

Lecture 5: Locomotion; Mobile Robot Control Architecture

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Zhi-Hong Mao
Professor of ECE and Bioengineering
University of Pittsburgh, Pittsburgh, PA

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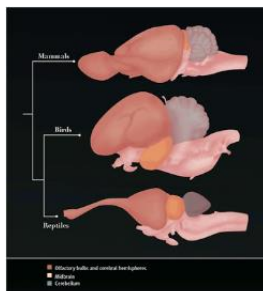
Outline

- Homework 3 (due Feb. 16)
- Locomotion
- Mobile robot components
- Behavior-based control of mobile robots
 - Go-to-goal behavior
 - Obstacle-avoidance behavior

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Locomotion

- Introduction
 - Locomotion refers to the way a robot moves from place to place
 - In nature, movement requires significant "brain power"



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Locomotion

- Introduction

- Locomotion refers to the way a robot moves from place to place
- In nature, movement requires significant “brain power”
- Moving the robot around becomes the first challenge for the robot “brains”—controllers
- Effectors and actuators
 - *Legs*, for walking, crawling, climbing, jumping, etc.
 - *Wheels*, for rolling
 - *Arms*, for swinging, crawling, climbing, etc.
 - *Wings*, for flying
 - *Flippers*, for swimming

Note: Legged locomotion is more difficult than wheeled locomotion due to the challenge of stability and larger number of degrees of freedom.

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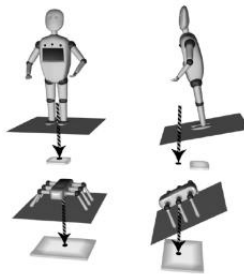
Locomotion

- Introduction

- Stability

- Static stability

- A *statically stable* robot can stand still without falling over
- The *center of gravity* of any body needs to be above *polygon of support*—the area that is covered by the ground points (legs or wheels)
- If the robot can walk while staying balanced at all times, we call this *statically stable walking*



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Locomotion

- Introduction

- Stability

- Static stability

- Dynamic stability

- In *dynamic stability*, the body must actively balance or move to remain stable
- A statically stable robot can use dynamically stable walking patterns, in order to be fast and efficient

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Locomotion

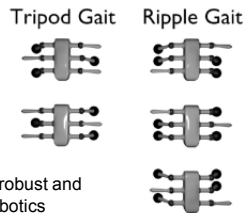
- Introduction
- Stability

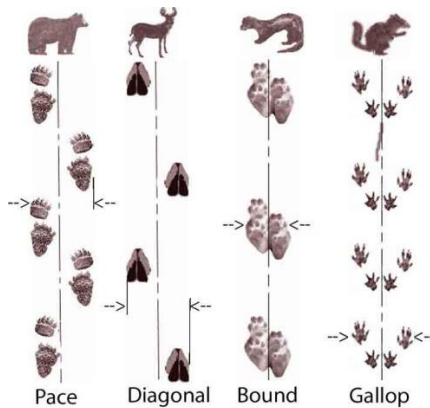
• Moving and gaits

- A *gait* is the particular way a robot moves, including the order in which it lifts and lowers its legs and places its feet on the ground
- Desirable robot gaits have the following properties

- *Stability*
- *Speed*
- *Energy efficiency*
- *Robustness*
- *Simplicity*

Note: Six-legged walking is highly robust and therefore common in nature and robotics





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Locomotion

- Introduction
- Stability
- Moving and gaits

• Wheels and steering

– Wheels in robotics

- Wheel-like structures are very rare in biological locomotion (but do appear in certain bacteria)
- Because of their efficiency and comparative simplicity of control, wheels are the major choice of locomotion effector in robotics
- Wheeled robots are usually designed to be *statically stable*, but they are not necessarily *holonomic* (meaning they cannot control all of their available degrees of freedom)



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Locomotion

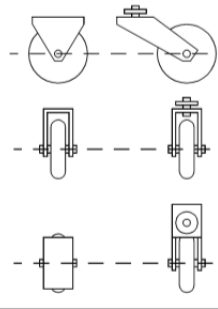
- Introduction
- Stability
- Moving and gaits

