# UC Santa Barbara, Department of Chemical Engineering
# ChE 180A: Chemical Engineering Laboratory

**Friction Factors and Velocity Profile**
Updated 5/30/19

## Overview

The purpose of this experiment is to measure both frictional factors and the velocity profile associated with turbulent fluid flow in circular pipes. This experiment utilizes a variable-speed, centrifugal pump to control the flow rate of water through circular test sections of pipe. Friction loss factors for various pipe geometries (straight, 90°, and 180° bends) and valves will be investigated. A turbulent velocity profile will be measured by traversing a pitot tube across the diameter of a circular pipe and compared with semi-empirical models.

## Objectives

- Understand the operation of centrifugal pumps and methods of regulating fluid flow
- To measure frictional losses for various pipe geometries and valves
- To be able to experimentally measure and model the velocity profile of turbulent flow
- Discuss any discrepancies between experimental data and tabulated values for frictional factors

## Introduction

When a fluid flows through a pipe, the fluid exerts a drag force on the pipe wall. This force is the manifestation of axial momentum being transmitted from the fluid to the wall. It may be viewed as fluid friction because the equal and opposite force of the wall on the fluid tends to retard the flow. The tangential drag force per unit area of fluid-solid interface is called the shear stress $\tau_0$. A steady-state, macroscopic force balance on a section of straight, horizontal, circular pipe of length L and radius R indicates that the fluid friction causes a pressure drop in the direction of flow:

$$2\pi R L \tau_0 = \pi R^2 (p_0 - p_l) \tag{1}$$

measure friction

where $p_0$ is the fluid pressure at the entrance of the length of pipe, and $p_l$ is the pressure at the exit.

For practical purposes, it is desirable to relate $\tau_0$ to the flow rate of the fluid so that Equation 1 may be used to calculate the relationship between flow rate and pressure drop in pipes. For Newtonian fluids, the shear stress at the wall is given by Newton's law of viscosity

$$\tau_0 = -\mu \frac{\partial v_z}{\partial r}\bigg|_{r=R} \tag{2}$$

Where $\mu$ is the fluid viscosity, $v_z$ is the fluid velocity in the axial direction, and r is the radial coordinate in the pipe. For laminar flow, one can solve for the steady-state velocity profile $v_z(r)$. This solution may then be used to calculate the total volumetric flow rate of fluid Q as well as the derivative in Equation 2. These two results may be combined to obtain the Hagen- Poiseuille law*:*

$$Q = \frac{\pi((p_0 - p_l)R^4}{8\mu L} \tag{3}$$

measure

transducer

which may be used to calculate either Q from $(p_0 - p_l)$ or *vice versa*.

However, the chaotic nature of turbulent flow and the radial transport of momentum by random eddies make it impossible to calculate the velocity profile from first principles, and it is even difficult to measure the velocity profile near the pipe wall very accurately. In this case Equation 2 is of limited utility, and it becomes necessary to use empirical methods to relate pressure drop to fluid flow rate and other system properties.

Even when the related differential equations cannot be solved, the physical principles that govern fluid flow may be used to define a quantity for correlating experimental fluid flow behavior in laminar or turbulent flow and in more complex geometries. This quantity is known as the *friction factor* and is calculated from measured values of pressure drops and flow rates. According to dimensional analysis, friction factor values may be presented in general dimensionless correlations that can be used for practical prediction of flow behavior. In this experiment, the pressure drop is measured as a function of flow rate for water passing through horizontal pipes. A general correlation of the friction factor is constructed from the data. Since piping systems are rarely entirely composed of long lengths of straight pipe, it is also useful to define a similar parameter to describe pressure loss in fittings such as bends and valves. This parameter is known as the *friction loss factor* and is represented by $e_v$. Tables of $e_v$ appear in *Transport Phenomena* (p. 207) as well as *Perry's Handbook.*

**Theory**

***Frictional Losses***

For fully developed steady-state flow the local time-smoothed shear stress at the wall $\tau_0$ (Equation 2) is independent of wall position because the time-smoothed velocity profile is independent of z, the axial direction, and $\theta$, the angle along the radial plane. A dimensionless friction factor $f$ for this case may be defined as the proportionality factor in the relation

$$\tau_0 = \left(\frac{1}{2}\rho\langle v_z\rangle^2\right) f \tag{4}$$

where $\langle v_z\rangle$ is the average velocity in the pipe, defined by

$$\langle v_z\rangle = \frac{Q}{\pi R^2} \tag{5}$$

This definition of $f$ is called the *Fanning friction factor* for pipe flow. Combination of Equation 4 with Equation 1 yields the relation

$$f = \frac{\pi^2 R^5}{\rho Q^2}\left(\frac{p_0 - p_l}{L}\right) \tag{6}$$

which enables calculation of $f$ from experimental pressure and flow rate measurements. Combination of Equation 4 with Equation 2 yields another expression for f:

$$f = \frac{-2\mu \frac{\partial v_z}{\partial r}\Big|_{r=R}}{\rho\langle v_z\rangle^2} \tag{7}$$

which provides a theoretical form for computing $f$ if the detailed velocity profile is known.

