ECE 2570 - Homework 3

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1. ECE 1673 Lecture 2 : 20 min

   ECE 2646 Lecture 3 : 15 min

   PID control supp. note : 15 min

2. $$\frac{d^2y(t)}{dt^2} + 4\frac{dy(t)}{dt} + 3y(t) = 6r(t)$$

$y(0) = 3$, $\dot{y}(0) = 0$, $r(t) = 1$, $t \geq 0$

Response $y(t)$ for $t \geq 0$

$$\frac{Y(s)}{R(s)} = \frac{6}{s^2 + 4s + 3}$$

(i) Zero state response:

$$Y_{zs}(s) = \frac{6}{s^2+4s+3} \cdot \frac{1}{s}$$

Laplace transform of input ($r = 1(t)$)

$$= \frac{6}{(s+3)(s+1)s}$$

PF expansion: $\frac{6}{s(s+1)(s+3)} = \frac{A_1}{s} + \frac{A_2}{s+1} + \frac{A_3}{s+3}$

Find A_1: $\frac{6}{(s+1)(s+3)} = A_1 + \frac{sA_2}{s+1} + \frac{sA_3}{s+3}$

$s = 0 \rightarrow A_1 = \frac{6}{(1)(3)} = 2$

Find A_2: $\frac{6}{s(s+3)} = \frac{A_1(s+1)}{s} + A_2 + \frac{A_3(s+1)}{s+3}$

$s = -1 \rightarrow A_2 = \frac{6}{(-1)(2)} = -3$

Find A_3: $\frac{6}{s(s+1)} = \frac{A_1(s+3)}{s} + A_2\frac{(s+3)}{(s+1)} + A_3$

$s = -3 \rightarrow A_3 = \frac{6}{(-3)(-2)} = 1$

$$= \frac{2}{s} + \frac{-3}{s+1} + \frac{1}{s+3}$$

$$\downarrow$$

$$y_{zs}(t) = 2 - 3e^{-t} + e^{-3t}$$

$\mathcal{L}\left[\frac{d^n f}{dt^n}\right] = s^n F(s) - s^{n-1} f(0^-) - \dots - f^{n-1}(0^-)$

$\mathcal{L}\left[\frac{d^2 f}{dt^2}\right] = s^2 y(s) - s\,y(0^-) - \dot{y}(0^-)$

(ii) Zero input response:

$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 3y = 6r \longrightarrow$ Redo LT including initial condition

$s^2 y(s) - \dot{y}(0) - sy(0) + 4[sy(s) - y(0)] + 3y(s) = 6r(s)$

$y(s)[s^2 + 4s + 3] = \dot{y}(0) + sy(0) + 4y(0)$

$= 3s + 12$

$y(s) = \frac{3s+12}{s^2+4s+3}$

PF expansion: $\frac{3s+12}{(s+3)(s+1)} = \frac{A_1}{s+3} + \frac{A_2}{s+1}$

Find A_1: $\frac{3s+12}{(s+1)} = A_1 + A_2\frac{(s+3)}{s+1)}$

$s = -3 \longrightarrow A_1 = \frac{3}{-2} = -1.5$

Find A_2: $\frac{3s+12}{(s+3)} = A_1\frac{(s+1)}{(s+3)} + A_2$

$s = -1 \longrightarrow A_2 = \frac{9}{2} = 4.5$

$y_{ZI}(s) = \frac{-1.5}{s+3} + \frac{4.5}{s+1}$

$\downarrow$

$y_{ZI}(t) = -1.5e^{-3t} + 4.5e^{-t}$

Use superposition to combine $y_{ZS}(t)$ and $y_{ZI}(t)$

$y(t) = 2 - 3e^{-t} + e^{-3t} - 1.5e^{-3t} + 4.5e^{-t}$

$= 2 + e^{-t}(4.5 - 3) + e^{-3t}(1 - 1.5)$

$= \boxed{2 + \frac{3e^{-t}}{2} - \frac{e^{-3t}}{2}}$

3. $G(s) = \dfrac{4s + 2}{s^2 + 2s + 5}$

Nominal Form:

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}, \ \zeta, \omega_n > 0$$

$\omega_n^2 = 5 \longrightarrow \omega_n = \sqrt{5}$

$2\zeta\omega_n = 2$

$\zeta(\sqrt{5}) = 1 \longrightarrow \zeta = \dfrac{1}{\sqrt{5}}$

Impulse Response: $h(t) = \mathcal{L}^{-1}\{H(s)\}$

$$= \mathcal{L}^{-1}\left\{\frac{4s+2}{s^2+2s+5}\right\} = \mathcal{L}^{-1}\left\{\frac{4s+4}{s^2+2s+5} - \frac{2}{s^2+2s+5}\right\}$$

$$= 4\,\mathcal{L}^{-1}\left\{\frac{s+1}{(s+1)^2+4}\right\} - 2\,\mathcal{L}^{-1}\left\{\frac{1}{(s+1)^2+4}\right\}$$

$$= 4 \cdot e^{-t}\cos 2t - 2 \cdot \tfrac{1}{2} e^{-t} \sin 2t$$

$$h(t) = 4e^{-t}\cos 2t - e^{-t}\sin 2t$$