

Towards a Multi-Purpose Agent-Based Model for the Brazilian Economy *

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This paper provides a full description of an agent-based model designed to reproduce stylized facts of the Brazilian economy. We focus on measuring the relative relevance of proposed reasons to the persistently high interest rates in Brazil, such as low saving rates, cheap loans to large companies made by the public development bank (BNDES), the risk associated with Brazilian debt (a consequence of a misconducted fiscal policy), inertial inflation, and high private default rates (which might be associated with heavy taxation). A preliminary model is used to study the role of savings in the dynamics of economic growth, especially its impact on interest rates. The model we present afterwards, which tries to answer the central questions of this work, is larger but is only an early version of what is to become a comprehensive multi-purpose model of the Brazilian economy.

*Replication files are available on the author's Github account (<http://github.com/talithafs>). **Current version:** outubro 28, 2018; **Corresponding author:** talitha.speranza@gmail.com

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1. Introduction

The objective of this work is to describe the design of a model for the Brazilian economy, as well as to present the results of simulations meant to capture the relative relevance of each proposed explanation for the high interest rates that prevail in Brazil. The approach for building the model derived from complexity theory and is known as agent-based modeling. The fact that this class of models is capable of reproducing a strikingly high number of stylized facts about complex economies ([Delli Gatti et al., 2005b](#)) allows us to devise extensions that could account for many other issues regarding the Brazilian economy.

Traditionally, an analytical or empirical model is constructed to demonstrate the validity of each suggested reason to the persistently high interest rates in Brazil. These models employ distinct assumptions and structures, which are not always compatible with each other. This poses a problem if we want to make a comparison between different propositions, so we need a multi-purpose model.

The agent-based framework, which we adopt in this work, is probably the most appropriate class of models for this purpose, especially when we consider the alternative, the dynamic stochastic general equilibrium (DSGE) type of models. Economists ([Blanchard, 2016](#); [Romer, 2016](#)) have pointed to severe flaws in these standard macroeconomic models. Criticism of the DSGEs gained momentum with the financial crisis in 2008 and repeated failure to address it kept the debate alive. Others ([Howitt, 2012](#); [Colander and Kupers, 2014](#)) argue that computational models such as agent-based models (ABMs) are powerful means for examining economic phenomena, since economies are complex, self-organizing, and analytically intractable systems. Preceding them, [Leijonhufvud \(2006\)](#) argued that agent-based methods were *the only way we could exploit complex economies*.

The literature on high interest rates in Brazil is extensive and dates back to the early 2000s, when scientists started to investigate why rates remained in such levels even after macroeconomic stabilization in 1994. Research methods differs significantly among contributions, but while some of them are purely statistical and did not allow for cause and effect analysis, others were small versions of DSGEs, carrying their serious deficiencies. These weaknesses were summarized by [Özge Dilaver et al. \(2018\)](#) as (i) the use of representative agents, which disregards the fallacy of composition; (ii) the equilibrium analysis and lack of concern for disequilibrium dynamics; (iii) examination of effects of imaginary shocks caused by no single agent as opposed to emergence of phenomena derived from the interaction of heterogeneous agents; (iv) assumption of rational expectations.

[Garcia and Brandao \(2003\)](#) is the first consistent study we could find. He estimated the exchange rate risk and Brazil's country risk through a Kalman Filter to investigate its relevance in the determination of domestic interest rates. The authors found that both are important and, since these risks have common causes, attacking such causes could reduce interest rates substantially. Soon after, [Arida et al. \(2005\)](#) constructed a simple version of the forward-looking short-term open macro model to illustrate how the distortions associated with jurisdictional uncertainty affect the interest rate set by the central bank. They discovered that unwinding the policy responses to the jurisdictional uncertainty reduces the short-term interest rate required to keep inflation on target.

[Muinhos and Nakane \(2006\)](#) tried a more flexible approach and experimented with different methodologies, such as IS curves and panel regressions compared Brazilian real interest rates with those of several other countries. They found that interest rates Granger-cause debt levels in Brazil and, that uncertainty associated with inflation may explain high real interest rates. [Gonçalves et al. \(2005\)](#) also used cross-country panel data analysis, but to test the conjectures associated with [Arida et al. \(2005\)](#) that jurisdictional uncertainty and currency inconvertibility are important determinants of short-term real interest rates in Brazil. Results were not favorable to these hypothesis and

their variants, indicating that other factors, such as monetary and fiscal explanations are more relevant.

[Bacha et al. \(2008\)](#), in order to examine another possible explanation to the high interest rate levels after Brazil's 1994 inflation stabilization, modeled the trade-offs involved in choosing between dollar and local currency denominated assets. The model showed that Brazilian real interest rate could fall if investment-grade status is achieved, which would require deep institutional changes. [Bacha \(2010\)](#) took a step further and presented statistical tests based on linear regressions that suggested that the observed gap between Brazilian interest rates and those practiced internationally persisted, even after the changes in macroeconomic policy in 1999. Tests also indicated that this persistence has roots in the country's hyperinflationary past, which reduces tolerance to public debt and hinders the monetary policy from achieving its goals.

[Lara Resende \(2011\)](#) defended the thesis that the root of high interest rates in Brazil is the incompatibility between consumer's low propensity to save and high levels of public spending. He estimates a basic Keynesian model in which the IS curve defines the macroeconomic balance between savings and investments. That same year, [Goldfajn and Bicalho \(2011\)](#) studied the dynamics of short-term and long-term factors of the equilibrium real interest rate after Brazilian economy's stabilization. Long-term analysis was conducted by estimating a vector auto-regressive (VAR) model and evaluating the effect of shocks, while a general specification of an IS curve was employed for short-term examinations. Results suggested that the public debt risk as a proportion of GDP and the increase in the proportion of credit contributed to the then prevailing falling trend of real interest rates. The authors also showed that the significant fall in world economic activity in 2008 substantially reduced real short-term equilibrium interest rates, but this was largely offset by fiscal expansion and credit directioning.

[Segura-Ubiergo \(2012\)](#) estimated a panel vector error correction model (VEC) using data from 15 emerging market economies, including Brazil, to study the determinants of short-term real interest rates. The author concluded that inflation volatility, domestic saving rates, the adoption of an inflation target regime and international financial conditions are strongly associated with short-term real interest rates in these countries. [Lopes \(2014\)](#) also examined the role of deficient private savings in the determination of high interest rates in Brazil. In addition, he measured the impact of large government deficit, excessive investment demand, inflation bias and fear of floating. The model, a simple two-equation Keynesian model, did not provide strong evidence to any of these explanations.

[Balliester Reis \(2016\)](#) performed a panel data analysis to compare Brazilian interest rates with other developing countries under inflation targeting regime. She assessed whether different proposed arguments for high policy real interest rates could be supported and concluded that neither hypothesis is sufficient to explain the Brazilian case. Among those arguments, she tried to find evidence for low level of savings, risk premia, convertibility risk, jurisdictional uncertainty, exchange rate volatility and pass-through, and BCB conservatism. The author's contribution had a similar objective to ours, but cause and effect analysis was not a concern in her work, as it is in ours.

An ABM constructed to investigate movements and subtleties of the Brazilian economy as a whole is an entirely new tool, as the work of [Furtado and Eberhardt \(2016\)](#) points out. Given the inherently modular and flexible nature of ABMs, the solid model we will uncover here will enable us to build upon it, extending its capabilities in the future.

2. Methodology

We aim at building a large model in an incremental fashion, so we will use a simpler model at first, to test design concepts and examine some preliminary issues on the dynamics of the Brazilian

economy. This model will not feature intelligent agents.