Even if the velocity profile cannot be computed from basic principles (for example in turbulent flow) the underlying differential equations may be subjected to dimensional analysis as described in Section 3.7 of *Transport Phenomena.* For steady, fully developed Newtonian flow in a straight pipe the velocity profile can be cast in a dimensionless form and that the solution of the basic equations becomes a general function of the form

$$\frac{V_z(r,\rho,\mu,R)}{\langle V_z\rangle} = V_Z^*\left(\frac{r}{R}; Re\right) \tag{8}$$

The second argument of this function Re is the Reynolds number, defined as

$$Re = \frac{D\langle V_Z\rangle\rho}{\mu} \tag{9}$$

where D is the pipe diameter, D=2R.

Substituting Equation 8 into Equation 7 leads to the conclusion that the friction factor function should be simply

$$f = f(Re) \tag{10}$$

for incompressible, Newtonian fluids in a straight, smooth pipes. More generally, the friction factor can depend on dimensionless geometrical ratios. For example, for shorter pipes in which the velocity profile is developing and $\tau_0$ depends on the distance downstream from an entrance, the expected relationship for the average friction factor is of the form

$$f = f(Re, \frac{L}{D}) \tag{11}$$

Or if the pipe wall is rough with a scale of roughness of average magnitude k, the correlation for $f$ in fully developed flow is expected to be

$$f = f(Re, \frac{k}{D}) \tag{12}$$

Values of $f$ can be calculated from flow measurement in straight pipes using Equation 6. The resulting values may be fitted into an empirical dimensionless correlation to obtain the dimensionless function indicated in Equation 10 (or 11 for short pipes). According to the principle of dimensional analysis, the resulting dimensionless function should be valid for all Newtonian fluids for pipes of all sizes. Thus, for a different fluid in a pipe of any size, when the flow rate is specified, one may calculate the Reynolds number and estimate f from the general correlation. Finally, the expected pressure drop in the pipe may be predicted using Equation 6. A similar approach is applied in the definition and use of heat or mass transfer coefficients, as explained in Chapters 13 and 21 of *Transport Phenomena.*

A general correlation of $f$ values has been constructed from a large body of data and is illustrated in Figure 6.2-2 of *Transport Phenomena.* This plot includes the theoretical relationship obtained from Equation 7 for fully developed laminar flow, for which the velocity profile is given by Equation 2.3-16 in *Transport Phenomena*. Also depicted are empirical curves for turbulent flow in pipes with various degrees of roughness, characterized by the scale dimension k. Although there are two possible solutions for f at any value of Re, corresponding to both laminar and turbulent flow, the former is stable only at Reynolds numbers lower than 2100. The turbulent-flow friction factor is always greater than that for laminar flow because turbulent mixing enhances momentum transport to the pipe wall. Similarly, finite values of L/D or k/D tend to increase the value of f.

In many cases the flow rate is not known, and the Reynolds number and friction factor cannot be directly calculated. For example, in the case where the pressure drop is imposed by external conditions, the flow rate and hence the Reynolds number are dependent variables to be determined from the f correlations. In this case one may rearrange the relation given by Equation 10 to solve for the quantity

$$Re\sqrt{f} = \frac{D\rho}{\mu}\sqrt{\frac{(p_0 - p_l)D}{2L\rho}} \tag{13}$$

which is independent of <$v_z$> to obtain a new correlation

$$f = f(Re\sqrt{f}) \tag{14}$$

For example, Figure 6.2-2 in *Transport Phenomena* may be replotted in the form of Equation 14. The quantity $Re\sqrt{f}$ can be calculated from a known pressure difference by means of Equation 13 and used to estimate the value of $f$. Then, Equation 6 may be used to predict the volumetric flow rate $Q$.

