ECE 1390/2390 Image Processing and Computer Vision – Fall 2021

PITT | SWANSON ENGINEERING
ELECTRICAL & COMPUTER

Camera Calibration - Extrinsic calibration

Ahmed Dallal

Quiz 3

- Wed 10/20
- Covers: Frequency domain analysis, camera models, and stereo geometry

Reading

- FP chapter 1.2 and 1.3
- Szeliski section 5.2, 5.3

- Today: Really using homogeneous systems to represent projection. And how to do calibration.

Extrinsic camera calibration

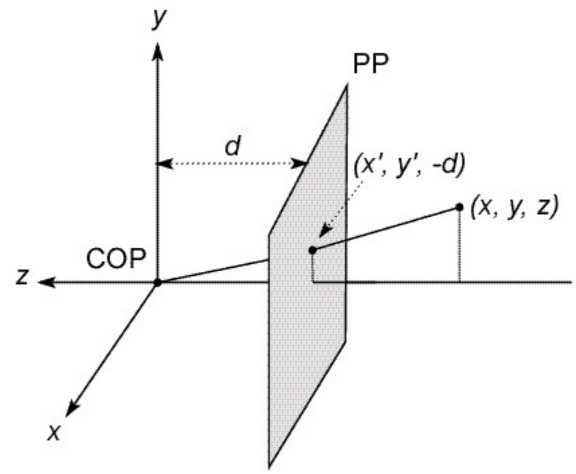
Recall: Modeling projection

Projection equations

- Compute intersection with Perspective Projection of ray from (x,y,z) to COP
- Derived using similar triangles

$$(X, Y, Z) \rightarrow (-d\frac{X}{Z}, -d\frac{Y}{Z}, -d)$$

(assumes normal Z negative – we'll change later)



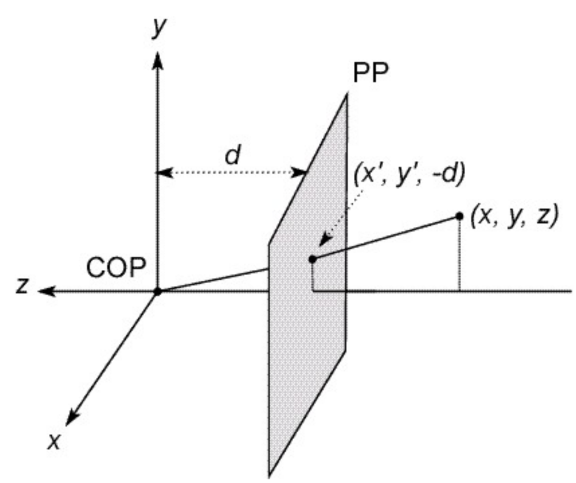
Recall: Modeling projection

Projection equations

$$(X, Y, Z) \rightarrow (-d\frac{X}{Z}, -d\frac{Y}{Z}, -d)$$

We get the projection by throwing out the last coordinate:

$$(x', y') = (-d\frac{X}{Z}, -d\frac{Y}{Z})$$



Recall: Homogeneous coordinates

Is this a linear transformation?

No – division by the (not constant) Z is non-linear

Trick: add one more coordinate:

$$(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \qquad (x, y, z) \Rightarrow \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

homogeneous image (2D) coordinates

homogeneous scene (3D) coordinates

Recall: Homogeneous coordinates

Converting *from* homogeneous coordinates:

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} \Rightarrow (x/w, y/w) \qquad \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \Rightarrow (x/w, y/w, z/w)$$

(this makes homogenous coordinates invariant under scale)

Recall: Perspective Projection

- Projection is a matrix multiply using homogeneous coordinates (and |z|):

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1/f & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ |z| \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ |z/f| \end{bmatrix} \Rightarrow \left(f\frac{x}{|z|}, f\frac{y}{|z|} \right)$$

$$\Rightarrow (u, v)$$

S. Seitz

But…

- In all this discussion we have the notion of a camera's coordinate system – an origin and an orientation.
- We put the center of projection at this origin and the optic axis down the z axis.
- So to do geometric reasoning about the world we need to relate the coordinate system of the world to that of the camera and the image.
- Today we'll do from the world to the camera, and next lesson from the camera to the image.

