In [83]:
```
using Plots
```

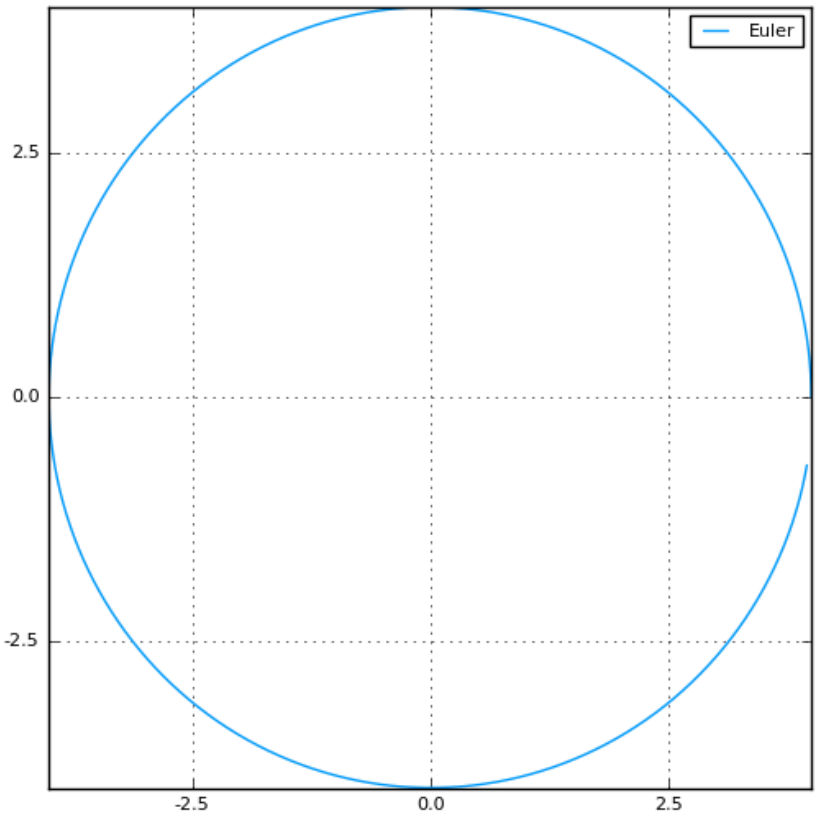
In [66]:
```
function runge_kutta(t,y,f,h)
    q1 = f(t, h*y)
    q2 = f(t + (h/2), y + (h/2) * q1)
    q3 = f(t + (h/2), y + (h/2) * q2)
    q4 = f(t + h, y + h*q3)
    return y + h/6 * (q1+ 2*q2 + 2*q3 + q4)
end
```

Out[66]: runge_kutta (generic function with 1 method)

In [77]:
```
# Euler Method
h = 0.0025
t_max = 50
n = length(1:h:t_max)

x_e = zeros(n,1)
y_e = zeros(n,1)
w_e = zeros(n,1)
z_e = zeros(n,1)

dz_dt = (x,y) -> (-x)/(sqrt(x^2 + y^2)^3)
dw_dt = (x,y) -> (-y)/(sqrt(x^2 + y^2)^3)

x_e[1] = 4
w_e[1] = 0.5

for t=2:n
    z_e[t] = z_e[t-1] + h*dz_dt(x_e[t-1],y_e[t-1])
    w_e[t] = w_e[t-1] + h*dw_dt(x_e[t-1],y_e[t-1])
    x_e[t] = x_e[t-1] + h*z_e[t-1]
    y_e[t] = y_e[t-1] + h*w_e[t-1]
end
```