In the present experiment the pressure drop is measured for several flow rates of water passing through horizontal pipes of various diameters. The Reynolds number and the friction factor f are calculated from this data and used to construct a correlation corresponding to Figure 6.2-2.

It is instructive to compare regions of the empirical friction-factor correlation with relations predicted from various forms of the velocity profile according to Equation 7. For example, the parabolic laminar velocity profile yields the theoretical relation

$$f = \frac{16}{Re} \tag{15}$$

which can be confirmed experimentally for Re<2100. If the time-smoothed velocity profile in fully-developed turbulent flow is fit by the power-law form given in Equation B.1-8 (See Problem 5.C in *Transport Phenomena.)*, the corresponding result from Equation 7 is

$$f = \frac{0.0791}{Re^{1/4}} \tag{16}$$

On the other hand, fitting the turbulent flow velocity profile to a slightly modified form of Equation B.1-3 (See Problem 6.J in *Transport Phenomena*.) yields a specific form of Equation 14:

$$\frac{1}{\sqrt{f}} = 4.0log_{10}[Re\sqrt{f}] - 0.40 \quad (17)$$

Equation 16 is generally valid up to $Re = 10^5$, while Equation 17 fits smooth-pipe turbulent-flow data up to $Re = 5 \times 10^6$.

***Velocity Profile***

When the fluid flow is fast relative to the fluid viscosity, it becomes unstable with respect to various disturbances, and the streamlines experience instantaneous fluctuations in both magnitude and direction known as turbulent eddies. For pipe flow this usually occurs when the Reynolds number is greater than approximately 2100. The velocity fluctuations provide a mechanism for momentum transport across the time-averaged streamlines, increasing the rate of momentum transport to the pipe wall. The enhancement of the momentum flux by turbulent eddies also modifies the time-smoothed velocity profile of the fluid, and in regions of effective turbulent transport steep velocity gradients are not needed to drive momentum transport by viscous forces. The time-smoothed velocity profile is therefore flatter than the corresponding laminar flow field. On the other hand, in regions such as near a solid wall where the velocity fluctuations are blocked or damped out, viscous forces must carry the momentum flux into the wall. A laminar sublayer exists near the wall in which the velocity gradient becomes steep according to Newton's law of viscosity.

Conservation of momentum and the associated equations of motion (Equation 3.2-10 in Transport Phenomena) are still valid in the case of turbulent flow. Equations describing the time-smoothed velocity profiles may be obtained by averaging over a time that is long compared with the frequency of the turbulent fluctuations. In addition to the usual viscous transport and inertial terms, these equations contain extra terms that arise from the mixing effects of the eddies. The extra terms are identified as the turbulent momentum flux, $\bar{\tau}^{(t)}$ , and referred to as Reynolds stresses. The general relationship between the turbulent momentum flux and the time-smoothed velocity gradient for turbulent flows may be expressed in a form analogous to Newton's law of viscosity, making it possible to solve the time-smoothed equations of motion to obtain the averaged velocity profile and the shear stress on the walls.

Section 5.3 of Transport Phenomena summarizes several the empirical relationships that have been proposed to describe the turbulent momentum flux, including those of Prandtl and Deissler that are useful in describing the velocity distributions for incompressible fluids flowing in tubes. These two empirical relationships, combined with the time-smoothed equation of motion and experimental data, yield the three following expressions for the time-smoothed velocity distributions in pipes at Reynolds numbers greater than 20,000:

$$v^+ = s^+ \quad for \quad 0 \le s^+ \le 5 \quad (17)$$

$$v^+ = \int_0^{s^+} \frac{ds^+}{1+0.0154v^+s^+[1-\exp(-0.0154v^+s^+)]} \quad for \quad 0 \le s^+ \le 26 \quad (18)$$

and

$$v^+ = \frac{1}{0.36}\ln(s^+) + 3.8 \quad for \quad s^+ \geq 26 \tag{19}$$

These equations are derived in Examples 5.3-1 and 2 of Transport Phenomena where the notation is defined. Specifically, the dimensionless velocity $v^+$ is defined as

$$v^+ = \frac{\bar{v}_z}{v_*} \tag{20}$$

where $v_* = \sqrt{\tau_0/\rho}$ and $\tau_0$ is the normal shear stress (or momentum flux) at the wall. Also,

$$s^+ = \frac{Sv_*\rho}{\mu} \tag{21}$$

where $s$ is the distance from the wall.

