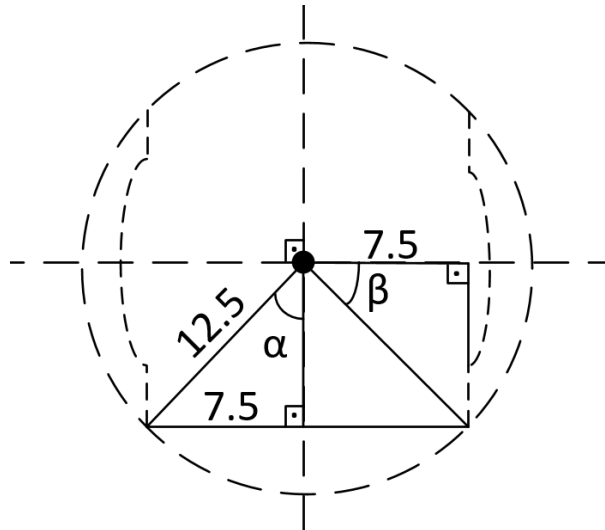


Figure 1: The certain angle that changes air-gap length

$$A = h(r + 0.5g)\theta \quad (2)$$

$$\mathcal{R} = \frac{2g}{\mu_r h(r + 0.5g)\theta} \quad (3)$$

Inductance is function of reluctance and number of turn. The relationship between them is given in Eq-4. If Eq-3 is placed into Eq-4, Eq-5 is derived as a function of  $\theta$ .

$$L = \frac{N^2}{\mathcal{R}} \quad (4)$$

$$L(\theta) = \frac{\mu_0 N^2 h(r + 0.5g)\theta}{2g} \quad (5)$$

b)

Torque is given in Eq-6. If Eq-5 is placed in the Eq-6, torque equality is obtained as shown in Eq-7. Generated torque as a function of rotation is plotted in Fig-3. It can also be verified by using inductance as a function of rotation which is plotted in Fig-2. There is ratio between derivative of inductance and generated torque which can be observable in Eq-6.

