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The Pobotics Primer, Chapter 19: 15 min Sliding mode control supplement: 25 min

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

(a)
$$g(x) = x_1 + ax_2$$
, $a > 0$

$$\frac{\partial g}{\partial x} = \begin{bmatrix} 05/\partial x_1 \\ 05/\partial x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ a \end{bmatrix}$$

(b)
$$\left(-\frac{1}{a}x_{2} - \frac{1}{a}\eta, g(x) = 0\right)$$

 $u = \left(-\frac{1}{a}x_{2} + \frac{1}{a}\eta, g(x) = 0\right)$

9 7 0

$$\dot{x} = \begin{cases} f_1(x) , g(x) & 20 \\ f_2(x) , g(x) & 60 \end{cases}$$

Since fi is $x_1 = x_2$ $x_2 = \begin{bmatrix} x_2 \\ x_3 = x \end{bmatrix}$ the one $x_1 = x_2$ $x_2 = \begin{bmatrix} x_2 \\ x_3 \end{bmatrix}$

that corresponds

$$f_1 = \begin{bmatrix} x_2 \\ -\frac{1}{4}x_2 - \frac{1}{4} \eta \end{bmatrix} \qquad f_2 = \begin{bmatrix} x_2 \\ -\frac{1}{4}x_2 + \frac{1}{4} \eta \end{bmatrix}$$

$$\begin{pmatrix}
x_{2} \\
-\frac{1}{a}x_{2} - \frac{1}{a}N
\end{pmatrix}, g(x) \ge 0$$

$$x = \begin{cases}
x_{2} \\
-\frac{1}{a}x_{2} + \frac{1}{a}N
\end{cases}, g(x) \ge 0$$

$$L_{f,g} = \left(\frac{J_{g}}{J_{x}}\right)^{T} f_{i} = \left[1 \text{ a}\right] \left(\frac{x_{2}}{-\frac{1}{a}x_{2} - \frac{1}{a}N}\right) = x_{2} - x_{2} - N = -N$$

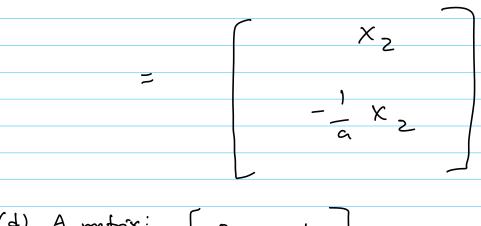
$$L_{f_2}g = \left(\frac{ds}{ds}\right)^T f_2 = \left[1 \quad \alpha\right] \left[\begin{array}{c} x_2 \\ -\frac{1}{\alpha}x_2 + \frac{1}{\alpha}y \end{array}\right] = x_2 - x_2 + y = +y$$

Sitisfies equations, so sliding mode exists

(c)
$$\sigma_{i} = \frac{l_{i}g}{l_{f_{i}}g - l_{f_{i}}g} = \frac{n}{n - (-n)} = \frac{1}{2}$$

$$\frac{1}{x} = \frac{1}{2} \left[\begin{array}{c} x_2 \\ -\frac{1}{a}x_2 - \frac{1}{a}y \end{array} \right] + \frac{1}{2} \left[\begin{array}{c} x_2 \\ -\frac{1}{a}x_2 + \frac{1}{a}y \end{array} \right]$$

$$= \begin{bmatrix} \frac{1}{2} x_2 + \frac{1}{2} x_2 \\ -\frac{1}{2a} x_2 - \frac{1}{2a} x_1 - \frac{1}{2a} x_2 + \frac{1}{2a} x_1 \end{bmatrix}$$



$$\lambda I - A = \begin{bmatrix} \lambda & -1 \\ 0 & \lambda^{\frac{1}{\alpha}} \end{bmatrix}$$

$$\lambda(\lambda + \frac{1}{2}) = 0$$

 $\lambda = 0$, $\lambda = -\frac{1}{a}$ Marsinally stable \rightarrow it might converge to 0 as $t \rightarrow \infty$, but this depends on the initial condition