Equations 17, 18, and 19 are plotted along with experimental data for turbulent flow in pipes in Figure 5.3-1 of Transport Phenomena. This plot is sometimes referred to as the universal velocity profile for turbulent flow. Although these equations and the corresponding plot fit data on the turbulent velocity profile in pipes at high Reynolds numbers, an awkward aspect of this approach is that Equation 19 and Figure 5.3-1 do not recognize the existence of the centerline of the pipe, where the velocity profile should be flat. The pipe radius $R$ does not appear in these correlations because they focus on the effect of the wall, namely the shear stress $\tau_0$, on the structure of the turbulent boundary layer.

The average shear stress at the wall can be determined from a macroscopic force balance on the pipe. For steady flow in a horizontal pipe the shear stress on the wall balances the net pressure force acting axially on the fluid. That is,

$$(p_0 - p_l)\pi R^2 = \tau_0 2\pi RL \tag{22}$$

Therefore, the wall shear stress is given by

$$\tau_0 = \frac{(p_0 - p_l)R}{2L} \tag{23}$$

As an alternative to fitting the turbulent momentum flux in order to derive the time-smoothed velocity profile in turbulent flow, one may simply correlate experimental data on the velocity profile in a particular geometry. For pipe flow at Reynolds numbers between $10^4$ and $10^5$, Prengle and Rothfus (1955) reported that

$$\frac{\bar{V}_z}{\bar{v}_{z,max}} = \left(1 - \frac{r}{R}\right)^{1/7} \tag{24}$$

Schlichting (1951) broadened the applicability of Equation 24 by making the exponent an empirical function of Reynolds number, resulting in the following empirical equation to describe the velocity distribution for steady flow in round tubes:

$$\bar{V}_z = \bar{v}_{z,max}\left(1 - \frac{r}{R}\right)^{1/n} \tag{25}$$

in which the constant $n$ depends on the Reynolds number as summarized in Table 1. Although in certain respects Equation 25 is unsatisfactory, it is simple and convenient. For example, it allows one to relate the maximum velocity in a pipe to the average velocity (See Problem 5.C in Transport Phenomena.), but it cannot be used to calculate shear stress at the wall nor pressure drop.

Table 1. Value of $n$ in equation 25 as a function of Reynolds number

| Re | $4 \times 10^3$ | $7.3 \times 10^4$ | $1.1 \times 10^5$ | $1.1 \times 10^6$ | $2.0 \times 10^6$ | $3.2 \times 10^6$ |
|---|---|---|---|---|---|---|
| n | 6.0 | 6.6 | 7.0 | 8.8 | 10 | 10 |

An impact tube is a convenient method for measuring point velocities within a flowing fluid. Impact tubes are also known as pitot tubes and are described in Chapter 8 of McCabe, Smith, and Harriott (1993). By conversion of kinetic energy head to static pressure head at the mouth of a tubular probe, the undisturbed velocity in an impinging streamline can be related to the rise in pressure within the impact tube above the static pressure in the fluid at the point of impact. The local velocity of the fluid impacting the mouth of the tube $v_n$ is related to the pressure difference by the relation

$$v_n = \sqrt{\frac{2\Delta p}{\rho_{H_2O}}} \tag{26}$$

where $v_n$ is fluid velocity normal to the mouth of the tube and $\Delta p$ is the pressure difference between the pitot tube and the static port. It is important to be consistent in dimensions and units when making these calculations.

## Apparatus

*Friction factors*

The apparatus for this experiment consists of a 55-gallon water supply tank connected to ¾ horsepower, 60 cycle, 220 volt, 3-phase electric motor equipped with a variable speed drive. Figure 1 shows the control panel for pump operation. The discharge of the pump is fitted with removable pipe sections before finally returning to the supply tank. A series of straight pipe sections, bends, globe valves, and unions equipped with pressure taps will be provided for testing friction factors. The components will need to be assembled as necessary to perform the three parts of the experiment, and the parts may be completed in any order. Pressures are measured using 4-20 mA electronic transducers. Four transducers are provided for the experiments (0-5 psi, 0-10 psi, 0-25 psi, 0-50 psi). The results are very sensitive to the experimental configuration, so ensure that the transducer lines are purged of air and the discharge pipe submerged below the water level in the supply tank. **The exact pipe dimensions necessary for calculations should be measured during the lab.**

