Lecture 1: Introduction, logistics, review

STATS 101: Foundations of Statistics

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About me

Takeda
ONCOLOGY

REMOTE AREA MEDICAL
RAM
VOLUNTEER CORPS

Google

WORST COOKS IN AMERICA

UCSF
University of California
San Francisco

Theta Hat

Cerebral Palsy • Vegetative State • Autism • Downs
Life Expectancy Project
Traumatic Brain Injury • Spinal Cord Injury

NATIONAL REGISTRY — EMERGENCY MEDICAL TECHNICIANS

About me

I'm here to teach you statistics!

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AKA Data science. AKA Big data. AKA Machine learning. AKA Artificial Intelligence. AKA ⟨`Next hype term here`⟩

Statistics

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Lay impression: the collection, summary, and analysis of data of any type.
My impression: The study of variability.

Data Scientist:

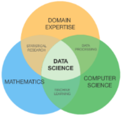
The Sexiest Job of the 21st Century

A.I. Researchers Are Making More Than $1 Million, Even at a Nonprofit

Earn a $1.5 Million Prize at 'Kaggle!' (American Applicants Only, Please.)

- **Once again, data scientist ranks as the best job in America, according to employees.**

Github tops 40 million developers as Python, data science, machine learning popularity surges



Course Series

Current plan

- STATS 101: Foundations of Statistics
- STATS 102: Introduction to Data Analysis
- STATS 103: Introduction to Statistical Learning

Course Series

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- STATS 101: Foundations of Statistics
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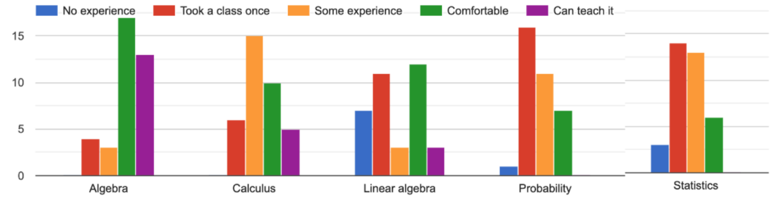
Wishful thinking

- STATS 204, 205, ...: Machine Learning, Deep Learning, etc.
- STATS 214, 215, ...: Survival analysis, clinical trial design, genomics, etc.
- STATS 224, 225, ...: Statistical theory, Semi-parametric efficiency, etc.