In [107]:
```
plot(x_e,y_e,size=(500,500),label="Euler")
```

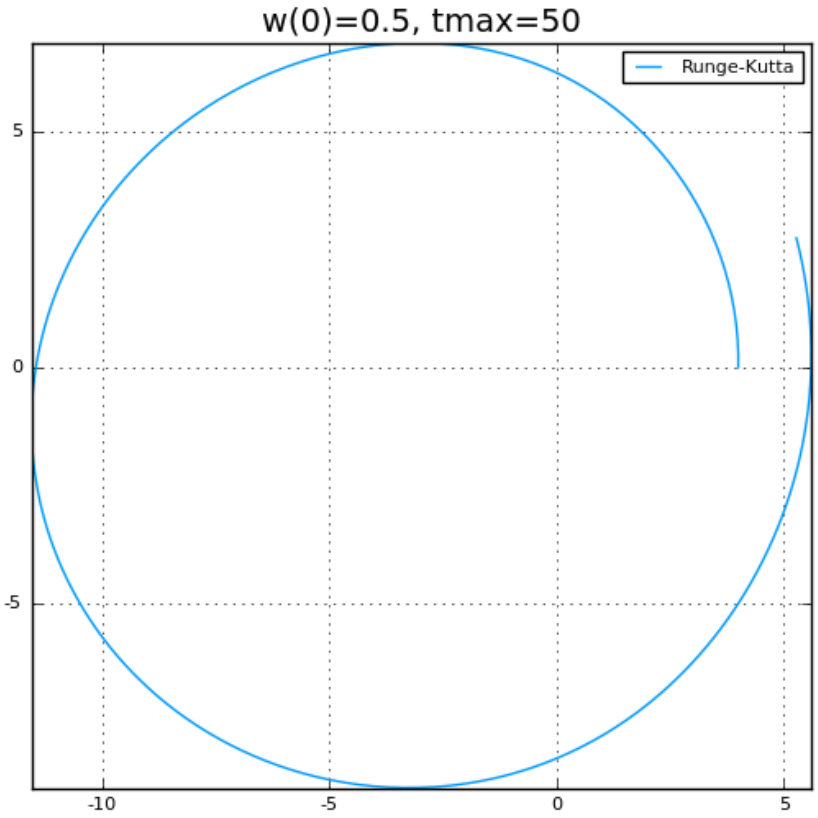
Out[107]:



In [85]:
```
# Runge Kutta
h = 0.25
n = length(1:h:t_max)

x_r = zeros(n,1)
y_r = zeros(n,1)
w_r = zeros(n,1)
z_r = zeros(n,1)

x_r[1] = 4
w_r[1] = 0.5

dz_dt = (x,y) -> (-x)/(sqrt(x^2 + y^2)^3)
dw_dt = (x,y) -> (-y)/(sqrt(x^2 + y^2)^3)

for t=2:n
    q1_z = dz_dt(x_r[t-1], h * y_r[t-1])
    q2_z = dz_dt(x_r[t-1]+ (h/2), y_r[t-1] + (h/2) * q1_z)
    q3_z = dz_dt(x_r[t-1]+ (h/2), y_r[t-1] + (h/2) * q2_z)
    q4_z = dz_dt(x_r[t-1]+ (h/2), y_r[t-1] + h * q3_z)

    q1_w = dw_dt(x_r[t-1], h * y_r[t-1])
    q2_w = dw_dt(x_r[t-1]+ (h/2), y_r[t-1] + (h/2) * q1_w)
    q3_w = dw_dt(x_r[t-1]+ (h/2), y_r[t-1] + (h/2) * q2_w)
    q4_w = dw_dt(x_r[t-1]+ (h/2), y_r[t-1] + h * q3_w)

    z_r[t] = z_r[t-1] + (h/6)*(q1_z + 2*q2_z + 2*q3_z + q4_z)#runge_kutta(x
_r[t-1], y_r[t-1], dz_dt, h)
    w_r[t] = w_r[t-1] + (h/6)*(q1_w + 2*q2_w + 2*q3_w + q4_w)#runge_kutta(x
_r[t-1], y_r[t-1], dw_dt, h)
    x_r[t] = x_r[t-1] + h*z_r[t-1]
    y_r[t] = y_r[t-1] + h*w_r[t-1]
end
```

In [89]:
```
plot(x_r,y_r,size=(500,500),label="Runge-Kutta",title="w(0)=0.5, tmax=50")
```

