Talis Biomedical Statistics Course - Homework 2

Due: 5 December 2019 11:59 PM

Name:	[your first and last name]
Collaborators:	[list all the people you worked with]
Date:	[date of submission]

By turning in this assignment, I agree by the **Stanford honor code** and declare that all of this is my own work.

The exercises can be found in the DeGroot & Schervish's textbook, available here.

Chapter 1

Problem 1

From DeGroot & Schervish:

(a) Section 1.4: Exercise 1

If $x \in A$, then $x \in B$ since $A \subset B$. However, this would contradict $x \in B^c$. Thus, $x \notin A$ and $x \in A^c$.

(b) Section 1.4: Exercise 6

a. Blue card numbered 2 or 4.

b. Blue card numbered 5 or greater.

c. Any blue card, or red card numbered 1, 2, 3, 4, 6, 8, or 10.

d. Even numbered blue card or red card numbered 2 or 4.

e. A red card numbered 5, 7, or 9.

(c) Section 1.4: Exercise 8

Blood type A is $A \cap B^c$, B is $A^c \cap B$, AB is AB,and O is $A^c \cap B^c$.

Problem 2

From DeGroot & Schervish:

(a) Section 1.5: Exercise 1

1 - 1/5 - 2/5 = 2/5

(b) Section 1.5: Exercise 3

(a) $P(B \cap A^c) = P(B) = 1/2$

(b) We have that $B = A \cup (B \cap A^c) \Rightarrow P(B) = P(A) + P(B \cap A^c)$. Thus, we have that $1/2 = 1/3 + P(B \cap A^c) \Rightarrow P(B \cap A^c) = 1/6$.

(c) Note that $B = (B \cap A) \cup (B \cap A^c) \Rightarrow P(B) = P(B \cap A) + P(B \cap A^c)$. Plugging in 1/2 and 1/8 gives us $P(B \cap A^c) = 3/8$.

(c) Section 1.5: Exercise 4

Let F_A be the event that student A fails, and F_B be the event that student B fails. $P(F_A \cup F_B) = P(F_A) + P(F_B) - P(F_A \cap F_B) = 0.5 + 0.2 - 0.1 = 0.6$

(d) Section 1.5: Exercise 6

$P((F_A \cap F_B^c) \cup (F_A^c \cap F_B)) = P(F_A \cap F_B^c) + P(F_A^c \cap F_B)$. Calcuating each of the probabilities separately, we have $P(F_A \cap F_B^c) = P(F_A) - P(F_A \cap F_B) = 0.5 - 0.1 = 0.4$. Likewise, we have $P(F_A \cap F_B^c) = 0.2 - 0.1 = 0.1$.

(e) Section 1.5: Exercise 14

(a) $P(AB) = 1 - P(A \cup B \cup O) = 1 - (0.5 + 0.34 + 0.12) = 0.04$. The probability of reacting with antigen A is $P(A \cup AB) = 0.34 + 0.04 = 0.38$. The probability of reacting with antigen B is $P(B \cup AB) = 0.12 + 0.04 = 0.16$.

(b) The probability of reacting with both antigens is $P(AB) = 0.04$.

Problem 3

From DeGroot & Schervish:

(a) Section 1.6: Exercise 1

Let result of the first die be x. The second die will result in either an odd or an even sum, irrespective of what the first die's number is (e.g. if x is even than the second die can be either 1, 3, or 5. If x is odd, then the second die can be either 2, 4, or 6). Furthermore, the probability of the first die is uniform. Consequently, $P(\text{Odd outcome}) = \sum_x P(x)P(\text{Odd outcome}|x) = \sum_x 1/6(1/2) = 1/2$.

(b) Section 1.6: Exercise 2

The sum will either be odd or even. We know from Exercise 1 that $P(\text{Odd outcome}) = 1/2$. Therefore, $P(\text{Even outcome}) = 1 - P(\text{Odd outcome}) = 1/2$.

(c) Section 1.6: Exercise 8

(a) S is the cartesian product of the sample space for flipping a coin (i.e. Heads, Tails) and for rolling a die (i.e. 1,2, ..., 6). Our sample space is therefore $S = \{(Head, 1), (Tail, 1), ..., (Head, 6), (Tail, 6)\}$.