- **Wheels and steering**

- Wheels in robotics

- **Wheel types**

- **Standard wheel:** two degrees of freedom; rotation around the wheel axle and the contact point
 - **Castor wheel:** two degrees of freedom; rotation around the wheel axle and an offset steering joint



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Locomotion

- Introduction
- Stability
- Moving and gaits

- **Wheels and steering**

- Wheels in robotics

- **Wheel types**

- *Standard wheel*
 - *Castor wheel*
 - **Swedish or Mecanum wheel:** three degrees of freedom; rotation around the wheel axle, around the rollers, and around the contact point



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Locomotion

- Introduction
- Stability
- Moving and gaits

- **Wheels and steering**

- Wheels in robotics

- **Wheel types**

- *Standard wheel*
 - *Castor wheel*
 - *Swedish or Mecanum wheel*
 - **Ball or spherical wheel:** realization technically difficult



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Locomotion

- Introduction
- Stability
- Moving and gaits
- **Wheels and steering**
 - Wheels in robotics
 - Wheel types
 - **Wheel configurations**
 - Two wheels

Bicycle

Rear wheel
for power



1st wheel
for steering

Segway



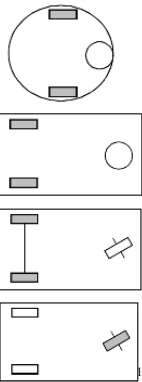
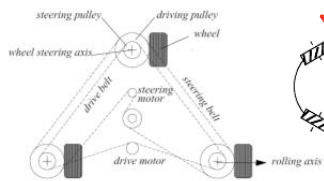
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cleaning robot

Locomotion

- Introduction
- Stability
- Moving and gaits
- **Wheels and steering**
 - Wheels in robotics
 - Wheel types
 - **Wheel configurations**
 - Two wheels
 - **Three wheels**

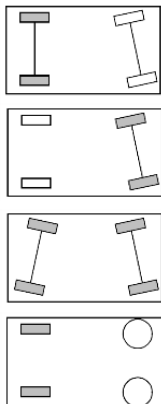
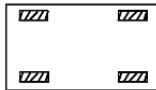
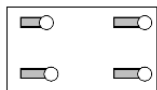
Swedish
wheel



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Locomotion

- Introduction
- Stability
- Moving and gaits
- **Wheels and steering**
 - Wheels in robotics
 - Wheel types
 - **Wheel configurations**
 - Two wheels
 - Three wheels
 - **Four Wheels**



Locomotion

- Introduction
- Stability
- Moving and gaits
- Wheels and steering
 - Wheels in robotics
 - Wheel types
 - Wheel configurations
- Maneuverability and “controllability”
 - There is generally an *inverse correlation* between maneuverability and “controllability” (e.g., controlling an omnidirectional robot for a specific direction of travel is more difficult and often less accurate when compared to less maneuverable designs)

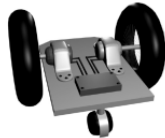
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Holonomic robot - harder to control
→ Underactuated robots are easier to work with

Locomotion

- Introduction
- Stability
- Moving and gaits
- Wheels and steering
 - Wheels in robotics
 - Wheel types
 - Wheel configurations
 - Maneuverability and “controllability”
- Differential drive and differential steering
 - The ability to drive wheels separately and independently, through the use of separate motors, is called a *differential drive*
 - Being able to steer wheels independently is called *differential steering*

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Locomotion

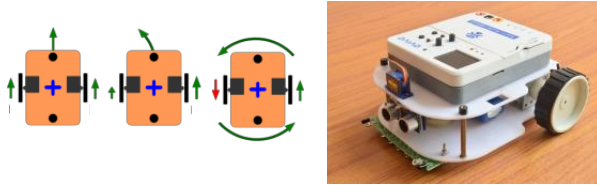
- Introduction
- Stability
- Moving and gaits
- Wheels and steering
- Navigation
 - *Navigation* refers to the problem of moving the robot's whole body to various destinations
 - Main issues
 - *Localization* and *map building*: to figure out where the robot is relative to some model of the environment
 - *Path planning*, also called *trajectory planning*: to search through all possible trajectories and evaluate them, in order to find one that will satisfy the requirements
 - *Motion control*: to get the robot to a particular location

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Dynamics to follow path planning