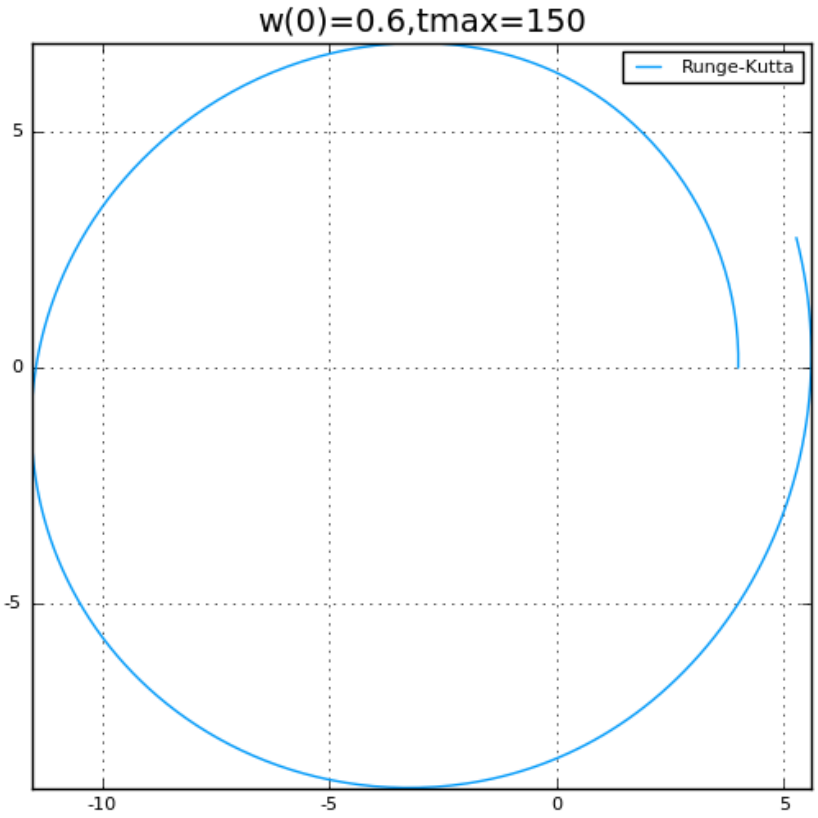
Out[89]:



In [87]:
```
# Runge Kutta
h = 0.5
t_max=150
n = length(1:h:t_max)

x_r = zeros(n,1)
y_r = zeros(n,1)
w_r = zeros(n,1)
z_r = zeros(n,1)

x_r[1] = 4
w_r[1] = 0.6

dz_dt = (x,y) -> (-x)/(sqrt(x^2 + y^2)^3)
dw_dt = (x,y) -> (-y)/(sqrt(x^2 + y^2)^3)

for t=2:n
    q1_z = dz_dt(x_r[t-1], h * y_r[t-1])
    q2_z = dz_dt(x_r[t-1]+ (h/2), y_r[t-1] + (h/2) * q1_z)
    q3_z = dz_dt(x_r[t-1]+ (h/2), y_r[t-1] + (h/2) * q2_z)
    q4_z = dz_dt(x_r[t-1]+ (h/2), y_r[t-1] + h * q3_z)

    q1_w = dw_dt(x_r[t-1], h * y_r[t-1])
    q2_w = dw_dt(x_r[t-1]+ (h/2), y_r[t-1] + (h/2) * q1_w)
    q3_w = dw_dt(x_r[t-1]+ (h/2), y_r[t-1] + (h/2) * q2_w)
    q4_w = dw_dt(x_r[t-1]+ (h/2), y_r[t-1] + h * q3_w)

    z_r[t] = z_r[t-1] + (h/6)*(q1_z + 2*q2_z + 2*q3_z + q4_z)#runge_kutta(x
_r[t-1], y_r[t-1], dz_dt, h)
    w_r[t] = w_r[t-1] + (h/6)*(q1_w + 2*q2_w + 2*q3_w + q4_w)#runge_kutta(x
_r[t-1], y_r[t-1], dw_dt, h)
    x_r[t] = x_r[t-1] + h*z_r[t-1]
    y_r[t] = y_r[t-1] + h*w_r[t-1]
end
```