$$T = \frac{1}{2} i^2 \frac{dL(\theta)}{d\theta} \quad (6)$$

$$T = \frac{\mu_0 N^2 i^2 h(r + 0.5g)}{4g} \quad (7)$$

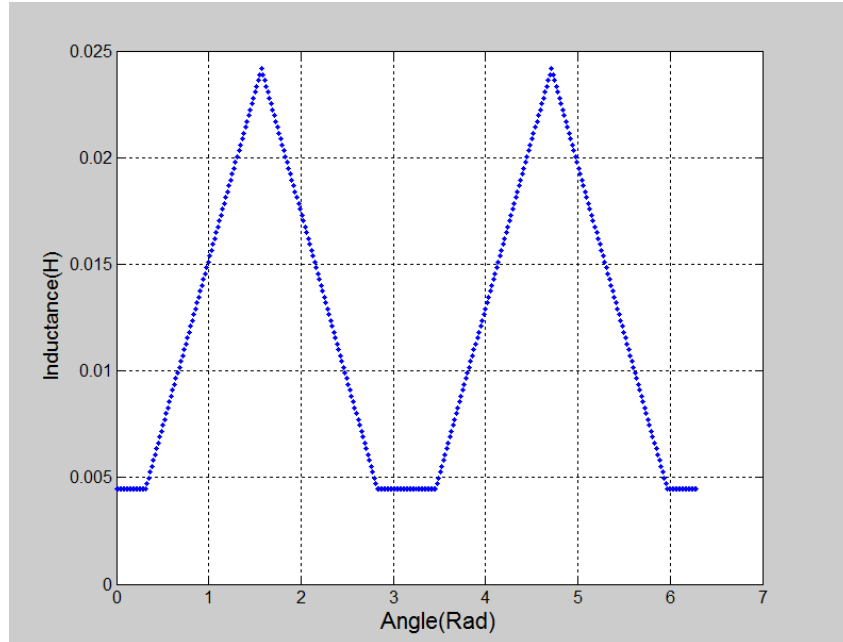


Figure 2: Inductance as function rotation

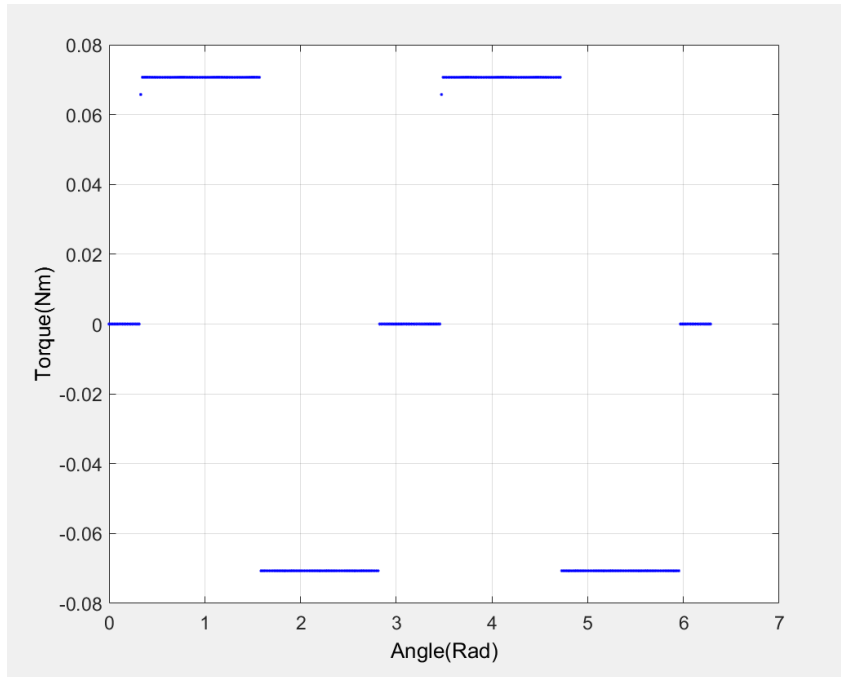


Figure 3: Torque as function rotation

c)

## Question 2

a)

b)

Table 1: Inductance and energy for linear material

Rotational Position	Inductance(H)	Stored Energy(J)
0°	0.0305	0.0015
45°	0.0200	0.0028
90°	0.0103	0.0052

c)

Table 2: Torque for linear material

Rotational Position	Torque(Nm)
0°	0.00006
45°	0.0716
90°	0.00013

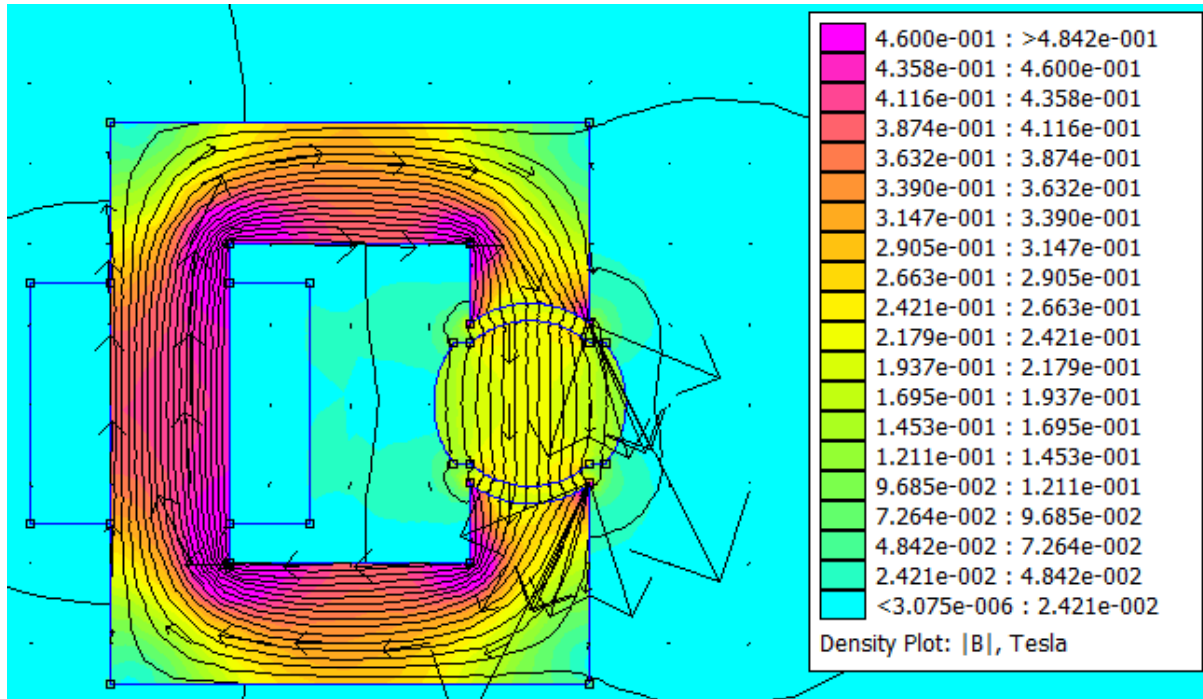


Figure 4: Linear material for  $\alpha=0$

### Question 3

- a)  
b)