Mobile robot components

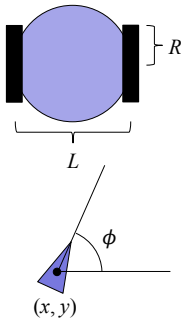
- Body and dynamics
 - Model for a differential drive wheeled robot



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Mobile robot components

- Body and dynamics
 - Model for a differential drive wheeled robot



State space model

$$\begin{aligned}\dot{x} &= \frac{R}{2}(v_r + v_l)\cos\phi \\ \dot{y} &= \frac{R}{2}(v_r + v_l)\sin\phi \\ \dot{\phi} &= \frac{R}{L}(v_r - v_l)\end{aligned}$$

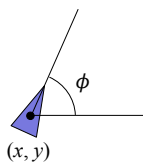
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State Space: x is function of (x, u)

Inputs are v_r and v_l (angular velocity at each wheel)

Mobile robot components

- Body and dynamics
 - Model for a differential drive wheeled robot
 - The "unicycle" model
 - It is not very natural to "think" in terms of wheel velocities
 - Instead, go directly for translational velocity (v) and angular velocity (ω)



$$\begin{aligned}\dot{x} &= v \cos\phi \\ \dot{y} &= v \sin\phi \\ \dot{\phi} &= \omega\end{aligned}$$

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Inputs: v, ω

ω : "heading's" angular velocity

Mobile robot components

- Body and dynamics
 - Model for a differential drive wheeled robot
 - The “unicycle” model
 - It is not very natural to “think” in terms of wheel velocities
 - Instead, go directly for translational velocity (v) and angular velocity (ω)
 - **Idea of design:** design for the “unicycle” model and then implement the differential drive model

$$\begin{aligned}
 \dot{x} &= \frac{R}{2} (v_r + v_l) \cos \phi \\
 \dot{y} &= \frac{R}{2} (v_r + v_l) \sin \phi \\
 \dot{\phi} &= \frac{R}{L} (v_r - v_l)
 \end{aligned}
 \quad
 \begin{aligned}
 v &= \frac{R}{2} (v_r + v_l) \\
 \omega &= \frac{R}{L} (v_r - v_l)
 \end{aligned}
 \quad
 \begin{aligned}
 \dot{x} &= v \cos \phi \\
 \dot{y} &= v \sin \phi \\
 \dot{\phi} &= \omega
 \end{aligned}$$

$$\begin{aligned}
 v_r &= \frac{2v + \omega L}{2R} \\
 v_l &= \frac{2v - \omega L}{2R}
 \end{aligned}$$

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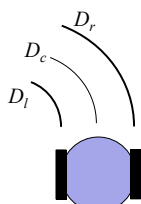
Mobile robot components

- Body and dynamics
- Sensors, actuators, and power
 - State estimation
 - The state of the robot is $[x, y, \phi]^T$
 - Coupling of proprioceptive and exteroceptive sensors for state estimation
 - Proprioceptive sensors
 - Orientation: compass
 - Position: accelerometers, gyroscopes, ...
 - Wheel encoders
 - Exteroceptive sensors
 - A “skirt” of range sensors: IR, ultra-sound, LIDAR, ...
 - Other standard exteroceptive sensors include vision, tactile, and “GPS”

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Mobile robot components

- Body and dynamics
- Sensors, actuators, and power
 - State estimation
 - Example of proprioceptive sensing: wheel encoders
 - Wheel encoders give the distance moved by each wheel
 - Assume the wheels are following an arc (short time scale)



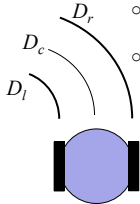
$$D_c = \frac{D_l + D_r}{2}$$

$$\begin{aligned}
 x' &= x + D_c \cos(\phi) \\
 y' &= y + D_c \sin(\phi) \\
 \phi' &= \phi + \frac{D_r - D_l}{L}
 \end{aligned}$$