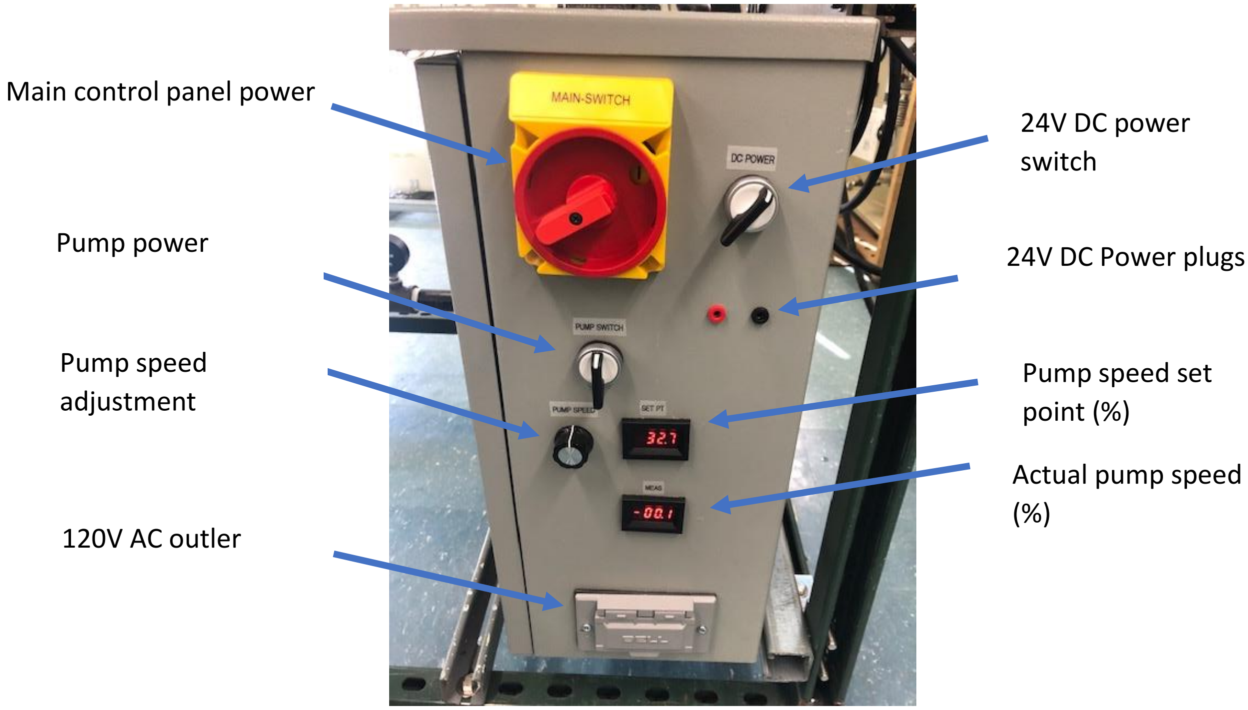


**Figure 1.** Control panel for centrifugal pump

*Pitot Tube*

The apparatus for this experiment consists of a cylindrical test section equipped with pressure taps at the inlet and outlet for measuring the pressure drop along the length of the pipe (used to determine $\tau_0$) and a traversing impact tube with a static pressure tap. The inside diameter of the test section is ~1 inch. Exact dimensions between the pressure taps and impact tube should be measured during the lab session. Use a 5 psid sensor to measure pressure drop along the test section and a 10 psid sensor to measure the impact pressure.

**Pre-lab**

- Familiarize yourself with the concepts of centrifugal pump performance curves, NPSHR and NPSHA.
- The electronic pressure transducers provided for pressure measurements output a 4-20 mA signal. A multimeter is suppled to measure this signal. Come prepared with an excel spreadsheet to quickly convert to the displayed current signal to the pressure reading. Assume a linear response for the transducer (i.e. for a 0-25 psi transducer 4 mA = 0 psi, 20 mA = 25 psi).
- Be able to calculate *Re* from measure lab flow rates in the same spreadsheet. Loop up any necessary constants prior to the lab session.
- Review the section on friction factor in Chapter 8 in McCabe, Smith, and Harriot (GauchoSpace).

- Collected data should be entered during the laboratory period and a preliminary log-log plot of friction factor *vs.* Reynolds number should be prepared.

**Procedure**
**Day 1**

**Frictional Losses**

This experiment requires measurement of pressure drops in three test sections: a straight pipe, a valve, and a 90/180° bend. The tests can be performed in any order. For each configuration, initially install the 25 psi sensor. If the voltage readings indicate that the pressure is within the range of a lower pressure sensor, switch to that sensor. Purge the sensor lines whenever air bubbles are present. When performing pressure drop measurements, connect a hose barb to the correct tee fittings and plug any remaining tees. The sensor port marked ‘high pressure’ should be connected to the hose barb furthest upstream. Flow rates should be measured using a bucket-and-stopwatch method where the flow will temporarily be diverted to a bucket which will be weighed. Set the pump speed to 60% (2060 rpm) and measure the outlet flow and pressure drop. Repeat this process for pump speeds of 80% (about 2760 rpm) and 100% (3450 rpm). Record all values in your lab notebook.

