

# Part 1:

## Probability, Bayes' theorem

$$\text{Bayes' theorem: } P(B/A) = \frac{P(A \setminus B) * P(B)}{P(A)}$$

### Q1: a

B=identical male twins(I.M.T)

A=male twins(M.T)

	male&male	male&female	female&male	female&female
Not identical $\frac{1}{125}$	$\frac{1}{4} = \frac{1}{500}$	$\frac{1}{4} = \frac{1}{250}$	$\frac{1}{4} = \frac{1}{250}$	$\frac{1}{4} = \frac{1}{500}$

	male&male	female&female
Identical $\frac{1}{300}$	$\frac{1}{2} = \frac{1}{600}$	$\frac{1}{2} = \frac{1}{600}$

$$P(I.M.T / M.T) = \frac{P(M.T \setminus I.M.T) * P(I.M.T)}{P(M.T)} = \frac{1 * \frac{1}{600}}{\frac{1}{600} + \frac{1}{500}} = \frac{\frac{1}{600}}{\frac{11}{3000}} = \frac{5}{6}.$$

### Q1: b

B=Chocolate (C)

A= bowl number 1 (1)

$$P(C/1) = \frac{P(1 \setminus C) * P(C)}{P(1)} = \frac{\frac{3}{4} * \frac{1}{2}}{\frac{5}{8}} = \frac{3}{5}.$$

## Q2

B= Yellow and Green (Y.G)

A= Yellow from 1994 (Y 94)

$$P(Y94/Y.G) = \frac{0.2*0.2}{0.2*0.2+0.14*0.10} = \frac{0.04}{0.054} = \frac{20}{27}$$

## Q3: a

B = Positive (p)

A= Sick (s)

$$P(P/S) = \frac{1 * \frac{1}{10,000}}{\frac{1}{10,000} + 0.01} = \frac{1}{101}$$

## Q3: b

$$P(P/S) = \frac{1 * \frac{1}{200}}{\frac{1}{200} + 0.01} = \frac{1}{3}$$

## Random variables

$$\text{Expected value} = \sum_{i=0}^n x_i * P(x_i)$$

## Q1

Options of Sum={2,3,4,5,6,7,8,9,10,11,12}

Sum divided by 3={3,6,9,12}=

P(sum 3)+P(sum 6)+P(sum 9)+P(sum 12)

$$\frac{2}{36} + \frac{5}{36} + \frac{4}{36} + \frac{1}{36} = \frac{1}{3}$$

$$P(\text{sum not divided by 3}) = 1 - \frac{1}{3} = \frac{2}{3}$$

$$E_{(X)} = 6 * P(\text{win 6\$}) - 3 * P(\text{lose 3\$})$$

$$= 6 * \left(\frac{1}{3}\right) - 3 * \left(\frac{2}{3}\right) = 2 - 2 = 0 \text{ (a fair game)}$$

## Q2

$$P(\text{sum bigger than 12}) = P(\text{sum 13}) + P(\text{sum 14}) + P(\text{sum 15})$$

$$= \frac{3}{25} + \frac{2}{25} + \frac{1}{25} = \frac{6}{25}$$

$$P(\text{sum equals to 12}) = \frac{4}{25}$$

$$P(\text{sum smaller than 12}) = 1 - \frac{6}{25} - \frac{4}{25} = \frac{3}{5}$$

$$E_{(X)} = 5 * \frac{6}{25} - 6 * \frac{3}{5} = -\frac{12}{5} \text{ (isn't a fair game)}$$

## Q3

The number of males in the company is:  $0.4 * 200 = 80$

$\mu = \text{sample size} * \text{Percentage of population in the sample}$

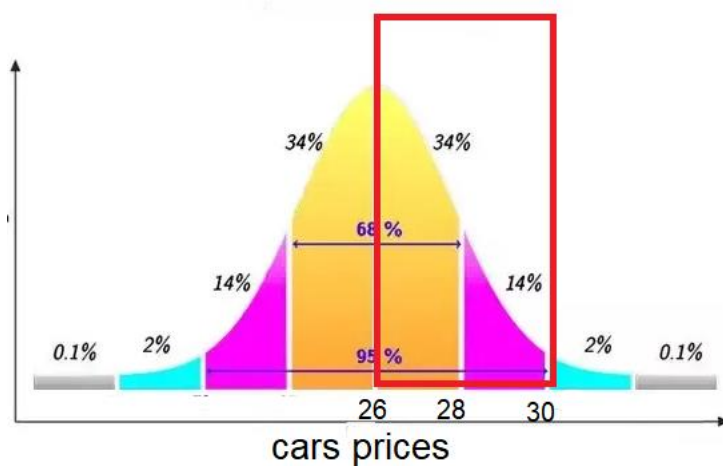
$$= 0.4 * 8 = 3.2 \text{ (mean)}$$

The Standard deviation of the sample =

$$\sigma = \sqrt{\frac{\sum_{i=1}^n (x_i - \mu)^2}{N}}$$

$$6 = \sqrt{\frac{0.4 * (1 - 0.4)}{8}} = 0.1732 \text{ (Standard deviation)}$$

**Q4**



$$X = 34\% + 14\% = 48\% = 0.48.$$

**Q5**

$$P(X > 3) = \frac{2 \cdot 0.4}{2} = 0.4 \text{ (area of the Triangle)}$$

**Q6**

$$P\left(\frac{\text{The number of results desired}}{\text{Number all possible outcomes}}\right) =$$

$$\frac{\binom{300}{3} * \binom{200}{1}}{\binom{500}{4}} = 0.3462.$$

**Q7**

$$E_{(X)} = -10 * 0.1 - 5 * 0.35 + 0 + 10 * 0.1 + 5 * 0.35 = 0.$$