Geometric Camera calibration

- We want to use the camera to tell us things about the world.
  - So we need the relationship between coordinates in the world and coordinates in the image: ***geometric camera calibration***

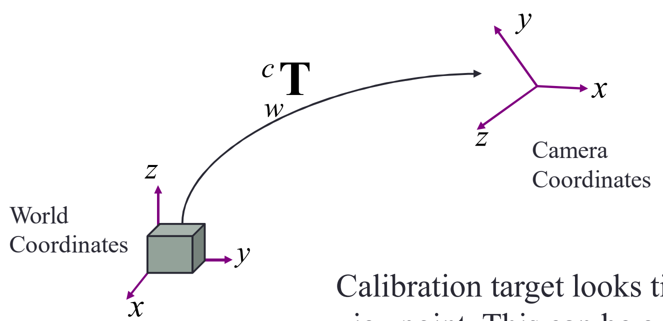
Geometric Camera calibration

Composed of 2 transformations:

- From some (arbitrary) world coordinate system to the camera's 3D coordinate system. *Extrinisic parameters (or camera pose)*
- From the 3D coordinates in the camera frame to the 2D image plane via projection. *Intrinisic parameters*

Camera Pose

In order to apply the camera model, objects in the scene must be expressed in *camera coordinates*.



Calibration target looks tilted from camera viewpoint. This can be explained as a difference in coordinate systems.

Rigid Body Transformations

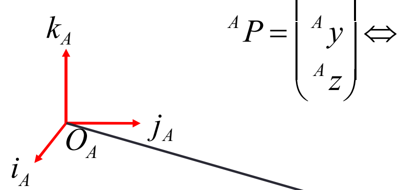
Need a way to specify the six degrees-of- freedom of a rigid body. Why 6?

A rigid body is a collection of points whose positions relative to each other can't change	Fix one point, 3 DOF	Fix second point, 2 more DOF (must maintain distance constraint)	Third point adds 1 more DOF, for rotation around line
	3	**+2**	**+1**

Notations (from F&P)

- Superscript references coordinate frame
- ${}^{A}P$ is coordinates of P in frame A
- ${}^{B}P$ is coordinates of P in frame B

$$ {}^{A}P = \begin{pmatrix} {}^{A}x \\ {}^{A}y \\ {}^{A}z \end{pmatrix} \Leftrightarrow \overline{OP} = \left({}^{A}x \bullet \overline{i_A}\right) + \left({}^{A}y \bullet \overline{j_A}\right) + \left({}^{A}z \bullet \overline{k_A}\right) $$

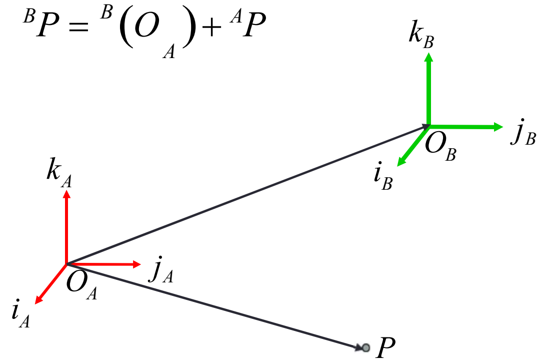


Translation Only

$$ {}^{B}P = {}^{A}P + {}^{B}(O_A) $$

$$ or $$

$$ {}^{B}P = {}^{B}(O_A) + {}^{A}P $$



Translation

- Using homogeneous coordinates, translation can be expressed as a matrix multiplication.

$$ {}^{B}P = {}^{A}P + {}^{B}O_{A} $$

I: 3x3 identity

$$ \boxed{\begin{bmatrix} {}^{B}P \\ 1 \end{bmatrix} = \begin{bmatrix} I & {}^{B}O_{A} \\ \mathbf{0}^{T} & 1 \end{bmatrix} \begin{bmatrix} {}^{A}P \\ 1 \end{bmatrix}} $$