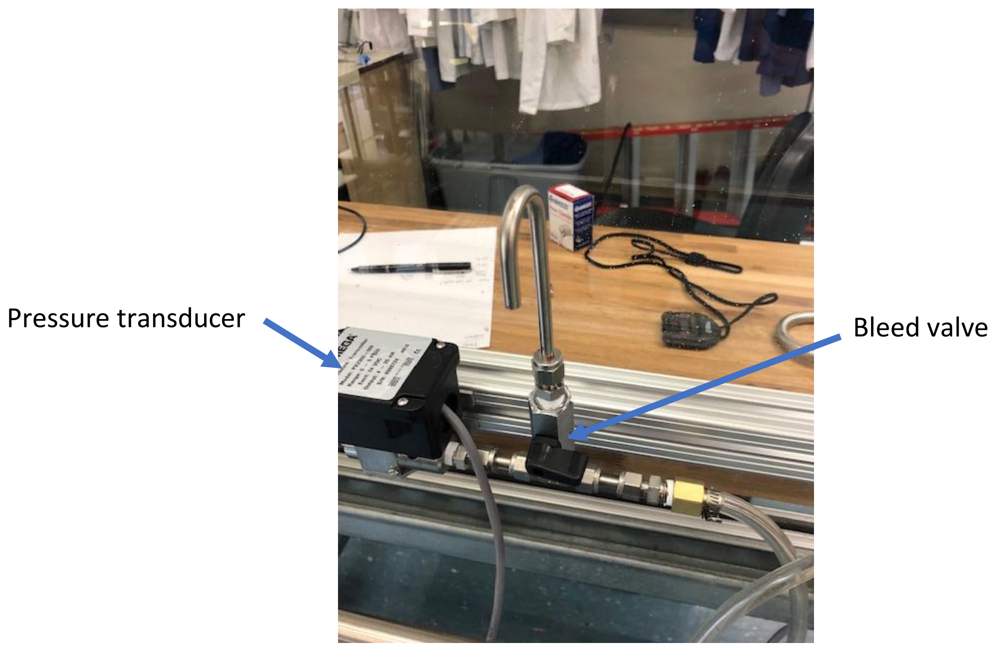


**Figure 2.** Bleed valve for electronic pressure transducer

Several types of fitting are employed in this lab. Pipe fittings (otherwise known as NPT fittings) are tapered and form an interference fit as they are assembled. To assure a seal, a layer of PTFE tape is applied to one of the fittings. Two layers is sufficient. Compression fittings (also known as Swagelok fittings) are used to connect tubing at relatively high pressures. They are very common in small scale chemical processes. These fittings do not require sealing tape. The end of the tube is inserted into the fitting until it stops, and the fitting nut is tightened **a maximum of 1¼ turns**. **Do not overtighten the fitting**, as this distorts the threads. The distortion increases the probability of leaks and prevents the fitting from being disassembled and re-used. The sealing is accomplished using two components known as ferrules that are squeezed against the tubing.

**Straight pipe section:** The straight pipe configuration is illustrated in Figure 3. Use a reducing union to convert the pump outlet to the desired tube size. Connect a 6" section of tubing. Connect a tee fitting to the other end, followed by an 18" section, a second tee, a second 18" section, a third tee, and a 6" section of tubing. Use the support bracket to secure the outlet end of the test section. Connect the outlet end to the return line. Measure the flow rate and pressure drop across the 18 and 36" at pump speeds 10%, 40%, 60%, 80%, and 100%. With the assistance of the TA, attempt to measure the laminar region by regulating the fluid flow with a gate valve at the pump discharge.

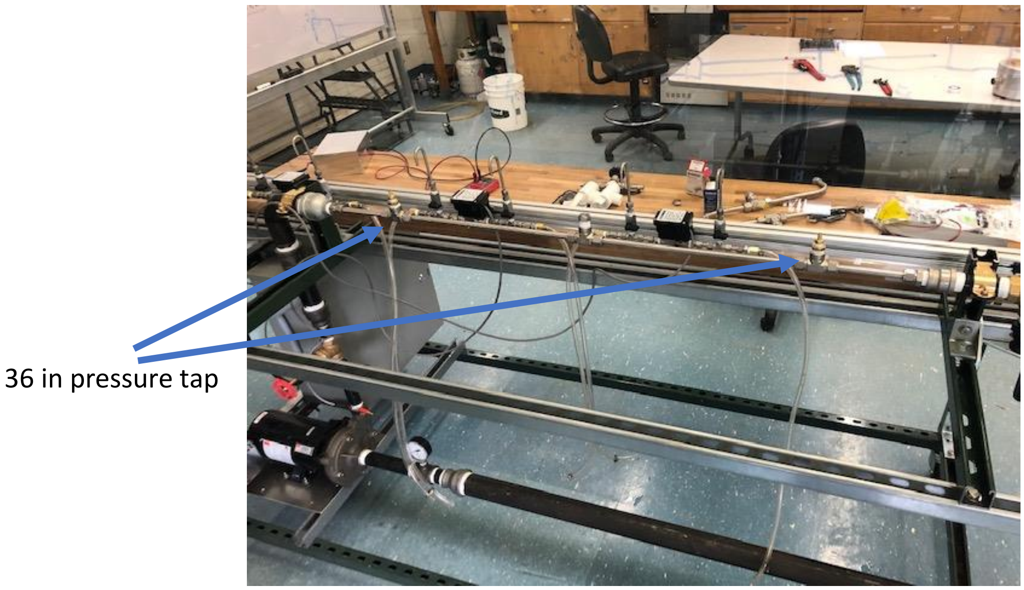


**Figure 3.** Straight pipe test section

**Valve test section:** Begin with the pipe nipple, bell reducer, and compression adapter used in the straight pipe tests. Insert an 18" straight section of tubing, followed by a tee, a 3" section, a globe valve, another 3" section, a second tee, and a second 18" section. Clamp the tubing to the support bracket and connect the return line to the downstream end. Measure the flow rate and pressure drop across the fully-open and 50%-opened valve at a pump speeds of 100%.

**90/180 test section:** The 90/180 test section is assembled according to the layout in Figure 4. Connect an 18" section to the compression adapter, followed by a tee. Insert a 90° bend. Connect a tee to the other end of the bend and insert the 180° piece. Add another tee and 90° section, followed by a fourth tee, an 18" straight section, and the return line. Measure the flow rate and pressure drop across the 90° and 180° bends at three pump speeds.

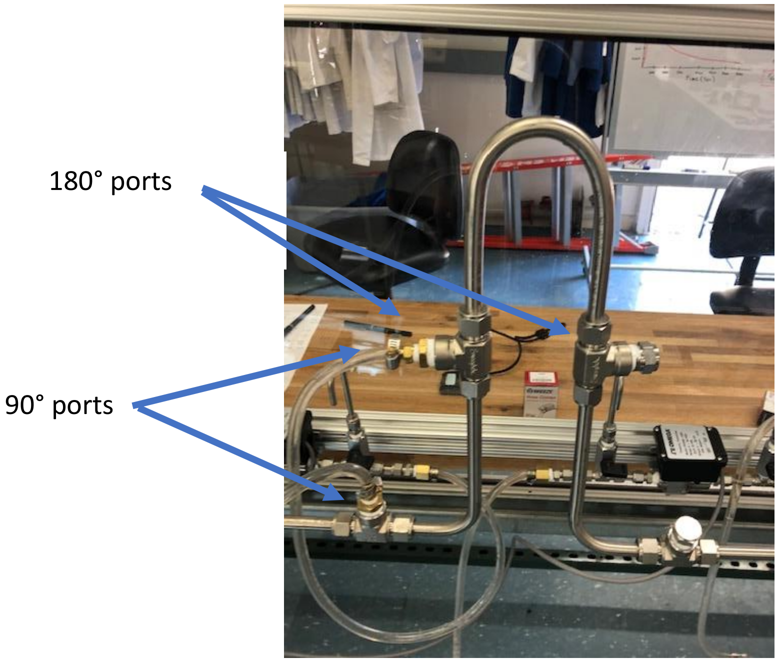


**Figure 4.** 90°/180° bend pipe test section

**Clean-up:**

Disassemble the fittings and remove stray PTFE tape from the pipe threads. Measure the inside diameter of the tubing using calipers. Reassemble the components to match the configuration at the start of the lab. Mop up any spilled water

**Day 2**

Measure the time-smoothed velocity profile in the pipe at three Reynolds numbers according to the following procedure.