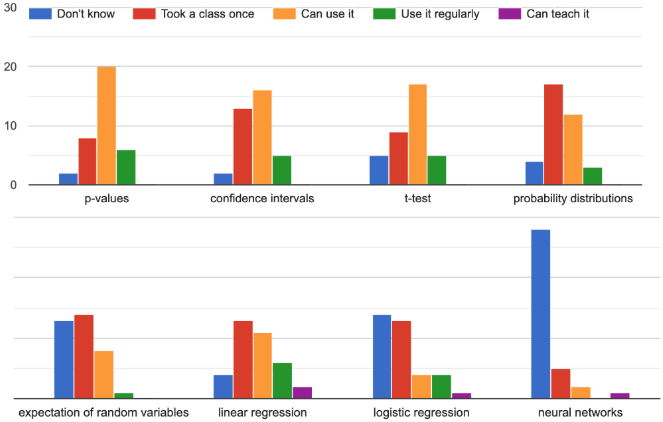
Preliminary survey

Math background



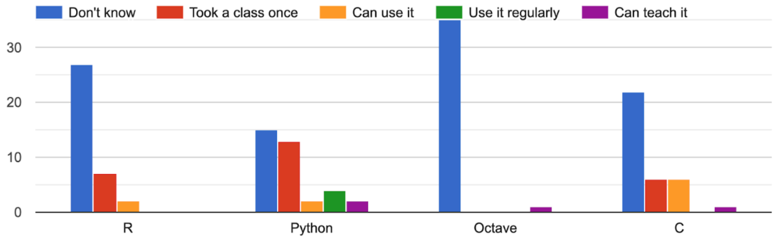
Preliminary survey

Statistical experience



Preliminary survey

Coding experience



Preliminary survey

Summary

- **Math**:
  - Some haven't really used algebra/calculus; many have
  - About half are weak on linear algebra
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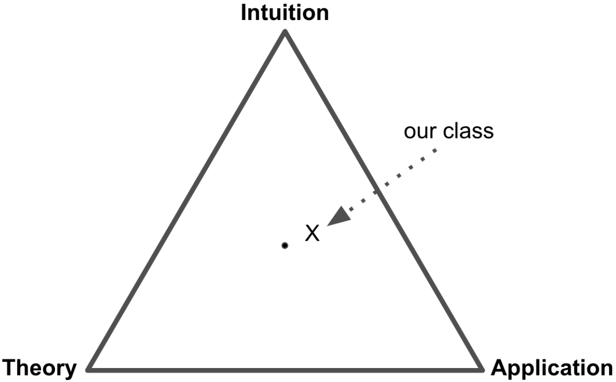
Conclusion: Split the class into two

1. Slower & more friendly class
2. Fast & more in-depth class

Course mixture

Proportion of concepts targeted



Course Information

- **Class website:** *talisstats.github.io*
- **Textbook:** *Introduction to Probability (Grinstead and Snell)*
- **Email policy:** Please use *Piazza* for most questions.
- **Homework:** Should be submitted to *Gradescope*.
- **Exams:** No exams will be administered.

Using math language

Warning: Math uses a lot of symbols!, e.g.

Α Alpha	Β Beta	Γ Gamma	Δ Delta	Ε Epsilon	Ζ Zeta
Η Eta	Θ Theta	Ι Iota	Κ Kappa	Λ Lambda	Μ Mu
Ν Nu	Ξ Xi	Ο Omicron	Π Pi	Ρ Rho	Σ Sigma
Τ Tau	Υ Upsilon	Φ Phi	Χ Chi	Ψ Psi	Ω Omega

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$\ldots$	\ldots	$\cdots$	\cdots	$\vdots$	\vdots
$\ddots$	\ddots	$\prime$	\prime	$\forall$	\forall
$\aleph$	\aleph	$\emptyset$	\emptyset	$\exists$	\exists
$\hbar$	\hbar	∇	\nabla	$\neg$	\neg
$\imath$	\imath	$\surd$	\surd	$\flat$	\flat
$\jmath$	\jmath	$\top$	\top	$\natural$	\natural
ℓ	\ell	$\bot$	\bot	$\sharp$	\sharp
$\wp$	\wp	$\backslash$	\backslash	$\angle$	\angle
$\Re$	\Re	∂	\partial	$\mho$	\mho
$\Im$	\Im	∞	\infty	$\Box$	\Box
$\Diamond$	\Diamond	$\triangle$	\triangle	$\clubsuit$	\clubsuit
$\diamondsuit$	\diamondsuit	$\heartsuit$	\heartsuit	$\spadesuit$	\spadesuit
LaTeX	\LaTeX				

Example:

$$\hat{K}(P_n) \triangleq \arg\min_{k} \mathbb{E}_{B_n} \int L(o, \hat{\Psi}_k(P^0_{n,B_n}))\partial P^1_{n,B_n}(o) \tag{1}$$

Review: algebra

Deals with equations and unknown quantities, e.g.

$$-x^2(x+2)(x+3) \tag{2}$$

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Sometimes written as a polynomial, i.e.

$$\begin{aligned} -x^2(x+2)(x+3) &= -x^2(x^2+5x+6) \qquad (3) \\ &= -x^4-5x^3-6x^2 \qquad (4) \end{aligned}$$

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Many times, will be defined as a function, i.e.

$$f(x) = -x^2(x+2)(x+3) \tag{5}$$

Also can be defined as another variable, i.e.

$$y = -x^2(x+2)(x+3) \tag{6}$$

Review: algebra

System of equations

$$x^2 + y^2 = 25 \tag{7}$$
$$y = x + 1 \tag{8}$$

Review: algebra

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Can solve them by combining/substituting equations, e.g.