In [88]:
```
plot(x_r,y_r,size=(500,500),label="Runge-Kutta",title="w(0)=0.6,tmax=150")
```

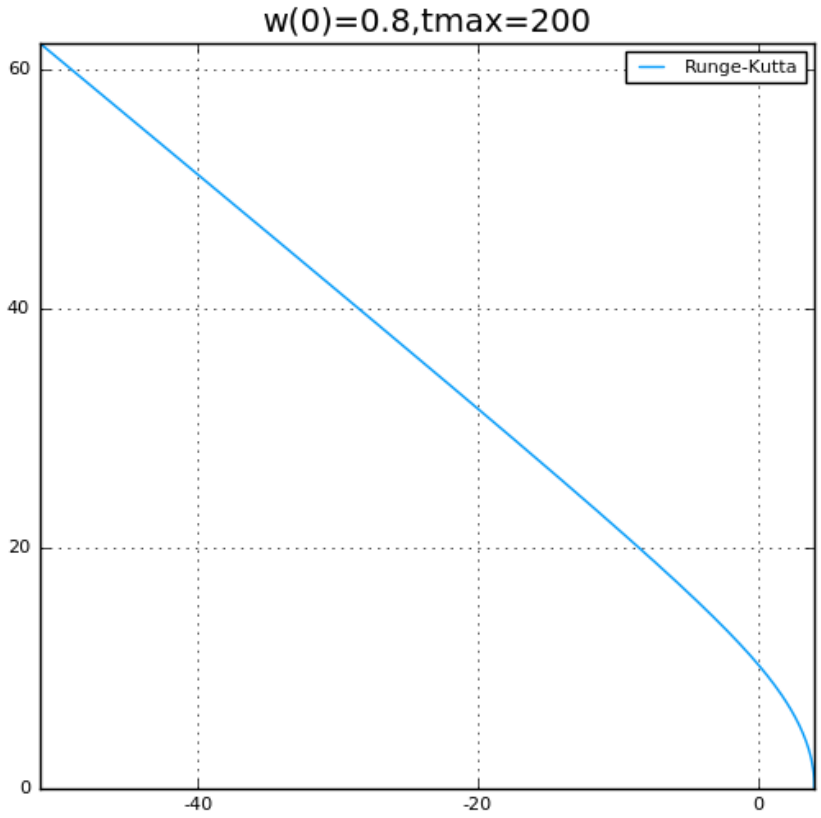
Out[88]:



In [93]:
```
# Runge Kutta
h = 0.5
t_max = 200
n = length(1:h:t_max)

x_r = zeros(n,1)
y_r = zeros(n,1)
w_r = zeros(n,1)
z_r = zeros(n,1)

x_r[1] = 4
w_r[1] = 0.8

dz_dt = (x,y) -> (-x)/(sqrt(x^2 + y^2)^3)
dw_dt = (x,y) -> (-y)/(sqrt(x^2 + y^2)^3)

for t=2:n
    q1_z = dz_dt(x_r[t-1], h * y_r[t-1])
    q2_z = dz_dt(x_r[t-1]+ (h/2), y_r[t-1] + (h/2) * q1_z)
    q3_z = dz_dt(x_r[t-1]+ (h/2), y_r[t-1] + (h/2) * q2_z)
    q4_z = dz_dt(x_r[t-1]+ (h/2), y_r[t-1] + h * q3_z)

    q1_w = dw_dt(x_r[t-1], h * y_r[t-1])
    q2_w = dw_dt(x_r[t-1]+ (h/2), y_r[t-1] + (h/2) * q1_w)
    q3_w = dw_dt(x_r[t-1]+ (h/2), y_r[t-1] + (h/2) * q2_w)
    q4_w = dw_dt(x_r[t-1]+ (h/2), y_r[t-1] + h * q3_w)

    z_r[t] = z_r[t-1] + (h/6)*(q1_z + 2*q2_z + 2*q3_z + q4_z)#runge_kutta(x
_r[t-1], y_r[t-1], dz_dt, h)
    w_r[t] = w_r[t-1] + (h/6)*(q1_w + 2*q2_w + 2*q3_w + q4_w)#runge_kutta(x
_r[t-1], y_r[t-1], dw_dt, h)
    x_r[t] = x_r[t-1] + h*z_r[t-1]
    y_r[t] = y_r[t-1] + h*w_r[t-1]
end
```