Each model should reflect real markets and be subdivided in well-defined parts so as to keep the code modular. The subsystems designed by [Deissenberg et al. \(2008\)](#) were considered general and intuitive enough to be followed by this work: (i) consumer goods market, (ii) capital goods market, (iii) credit market and (iv) labor market.

The construction of the models will follow an ordering based on [Helbing \(2012\)](#):

- i. Definition of stylized facts that the model must reproduce, according to well-established macroeconomic evidence. The literature on this subject is vast, but the works of [Delli Gatti et al. \(2005b\)](#), [Dosi et al. \(2010\)](#) and [Dosi et al. \(2012\)](#) will be the starting points.
- ii. Choice and design of agents. The behavior of agents (consumers, producers, banks, central bank, and others) must be compatible with what is described by Hayek, Menger and other precursors of the complexity approach in economics ([Barbieri, 2013](#)). This means that agents must be heterogeneous, have limited rationality, learn from experience, and act not according to predefined functions, but to a set of rules. To emulate adaptive learning and bounded rationality in consumers and firms, behavioral functions parameters will evolve genetically, as suggested by [Mandel \(2012\)](#), with an algorithm inspired in [Delli Gatti et al. \(2005a\)](#), and reinforcement learning algorithms, based on [Tsfatsion \(2006\)](#). The central bank will use a fuzzy rule extracted from real data to make its decisions.
- iii. Adjust parameters using real data, empirical findings or search for optimal settings. In the case of calibration through search, we will utilize genetic algorithms as a global optimization strategy, a procedure proposed by [Judd \(2006\)](#).
- iv. Model validation: comparison of simulation results with empirical evidence, i.e. the stylized facts, following [Tsfatsion \(2014\)](#).
- v. Simulations for different sets of initial conditions. Generation of statistics and confidence levels through Monte-Carlo distributions ([Fagiolo and Roventini, 2012](#)).

2.1. Stylized Facts

In order to be used, the models must be able to qualitatively reproduce facts that are valid for real-world economic data:

- Okun's Law: negative correlation between the unemployment rate and the GDP's percentage deviation from its potential.
- Beveridge Curve: negative correlation between the proportion of vacancies in the labor market and the unemployment rate.
- Business Cycles: periodic oscillations of GDP growth.
- Procyclical correlations: positive correlation between (i) GDP growth and employment, (ii) nominal base interest rate (SELIC), and (iii) the proportion of job vacancies
- Countercyclical correlations: negative correlation between GDP growth and unemployment rate.

2.2. Reference Models

3. First Model

The model we present in this section is a tool to explore the effects of varying the household saving rate on the Brazilian economy. The impact of savings on interest rates is of particular interest, since lower levels of interest are needed to sustain a steady growth.

According to the [International Monetary Fund](#), Brazilian households today save slightly less than 16% of GDP, 6 percentage points less than the average for OECD high income countries and almost 4 percentage points less than the average for Latin America. The low level of savings strangles the development of the country, since it is one of the main reasons for high interest rates.

Decades of government policies focused on stimulating consumption obscured a fundamental principle: consumption, by itself, does not fuel growth. First, it is necessary to obtain the resources to exchange for consumption, and saving plays a fundamental role in the process of generating wealth. Entrepreneurs who invest in productive activity are financed through loanable funds, which increase when consumers save more money. The increase in the supply of loanable funds, in turn, makes them cheaper, i.e. it reduces the interest rates of the economy. Lower interest rates would make longer-term investments more feasible, and these are typically more ambitious projects that create more jobs and boost productivity, thus heightening wages. Moreover, saving means postponing consumption, so more savings implies greater future demand for production resulting from investments made today. These are all facts that our model seeks to reproduce and further describe, paving the way for creating extensions to explore many other topics.

3.1. Components

In this section, we expound the attributes of the agents. Behavioral rules will be explained in detail in the following sections, when we present comprehensive information about the way these agents are initialized and how they interact in each time step.

The model consists of three types of agents - consumers/workers, banks, and firms - that can be connected in three different ways - accounts, loans, and jobs.

3.1.1. Types of Agents

- **Banks:** Receive deposits from consumers; lend money to firms and consumers; charges interest on loans; pay interest on deposits. Some of their properties include assets, liabilities, and a list of interest rates charged to each kind of customer.
- **Firms:** Borrow money from banks; sell homogenous goods to consumers/workers; pay wages to consumers/workers. Each is distinguished by its wealth (capital), employees, production, debt, and prices charged for goods.
- **Consumers/Workers:** Buy homogenous goods from firms; receive wages from firms if employed or search for employment if unemployed; save money in banks; borrow money from banks. They hold properties such as wealth (savings), wage (monthly income), reservation wage, and debt.

3.1.2. Types of Links (Relationships)

- **Accounts:** Link between a bank and a firm or a bank and a worker. Contain a money deposit and the associated interest paid by the bank.

- **Loans:** Link between a bank and a firm or a bank and a worker. Bear the amount lent and the associated interest that the borrower must pay.
- **Jobs:** Link between a firm and a worker. Carry the wage the firm pays to the worker.

In the next model, we will also incorporate the government, the Bank of Social and Economic Development (BNDES) and the Central Bank. They all play very significant roles in the Brazilian economy, but were not included in this version for simplicity.

3.2. Data and Initialization

In this section we describe the way agent's properties were initialized and indicate the sources of data. Initial values and parameters were mostly calibrated from real Brazilian data. We chose the year 2015 to retrieve values from, but we could also have selected another year or a range of years and taken the mean/median. The reason not to do so was to keep calculations simple, especially for income and wealth. In any case, we later shall have the opportunity to test for robustness using data from different years.

3.2.1. Number of Agents

The number of agents of each type in relation to other types was based on the real proportions we observe in Brazilian data. The number of consumers, from now on E , is fixed in 2000 and the quantity of other agents is calculated from this number. Data were taken from [Central Register of Enterprises](#), a broad official record of Brazilian firms produced by the Brazilian Institute for Geography and Statistics (IBGE).

According to this dataset, in 2015 Brazil had approximately 4.7 million non-financial private firms, which employed around 43.2 million workers, and there were about 88.96 thousand private banks and other financial firms. The number of consumption goods firms was approximately 4.3 million, whereas the number of capital goods firms was around 0.4 million.

We need to consult the unemployment rate series to calculate the total number of workers available in 2015. That year, the unemployment rate was 8.3% in 2015, as disclosed by IBGE's [Monthly National Household Survey](#), so the total number of workers can be approximated to 47.14 million.

Let F_1 be the number of consumption goods firms, F_2 the number of capital goods firms, and B the number of banks within the model. Then:

$$F_1 \approx E \cdot \left(\frac{4.3}{47.1} \right) \approx 2000 \cdot 0.092 \approx 184 \quad (1)$$

$$F_2 \approx E \cdot \left(\frac{0.4}{47.1} \right) \approx 2000 \cdot 0.008 \approx 16 \quad (2)$$

$$B \approx E \cdot \left(\frac{0.1}{47.1} \right) \approx 2000 \cdot 0.002 \approx 4 \quad (3)$$

Table 1 shows a list of the parameters already discussed.

Table 1: Number of agents of each type

Description	Value
Number of Consumers	2000
Number of Unemployed Consumers	166
Number of Consumption Goods Firms	184
Number of Capital Goods Firms	16
Number of Banks	4

Table 2: GDP and its components.

