Talis Biomedical Statistics Course - Homework 3

Due: 11 December 2019 11:59 PM

Name: [your first and last name]
Collaborators: [list all the people you worked with]
Date: [date of submission]

By turning in this assignment, I agree by the **Stanford honor code** and declare that all of this is my own work.

Linear algebra

Problem 1

When is it true? Fill in each blank with '*always*', '*sometimes*', or '*never*'. Justify your choice.

(a) A nonsingular matrix is ________________________ invertible.

(b) A square matrix is ________________________ full-rank.

(c) If $\mathbf{AB} = 0$, then $\mathbf{BA}$ is ________________________ a zero matrix.

(d) The rank of $\mathbf{A} + \mathbf{B}$ is ________________________ greater than $rank(A)$.

(e) If $\mathbf{A}^2$ is invertible, then $\mathbf{A}$ is ________________________ invertible.

(f) If the linear equation $\mathbf{y} = \mathbf{Ax}$ has a unique solution, then $\mathbf{A}$ is ________________________ square.

Problem 2

True or False. Fill in each blank with '*True*' or '*False*'. Justify your answer.

(a) A diagonalizable matrix $\mathbf{A}$ is nonsingular. ________________________

(b) A nonsingular matrix $\mathbf{A}$ is diagonalizable. ________________________

(c) A positive square matrix $\mathbf{A}$ is positive definite. ________________________

(d) A square matrix $\mathbf{A}$ with real and positive eigenvalues is positive definite. ________________________

Problem 3

$$\mathbf{A} = \begin{bmatrix} 2 & 0 & -1 \\ 4 & -5 & 2 \end{bmatrix} \quad , \quad \mathbf{B} = \begin{bmatrix} 7 & -5 & 1 \\ 1 & -4 & -3 \end{bmatrix}$$
$$\mathbf{C} = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix} \quad , \quad \mathbf{D} = \begin{bmatrix} 3 & 5 \\ -1 & 4 \end{bmatrix}, \mathbf{E} = \begin{bmatrix} -5 \\ 3 \end{bmatrix}$$

Given the matrices above, compute each matrix operation (if it is defined). If an expression is undefined, explain why.

(a) $-2\mathbf{A}$

(b) $\mathbf{B} - 2\mathbf{A}$

(c) $\mathbf{AC}$

(d) $\mathbf{CD}$

Problem 4

(a) Find the inverse of $\mathbf{A} = \begin{bmatrix} 8 & 6 \\ 5 & 4 \end{bmatrix}$.

(b) Let $\mathbf{A} = \begin{bmatrix} 5 & 7 \\ -3 & -6 \end{bmatrix}$. Is $\mathbf{A}$ invertible?

Problem 5

Compute the following determinants.

(a) $\begin{bmatrix} 4 & -1 \\ -2 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 3 & 0 & 4 \\ 2 & 3 & 2 \\ 0 & 5 & -1 \end{bmatrix}$

Problem 6

Let $\mathbf{A} = \begin{bmatrix} 1 & 6 \\ 5 & 2 \end{bmatrix}$, $\mathbf{u} = \begin{bmatrix} 6 \\ -5 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$. Are $\mathbf{u}$ and $\mathbf{v}$ eigenvectors of $\mathbf{A}$?

Problem 7

Let $\mathbf{A} = \mathbf{PDP}^{-1}$, where

$$\mathbf{P} = \begin{bmatrix} 5 & 7 \\ 2 & 3 \end{bmatrix}, \mathbf{D} = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$$

. Compute $\mathbf{A}^4$

Problem 8

Find the singular values of $\mathbf{A} = \begin{bmatrix} 1 & -1 \\ -2 & 2 \\ 2 & -2 \end{bmatrix}$.