In [94]:
```
plot(x_r,y_r,size=(500,500),label="Runge-Kutta",title="w(0)=0.8,tmax=200")
```

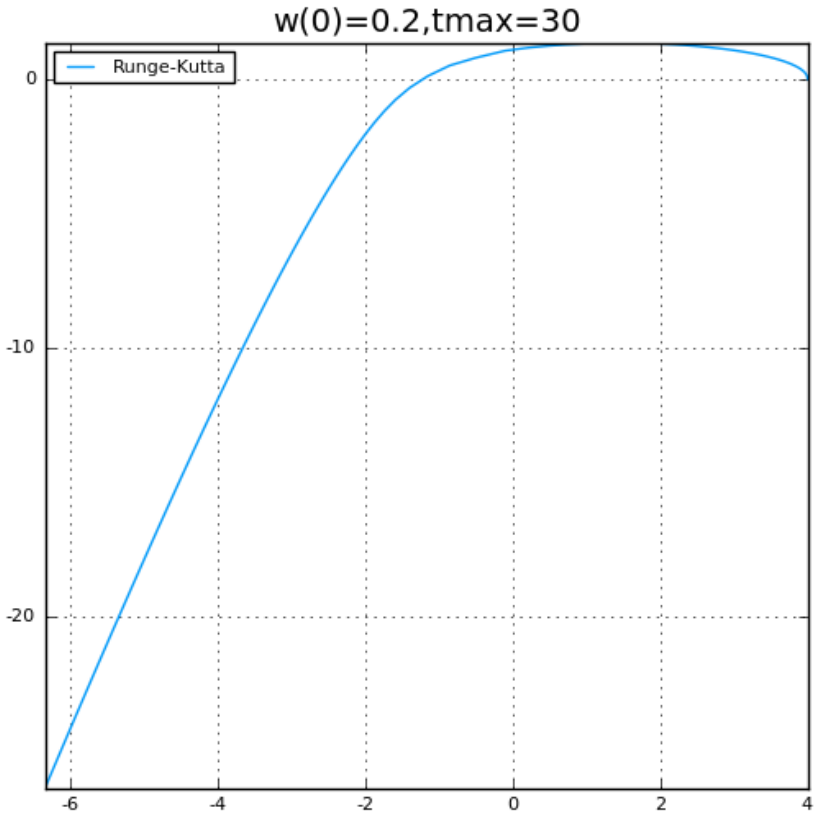
Out[94]:



In [97]:
```
# Runge Kutta
h = 0.25
t_max = 30
n = length(1:h:t_max)

x_r = zeros(n,1)
y_r = zeros(n,1)
w_r = zeros(n,1)
z_r = zeros(n,1)

x_r[1] = 4
w_r[1] = 0.2

dz_dt = (x,y) -> (-x)/(sqrt(x^2 + y^2)^3)
dw_dt = (x,y) -> (-y)/(sqrt(x^2 + y^2)^3)

for t=2:n
    q1_z = dz_dt(x_r[t-1], h * y_r[t-1])
    q2_z = dz_dt(x_r[t-1]+ (h/2), y_r[t-1] + (h/2) * q1_z)
    q3_z = dz_dt(x_r[t-1]+ (h/2), y_r[t-1] + (h/2) * q2_z)
    q4_z = dz_dt(x_r[t-1]+ (h/2), y_r[t-1] + h * q3_z)

    q1_w = dw_dt(x_r[t-1], h * y_r[t-1])
    q2_w = dw_dt(x_r[t-1]+ (h/2), y_r[t-1] + (h/2) * q1_w)
    q3_w = dw_dt(x_r[t-1]+ (h/2), y_r[t-1] + (h/2) * q2_w)
    q4_w = dw_dt(x_r[t-1]+ (h/2), y_r[t-1] + h * q3_w)

    z_r[t] = z_r[t-1] + (h/6)*(q1_z + 2*q2_z + 2*q3_z + q4_z)#runge_kutta(x
_r[t-1], y_r[t-1], dz_dt, h)
    w_r[t] = w_r[t-1] + (h/6)*(q1_w + 2*q2_w + 2*q3_w + q4_w)#runge_kutta(x
_r[t-1], y_r[t-1], dw_dt, h)
    x_r[t] = x_r[t-1] + h*z_r[t-1]
    y_r[t] = y_r[t-1] + h*w_r[t-1]
end
```