Table 3: Inductance and energy for nonlinear material

Rotational Position	Inductance(H)	Stored Energy(J)
0°	0.0082	0.0033
45°	0.0089	0.0058
90°	0.0093	0.0072

- c)

Table 4: Torque for linear material

Rotational Position	Torque(Nm)
0°	0.000008
45°	0.0185
90°	0.0.00008

### Conclusion

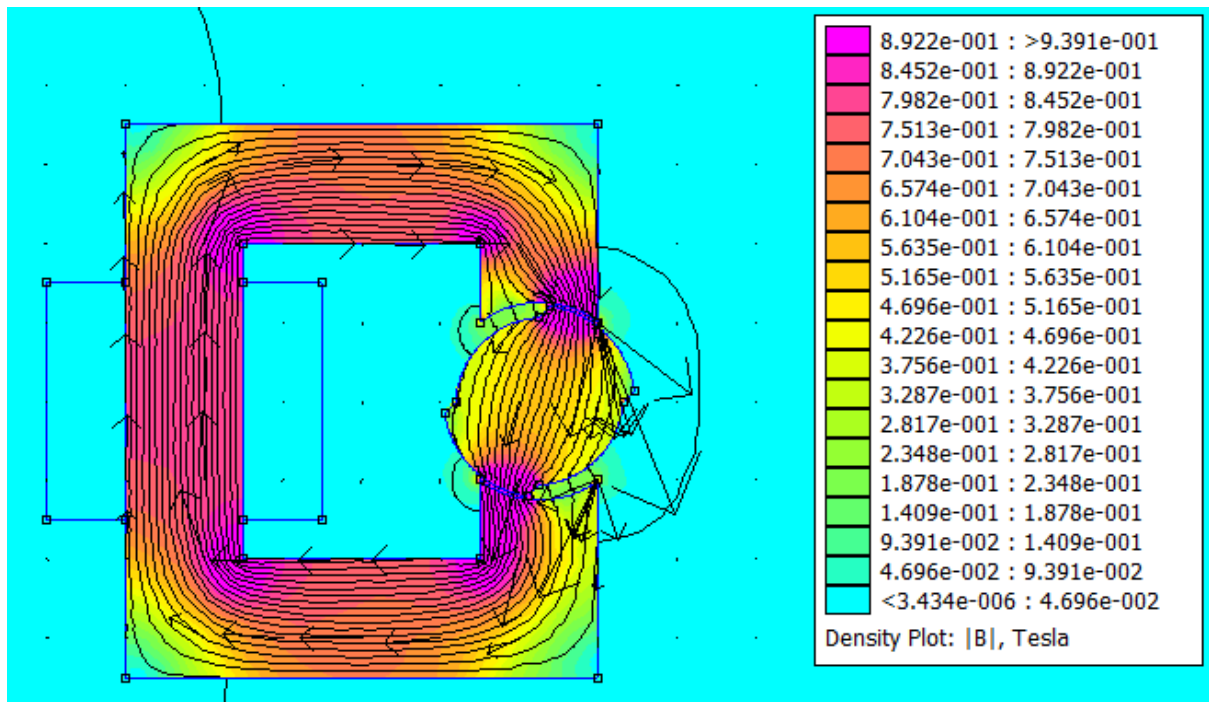


Figure 5: Linear material for  $\alpha=45$

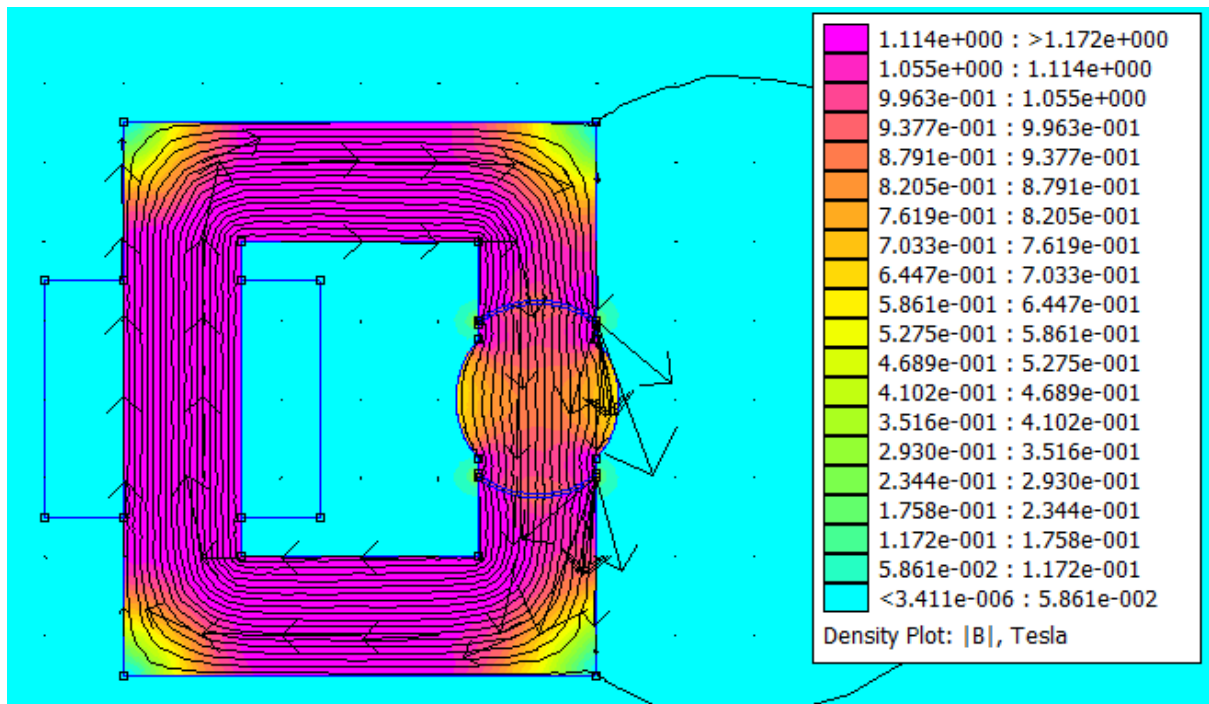


Figure 6: Linear material for  $\alpha=90$

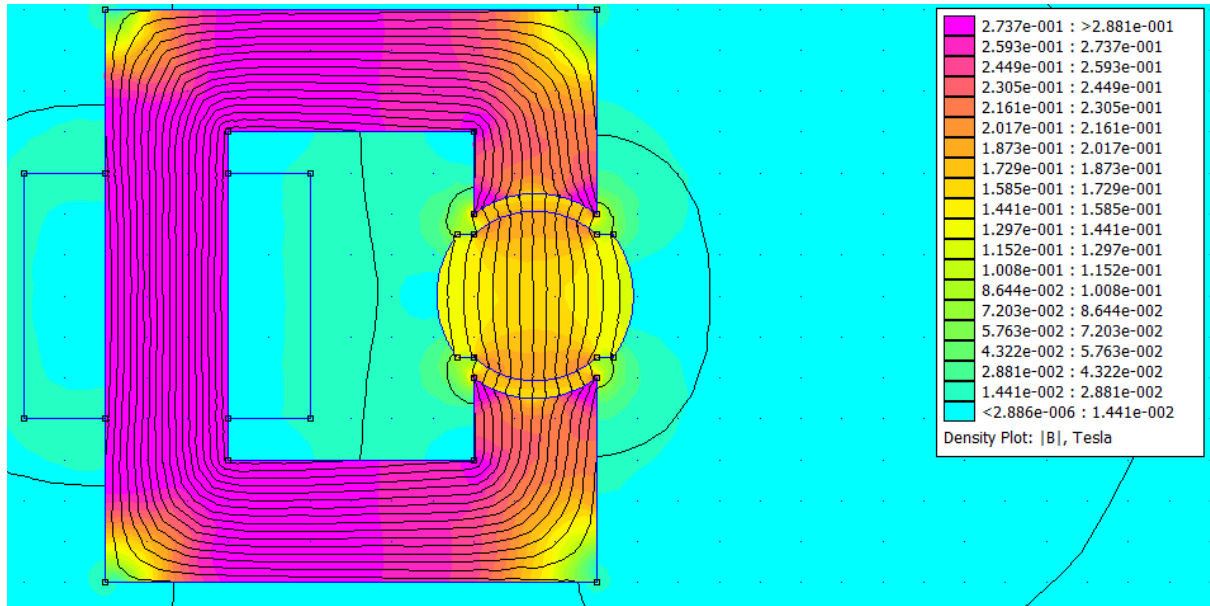