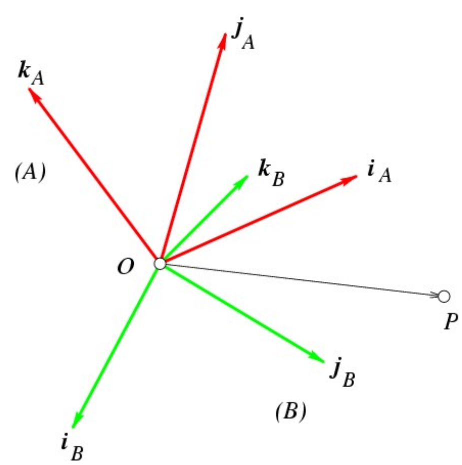
(Translation is commutative)

Rotation

$$ \overline{OP} = (i_A \quad j_A \quad k_A) \begin{pmatrix} {}^{A}x \\ {}^{A}y \\ {}^{A}z \end{pmatrix} = (i_B \quad j_B \quad k_B) \begin{pmatrix} {}^{B}x \\ {}^{B}y \\ {}^{B}z \end{pmatrix} $$

$$ {}^{B}P = {}^{B}_{A}R \; {}^{A}P $$

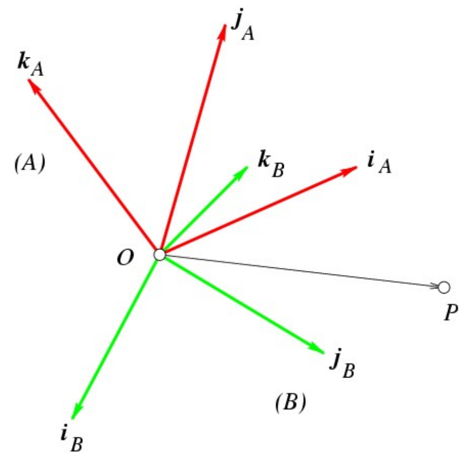
${}^{B}_{A}R$ means describing frame A in the coordinate system of frame B



Rotation

$$
{}^{B}_{A}R = \begin{bmatrix} \mathbf{i}_A . \mathbf{i}_B & \mathbf{j}_A . \mathbf{i}_B & \mathbf{k}_A . \mathbf{i}_B \\ \mathbf{i}_A . \mathbf{j}_B & \mathbf{j}_A . \mathbf{j}_B & \mathbf{k}_A . \mathbf{j}_B \\ \mathbf{i}_A . \mathbf{k}_B & \mathbf{j}_A . \mathbf{k}_B & \mathbf{k}_A . \mathbf{k}_B \end{bmatrix}
$$

$$
= \begin{bmatrix} {}^{\mathbf{B}}\mathbf{i}_{\mathbf{A}} & {}^{\mathbf{B}}\mathbf{j}_{\mathbf{A}} & {}^{\mathbf{B}}\mathbf{k}_{\mathbf{A}} \end{bmatrix}
$$



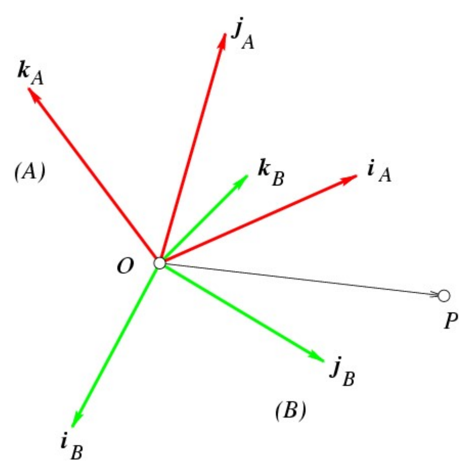
The columns of the rotation matrix are the axes of frame A expressed in frame B.