Description	Unit	Value
Real GDP	goods	77094000
Real GDP per employed worker (last 12 months)	goods	42036
Percentage of salaries in GDP	percentage	44
Profit per employed worker (last 12 months)	goods	23540

3.2.2. Gross Domestic Product

The initial Gross Domestic Product (GDP) of the model was defined in terms of the population of consumers, just as every other model parameter. First, we collect the values of the real¹ and the nominal² GDP. In 2015, real GDP was around R\$ 1.8 trillion. Nominal GDP, provided by the [Department of Economics](#) of the Central Bank of Brasil (BCB/Depec), was about R\$ 6 trillion in 2015.

If y is the real GDP per worker in 2015, then the model's initial GDP, Y , can be written as

$$Y = y \cdot E \approx \left(\frac{1.8e + 06}{47.1} \right) \cdot 2000 \approx (3.9e + 04) \cdot 2000 \approx 7.7e + 07 \quad (4)$$

At this point we also need to obtain the percentage of salaries in GDP³. The most recently published data on detailed composition of Brazilian GDP are from 2014's [National Accounts System](#) (IBGE), but these are surely good approximations for 2015's values. In 2014, wages accounted for 44% of the GDP. Hence, profits were 56% of the GDP that same year. These figures allowed us to set some other parameters, as shown in table 2.

Initial profits were assigned to each firm according to its number of workers. The unit of profit, π_u , is shown in the table above (*Profit per employed worker*). If a firm has N employees, its starting profit is $N \cdot \pi_u \cdot x$, where $x \sim U(0, 2)$. This way, we introduce more heterogeneity into the system, by allowing some firms to begin with littler or no profits and other with outstanding past results. Given that the mean of x is 1, the average initial profit per worker matches real data.

¹World Bank, [World Development Indicators](#) (2017)

²Central Bank of Brasil (BCB), [Department of Economics](#)

³The GDP can be defined as the total sum of salaries and profits in an economy

Table 3: Aggregated data for capital goods firms, by brackets. Some columns are not shown, because they are not going to be useful in the following calculations.

Bracket	Units	Employees
0 a 4	277072	500346
10 a 19	26293	347908
100 a 249	1871	263939
20 a 29	7308	166033
250 a 499	629	180062
30 a 49	5229	179502
5 a 9	56226	349936
50 a 99	3641	250889
500 ou mais	616	1189663

Table 4: Number of firms and employees per bracket.

Bracket	Units	Employees	units	employees
0 a 4	277072	500346	12	22
10 a 19	26293	347908	2	15
100 a 249	1871	263939	1	12
20 a 29	7308	166033	1	8
250 a 499	629	180062	1	8
30 a 49	5229	179502	1	8
5 a 9	56226	349936	3	15
50 a 99	3641	250889	1	11
500 ou mais	616	1189663	1	51

3.2.3. Number of Employees Per Firm

The *Central Register of Enterprises* also serves the purpose of calculating how many employees each firm must have. Before all else, we extract two groups from the dataset: consumption goods firms and capital goods firms. Then, we aggregate data by brackets, adding grouped columns for each subset. For instance, capital goods subset would look like the table shown below (table 3).

The next step is to divide each element from the column *units* by the sum of all elements, so as to find the percentages they represent. On the other hand, elements of *employees* are divided by 43230386, which is the total number of workers registered in the original dataset (the sum of employees from capital and consumption goods firms). This way, this column will represent percentages of the total number of workers. Having these columns of percentages, we multiply the first by the number of the associated type of firm within the model (16 capital goods firms and 184 consumption goods firms) and the second by the number of employed consumers (1834). The results are presented in table 4.

Note that the second column, *employees* represent the total number of employees in all units. Therefore, we find the average number of employees per firm and round it down to the nearest integer. Some firms will have this number of employees, while others will have one more, in order to absorb the remainder. For instance, if *employees* is 535 and *units* is 100, 65 firms will

Table 5: Distribution of employees per firm. The first table presents results for capital goods firms. The second, for consumption goods firms.

N. Firms	N. Employees	Tot. Employees	N. Firms	N. Employees	Tot. Employees
3	1	3	34	1	34
9	2	18	99	2	198
1	4	4	22	6	132
2	5	10	7	7	49
5	28	35	11	12	132
1	10	10	4	13	52
1	11	11	2	19	38
1	50	50	2	20	40
			3	31	93
			1	54	54
			1	55	55
			1	82	82
			1	121	121
			1	507	507

have 5 employees and 35 firms, 6 employees, since $\lceil 535/100 \rceil = 5$ and the remainder is 35, which imposes that 35 firms have $5 + 1 = 6$ employees. We now have the initial distribution of employees per firm. The result of such calculations are shown in table 5.

3.2.4. Distribution of Wealth and Income

Workers belong to social classes that are defined by income brackets. Each of these brackets concentrates a certain amount of wealth and is delimited by monthly earnings in terms of minimum wages. In this session we show how we attributed a social class to every worker and set his income and wealth accordingly. Data on the distribution of wealth and income are yearly disclosed by the [Federal Revenue Office](#). The dataset is presented below (table 6).

Wealth is on the last column of this dataset, and totals on the last row. Hence, total wealth is element (18, 6) of table 6 multiplied by 1 million (data are in R\$ millions). To find the model's total wealth proportional to its GDP, we have to divide it by the nominal GDP, since income and wealth are in nominal values, and then multiply it by the model's initial GDP. By doing so, we find that the model initial total wealth is 92.4 million.

Given that the table separates income that is exempt from taxes (column 5) from income that is not exempt (column 3), we collapse the two values by adding them. Column 4 is removed because it contains income from investments and real-state, but the current version of the model does not incorporate these.

[IBGE](#) divides the population in social classes according to the number of minimum salaries workers earn each month. So we aggregate the lines of table 6 following this classification.

First, we found the percentage of people in each class in the Brazilian population. Then, we applied those percentages to the model and, using the unemployment rate, calculated the numbers of employed and unemployed workers within a class. For instance, 8.45% of the Brazilian population belongs to class A. Hence, $169 (= 2000 \cdot 0.845)$ workers in our model must belong to this class, $14 (= 169 \cdot 0.83)$ being initially unemployed and $155 (= 169 - 14)$, employed. Results of

Table 6: Income and wealth distribution.

Bracket	Population	Income	Not Exempt	Exempt	Wealth
Até 1/2	1301366	254	46	113	136273
Mais de 1/2 a 1	573674	4487	92	341	38903
Mais de 1 a 2	1227268	14525	599	2553	135712
Mais de 2 a 3	3278035	73567	2159	6323	268682
Mais de 3 a 5	7403868	228922	16832	29606	526420
Mais de 5 a 7	4339708	192783	16498	32910	443328
Mais de 7 a 10	3352450	202073	18801	42627	496954
Mais de 10 a 15	2536352	211127	21922	58535	604905
Mais de 15 a 20	1180520	130938	15647	45710	445973
Mais de 20 a 30	1086611	157914	21739	69414	622922
Mais de 30 a 40	489421	92454	14777	51599	426299
Mais de 40 a 60	389811	89905	18318	69382	524434
Mais de 60 a 80	142916	37610	10550	44527	303922
Mais de 80 a 160	141451	40987	18427	84343	533681
Mais de 160 a 240	32329	11540	8269	39315	245037
Mais de 240 a 320	13753	6063	5447	24337	151526
Mais de 320	29311	27541	62826	207572	1288419
Total	27518844	1522690	252949	809206	7193391

Table 7: Wealth and income of our model's consumers.