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x^2 + y^2 &= 25 && (9)\\
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2x^2 + 2x + 1 &= && (11)
\end{aligned}
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Solving for x gives us

$$
\begin{aligned}
2x^2 + 2x - 24 &= 0 \qquad (12)\\
x^2 + x - 12 &= \qquad (13)\\
(x+4)(x-3) &= \qquad (14)\\
&\implies x = -4 \text{ or } x = 3 \qquad (15)
\end{aligned}
$$

Logarithms

Deals with bases and exponentials, e.g.

$$\log_b a = x \tag{16}$$

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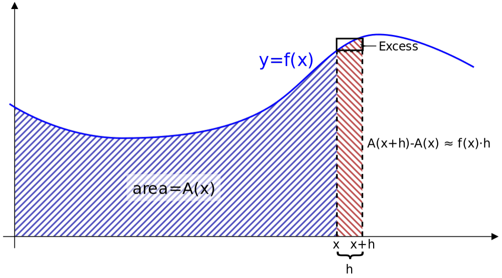
Has lots of properties, e.g.

$$\begin{aligned}
\log_b(x \cdot y) &= \log_b(x) + \log_b(y) && (18)\\
\log_b(x/y) &= \log_b(x) - \log_b(y) && (19)\\
\log_b(x^y) &= y \cdot log_b(x) && (20)\\
\log_b(c) &= 1/\log_c(b) && (21)\\
\log_b(x) &= \log_c(x)/\log_c(b) && (22)\\
f(x) &= \log_b(x) \implies f'(x) = 1/(x\log(b)) && (23)
\end{aligned}$$

Review: calculus

The fundamental theorem of calculus

1. $\int_a^b f(x)dx = F(b) - F(a)$
2. if $F(x) = \int_a^x f(t)dt$, then $F'(x) = f(x)$



Derivatives and integrals

Given a (possibly multivariate function), e.g.

$$f(x, y) = x^2 y + \sin(y) \tag{24}$$

Review: calculus

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$$\frac{\partial}{\partial x} f(x, y) = 2xy \tag{25}$$

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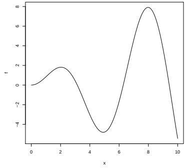
We can take an integral, e.g. wrt x

$$\int f(x,y)dx = \frac{1}{3}x^3 y + \sin(y) + C(y) \tag{26}$$

Example:

Say, $f(x) = x\sin(x)$ represents a position function, i.e.



Review: calculus

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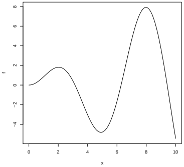

We can calculate the velocity by $\frac{\partial}{\partial x}f(x)$ (by applying the product rule)

$$
\begin{aligned}
\frac{\partial}{\partial x}f(x) &= \frac{\partial}{\partial x}(x)\cdot\sin(x) + x\cdot\frac{\partial}{\partial x}(\sin(x)) && (27)\\
&= \sin(x) + x\cos(x) && (28)
\end{aligned}
$$

Review: calculus

Example: $f(x) = x\sin(x)$
What about acceleration?

Review: calculus

Example: $f(x) = x\sin(x)$
What about acceleration?

Take the second derivative, i.e. $\frac{\partial^2}{\partial x^2} f(x) = \frac{\partial}{\partial x} f'(x)$ (by again applying the product rule).

$$
\begin{aligned}
\frac{\partial}{\partial x} f'(x) &= \frac{\partial}{\partial x} \sin(x) + x\cos(x) && (29)\\
&= \cos(x) + (\cos(x) - x\sin(x)) && (30)\\
&= 2\cos(x) - x\sin(x) && (31)
\end{aligned}
$$

Review: calculus

Gradients

Given a (possibly multivariate function), e.g.

$$f(x, y) = x^2 y + \sin(y) \tag{32}$$

the *gradient* is a vector of partial derivatives of the function, e.g.

$$\begin{aligned} \nabla f(x, y) &= \begin{bmatrix} \frac{\partial}{\partial x} f(x, y) \\ \frac{\partial}{\partial y} f(x, y) \end{bmatrix} && (33) \\ &= \begin{bmatrix} 2xy \\ x^2 + \cos(y) \end{bmatrix} && (34) \end{aligned}$$

The Jacobian matrix

Given multiple multivariate functions, e.g.

$$f_1(x,y) = x^2y + \sin(y) \quad (35)$$
$$f_2(x,y) = x^2\sin(y) \quad (36)$$
$$\quad (37)$$

the *Jacobian matrix* is a vector of all partial derivatives of the functions, e.g.

