Predicting S&P 500 Direction with Ensemble Methods

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Agenda

Motivation

"Stock returns are predictable, but not by much."

— John H. Cochrane, Asset Pricing (2005)

- Forecasting stock market direction is one of the most classical and challenging tasks in finance.
- Weak predictability can matter for portfolio allocation and risk management.

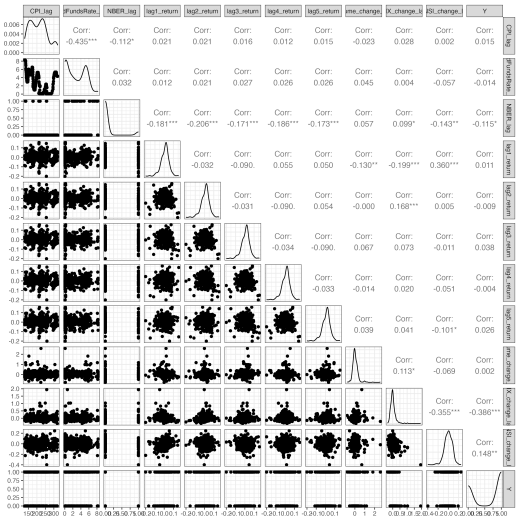
Our Set-Up:

- We focus on predicting whether the S&P 500 index goes **Up** or **Down** next month.
- We evaluate statistical learning methods in a time-series context.
- Introduce the role of regularization and nonlinear models.
- Compare linear vs. nonlinear classification models (Elastic Net vs. Random Forest).

Data Overview

- We created our own monthly dataset from S&P 500, FRED, and FRBSF data banks (1990–today).
- **Target:** Monthly S&P 500 market direction (**UP** or **DOWN**) in $t + 1$.
- **Predictors:**
  1. **Market data:** Lagged S&P 500 returns (up to five months) and trading volume changes.
  2. **Macroeconomic indicators:** CPI, Federal Funds Rate, NBER recession dummy.
  3. **Volatility:** Lagged changes in the VIX index.
  4. **Sentiment:** Daily News Sentiment Index.
- All features are lagged to avoid look-ahead bias.

Exploratory Analysis: Feature Relationships



Pairwise correlations and marginal distributions.

Why these predictors may matter:

- **Macro indicators** (CPI, Fed Funds Rate, NBER) affect discount rates and expected returns.
- **Lagged returns** reflect momentum and reversal effects.
- **Volatility** (VIX) captures uncertainty and risk sentiment.
- **Sentiment** (DNSI) reflects investor expectations and behavioral biases.

Empirical observation:

- Weak linear correlations $\Rightarrow$ low-signal, non-linear problem.
- Several predictors show significant **multicollinearity**.
- Motivates using **Elastic Net**, **Random Forests**, and **Gradient Boosting**.

Stationarity Considerations

- Most macro-financial time series are **non-stationary** in levels.
- We therefore use:
  - **Lagged returns** instead of prices.
  - **Changes** in VIX and sentiment rather than levels.
  - **Changes in macro variables** to preserve temporal causality.
- This transformation makes features approximately stationary, ensuring:
  - Stable model coefficients over time.
  - Valid cross-validation across time periods.

Hyperparameter Tuning: Time-Series Cross-Validation

Why not standard cross-validation?

- Random k-fold CV assumes i.i.d. data.
- Time-series data exhibit autocorrelation $\Rightarrow$ temporal dependence.
- Random shuffling lets the model see the future $\Rightarrow$ data leakage and over-optimism.

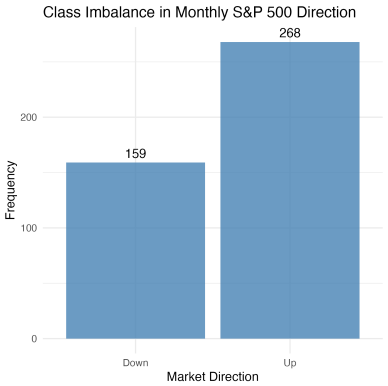
Our method: Rolling-window time-series CV

- Construct a hyperparameter grid:
  - Elastic Net: α and λ grid.
  - Random Forest: `mtry`, `min.node.size`, `max.depth`, `sample.fraction`.
- Use a chronological window structure:
  - Initial training window: 60 months (5 years).
  - Validation horizon: 12 months (1 year).
  - Fixed-length rolling window moving forward in time.
- For each fold:
  - Fit model only on past data and predict the next 12 months.
  - Record accuracy and AUC on the validation slice.

