Talis Biomedical Statistics Course - Homework 8

Due: 5 February 2020 9:00 AM

Name: [your first and last name]
Collaborators: [list all the people you worked with]
Date: [date of submission]

By turning in this assignment, I agree by the **Stanford honor code** and declare that all of this is my own work.

The exercises can be found in the DeGroot & Schervish's textbook, available here.

Problem 1

Section 9.1. Exercise 2.

(a) Recall that the cdf for Y_n is

$$P(Y_n \leq y) = \begin{cases} \left(\frac{y}{\theta}\right)^n & \text{if } 0 < y < \theta \\ 1 & \text{if } y \geq \theta \end{cases} \tag{1}$$

Thus, we have that

$$\pi(\theta) = P(Y_n \leq 1.5) = \begin{cases} 1 & \text{if } \theta \leq 1.5 \\ \left(\frac{y}{\theta}\right)^n & \text{if } \theta > 1.5 \end{cases} \tag{2}$$

(b) The size of the test is

$$\alpha = \sup_{\theta \geq 2} \pi(\theta) = \sup_{\theta \geq 2} \left(\frac{y}{\theta}\right)^n = \left(\frac{1.5}{2}\right)^n = \left(\frac{3}{4}\right)^n \tag{3}$$

Problem 2

Section 9.2. Exercise 2.

(a) Theorem 9.2.1 can be applied with $a = 1$ and $b = 2$. This leads us to accept H_0 if $f_1(x)/f_0(x) < 1/2$. Since $f_1(x)/f_0(x) = 2x$, the procedure is then to accept H_0 if $x < 1/4$ and to reject H_0 if $x \geq 1/4$.

(b) For the procedure,

$$\alpha(\delta) = P(\text{Reject } H_0|f_0) = \int_{1/4}^{1} f_0(x)dx = \frac{3}{4}\beta(\delta) = P(\text{Accept } H_0|f_1) = \int_0^{1/4} f_1(x)dx = \frac{1}{16} \tag{4}$$

Therefore, $\alpha(\delta) + 2\beta(\delta) = 7/8$.

Problem 3

Section 9.4. Exercise 4.

(a) Choosing c_1 and c_2 to be symmetric with respect to the value 0.15, we have that $\pi(0.1|\delta) = \pi(0.2|\delta)$. Accordingly, let $c_1 = 0.15 - k$ and $c_2 = 0.15 + k$. Letting $\mu = 0.1$, the random variable $Z = 5(\bar{X}_n - \mu)$ has a standard normal distribution. Therefore,

$$\begin{aligned} \pi(0.1|\delta) &= P(\bar{X}_n \le c_1|0.1) + P(\bar{X}_n \ge c_2|0.1) && (5) \\ &= P(Z \le 0.25 - 5k) + P(Z \ge 0.25 + 5k) && (6) \\ &= \Phi(0.25 - 5k) + \Phi(-0.25 - 5k) && (7) \end{aligned}$$

We must choose k such that $\pi(0.1|\delta) = 0.07$. Letting $5k = 1.867$ results in

$$\pi(0.1|\delta) = \Phi(-1.617) + \Phi(-2.117) = 0.0529 + 0.0171 = 0.07. \tag{8}$$

Thus, we have that $k = 0.3734$.

Problem 4

Section 9.5. Exercise 2.

Note that letting $\mu_0 = 20$, we can use the U statistic provided by Equation 9.5.2. In our setting, it follows a t-distribution with $n - 1 = 9 - 1 = 8$ degrees of freedom. We therefore have that

$$U = n^{1/2}\frac{\bar{X}_n - \mu_0}{\sigma'} = 2 \tag{9}$$

(a) At a level of significance of 0.05, we reject the null if $U > 1.860$. Therefore, we reject H_0.

(b) We reject the null if $U \le -2.306$ or $Y \ge 2.306$. Therefore, we accept H_0.

(c) Note that a 95% CI corresponds to values of $-2.306 \le U \le 2.306$. Solving for μ_0 gives us $19.694 \le \mu_0 \le 24.306$.

Problem 5

Section 9.6. Exercise 2.

In this exercise, $m = 8, n = 6, \bar{X}_m = 1.5125, \bar{Y}_n = 1.6683, S_X^2 = 0.18075$, and $S_Y^2 = 0.16768$. When $\mu_1 = \mu_2$, the statistic U defined by equation 9.6.3 will have a t distribution with 12 degrees of freedom. We're testing the hypothesis

$$\begin{aligned} H_0: &\quad \mu 1 \geq \mu_2 \\ H_1: &\quad \mu 1 < \mu_2 \end{aligned}$$

As the inequalities are reversed from the ones provided in 9.6.1, we reject H_0 if $U < c$. We can look up c from the table in the back of the book as $c = -1.356$. Plugging in our exercise values for 9.6.3 gives us $U = -1.692$. We therefore reject H_0.

Problem 6

Section 9.7. Exercise 7.

(a) We have $\bar{X}_m = 84/16 = 5.25$ and $\bar{Y}_n = 18/10$. Therefore, $S_1^2 = \sum_{i=1}^{16} X_i^2 - 16(\bar{X}_m^2) = 122$ and $S_2^2 = \sum_{i=1}^{10} Y_i^2 - 16(\bar{Y}_n^2) = 39.6$. It follows that

$$\hat{\sigma}_1^2 = \frac{1}{16} S_1^2 = 7.625 \tag{10}$$

$$\hat{\sigma}_2^2 = \frac{1}{10} S_2^2 = 3.96 \tag{11}$$

(b) If $\sigma_1^2 = \sigma_2^2$, then the following statistic will have an F distribution with 15 and 9 degrees of freedom

$$V = \frac{S_1^2/15}{S_2^2/9} \tag{12}$$

If the test is carried out at the level of significance of 0.05, then H_0 should be rejected if $V > 3.01$. In our exercise, we get $V = 1.848$. We therefore do not reject H_0.