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Mobile robot components

- Body and dynamics
- Sensors, actuators, and power
 - State estimation
 - Example of proprioceptive sensing: wheel encoders
 - Wheel encoders give the distance moved by each wheel
 - Assume the wheels are following an arc (short time scale)
 - How do we know how far each wheel has moved?
 - Assume each wheel has N "ticks" per revolution
 - Most wheel encoders give the total tick count since the beginning
 - For both wheels: $\Delta \text{tick} = \text{tick}' - \text{tick}$



$$D_l = 2\pi R \frac{\Delta \text{tick}_l}{N}$$

$$D_r = 2\pi R \frac{\Delta \text{tick}_r}{N}$$

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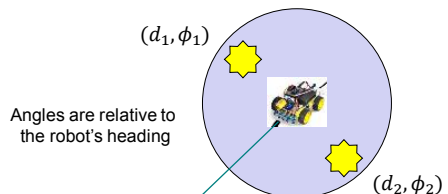
Mobile robot components

- Body and dynamics
- Sensors, actuators, and power
 - State estimation
 - Example of proprioceptive sensing: wheel encoders
 - Wheel encoders give the distance moved by each wheel
 - Assume the wheels are following an arc (short time scale)
 - How do we know how far each wheel has moved?
 - A major issue: **DRIFT!**

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Mobile robot components

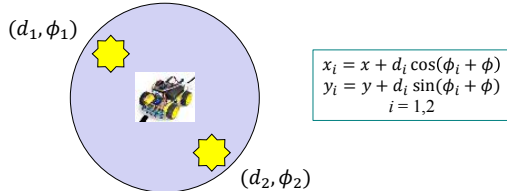
- Body and dynamics
- Sensors, actuators, and power
 - State estimation
 - Example of proprioceptive sensing: wheel encoders
 - Example of exteroceptive sensing: disk abstraction
 - Assumption: instead of worrying about the resolution of the sensors, assume we know the distance and direction to all obstacles around us (that are close enough)



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Mobile robot components

- Body and dynamics
- Sensors, actuators, and power
 - State estimation
 - Example of proprioceptive sensing: wheel encoders
 - Example of exteroceptive sensing: disk abstraction
 - Assumption
 - Coordinates of the obstacles (if we know our own position and orientation)



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Mobile robot components

- Body and dynamics
- Sensors, actuators, and power
 - State estimation
 - Example of proprioceptive sensing: wheel encoders
 - Example of exteroceptive sensing: disk abstraction
 - Assumption
 - Coordinates of the obstacles
 - A case study



Multi-robot rendezvous task
(Li, Sun, and Yang, 2013)

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Mobile robot components

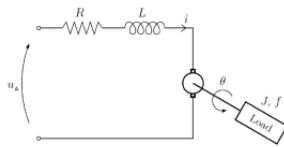
- Body and dynamics
- Sensors, actuators, and power
 - State estimation
 - Example of proprioceptive sensing: wheel encoders
 - Example of exteroceptive sensing: disk abstraction
- Actuation
 - Gearing
 - If the output gear is larger than the input gear, the torque increases but the speed decreases
 - If the output gear is smaller than the input gear, the torque decreases but the speed increases



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Mobile robot components

- Body and dynamics
- Sensors, actuators, and power
 - State estimation
 - Example of proprioceptive sensing: wheel encoders
 - Example of exteroceptive sensing: disk abstraction
- Actuation
 - Gearing
 - Dynamics and time constant of a DC motor
 - About model order reduction



$$u_s(t) = L \frac{di(t)}{dt} + Ri(t) + k_v \omega(t)$$

$$k_a i(t) = J \dot{\omega}(t) + f \omega(t)$$

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Mobile robot components

- Body and dynamics
- Sensors, actuators, and power
 - State estimation
 - Example of proprioceptive sensing: wheel encoders
 - Example of exteroceptive sensing: disk abstraction
 - Actuation
- Power management



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Control Architectures:

Reactive	Behavior-based
Deliberative	Hybrid

Mobile robot components

- Body and dynamics
- Sensors, actuators, and power
- Control architecture: behavior-based control
 - Motivation
 - The world is fundamentally unknown and changing
 - It does not make sense to over-plan
 - *Key idea:* develop a library of useful controllers (behaviors) and switch among controllers in response to environmental changes

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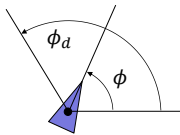
Mobile robot components

- Body and dynamics
- Sensors, actuators, and power
- Control architecture: behavior-based control
 - Motivation
 - Behaviors
 - Go-to-goal behavior
 - Avoid-obstacles behavior
 - Follow-wall behavior
 - Track-target behavior
 - ...