Figure 7: Nonlinear material for  $\alpha=0$

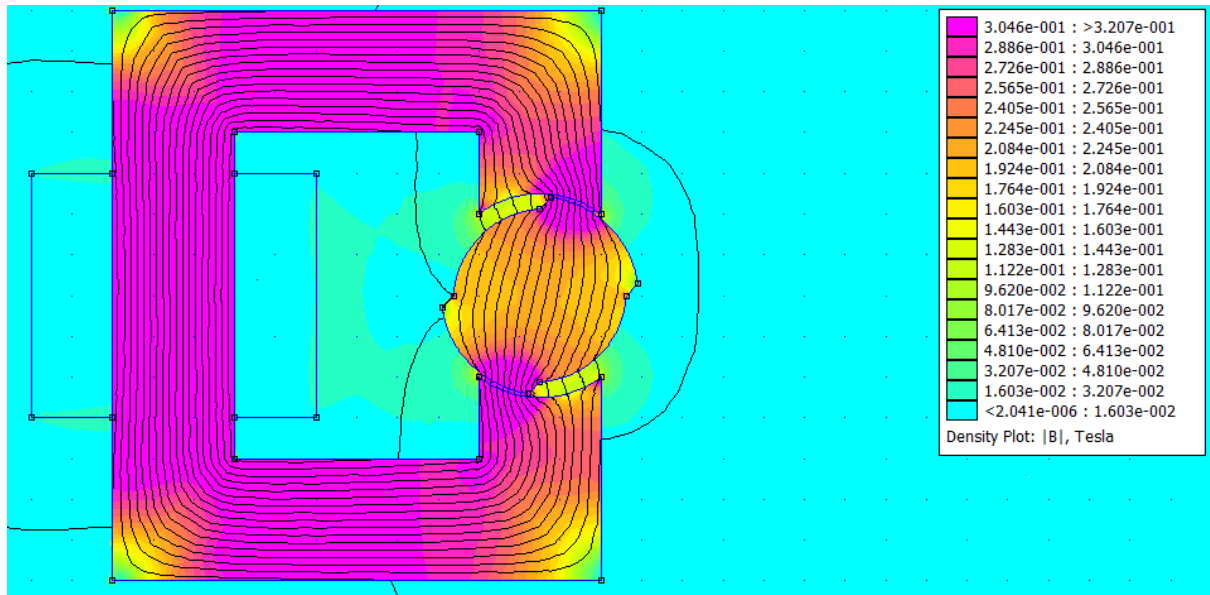


Figure 8: Nonlinear material for  $\alpha=45$

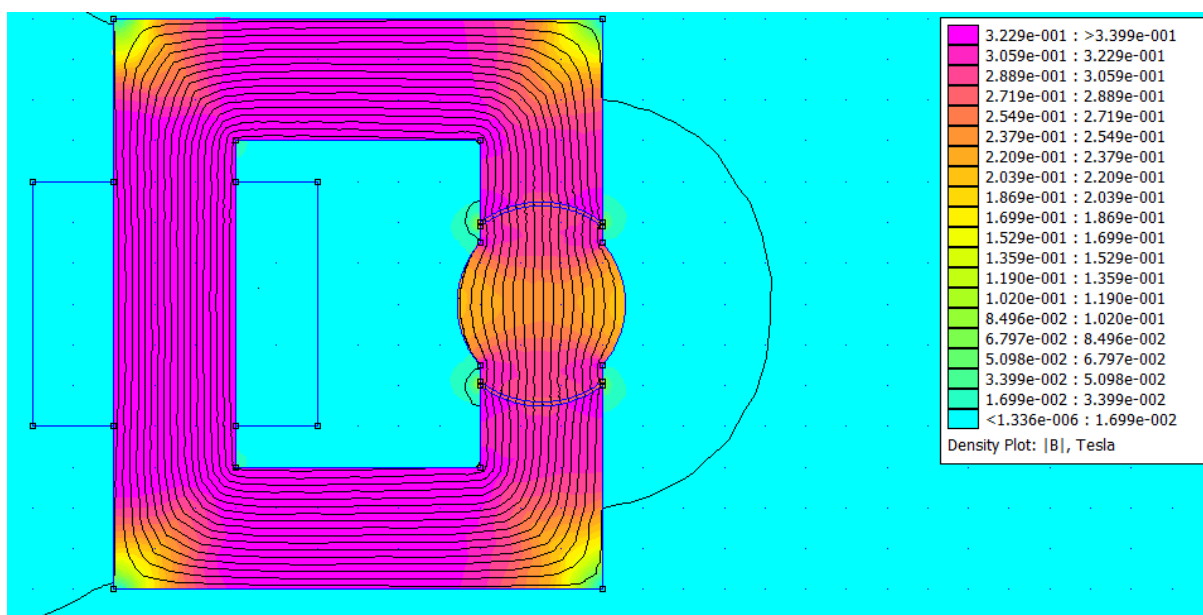


Figure 9: Nonlinear material for  $\alpha=90$