	Employed	Unemployed	Monthly Wage	Wealth
A	155	14	8571	311371
B	248	22	2134	49981
C	513	46	1078	21609
D	712	64	549	13158
E	207	19	122	17715

Table 8: Debt parameters

Description	Unit	Value
Mean household debt (% last 12 months earnings)	goods	27.17
Mean firms debt (% GDP)	goods	23.00
Firms debt per employee	goods	9668.28

these computations are exhibited above (table 7).

3.2.5. Debt

Non-corporate debt is expressed as a percentage of household income⁴. Since families in our model are composed of one member only, this percentage represents approximately how much debt each worker has in relation to his income, in the first run. Actually, non-corporate debt rate was assigned to consumers following a $U(0, 54.34)$ distribution, whose mean is exactly the percentage found in Brazilian data (27.17, as shown in the table below). We did so because some individuals might not hold debt at all, but others may be severely indebted.

Corporate debt⁵ is split among firms roughly in proportion to the their number of employees. It means that the whole amount of debt is divided into the total number of workers, and the result of this division is the unit of debt, which is then multiplied by a uniform random variable between 0 and 2 (such that the mean is exactly the unit of debt). For instance, a firm with 5 workers has an initial debt of 5 times this randomized unit. Again, the idea of using a random element is to introduce heterogeneity and account for different levels of indebtedness. This level, however, refer only to the long-term debt. We will talk about short-term debt later.

As usual, real values were those of 2015 and compatibilization with model units was achieved through simple proportions.

3.2.6. Interest Rates

Interest rates charged on loans are different across types of agents. The risk of a worker defaulting on his debt is greater than the risk of a firm going bankrupt. Therefore, a worker must pay higher interests on loans. The same logic applies to firms of different sizes. Smaller firms offers more risk, so they must be charged higher interests.

In view of the lack of data on the dimensions of the differences among these charges, interest rates will be calibrated along with several other parameters in the second model. However, in the first model, banks choose initial values for each type of borrower obeying a very simple rule. A mean rate, which is the interest demanded from medium firms, is attributed to each bank accoring to a normal distribution, whose mean is the real mean of interest rates on new credit operations⁶ in 2015. Banks add or subtract a uniformly random value between 0 and 0.5 to set other interest rates. For example, if the interest charged to medium firms is 2.08, then to small firms it could be 2.29 ($= 2.08 + 0.21$), to micro firms, 2.43 ($= 2.29 + 0.14$), and to large firms, 1.65 ($= 2.08 - 0.43$).

⁴BCB, Household Debt without Mortgage Loans (Series 20400)

⁵BCB, Credit operations outstanding by type of borrower - Private sector (Series 22047)

⁶BCB-Dstat, Monthly average interest rate of nonearmarked new credit operations - Non-financial corporations (Series 25437) and Households (Series 25462)

Table 9: Interest rates

Description	Unit	Value
Mean interest rate on loans - workers	monthly yield	3.95
Mean interest rate on loans - firms	monthly yield	2.08
Mean interest rate on savings	monthly yield	0.65
Bankruptcy rate	yearly %	3.10

3.3. Protocol and Behavioral Rules

In this section we present the logical flow of the simulator that runs the model. We used and added upon features of mainly four previous models, found in [Ashraf et al. \(2015\)](#), [Riccelli et al. \(2012\)](#), [Delli Gatti et al. \(2005a\)](#) and [Tefatsion \(2006\)](#). None of these works conceived interest rates as varying according to the type of client, so all structures relating to these variables are novel. [MANY MORE NEW FEATURES!]

Some functions contain unknown parameters represented by greek letters. These will be addressed in a later section. [WHY?]

3.3.1. Planning

There are F_1 firms j that produce an homogenous consumption good and F_2 capital goods firms k , which produce heterogenous machines, where $j \in [1, F_1]$, $k \in [F_1 + 1, F_1 + F_2]$, and the set F of firms' indexes is defined as

$$F = \{f \mid f \in \mathbb{N}, f \in [1, F_1 + F_2]\} \quad (5)$$

Furthermore, there are E workers indexed by i . At the beginning of the simulation, each is assigned to a single production sector - consumption or capital - and it does not change afterwards.

I. Consumption Goods Firms

Some of firm's j properties, at month t , are:

- $y_j(t)$: quantity of goods produced;
- $Q_j(t)$: value of actual consumer demand;
- $E_j(t)$: number of employees;
- $W_j(t) = \{w_j^1, w_j^2, \dots, w_j^v\}$: vector of wages, where $v = E_j(t)$;
- $D_j^b(t)$: debt with bank b ;
- $D_j(t)$: total liabilities (sum of $D_j^b(t)$ for all b);
- $K_j(t)$: installed capacity, i.e. capital (assets);
- $U_j^K(t)$: used capacity (therefore, $U_j^K(t) \leq K_j(t)$);
- $NW_j(t)$: net worth (assets minus liabilities);
- $M_j(t) = \{M_j^1(t), M_j^2(t), \dots, M_j^m(t)\}$: a vector of machines, where $m > 0$.

Each machine $M_j^m(t)$ is defined as the triple $[\hat{l}^m, \hat{w}^m, l^m(t)]$, where \hat{l}^m is its capacity of in units of consumption goods it can produce per month, \hat{w}^m is the maximum quantity of minimum wages the firm can pay for the operation of the machine, per month, $l^m(t)$ is the number of goods it is producing currently (the capacity that is being used at t).

From these definitions, we get that $K_j(t) = \sum_{m=1}^{|M_j(t)|} \hat{l}^m$ and $U_j^K(t) = \sum_{m=1}^{|M_j(t)|} l^m(t)$.

- a. Firm j determines of $q_j^e(t)$, the expected demand for month t in units of consumption goods.

$$q_j^e(t) = f(q_j(t-1), q_j(t-2), \dots) \quad (6)$$

Function f was not yet defined. It could be a SARIMA or a exponential smoothing model. This function might contain parameters to be calibrated or genetically evolved.

- b. Firm j calculates $y_j^D(t)$, the desired production at month t , in units of consumption goods.

Let $\hat{y}_j(t)$ be firm j 's consumption goods inventory at month t , then:

$$y_j^D(t) = q_j^e(t) + y_j^*(t) - \hat{y}_j(t-1) \quad (7)$$

where the optimum inventory level is defined as

$$y_j^*(t) = \beta_j q_j^e(t) : \beta_j \in \mathbb{R}, \beta_j \in [0, 1] \quad (8)$$

β_j evolves according to the genetic algorithm defined in section 10, where the behaviour of all parameters that adapt likewise is described.

- c. Firm j sets its price $p_j(t)$, evaluating the percentage change in inventories, i.e. $\Delta \hat{y} = 1 - \hat{y}_j(t-2)/\hat{y}_j(t-1)$. This is a subtle way of assessing excessive demand.

$$p_j(t) = p_j(t-1) - \kappa_j(\Delta \hat{y}) \quad (9)$$

κ_j is the price sensibility to inventory variation, which evolves according to the genetic algorithm defined in section 10. This parameter indicates that although produced goods are homogenous, the system is heterogeneous and prices are allowed to differ. This mimics asymmetric information and search costs.

- d. Firm j decides how many employees it wants to hire (or fire) and which machines are desired for the next period.

Let h be a function that calculates the productivity of a machine in units of goods it can produce in a month per minimum salary to operate it. We denote by $\bar{M}_j(t)$ the set with the same elements as $M_j(t)$ but in ascending order.

$$h(M_j^a) = h(\hat{l}^a, \hat{w}^a) = \frac{\hat{l}^a}{\hat{w}^a} \quad (10)$$

Three scenarios are possible⁷:

- i. $y_j^D(t) < U_j^K(t)$:

Firm j does not invest and fires the workers who control the most inefficient machines first, following $\bar{M}_j(t)$. It keeps firing until $U_j^K(t) \leq y_j^D(t)$. In this case, $y_j(t) = U_j^K(t)$.