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Mobile robot components

- Body and dynamics
- Sensors, actuators, and power
- Control architecture: behavior-based control
 - Motivation
 - Behaviors
 - Example of a behavior
 - Assume we have a differential-drive wheeled mobile robot driving at a constant speed
 - Want to drive in a desired heading



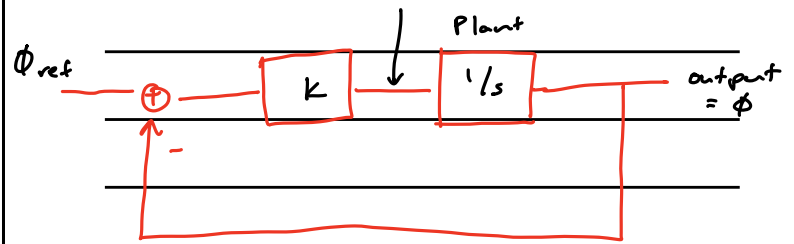
$$\begin{aligned}\dot{x} &= v_0 \cos \phi \\ \dot{y} &= v_0 \sin \phi \\ \dot{\phi} &= \omega\end{aligned}$$

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input: ω

output: ϕ

ω (input to plant)



$1/s$ because it is LT of diff eq.
Don't need PI because system itself is integrator

Mobile robot components

- Body and dynamics
- Sensors, actuators, and power
- Control architecture: behavior-based control
 - Motivation
 - Behaviors
 - Example of a behavior
 - Assume we have a differential-drive wheeled mobile robot driving at a constant speed
 - Want to drive in a desired heading
 - Use PID control

$$r = \phi_d$$

$$e = \phi_d - \phi$$

$$\dot{\phi} = \omega$$

$$\omega(t) = k_p e(t) + k_I \int_0^t e(\tau) d\tau + k_D \frac{de(t)}{dt}$$

or

$$\Omega(s) = \left(k_p + \frac{k_I}{s} + k_D s\right) E(s)$$

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Slide 35:

$$T(s) = \frac{k/s}{1+k/s} = \frac{k}{s+k} \quad (\text{1st order system})$$

DC Gain = 1 (replace s with 0)

$$e_{ss} = 0$$

$$\tau = 1/k$$

Mobile robot components

- Body and dynamics
 - Sensors, actuators, and power
 - Control architecture: behavior-based control
 - Motivation
 - Behaviors
 - Example of a behavior
 - Assume we have a differential-drive wheeled mobile robot driving at a constant speed
 - Want to drive in a desired heading
 - Use PID control
- Caution:** This typically will not work as we are dealing with angles!
- $$\phi_d = 0, \quad \phi = 100\pi \Rightarrow e = -100\pi$$

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Robot may spin like crazy

Mobile robot components

- Body and dynamics
 - Sensors, actuators, and power
 - Control architecture: behavior-based control
 - Motivation
 - Behaviors
 - Example of a behavior
 - Assume we have a differential-drive wheeled mobile robot driving at a constant speed
 - Want to drive in a desired heading
 - Use PID control
- Caution:** This typically will not work as we are dealing with angles!

Solution: Ensure that $e \in (-\pi, \pi]$. Standard trick is to use `atan2`:

$$e' = \text{atan2}(\sin(e), \cos(e)) \in (-\pi, \pi]$$

(MATLAB)

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Go-to-goal behavior

- How to drive a robot to a goal location?

$$\begin{aligned} \dot{x} &= v_0 \cos \phi & r &= \phi_d \\ \dot{y} &= v_0 \sin \phi & e &= \phi_d - \phi \\ \dot{\phi} &= \omega & \omega &= \text{PID}(e') \end{aligned}$$

Start location: (x, y)

Goal location: (x_g, y_g)

$$\phi_d = \arctan\left(\frac{y_g - y}{x_g - x}\right)$$

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Go-to-goal behavior

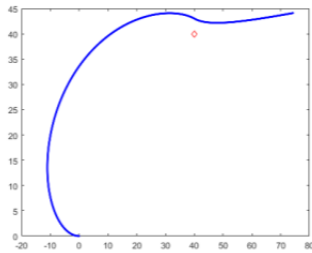
- How to drive a robot to a goal location?

Attempt 1

$$\phi_d = \arctan\left(\frac{y_g - y}{x_g - x}\right)$$

$$e = \phi_d - \phi$$

$$\omega = k_p e$$



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Didn't use atan2 → robot

goes well past target

Not consistent with our estimation
because it doesn't know difference
between 0° and 180°

Go-to-goal behavior

- How to drive a robot to a goal location?

Attempt 1

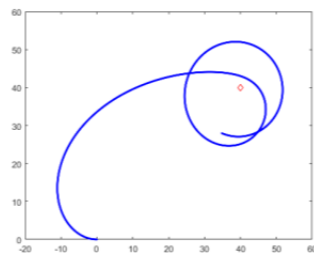
Attempt 2

$$\phi_d = \text{atan2}(y_g - y, x_g - x)$$

$$e = \phi_d - \phi$$

$$e' = \text{atan2}(\sin(e), \cos(e))$$

$$\omega = k_p e'$$



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Still not what we want

There is also an outer loop that

converts $(x, y) \leftrightarrow \phi$

ϕ_d is not unit step because it depends
on relative position of robot

ϕ_d is a ramp

Go-to-goal behavior

- How to drive a robot to a goal location?

Attempt 1

Attempt 2

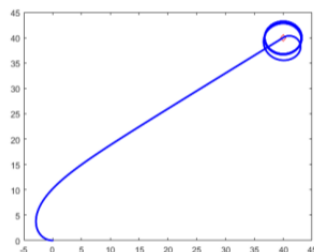
Attempt 3

$$\phi_d = \text{atan2}(y_g - y, x_g - x)$$

$$e = \phi_d - \phi$$

$$e' = \text{atan2}(\sin(e), \cos(e))$$

$$\omega = k_{Big} e'$$

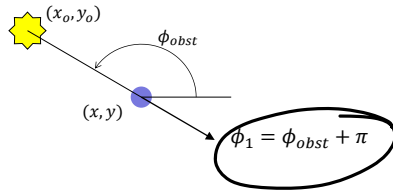


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Error is getting smaller

Obstacle-avoidance behavior

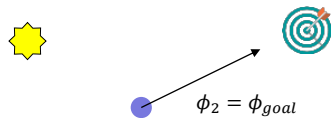
- How to avoid driving into obstacles?
 - We can use the same idea by defining a desired heading
 - “Pure” avoidance



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Obstacle-avoidance behavior

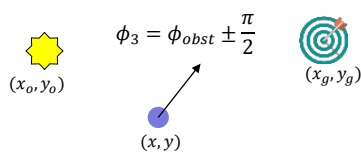
- How to avoid driving into obstacles?
 - We can use the same idea by defining a desired heading
 - “Pure” avoidance
 - “Pure” go-to-goal



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Obstacle-avoidance behavior

- How to avoid driving into obstacles?
 - We can use the same idea by defining a desired heading
 - “Pure” avoidance
 - “Pure” go-to-goal
 - Not “pure” but “blended” (choice of + or – depends on direction to goal)

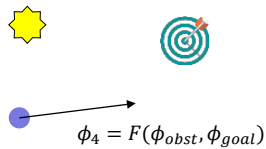


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Sensor measures both goal and obstacle - how to combine $\phi_{obstacle}$ and ϕ_{goal}

Obstacle-avoidance behavior

- How to avoid driving into obstacles?
 - We can use the same idea by defining a desired heading
 - "Pure" avoidance
 - "Pure" go-to-goal
 - Not "pure" but "blended"



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Obstacle-avoidance behavior

- How to avoid driving into obstacles?
- Behavior coordination
 - The previous example illustrates two fundamentally different mechanisms for behavior coordination
 - Arbitration (winner-takes-all, hard switches)
 - Fusion (blending, combining behaviors)
 - Both approaches have merit in different situations
 - Performance
 - Analysis
 - We will see how to design systematic behaviors and behavior coordination mechanisms in the following lectures of Module II

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