In [98]:
```
plot(x_r,y_r,size=(500,500),label="Runge-Kutta",title="w(0)=0.2,tmax=30")
```

Out[98]:



In [99]:
```
using ODE
```

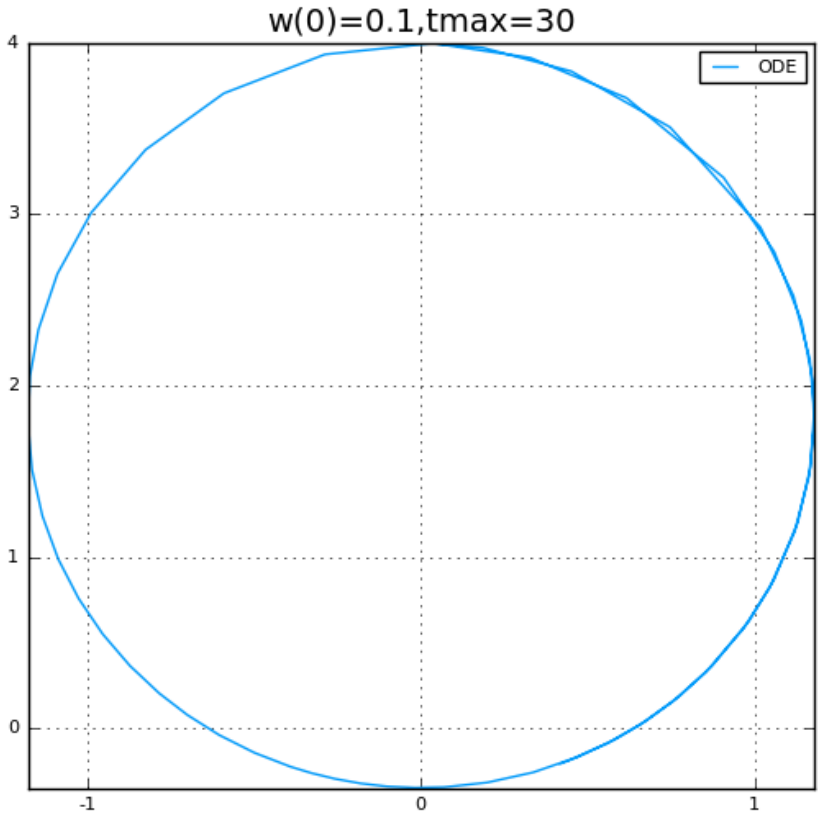
In [103]:
```
function gm(t, f)
    (x, y, z, w) = f
    dy_dt = z
    dx_dt = w
    dz_dt = -x/((sqrt(x^2+y^2)^3))
    dw_dt = -y/((sqrt(x^2+y^2)^3))

    [dy_dt; dx_dt; dz_dt; dw_dt]
end;
```

In [104]:
```
start = [4.0; 0.0; 0.0; 0.2]

ts = [0.0; 30.0];
```

In [105]:
```
t, res = ode45(gm, start, ts)

y = map(h -> h[1], res)
x = map(h -> h[2], res);
```

In [106]:
```
plot(x,y,size=(500,500),label="ODE",title="w(0)=0.1,tmax=30")
```

Out[106]:



In [ ]: