**Thailand October Camp 2016 - Day 1**
**IPST**
**18 October 2016**

## Morning (3 hours)

1. In $\triangle ABC$, $D, E, F$ are the midpoints of $AB, BC, CA$ respectively. Denote by $O_A, O_B, O_C$ the incenters of $\triangle ADF, \triangle BED, \triangle CFE$ respectively. Prove that $O_AE, O_BF, O_CD$ intersect in one point.

2. Let $m, n \in \mathbb{N}$ satisfies
$$\phi(5^m - 1) = 5^n - 1.$$
Prove that $\gcd(m, n) \neq 1$.

3. In $\triangle ABC$ with $AB > AC$, the tangent to the circumcircle at $A$ intersect line $BC$ at $P$. $Q$ is the point on $AB$ such that $AQ = AC$ and $A$ lies between $B$ and $Q$. $R$ is the point on ray $AP$ such that $AR = CP$. $X, Y$ are the midpoints of $AP, CQ$ respectively. Prove that $CR = 2XY$.

## Afternoon (3 hours)

4. Suppose that $m, n, k$ are positive integers satisfying
$$3mk = (m + 3)^n + 1.$$
Prove that $k$ is odd.

5. Let $\omega_1, \omega_2$ be two circles with different radius, and $H$ be the exsimilicenter of the two circles. $X$ is a point not inside either of the circles. The tangents from $X$ to $\omega_1$ touches $\omega_1$ at $P, Q$, and the tangents from $X$ to $\omega_2$ touches $\omega_2$ at $R, S$. If $PR$ passes through $H$ and is not a common tangent line of $\omega_1, \omega_2$, prove that $QS$ also passes through $H$.

6. $A$ and $B$ plays a game, with $A$ choosing a positive integer $n \in \{1, 2, \ldots, 1001\} = S$. $B$ must guess the value of $n$ by choosing several subsets of $S$, then $A$ will tell $B$ how many subsets $n$ is in. $B$ will do this three times selecting $k_1, k_2$ then $k_3$ subsets of $S$ each. What is the least value of $k_1 + k_2 + k_3$ that guarantees that $B$ will correctly guess the value of $n$ whatever $A$ chooses.

**Thailand October Camp 2016 - Day 2**
**IPST**
**27 October 2016**

**Time: 4.5 hours**

1. For 1.1-1.3 each problem must be written within 1 page

    1.1 Let $f(A)$ denote the difference of maximum and minimum value in set $A$. Find the sum of $f(A)$ when $A$ is the subset of $\{1, 2, ..., n\}$

    1.2 Let a $8 \times 8$ board be all white initially. We have an operation of changing the color from black to white and white to black on any 13 or 31 rectangle. Determine whether the stage that all 64 cells are black could occur.

    1.3 Prove that for all positive integers $m$ there exists a positive integer $n$ such that the set $\{n, n+1, n+2, \ldots, 3n\}$ contains exactly $m$ perfect squares.

2. Let $f, g$ be bijections on $1, 2, 3, ..., 2016$. Determine the value of

$$\sum_{i=1}^{2016} \sum_{j=1}^{2016} [f(i) - g(j)]^{2559}$$

.

3. Let $f$ be a function on $X$. Prove that

$$f(X - f(X)) = f(X) - f(f(X))$$

where for a set $S$, the notation $f(S)$ means $\{f(a)|a \in S\}$.

4. Find all functions $f : \mathbb{N} \to \mathbb{N}$ such that for all $n \in \mathbb{N}$, the following holds:

$$f(1)^3 + f(2)^3 + ... + f(n)^3 = (f(1) + f(2) + ... + f(n))^2 .$$

5. Prove that for all polynomials $P \in \mathbb{R}[x]$, $P(x) - x | P^n(x) - x$.

6. Find all polynomials $f$ with real coefficients such that for all reals $x, y, z$ such that $x + y + z = 0$ we have the following relation:

$$f(xy) + f(yz) + f(zx) = f(xy + yz + zx).$$

**Thailand October Camp 2016 - Day 3**
**IPST**
**28 October 2016**

**Time: 4.5 hours**

1. Find all functions $f : \mathbb{Z} \to \mathbb{Z}$ such that

$$f(m+n) + f(mn-1) = f(m)f(n) + 2$$

for all $m, n \in \mathbb{Z}$.

2. (Serbia MO 2016 P1)

   (i) Does there exist a positive integer $m > 2016^{2016}$ such that $\frac{2016^m - m^{2016}}{m+2016}$ is a positive integer?

   (ii) Does there exist a positive integer $m > 2017^{2017}$ such that $\frac{2017^m - m^{2017}}{m+2017}$ is a positive integer?

3. Let $a, b, c \in \mathbb{R}^+$. Prove that

$$\sum_{cyc} ab\left(\frac{1}{2a+c} + \frac{1}{2b+c}\right) < \sum_{cyc} \frac{a^3+b^3}{c^2+ab}.$$

4. We color some squares of a $8 \times 8$ table black (and others white) such that each row has different number of black squares, and all columns have the same number of black squares. What is the maximum number of pairs of adjacent cells with different color?

5. Let $a, b, c \in \mathbb{R}^+$ such that $a + b + c = 3$. Prove that

$$\sum_{cyc} \left(\frac{a^3+1}{a^2+1}\right)^4 \geq \frac{1}{27}\left(\sqrt{ab} + \sqrt{bc} + \sqrt{ca}\right)^4.$$