Result:

- Select the hyperparameters achieving the **highest mean AUC across all rolling folds**.
- Prevents look-ahead bias and mimics real-time forecasting.

Class Imbalance, Threshold Adjustment & AUC



Class imbalance in monthly S&P 500 direction

- Dependent variable `UP_DOWN` is imbalanced.
- A naive classifier that would always predict "Up" would achieve high accuracy.
- A simple threshold of 0.5 reduces the accuracy.
  $\Rightarrow$ **Accuracy alone is misleading.**

Threshold adjustment (Youden's J):

$$t^* = \arg\max_t \left\{ \text{Sensitivity}(t) + \text{Specificity}(t) - 1 \right\}$$

- We tune t^* on the **training set** using the ROC curve.
- Then apply this fixed threshold to the **test set** for true OoS evaluation.

AUC is robust here:

- ROC/AUC depends on the *ranking* of predicted probabilities, not on class proportions.
  $\Rightarrow$ **AUC is invariant to class imbalance** and a suitable metric for our setting.

Elastic Net Logistic Regression

Model Intuition:

- We model the probability of an **Up** move using a logistic function:

$$P(Y_t = 1 \mid X_t) = \frac{1}{1 + e^{-(\beta_0 + X_t \beta)}}$$

- Coefficients β are maximum likelihood estimates under a regularization penalty to prevent overfitting and perform variable selection.

Elastic Net Regularization:

$$\min_{\beta} \left[-\ell(\beta) + \lambda \left((1-\alpha) \frac{\|\beta\|_2^2}{2} + \alpha \|\beta\|_1 \right) \right]$$

- $\ell(\beta)$: log-likelihood of the logistic model.
- λ: overall penalty strength controlling coefficient shrinkage.
- α: mixes the two types of regularization:
  - Ridge (L2): smooth shrinkage.
  - Lasso (L1): sets some coefficients exactly to zero.

Elastic Net: Out-of-Sample Performance

Predictive performance (test set):

- Accuracy: **64.3%**
- AUC: **0.79** (strong probability ranking ability)
- Threshold tuned : $t^* = 0.6743 \rightarrow$ The model avoids overly optimistic Up predictions

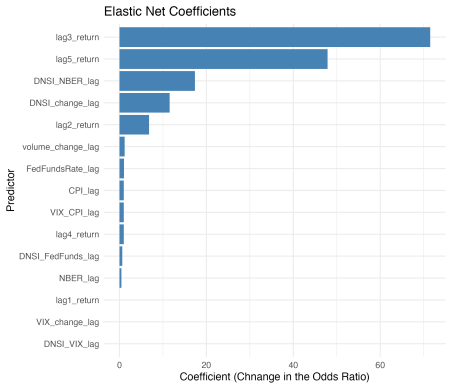
Confusion matrix:

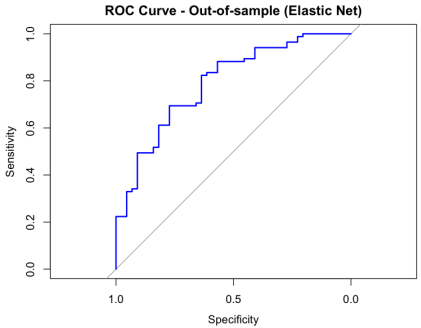
	Actual 0	Actual 1
Predicted 0	36	38
Predicted 1	8	47

Interpretation:

- **Specificity** (correct Down predictions): $36/44 \approx$ **82%** $\Rightarrow$ we correctly flag most Down months.
- **Sensitivity** (correct Up predictions): $47/85 \approx$ **55%** $\Rightarrow$ we still capture the majority of Up months.
- The model trades a small loss in raw accuracy for a much better **balance** between detecting costly Down markets and still identifying Up markets, which is desirable in an asset-management context.