$$\nabla \begin{bmatrix} f_1(x,y) \\ f_2(x,y) \end{bmatrix} = \begin{bmatrix} \frac{\partial}{\partial x} f_1(x,y) & \frac{\partial}{\partial y} f_1(x,y) \\ \frac{\partial}{\partial x} f_2(x,y) & \frac{\partial}{\partial y} f_2(x,y) \end{bmatrix} \quad (38)$$

$$= \begin{bmatrix} 2xy & x^2 + \cos(y) \\ 2x\sin(y) & x^2\cos(y) \end{bmatrix} \quad (39)$$

Review: calculus

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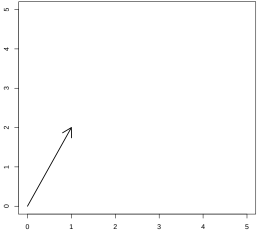
$$= \begin{bmatrix} 2xy & x^2 + \cos(y) \\ 2x \sin(y) & x^2 \cos(y) \end{bmatrix} \quad (39)$$

n.b. The matrix of second partial derivatives is called the *Hessian matrix*.

Review: linear algebra

Deals with vectors and spaces, e.g.

$$V = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \tag{40}$$



Review: linear algebra

Vectors can be added together, e.g.

$$V = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, W = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{41}$$

$$V + W = \begin{bmatrix} 1 \\ 2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix} \tag{42}$$

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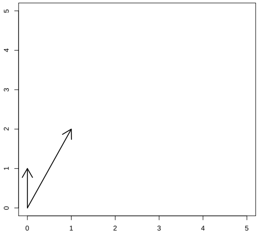

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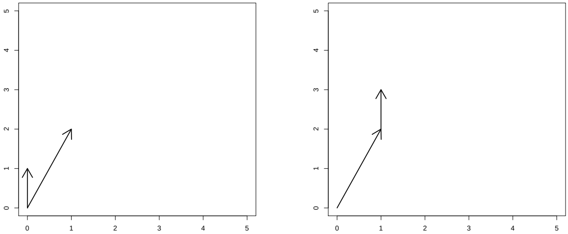

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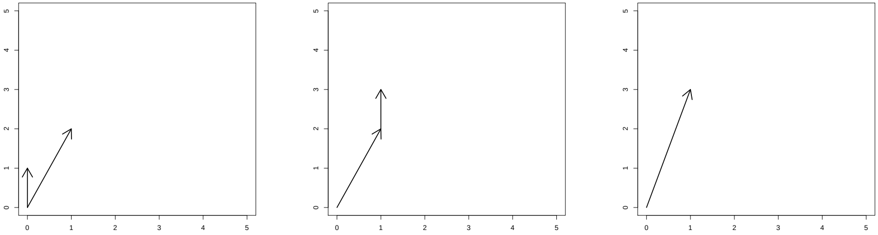

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Review: linear algebra

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This can be generalized to **inner product** (in functional spaces)

Review: linear algebra

Determinants:

Tell us the amount of volume scaling from transformations created by the matrix, e.g. for the matrix

$$A = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix} \tag{45}$$

The determinant ($|A| = 6$) results in the following transformation

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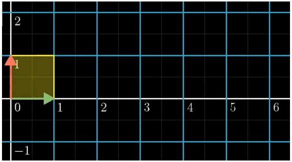

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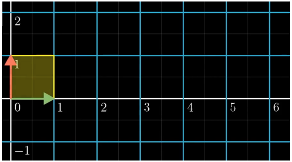

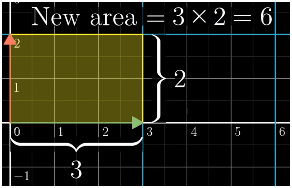

Review: linear algebra

Determinants:

How to calculate determinants:

For a 2x2 matrix:

$$|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc \tag{46}$$

For a 3x3 matrix:

$$\begin{aligned} |A| &= \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} && (47) \\ &= a \begin{vmatrix} e & f \\ h & i \end{vmatrix} - b \begin{vmatrix} d & f \\ g & i \end{vmatrix} + c \begin{vmatrix} d & e \\ g & h \end{vmatrix} && (48) \\ &= aei + bfg + cdh - ceg - bdi - afh && (49) \end{aligned}$$

Takeaway

Main points to keep in mind:

- This course establishes a feedback loop

Takeaway

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- This course is separate from your work

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- This course requires commitment
- This course is meant for you