

Talis Biomedical Statistics Course - Homework 0

Due: 20 November 2019 11:59 PM

Name: [your first and last name]
Collaborators: [list all the people you worked with]
Date: [date of submission]

By turning in this assignment, I agree by the **Stanford honor code** and declare that all of this is my own work.

Algebra

Problem 1

Simplify/factor the following expressions.

(a) Simplify $5z^2(7z^2 + 3z - 1)$

$$5z^2(7z^2 + 3z - 1) = 5z^2 * 7z^2 + 5z^2 * 3z - 5z^2 * 1 \quad (1)$$

$$= 35z^4 + 15z^3 - 5z^2 \quad (2)$$

(b) Simplify $(1 + x)(x^2 - 5x - 6)$

$$(1 + x)(x^2 - 5x - 6) = (1 * x^2 - 1 * 5x - 1 * 6) + (x * x^2 - x * 5x - x * 6) \quad (3)$$

$$= x^2 - 5x - 6 + x^3 - 5x^2 - 6x \quad (4)$$

$$= x^3 - 4x^2 - 11x - 6 \quad (5)$$

(c) Simplify $(x + 2x^2)^2$

$$(x + 2x^2)^2 = (x + 2x^2) * (x + 2x^2) \quad (6)$$

$$= x^2 + 2x^3 + 2x^3 + 4x^4 \quad (7)$$

$$= 4x^4 + 4x^3 + x^2 \quad (8)$$

(d) Simplify $\frac{2x^3 - x^2 - 12}{x + 3}$

$$\frac{2x^3 - x^2 - 12}{x + 3} = \frac{(x + 3)(2x^2 - 7x + 21) - 75}{x + 3} \quad (9)$$

$$= 2x^2 - 7x + 21 - \frac{75}{x + 3} \quad (10)$$

(e) Factor $(x - 5)^2 + 2y^3(x - 5) + y^6$

Let $U = (x - 5)$ and $V = y^3$. Then

$$(x - 5)^2 + 2y^3(x - 5) + y^6 = ((x - 5) + y^3)^2 \quad (11)$$

$$= (U + V)^2 \quad (12)$$

$$= (x - 5 + y^3)^2 \quad (13)$$

(f) Factor $16x^3 + 24x^2 + 9x$

$$16x^3 + 24x^2 + 9x = x(16x^2 + 24x + 9) \quad (14)$$

$$= x((4x)^2 + 2 \cdot 4 \cdot 3x + 3^2) \quad (15)$$

$$= x(4x + 3)^2 \quad (16)$$

(g) Factor $x^2 - 49y^2$

$$x^2 - 49y^2 = (x + 7y)(x - 7y) \quad (17)$$

$$(18)$$

(h) The polynomial $3x^3 - 20x^2 + 37x - 20$ has a known factor of $(x - 4)$. Factor it.

$$3x^3 - 20x^2 + 37x - 20 = (x - 4)(3x^2 - 8x + 5) \quad (19)$$

Problem 2

Logarithms and exponents.

(a) $3^a = \sqrt[5]{3^2}$. Solve for a .

$$(3^a)^5 = (\sqrt[5]{3^2})^5 \quad (20)$$

$$3^{5a} = 3^2 \quad (21)$$

$$\implies 5a = 2 \quad (22)$$

$$a = 2/5 \quad (23)$$

(b) $26^{9x+5} = 1$. Solve for x .

$26^0 = 1$. Therefore, we want $9x + 5 = 0$. Solving for x gives us $x = -5/9$.

(c) $2^{3x+5} = 64^{x-7}$. Solve for x .

$$2^{3x+5} = 64^{x-7} \quad (24)$$

$$= (2^6)^{x-7} \quad (25)$$

$$= 2^{6x-42} \quad (26)$$

Therefore, we want to have $3x + 5 = 6x - 42$. Solving for x gives us $x = 47/3$.

(d) Rewrite $\log_3 27x$ as a sum of a constant and (a function of) a variable.

$$\log_3 27x = \log_3 27 + \log_3 x \quad (27)$$

$$= 3 + \log_3 x \quad (28)$$

(e) Rewrite $\log_5 \frac{25^x}{y}$ as a sum of functions of two variables.

$$\log_5 \frac{25^x}{y} = \log_5 25^x - \log_5 y \quad (29)$$

$$= 2x - \log_5 y \quad (30)$$

(f) Solve $\log_c 16 \cdot \log_2 c$ where c is an unknown constant.

$$\log_c 16 \cdot \log_2 c = \frac{\log_{10} 16}{\log_{10} c} \cdot \frac{\log_{10} c}{\log_{10} 2} \quad (31)$$

$$= \frac{\log 16}{\log 2} \quad (32)$$

$$= \log_2 16 = 4 \quad (33)$$

(g) Solve the equation for t and express the answer in terms of base 10 logarithm.

$$10^{2t-3} = 7 \quad (34)$$

$$\log_{10} 7 = 2t - 3 \quad (35)$$

$$\log_{10} 7 + 3 = 2t \quad (36)$$

$$\frac{\log_{10} 7 + 3}{2} = t \quad (37)$$

Multivariable calculus

Problem 3

Calculate the following gradients.

- (a) Let $f(x, y) = x^2 - xy$. What is $\nabla f(x, y)$?

$$\begin{bmatrix} 2x - y \\ -x \end{bmatrix} \quad (38)$$

- (b) What is the gradient of $f(x, y) = -x^4 + 4(x^2 - y^2) - 3$?

$$\begin{bmatrix} -4x^3 + 8x \\ -8y \end{bmatrix} \quad (39)$$

- (c) What is the gradient of $f(x, y, z) = x - xy + z^2$?

$$\begin{bmatrix} 1 - y \\ -x \\ 2z \end{bmatrix} \quad (40)$$

- (d) Find $\frac{\partial}{\partial t}(\cos(t))^2 \sin(t)$.

We begin by defining $x(t) = \cos(t)$ and $y(t) = \sin(t)$. Then $f(x, y) = x^2 y$. We can then apply the multivariable chain rule to get the answer, i.e.

$$\frac{\partial}{\partial t} f(x, y) = \frac{\partial}{\partial y} f(x, y) \cdot \frac{\partial y}{\partial t} + \frac{\partial}{\partial x} f(x, y) \cdot \frac{\partial x}{\partial t} \quad (41)$$

$$= \cos^2(t) \cos(t) + 2 \cos(t) \sin(t)(-\sin(t)) \quad (42)$$

$$= \cos^3(t) - 2 \cos(t) \sin^2(t) \quad (43)$$

- (e) Find the Jacobian of $\begin{bmatrix} f_1(x, y) \\ f_2(x, y) \end{bmatrix} = \begin{bmatrix} x + \sin(y) \\ y + \sin(x) \end{bmatrix}$

$$\mathbf{J}_F = \begin{bmatrix} 1 & \cos(y) \\ \cos(x) & 1 \end{bmatrix} \quad (44)$$

- (f) Let $f(a, b, c) = \cos(ab) \sin(b) + c$. Evaluate $\frac{\partial f}{\partial a}(\frac{1}{2}, \frac{\pi}{3}, 7)$

$$\frac{\partial}{\partial a} f(a, b, c) = -b \sin(b) \sin(ab) \quad (45)$$

$$\text{and } \frac{\partial f}{\partial a}(\frac{1}{2}, \frac{\pi}{3}, 7) = -\frac{\pi}{3} \sin\left(\frac{\pi}{3}\right) \sin\left(\frac{1}{2} \cdot \frac{\pi}{3}\right) = -\frac{\sqrt{3}}{12} \pi$$

Problem 4

Calculate the following integrals.

(a) $\int x \log(x) dx$

We apply integration by parts. Let $u = \log(x)$ and $dv = x dx$. Then

$$\int u dv = uv - \int v du + c \quad (46)$$

$$= \frac{1}{2} x^2 \log(x) - \frac{1}{2} \int x dx + c \quad (47)$$

$$= \frac{1}{4} x^2 (2 \log(x) - 1) + c \quad (48)$$

(b) $\int x^2 \sin(x) dx$

We apply integration by parts. Let $u = x^2$ and $dv = \sin(x) dx$. Then

$$\int u dv = uv - \int v du + c \quad (49)$$

$$= -x^2 \cos(x) + 2 \int x \cos(x) dx \quad (50)$$

$$= 2x \sin(x) - (x^2 - 2) \cos(x) + c \quad (51)$$

(c) $\int_{0.5}^{2.5} \int_{0.5}^{3.5} \sin(xy) + \frac{6}{5} dx dy$

The inner integral is

$$\int_{0.5}^{3.5} \sin(xy) + \frac{6}{5} dx = \frac{-\cos(xy)}{y} + \frac{6x}{5} \Big|_{0.5}^{3.5} = \frac{-\cos(3.5y)}{y} + \frac{\cos(0.5y)}{y} + \frac{18}{5} \quad (52)$$

The integral of this function does not have a closed form. Consequently, we'd have to approximate it using e.g. the Riemann integral.

(d) $\int_0^1 \int_0^{x^2} x + 2y^2 dy dx$

$$\int_0^1 \int_0^{x^2} x + 2y^2 dy dx = \int_0^1 xy + \frac{2}{3} y^3 \Big|_0^{x^2} dx \quad (53)$$

$$= \int_0^1 x^3 + \frac{2}{3} x^6 dx \quad (54)$$

$$= \frac{x^4}{4} + \frac{2}{21} x^7 \Big|_0^1 \quad (55)$$

$$= \frac{1}{4} + \frac{2}{21} = \frac{29}{84} \quad (56)$$