⁷Notice that the condition $y_j^D(t) < U_j^K(t)$ implies $y_j^D < K_j(t)$, since $U_j^K(t) \leq K_j(t)$, $\forall t$

ii. $y_j^D(t) > U_j^K(t)$ and $y_j^D(t) > K_j(t)$

Firm j attempts to invest and hire workers. It has access to catalogs from the capital goods firms it has recently traded with, as well as an additional subset of catalogs. The size of this subset, n , is proportional to firm j 's size and the owners of these supplementary catalogs are randomly selected.

$$n = \frac{E_j(t)}{E} \cdot F_2 \quad (11)$$

Catalogs $C_k(t)$ are ordered as $\bar{M}_j(t)$. Firm j inspects the catalogs sequentially and tries to buy the machine whose capacity is more similar⁸ to the intended production. It chooses one machine per catalog and tries to buy the most efficient among these, adding it to $M_j^C(t)$, the set of machines firm j wants to buy at t .

In this scenario, the production will be proportional to the number of job vacancies that are filled, because machines are delivered in $t + 1$. Therefore, $y_j(t) < y_j^D(t)$. If all vacancies are filled, we will see that $y_j(t) = K_j(t)$.

Since firm j wants to use all the free capacity of current machines, it calculates the wages to be offered to new workers based on this number:

$$\tilde{w}^m = \hat{w}^m \cdot \left(1 - \frac{l^m(t)}{\hat{l}^m}\right), \forall m \in [1, |M_j(t)|] \quad (12)$$

$$W_j^N(t) = \{\tilde{w}_m \mid \tilde{w}_m \neq 0\} \quad (13)$$

$W_j^N(t)$ is the vector of job offerings that firm j wants to post, but before posting it needs to verify if there is available credit.

iii. $y_j^D > U_j^K(t)$ and $y_j^D(t) \leq K_j(t)$

Firm j does not invest, but attempts to hire workers. Job offerings are defined as in equations 12 and 13.

II. Capital Goods Firms

Firms k possess the same properties as consumption goods firms, except for $K(t)$ and $U^K(t)$, and some properties are defined differently:

- $M_k^m = [l^m, w^m, p^m(t), \hat{y}^m(t)]$, where $\hat{y}^m(t)$ is the quantity in stock and $p^m(t)$ is the price of machine m ;
 - $q_k^m(t)$: demand for machine m ;
 - $Q_k(t) = \sum_{m=1}^{|M_k(t)|} q_k^m(t) p_k^m(t)$.
- a. Firm k determines the value that it wants to invest in research and development (R&D), $RD_k(t)$.

$$RD_k(t) = v_k \cdot Q_k(t - 1) \quad (14)$$

v_k evolves according to the genetic algorithm defined in section 10.

⁸Footnote explaining what similar is

b. Firm k decides how many workers it should hire or fire.

$$\hat{w}_k(t) = \sum_{v=1}^{E_k(t-1)} w_k^v + RD_k(t) - RD_k(t-1) \quad (15)$$

$\hat{w}_k(t)$ is the desired payroll for month t . Three scenarios are possible:

i. $RD_k(t) = RD_k(t-1)$

In this case, firm k does not hire or fire any employees.

ii. $RD_k(t) > RD_k(t-1)$

Firm k tries to hire workers. It creates job offerings that are copies of its current jobs, choosing randomly from $W_k(t)$ and observing that the total amount of salaries must not be greater than $RD_k(t) - RD_k(t-1)$.

iii. $RD_k(t) < RD_k(t-1)$

Firm k fires some workers, whose salaries, together, must not exceed $RD_k(t-1) - RD_k(t)$. It begins with the employees who are paid less, since those are more common than workers who receive high wages⁹.

III. Consumers

Some of workers' properties are:

- $w_i(t)$: wage;
- $w_i^R(t)$: reservation wage;
- $D_i^b(t)$: debt with bank b (liabilities);
- $D_j(t)$: total liabilities (sum of $D_j^b(t)$ for all b);
- $S_i(t)$: savings (assets);
- $NW_i(t)$: wealth (assets minus liabilities).

a. Consumer i decides how much to spend.

First, consumer i calculates disposable income, $w_i^d(t)$. We introduce $w_i^e(t-1)$, the effective wage paid to worker i at $t-1$. This intermediate variable is needed because the worker might be unemployed, so that $w_i^e(t-1) = 0$, or the firm that employs i might not have paid the whole wage if it was facing financial issues. Let ζ_i be the decimal representation of the percentage consumer i saves from its salary each month and η_i the percentage of past consumption a worker with $w_i^e(t-1) = 0$ wants to maintain. Then

$$w_i^d(t) = \begin{cases} (1 - \zeta_i)w_i^e(t-1), & \text{if } w_i^e(t-1) \neq 0 \\ \eta_i(1 - \zeta_i)w_i^R(t), & \text{otherwise.} \end{cases} \quad (16)$$

But consumer's choice depends not only on the wage (monthly income), but also on wealth. Let τ be the propensity to consume wealth, and T a random variable such that $T \sim U(0, 1)$. Then, the value $v_i(t)$ that consumer i wants to spend at t is:

$$v_i(t) = \begin{cases} w_i^e(t-1), & \text{if } NW_i \leq 0 \text{ or } T > \tau \\ w_i^e(t-1) + \tau NW_i, & \text{if } NW_i > 0 \text{ and } T \leq \tau \end{cases} \quad (17)$$

⁹As in the case of consumption goods firms, maybe firm k should hire some workers if it fires more than the necessary.

b. Consumer i chooses where to buy goods.

It is a random choice, but the lower the price posted by a firm, the higher the probability of selecting this firm to buy goods from. We use the constant ϕ to represent the difficulty to obtain information about prices.

$$Pr_i(t)\{F = j\} = \frac{\exp(-P_j(t)/\phi)}{\sum_{l=1}^I \exp(-P_l(t)/\phi)} \quad (18)$$

ϕ can be understood as the degree to which information is dispersed. It is a fixed value, calibrated before running experiments.

b. Consumer i calculates how much he is willing to buy from the chosen firm.

The quantity of goods $q_{iF}(t)$ consumer i wants to buy from the chosen firm F is

$$q_{iF}(t) = v_i(t)/P_F(t) \quad (19)$$

3.3.2. Credit Market

There are B banks indexed by b . Some of their properties are:

- $L_b(t)$: total value of loans granted by b ;
- $D_b(t)$: total value of agents' deposits on b ;
- $r_b^a(t)$: interest rate charged on agent a 's loans;
- r_b^S : interest rate paid for deposits (savings);
- \hat{r}_b^a : risk premium charged on agent's a loans;
- $NW_b(t)$: net worth;
- $r_b(t)$: interest rate's component which depends on $NW_b(t)$.

a. Bank b determines the maximum total credit it can provide at month t , $L_b^T(t)$.

$$L_b^T(t) = \rho_b(t) \cdot D_b(t-1) \quad (20)$$

$$\rho_b(t) = 1 - \frac{\hat{A}_b(t-1)}{A_b(t-1)} \quad (21)$$

$\hat{A}_b(t-1)$ is the sum of debts of bankrupt agents in $t-1$ and $A_b(t-1)$ is the sum of bank b 's assets. For now, $A_b(t) = L_b(t)$, but we plan on including other assets.

b. Firm f calculates its last period's profits, $\pi_f(t-1)$.

$$\pi_f(t-1) = Q_f(t-1) - W_f^T \quad (22)$$

$$W_f^T = \sum_{v=1}^{E_f(t-1)} w_f^v \quad (23)$$

c. Firm f determines how much it wants to borrow to pay wages.