(b) All outcomes are equally likely. Three of the 12 possible outcomes have head and an odd number, so the probability is $3/12 = 1/4$.

Problem 4

From DeGroot & Schervish:

(a) Section 1.7: Exercise 1

This is just the cartesian product of the 'day of week' sample space (i.e. Sunday, Monday, ...) and the 'leap year' sample space (i.e. Yes, No). Thus, there are a total of $7 * 2 = 14$ different possible calendars.

(b) Section 1.7: Exercise 4

24.

(c) Section 1.7: Exercise 8

There are 7^5 different possible combinations that the 5 passengers can get off the elevator. Let us consider the number of ways that the passengers can all get off on different floors. The first passenger can get off on any of the 7 floors. Given the selected floor for the first passenger, the second passenger can get off on any of the remaining 6 floors. This gives us 7*6 different ways that the two passengers can get off on different floors. Continuing on to the third passenger, there are 5 different ways of getting off on a floor that wasn't chosen by the first two passengers. We continue until the last passenger, whom has only 3 ways of getting off on a different floor. Thus, our probability is $7 * 6 * 5 * 4 * 3/(7^5) = 360/2401$.

(d) Section 1.7: Exercise 10

(a) $r/100$.

(b) $r/100$. You can imagine this as randomly mixing the balls into a random order, and then just counting up to the 50^{th} index. Given that we don't know anything else about the first 49 balls, the probability doesn't change. This would be different, however, if say we were told the colors of the first 49 balls.

(c) $r/100$. The explaination is the same as the previous part.

Problem 5

From DeGroot & Schervish:

(a) Section 1.8: Exercise 1

We can think of this as different ways of choosing 10 houses from 20 (since the remaining 10 would be defaulted to the remaining pollster), resulting in $\binom{20}{10}$ combinations.

(b) Section 1.8: Exercise 2

$\binom{93}{30} = \frac{93!}{30!63!}$, while $\binom{93}{31} = \frac{93!}{31!62!}$. To see which is bigger, we can simply take the ratio and see if it's greater or less than 1. Thus, since $\frac{93!}{30!63!} \div \frac{93!}{31!62!} = 31/63$ which is < 1, we know that $\binom{93}{31}$ is larger.

(c) Section 1.8: Exercise 6

Of the n available seats, there are only $n-1$ ways that A and B can occupy such that they are sitting next to each other. We also know that there are $\binom{n}{2}$ ways that 2 people can be arranged in the n seats. Thus, the probability that they sit together is $(n-1)/\binom{n}{2} = 2/n$.

(d) Section 1.8: Exercise 8

We can approach this similar to the last question, with the exception that it is a circle. This means that there is no start or end, meaning that there are n ways of having k people sit adjacent to one another. The overall number of ways that k people can sit in n chairs is $\binom{n}{k}$. Thus, the probability is $n/\binom{n}{k} = \frac{(n-k)!k!}{(n-1)!}$.

(e) Section 1.8: Exercise 12

There are $\binom{35}{10}$ ways of choosing 10 people for one team (resulting in the remaining people being on the other team). If A and B are both on the first team, there are a remaining 8 spots to fill and $\binom{33}{8}$ ways of filling them. Conversely, if A and B are on the second team, then there are $\binom{33}{10}$ ways of putting the remaining people on the first team. The overall probability is then

$$\frac{\binom{33}{8} + \binom{33}{10}}{\binom{35}{10}} = 0.5798 \quad (1)$$

Problem 6

From DeGroot & Schervish:

(a) Section 1.9: Exercise 4

There are $\binom{10}{3,3,2,1,1}$ ways of arranging the 10 instances of the 5 letters, and only one way in which they spell '*statistics*'. The probability is therefore $1/\binom{10}{3,3,2,1,1}$.

(b) Section 1.9: Exercise 6

There are 6^7 total ways that the seven dice can land. To get the number of ways that 6 different numbers can show, note firstly that with 7 dice one number will be repeated. Thus, if we just consider, for example, that the number 1 shows twice, we have $\binom{7}{2,1,1,1,1,1}$ ways this can occur. Continuing the argument for $2, 3, ..., 6$ we similarly have $\binom{7}{2,1,1,1,1,1}$ ways that this can occur. The probability is therefore $6 * \binom{7}{2,1,1,1,1,1}/6^7 = \frac{7!}{2(6^6)}$.