Elastic Net: Selected Predictors and Probability Ranking





- Coeff. represent changes in the **odds** of an "Up" month.
- Elastic Net highlights variables with consistent signal.
- Momentum is a major driver of Up probabilities.
- Sentiment matters more during recession periods.

- **AUC = 0.79**: strong discriminative ability despite low-signal, noisy financial data.
- ROC curve well above diagonal $\Rightarrow$ model ranks Up vs. Down months effectively.

Random Forest: Out-of-Sample Performance

Predictive performance (test set):

- Accuracy: **71.3%**
- AUC: **0.753** (strong non-linear discriminative ability)
- RF produces well-calibrated probability scores without threshold tuning

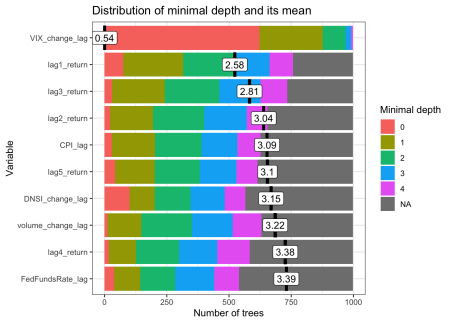
Confusion matrix:

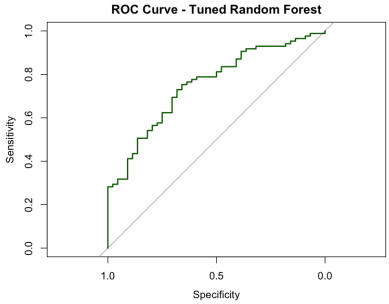
	Actual 0	Actual 1
Predicted 0	29	22
Predicted 1	15	63

Interpretation:

- **Specificity** (correct Down predictions): $29/44 \approx$ **66%**. RF detects a meaningful fraction of negative weeks.
- **Sensitivity** (correct Up predictions): $63/85 \approx$ **74%**. RF identifies most positive-return weeks.
- **Overall**: RF captures important **nonlinear return patterns**, outperforming Elastic Net in accuracy and maintaining a strong AUC.

Random Forest: Minimal Depth, Interpretation, and AUC





Explanation and Interpretation of Minimal Depth

- Min. depth measures how **early** a variable is used in the tree.
- **VIX_change_lag** is the dominant signal
- **Lagged returns** are meaningful momentum predictors.
- Overall: RF relies mainly on **volatility shocks** and **recent price dynamics**.

AUC

- **AUC = 0.753** $\Rightarrow$ strong discriminative ability and reliable ranking of Up vs. Down weeks.

Takeaway

- Random Forest effectively captures nonlinear interactions between **volatility movements** and **short-term momentum**.

Gradient Boosting Machine (GBM)

Model Intuition:

- A powerful **boosting** ensemble method, as discussed in class.
- It builds trees **sequentially**, not in parallel like Random Forest.
- Each new, simple tree (a "weak learner") is trained on the residuals (the errors) of the previous trees.
- It's a "slow" learner that "boosts" the signal over many iterations, fitting an additive model.
- We use `distribution = "bernoulli"` for binary classification, which optimizes the deviance (log-loss).

Tuned Hyperparameters (from Time-Series CV):

- `n.trees = 200` (The number of iterations M).
- `interaction.depth = 1` (Tree size $J = 1$, making this an additive model with no interactions).
- `shrinkage = 0.01` (The learning rate ν, for regularization).

GBM: Out-of-Sample Performance and Importance

Predictive performance (test set):

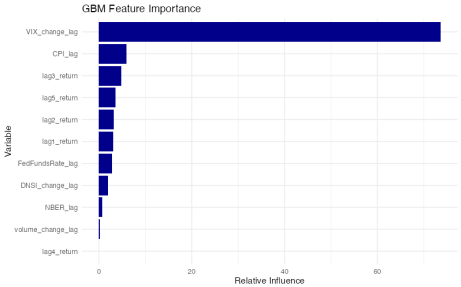
- Accuracy: **0.674**
- AUC: **0.783**
- Log-Loss: **0.5531**
- Tuned Threshold: **0.6962** (via training ROC)

Confusion matrix:

	Actual 0	Actual 1
Predicted 0	33	31
Predicted 1	11	54

Interpretation:

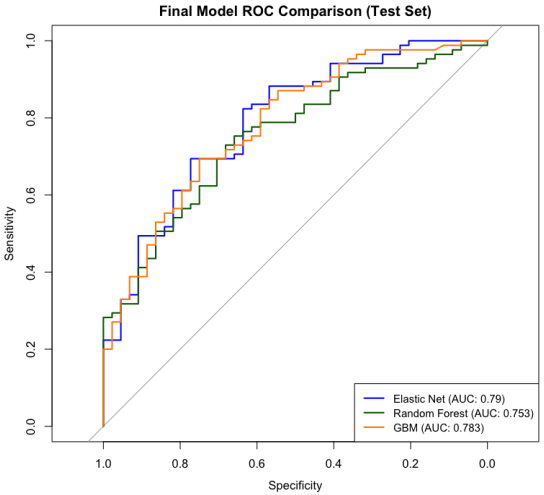
- **Specificity**: $33/(33+11) \approx$ **75%**
- **Sensitivity**: $54/(54+31) \approx$ **64%**
- The additive GBM model shows a good balance, effectively capturing both "Up" and "Down" markets.



Feature Importance Interpretation:

- The model is **dominated** by a single predictor: `VIX_change_lag` (73.6% relative influence).
- The best-tuned model was additive (`interaction.depth = 1`), so it only finds main effects.
- Other macro (`CPI_lag`) and momentum (`lag3_return`) features provide minor corrective adjustments to the main volatility signal.

Final Model Comparison



Final Model Performance on Hold-Out Test Set			
Model	Test_AUC	Test_Accuracy	Test_LogLoss
Elastic Net (Tuned)	0.7904	0.6434	0.5154
Gradient Boosting (Tuned)	0.7828	0.6744	0.5531
Random Forest (Tuned)	0.7532	0.7132	0.5644

Final performance metrics on the hold-out test set

- All models performed significantly better than chance (all AUC greater than 0.75).
- The linear **Elastic Net** had the highest Test AUC (0.790), narrowly beating the GBM (0.783).
- The **Random Forest** had the highest Test Accuracy (0.713), but the lowest AUC.
- This suggests the regularized linear model was best at ranking probabilities, while the RF was best at classification after threshold tuning.

Conclusion

What We Did:

- We built 3 distinct models (Regularized GLM, Bagging, Boosting) to predict S&P 500 direction.
- We correctly used a **rolling-window cross-validation** for hyperparameter tuning to respect the time-series nature of the data.
- We addressed **class imbalance** by tuning the probability threshold on the training set (Youden's J).

What We Found:

- The **Elastic Net (Tuned)** was the winning model based on our primary metric, Test AUC (0.790).
- All models found that **volatility** (e.g., 'VIX_change_lag') and **momentum** (e.g., 'lag1_return') were highly predictive.
- Methodology is critical: standard CV would have failed, and using a 0.5 threshold would have been misleading.

Thank you!

Questions & Discussion

Appendix: Hyperparameter Grids

Random Forest Grid:

- `mtry`
- `min.node.size`
- `max.depth`
- `sample.fraction`

Total combinations tested: 400

Gradient Boosting (GBM) Grid:

- `n.trees`
- `interaction.depth`
- `shrinkage`

Total combinations tested: 18

References

- Cochrane, John H. (2005). *Asset Pricing*. Princeton University Press.
- Federal Reserve Economic Data (FRED). *St. Louis Fed*. Retrieved 2025.
- Federal Reserve Bank of San Francisco. *Daily News Sentiment Index*. Retrieved 2025.
- Yahoo Finance. *S&P 500 ($\hat{GSPC}$) & CBOE VIX ($\hat{VIX}$) Data*. Retrieved 2025.
- Friedman, J., Hastie, T., & Tibshirani, R. (2010). Regularization Paths for Generalized Linear Models via Coordinate Descent. *Journal of Statistical Software, 33*(1), 1-22.
- Wright, M. N. & Ziegler, A. (2017). ranger: A Fast Implementation of Random Forests... *Journal of Statistical Software, 77*(1), 1-17.
- Ridgeway, G. (2007). Generalized Boosted Models: A guide to the gbm package.