Denote by $L_f^W(t)$ the desired loan and by $V_f^e(t)$ the value firm f expects to withdraw from its own funds to pay for its payroll. Whatever is left to pay, firm f tries to borrow. The distribution between these two options of financing is determined by the parameter α_f , which evolves according to the genetic algorithm defined in section 10. According to profits and net worth, two scenarios are possible:

$$\text{i. } \pi_f(t-1) \geq 0 \text{ or } NW_f(i-1) + \pi_f(t-1) \leq 0$$

$$L_f^W(t) = 0 \quad (24)$$

$$V_f^e(t) = \begin{cases} Q_f(t-1), & \text{if } \pi_f(t-1) \geq 0 \\ Q_f(t-1) + |\pi_f(t-1)|, & \text{otherwise.} \end{cases} \quad (25)$$

$$\text{ii. } \pi_f(t-1) < 0 \text{ and } NW_f(i-1) + \pi_f(t-1) > 0$$

$$L_f^W(t) = (1 - \alpha)|\pi_f(t-1)| \quad (26)$$

$$V_f^e(t) = Q_f(t-1) + \alpha|\pi_f(t-1)| \quad (27)$$

d. Consumption goods firm j calculates how much it wants to borrow to pay for desired investment.

Total desired investment at t is

$$I_j^d(t) = \sum_{m=1}^{|M_j^C(t)|} p^m(t) \quad (28)$$

$$L_j^I(t) = \begin{cases} (1 - \alpha)I_j^d(t) & \text{if } NW_j(t-1) + \pi_j(t-1) > (1 - \alpha)I_j^d(t) \\ 0, & \text{otherwise.} \end{cases} \quad (29)$$

b. Bank b calculates the maximum credit it is authorized to offer to each agent a , $L_b^a(t)$.

$$L_b^a(t) \leq \gamma L_b^T(t) \quad (30)$$

γ is an institutional parameter.

3.3.3. Labor Market

c. Bank b , in which firm f holds deposits, calculates the probability of firm f going bankrupt.

Bankruptcy is defined as $NW_f(t) < 0$. From the bank's point of view, $NW_f(t)$ can be estimated based on information firm f provides.

i. Consumption Goods Firms

$$NW_j^e(t) = NW_j(t-1) + p_j(t-1) \cdot X_j(t) - W_j(t-1) \quad (31)$$

where $NW_j^e(t)$ is an estimate of $NW_j(t)$ and $X_j(t) \sim \mathcal{U}(0, K_j(t))$, i.e. the production of firm j is seen as an uniform random variable between 0 and the full production capacity. The firm goes bankrupt if

$$NW_j(t-1) + p_j(t-1) \cdot \Psi_j(t) \cdot K_j(t) - W_j^T(t-1) < 0$$

$$\Psi_j(t) < \frac{W_j^T(t-1) - NW_j(t-1)}{K_j(t) \cdot p_j(t-1)} \quad (32)$$

where $\Psi_j(t) \sim \mathcal{U}(0, 1)$, since $X_j(t)$ can be written as $\Psi_j(t) \cdot K_j(t)$. Therefore, the probability of f going bankrupt at t is

$$P\{NW_j^e(t) < 0\} = \theta_j^{br} = \begin{cases} \Psi_j(t), & \text{if } \Psi_j(t) > 0 \\ 0, & \text{if } \Psi_j(t) \leq 0 \end{cases} \quad (33)$$

ii. Capital Goods Firms

To be completed.

iii. Firms in General

A loan might be authorized only if P_f^{br} does not exceed a maximum:

$$\theta_f^{br} \leq \psi_b \quad (34)$$

ψ_b evolves according to the genetic algorithm defined in section 10.

A loan must also comply with another condition: the total debt of f in b , $D_f^b(t)$, must not exceed $L_b^f(t)$, defined in equation 30.

$$\hat{L}_f^W(t) + D_f^b(t) \leq \gamma L_b^T(t) \quad (35)$$

Therefore, the maximum loan firm f can make in bank b at t is

$$L_f^b(t) = \begin{cases} \hat{L}_f^W(t), & \text{if conditions 34 and 35 are satisfied} \\ \gamma L_b^T(t) - D_f^b(t), & \text{if condition 34 is satisfied, but 35 is not, and } D_f^b(t) < \gamma L_b^T(t) \\ 0, & \text{if condition 34 is not satisfied or } D_f^b(t) \geq \gamma L_b^T(t) \end{cases} \quad (36)$$

If firm f wants a loan, it makes one, and $L_f^W(t) = L_f^b(t)$.

d. Firm f decides how many workers to hire, based on bank b 's decision.

Three scenarios are possible:

$$\text{i. } L_f^W(t) = \hat{L}_f^W(t)$$

Firm f tries to hire all workers it intended to.

ii. $L_f^W(t) < \hat{L}_f^W(t)$ and $L_f^W(t) \neq 0$

Firm f cannot hire all workers it intended to, because it is credit rationed. It assumes that, if the required loan cannot be granted, expenses with extra employees next period might not be covered. It removes elements from $W_f^N(t)$, lower wages first, until the sum of remaining wages is less or equal to $L_f^b(t)$.

iii. $L_f^W(t) = 0$

Firm f is credit rationed and cannot hire any workers. $W_f^N(t)$ is set to \emptyset (empty set).

e. Firm f pays its current employees.

As a last resort, if firm f is credit rationed, it checks its own funds $NW_f(t-1) + V_f^1(t)$ to see how much of the difference $\hat{L}_f^W(t) - L_f^b(t)$ can be paid. Unpaid workers will have their productivities reduced by a factor of ν , $0 \leq \nu \leq 1$. $y_f(t)$ must be updated accordingly. If the total amount of unpaid wages is w_f^R , then

$$y_f(t) = \nu \cdot \left(1 - \frac{w_f^R}{W_f^T(t)}\right) \cdot y_f(t) \quad (37)$$

f. Firm f posts job offerings $W_f^N(t)$, if $W_f^N(t) \neq \emptyset$.

g. Unemployed worker i applies for jobs offered in his field (capital or consumption).

He randomly finds a firm with vacancies and verify its whole list of offerings. Unemployed worker i accepts an offering if the salary, w , satisfies

χ_i evolves according to the genetic algorithm defined in section X.

$$w_i^R(t) \leq w \leq \chi_i \cdot w_i^R(t), \chi_i \in \mathbb{R} \quad (38)$$

If consumer i accepts the offering, $W_j(t) = w \cup W_j(t-1)$ and W_j^T is updated accordingly. Unemployed consumer i tries to find a job in up to three firms per month.

h. Consumer i adjusts his reservation wages.

$$w_i^R(t+1) = \begin{cases} w_i^R(t) \cdot (1 - X), & \text{if } i \text{ is unemployed} \\ w_i(t), & \text{if } i \text{ is employed} \end{cases} \quad (39)$$

Where $X \sim \mathcal{U}(0, 0.1)$.

i. Firm k 's debt is updated.

$$D_k(t) = D_k(t-1) + L_k^W(t) \quad (40)$$

At this point, only firms k can update debt, because they do not borrow any further. Firms j , instead, borrow to make investments.

3.3.4. Investment

Investment happens only under the circumstances described in section 2.d.ii. and if the firm is able to finance it.

- a. Firm j decides whether to replace some machines.

The probability of trying to substitute any given machine is

$$\theta_j^R(t) = 1 - e^{-\epsilon NW_j(t-1)}, 0 \leq \epsilon \leq 1 \quad (41)$$

$\theta_k^m(t)$ is a Bernoulli random variable that indicates whether firm k will try to replace machine m :

$$\Theta_k^m(t) = \begin{cases} 1 & \text{with probability } \theta_k^R \\ 0 & \text{with probability } 1 - \theta_k^R \end{cases} \quad (42)$$

Machine m is compared with similar machines listed in the catalogs firm j has access to. Possible replacements - one per catalog - are chosen based on its productivity, which must be greater than machine m 's and also the closest among all machines in the catalog. Whichever machine costs less, firm j decides to buy and adds it to $M_j^C(t)$.

- b. Firm j calculates desired investment $I_j^D(t)$.

$$M^z = [I^z, w^z, p^z(t)], z \in \mathbb{N} \quad (43)$$

Then,

$$I_j^D(t) = \sum_{z=1}^{|M_j^C(t)|} p^z(t) \quad (44)$$

Let $\widetilde{NW}_j(t)$ be the partial value of j 's net worth at t :

$$\widetilde{NW}_j(t) = NW_j(t-1) + V_j^1(t) - D_j(t-1) - L_j^W(t) \quad (45)$$

We define $V_k^2(t)$ as the amount of own funds firm f uses to invest. Since firms k do not invest, $V_k^2(t) = 0$, by definition. Regarding firms j , two situations are possible:

- i. $\widetilde{NW}_j(t) \leq (1 - \alpha_f) \cdot I_j^D(t)$

Firm j cannot use its own funds to finance investment. Hence, the desired loan value is $\hat{L}_j^I(t) = I_j^D(t)$ and $V_j^2(t) = 0$.

- ii. $\widetilde{NW}_j(t) > (1 - \alpha_f) \cdot I_j^D(t)$

Firm j 's desired loan for investment is $\hat{L}_j^I(t) = (1 - \alpha_j) \cdot I_j^D(t)$ and $V_j^2(t) = -\alpha_f \cdot I_j^D(t)$

- c. Bank b , in which firm j holds its deposits, decides the amount of credit $\tilde{L}_b^j(t)$ it is willing to lend to firm j .

- d. Firm j makes a loan to invest. Then, $L_j^I(t) = \tilde{L}_b^j(t)$.

e. Firm j decides how much to invest.

i. $L_j^I(t) = \hat{L}_j^I(t)$

Firm j buys all the machines it intended to.

ii. $L_j^I(t) < \hat{L}_j^I(t)$ and $\tilde{L}_j^b(t) \neq 0$

Firm j cannot buy all the machines it intended to, because it is credit rationed. It removes elements from $M_j^C(t)$, lower productivities first, until $\hat{L}_j^I(t) \leq \tilde{L}_j^b(t)$.

iii. $L_j^I(t) \neq 0$

Firm j is credit rationed and cannot buy any machines. $M_j^C(t)$ is set to \emptyset (empty set).

f. If $M_j^C(t) \neq \emptyset$, $M_j(t+1) = M_j(t) \cup M_j^C(t)$

g. Firm j 's debt is updated.

$$D_j(t) = D_j(t-1) + L_j^W(t) + L_j^I(t) \quad (46)$$

3.3.5. Production and Innovation

i. Consumption Goods Firms

Firm j produce goods according to current employees - including the ones it hired at t - and machines - not including the ones it bought at t .

ii. Capital Goods Firms

a. Firm k tries to innovate, i.e. to modify a machine according to its investment in R&D.

Let $\theta_k^{IN}(t)$ be the probability of innovating at t . If $\Theta_k^{IN}(t)$ is a Bernoulli random variable which indicates whether firm k will innovate, then

$$\theta_k^{IN}(t) = 1 - e^{-\xi RD_k(t)} \quad (47)$$

$$\Theta_k^{IN}(t) = \begin{cases} 1 & \text{with probability } \theta_k^{IN}(t) \\ 0 & \text{with probability } 1 - \theta_k^{IN}(t) \end{cases} \quad (48)$$

If $\Theta_k^{IN} = 1$, one (or more) machine(s) is (are) randomly chosen and improved. Let m be a chosen machine and $V = |M_k(t)|$. The first new machine is:

$$M_k^{V+1} = [l^m \cdot (1 + Z^l(t)), w^m \cdot (1 - Z^w(t)), p^v(t) \cdot (1 + Z^p(t))] \quad (49)$$

$$M_k(t+1) = M_k(t) \cup M_k^{V+1} \cup M_k^{V+2} \cup \dots \cup M_k^{V+n} \quad (50)$$

where n is the total number of new machines.

b. Firm k tries to imitate firms with similar technologies.

Technological distance between each pair of firms (a, b) is calculated in the following way:

$$\Lambda(a, b) = ((\bar{l}_a - \bar{l}_b) + (\bar{w}_a - \bar{w}_b))^{1/2} \quad (51)$$

where $\bar{l}_k = \sum_{\forall v} l^v / V$ and $\bar{w}_k = \sum_{\forall v} w^v$, $v : M_k^v \in M_k(t)$.

Imitation occurs randomly. The probability to imitate is

$$\theta_k^{IM}(t) = 1 - e^{-(1-\xi)RD_k(t)} \quad (52)$$

$\theta_k^{IM}(t)$ is a Bernoulli random variable which indicates whether firm k will innovate:

$$\Theta_k^{IM}(t) = \begin{cases} 1 & \text{with probability } \theta_k^{IM} \\ 0 & \text{with probability } 1 - \theta_k^{IM} \end{cases} \quad (53)$$

If $\Theta_k^{IM}(t) = 1$, firm k copies a machine from the most similar firm in terms of technology. The machine it imitates is the one which sold the most last month. If firm k already possess this machine, it copies the second most sold, and so on.

3.3.6. Trade

c. Employed consumer i deposits his savings in a bank and try to pay part of his debts.

Savings at t amount to $\zeta_i \cdot w_i(t)$. Bank b , where the total of i 's savings is deposited, is chosen randomly in the first run.

$$NW_i(t) = NW_i(t) + \zeta_i \cdot w_i(t) \quad (54)$$

Here, indebted employed consumers examine their wealth to determine if they are able to amortize part of their debt. If employed consumer i is indebted at t ,

$$D_i(t) = \begin{cases} D_i(t-1) - \delta_i NW_i(t-1), & \text{if } NW_i(t-1) > 0 \\ D_i(t-1), & \text{otherwise.} \end{cases} \quad (55)$$

d. Unemployed consumer i determines how to pay for consumption goods.

$$D_i(t) = \begin{cases} D_i(t-1) + \eta_i w_i^R(t), & \text{if } NW_i(t-1) < \eta_i w_i^R(t) \\ D_i(t-1), & \text{otherwise.} \end{cases} \quad (56)$$

$$NW_i(t) = \begin{cases} NW_i(t-1) - \eta_i w_i^R(t), & \text{if } NW_i(t-1) \geq \eta_i w_i^R(t) \\ NW_i(t-1), & \text{otherwise.} \end{cases} \quad (57)$$

e. Transactions take place.

3.3.7. Accounting

- a. Firm f calculates its net worth

$$NW_f(t) = NW_f(t-1) + V_f^1(t) + V_f^2(t) + D_f(t) \quad (58)$$

By definition, $V_k^2(t) = 0$ for all k , since capital goods firms do not invest in machinery.