1. Locate all necessary controls for operation of the experiment, including the pressure sensor purge valves, the bypass valve for obtaining mass flow samples, and the motor controls. Verify that the high-pressure side of the 5 psi sensor is connected to the upstream pressure tap and the low pressure side is connected to the downstream tap. The high-pressure side of the 10 psi sensor should be connected to the pitot tube and the low pressure side should be connected to the static port.

2. Fill the supply tank with enough water to cover the discharge pipe from the return line. If the discharge pipe is exposed air will be drawn into the system.

3. Remove the air from the sensor lines (Figure 2). This may need to be repeated if air is admitted to the test section.

  a) Turn the pump on at approximately 80% of maximum speed.
  b) Hold a beaker under the purge valve outlet for the line to be purged.
  c) Open the purge valve until the air has been removed from the line. Small bubbles are OK, but large sections of air will affect the sensor readings.
  d) Repeat this procedure for the other sensor lines.

4. Turn on the pump and adjust the speed to provide the desired flow rate. Suggested speeds for this experiment are 100, 80, and 60%.

5. Record the pressure drop along the length of the test section

6. Determine the fluid velocity at 10 positions across the diameter of the test section by measuring the pressure differential between the impact tube and the static pressure tap. Record the position of the probe for each pressure measurement. Make measurements on both sides of the centerline of the pipe to obtain a complete velocity profile. In particular, take readings at the extreme positions of the probe, close to the walls, in order to identify the relationship between the scale on the probe-positioning device and the coordinates within the pipe. Do not, however, use excessive force in positioning the probe against the pipe wall.

7. Perform Step 5 once for each flow rate.

8. Immediately mop up any spilled water from the floor.

**Suggested Data Analysis**

***Friction and Friction Loss Factors***

For straight pipes, convert the pressures directly to friction factors and calculates mass flow rates and Reynolds numbers. These values should be plotted on log-log graph to demonstrate the varying dependence of pressure drop on flow rate as one moves from laminar flow to increasingly turbulent conditions. It may be difficult to measure the laminar range as the pump does not operate efficiently at low flow rates.

In addition to a log-log plot of friction factor *versus* Reynolds number, the plot should also include lines to represent friction factor values calculated using equations 15, 16, or 17 as appropriate to Re. In the text, discuss whether the data would be better predicted if the constants in the equations were modified. Include a table of friction loss factors for the valves and bends, and calculate the equivalent length of straight tubing producing the same friction loss as each fitting. Discuss any trends in pressure drop with respect to Re, diameter, or flow rate.

***Velocity Profile***

Ensure that you are using a consistent set of units for your calculations, preferably by performing a dimensional analysis. From the pressure drop measurements obtained during each trial, use Equation 23 to calculate $\tau_0$. Calculate the mass flow rate and average water velocity for each run. Calculate the velocity profiles for each flow rate and plot them as $\frac{\overline{v}_z(r)}{\overline{v}_{z,max}}$ versus $r/R$.

Integrate the velocity profile numerically (as we learned in 132B) for each run to compute the volumetric flow rate of water. Tabulate the mass flow rates obtained from weighing and from this integration and calculate the average velocities $< \overline{v}_z >$ and Reynolds numbers using the integrated value.

For each run use your value of $\tau_0$ and Equation 17, 18, or 19, as appropriate (i.e., use Figure 5.3-1 from BSL on GauchoSpace), to estimate the maximum velocity in the pipe from this general correlation. Compare the results with your experimental values.

Fit your velocity data to Equation 25 by determining the apparent value of $n$ for each run and compare the results with values reported in Table 1. Calculate the average velocity in the pipe for each run from the resulting equations and compare with those determined by weighing.

For the intermediate flow rate, plot the velocity profile in the form $v^+$ vs. $y^+$ using the calculated value of $\tau_0$. On the same graph, plot the universal velocity profile. Check whether the shear stress and pressure drop can be calculated from Equation 25.

**References**

Bird, R.B., W.E. Stewart, and E.N. Lightfoot, Transport Phenomena, John Wiley and Sons, New York (1960).

McCabe, W.L., J.C. Smith, and P. Harriott, Unit Operations of Chemical Engineering, Fifth Edition, McGraw-Hill, Inc., New York (1993).

Perry, R.H., and D. Green (eds.), *Perry's Chemical Engineers' Handbook,* 6th ed., McGraw-Hill Book Co., New York, 1984.