- b. Firm f pays part of its debt

$$D_f(t+1) = \begin{cases} (1 + r_f^b(t-1)) \cdot D_f(t) - \delta_f NW_f(t), & \text{if } NW_f(t) > 0 \\ (1 + r_f^b(t-1)) \cdot D_f(t), & \text{otherwise.} \end{cases} \quad (59)$$

- c. Consumer i receives a small interest for his deposits at bank b .

$$NW_i(t) = NW_i(t) \cdot (1 + r_b^S(t)) \quad (60)$$

- d. Bank b determines the interest rates it is going to charge from each of its clients next month.

$$r_b^a(t+1) = r^{CB}(t) + r_b(t) + \hat{r}_b^a(t) \quad (61)$$

where a is a client and $r_b(t)$ is

$$r_b(t) = \begin{cases} r_b(t-1) \cdot (1 - \mu_b Z), & \text{if } \sum_{\forall a} L_b^a(t) < L_b^T(t) \\ r_b(t-1) \cdot (1 + \mu_b Z), & \text{otherwise.} \end{cases} \quad (62)$$

$Z \sim \mathcal{U}(0, 1)$ and $\mu_b \in \mathbb{R}$. The risk premium, $\hat{r}_b^a(t)$, is different for each kind of agent:

- i. Firms

$$\hat{r}_b^a(t) = \iota \left(1 - \frac{|W_f^T(t-1)|}{E} \right) \cdot \frac{D_j(t-1)}{NW_j(t-1)} \quad (63)$$

- ii. Workers

To be determined.

- e. Wages are updated according to GDP growth and inflation.

3.4. Calibration

The parameters to be calibrated are the prices' sensibility to profits (κ), the labor elasticity (β), the amortization rate (δ), the information cost (ϕ), the firms' strength in bargaining (γ), and the consumption inertia (η). We are going to run the simulation repeatedly until a configuration of these parameters is found such that the mean of some variables match real-world means. The number of variables must be equal to the number of parameters. We could use, for example, the inflation rate, real bonds yield, the unemployment rate, GDP gap volatility, inflation volatility, and unemployment duration.

3.5. Interface

3.5.1. Inputs

1. Saving percentage (ζ) mean: Consumers save a proportion of their wages every period. This proportion is going to be normally distributed across the population of Consumers and this parameter define the mean of the distribution (standard deviation will be a quarter of the mean).

3.5.2. Outputs

1. Output growth
2. Unemployment rate
3. Interest on loans (mean)
4. Mean prices
5. GDP per worker
6. Interest on deposits (mean)
7. Debt-to-equity

4. Second Model

4.1. Monetary Policy

4.2. Genetic Evolution

Consumption goods firms need to adapt the following parameters:

- β_j : proportion between inventory and production;
- δ_j : amortization rate;
- α_j : proportion between internal funds and loans for financing;
- κ_j : prices' sensibility to excessive demand.

The genetic selection of these values depends on basically three other parameters:

- P : population maximum size;
- N : number of other firms from which to copy chromossomes;
- T : number of lags to be stored.

A chromosome is defined as $c_f^l = [\beta_f^l, \delta_f^l, \alpha_f^l, \kappa_f^l]$, where f is the firm index and l is the time index. Therefore, $f \in \{j, f_1, f_2, \dots, f_N\}$ and $l \in \{t-1, t-2, \dots, t-T\}$. There are two objective functions in the case of these firms: $\Pi(c_f^l)$, realized profits, and $L(c_f^l)$, total value of loans. The first must be maximized, the second, minimized.

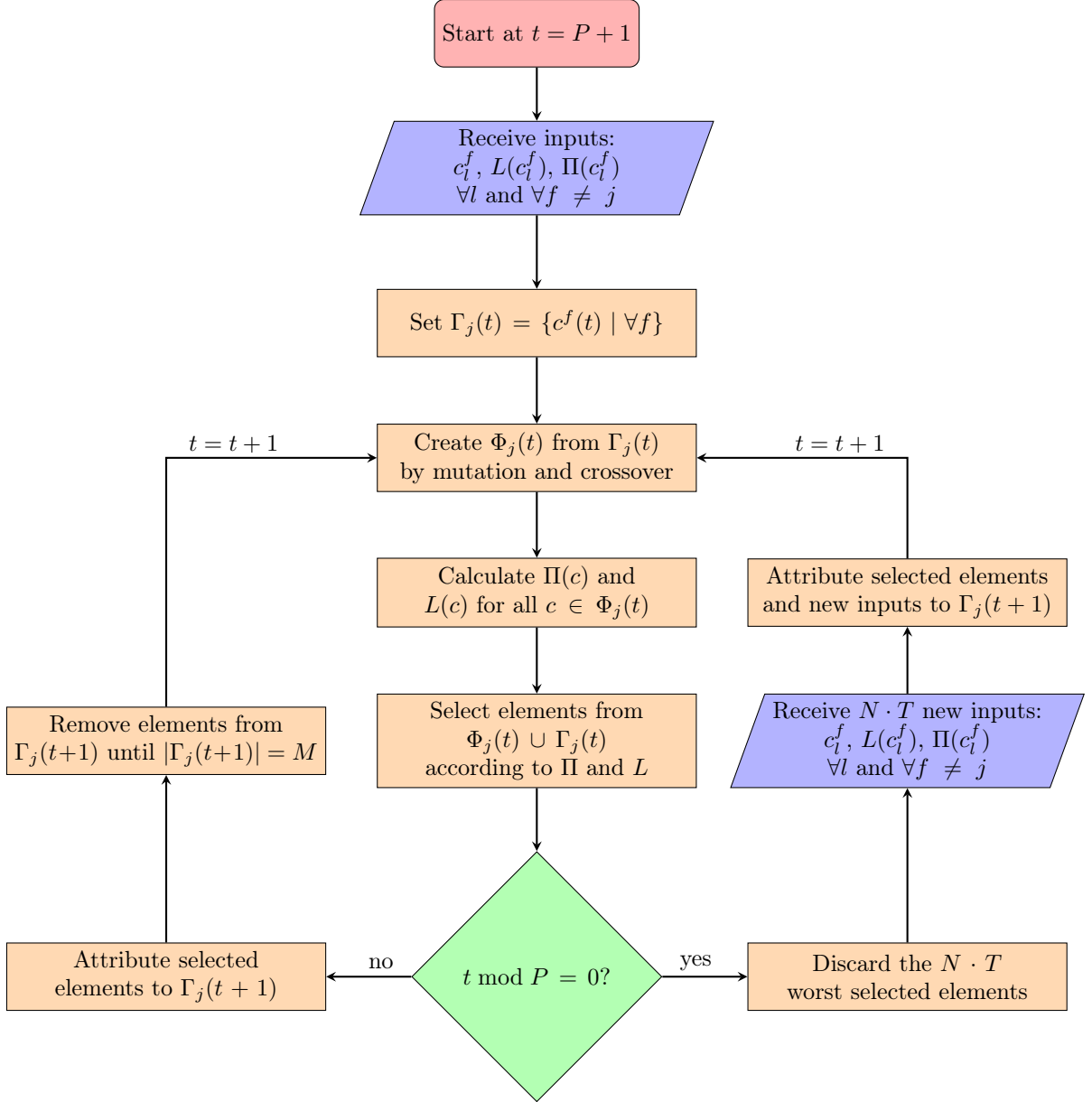


Figure 1: Flowchart of the genetic algorithm agents use to adapt their behaviour.

5. Results

To be completed

6. Conclusion

To be completed

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