Rotation

$$
{}^{B}_{A}R = \begin{bmatrix} \mathbf{i}_A . \mathbf{i}_B & \mathbf{j}_A . \mathbf{i}_B & \mathbf{k}_A . \mathbf{i}_B \\ \mathbf{i}_A . \mathbf{j}_B & \mathbf{j}_A . \mathbf{j}_B & \mathbf{k}_A . \mathbf{j}_B \\ \mathbf{i}_A . \mathbf{k}_B & \mathbf{j}_A . \mathbf{k}_B & \mathbf{k}_A . \mathbf{k}_B \end{bmatrix}
$$

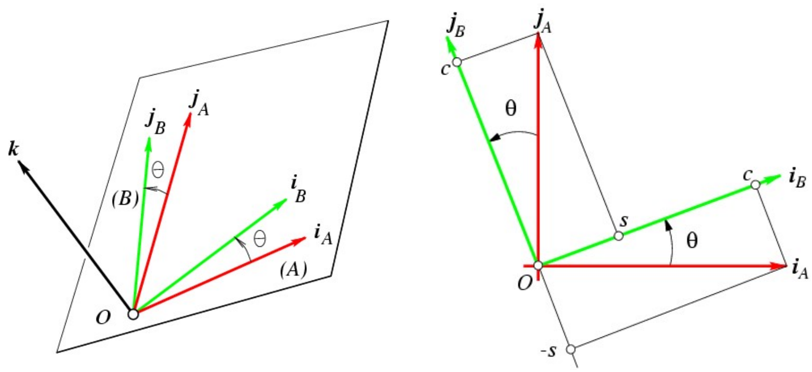
$$
= \begin{bmatrix} {}^{\mathbf{B}}\mathbf{i}_{\mathbf{A}} & {}^{\mathbf{B}}\mathbf{j}_{\mathbf{A}} & {}^{\mathbf{B}}\mathbf{k}_{\mathbf{A}} \end{bmatrix}
$$

$$
= \begin{bmatrix} {}^{\mathbf{A}}\mathbf{i}_{\mathbf{B}}^{T} \\ {}^{\mathbf{A}}\mathbf{j}_{\mathbf{B}}^{T} \\ {}^{\mathbf{A}}\mathbf{k}_{\mathbf{B}}^{T} \end{bmatrix}
$$



Orthogonal matrix!

Example: Rotation about z axis



What is the rotation matrix?

$$R_z(\theta) = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Combine 3 to get arbitrary rotation

$$R_X(\varphi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\varphi) & -\sin(\varphi) \\ 0 & \sin(\varphi) & \cos(\varphi) \end{bmatrix}$$

$$R_z(\theta) = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_Y(\kappa) = \begin{bmatrix} \cos(\kappa) & 0 & -\sin(\kappa) \\ 0 & 1 & 0 \\ \sin(\kappa) & 0 & \cos(\kappa) \end{bmatrix}$$

Rotation in homogeneous coordinates

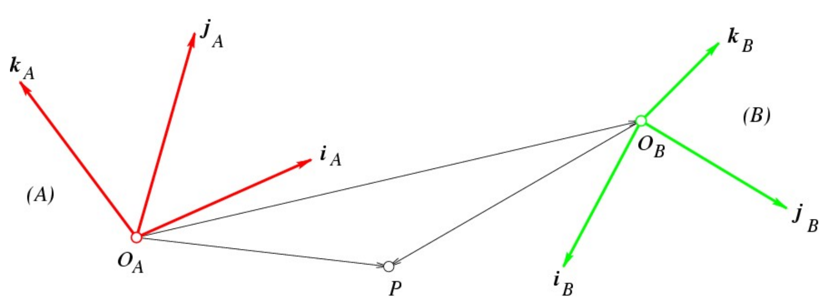
- Using homogeneous coordinates, rotation can be expressed as a matrix multiplication.

$$^{B}P = {}^{B}_{A}R \, {}^{A}P$$

$$\begin{bmatrix} ^{B}P \\ 1 \end{bmatrix} = \begin{bmatrix} ^{B}_{A}R & 0 \\ \mathbf{0}^{T} & 1 \end{bmatrix} \begin{bmatrix} ^{A}P \\ 1 \end{bmatrix}$$

- Rotation is ***not*** commutative

Rigid transformations



$$^{B}P = {}^{B}_{A}R \, {}^{A}P + {}^{B}O_{A}$$

Rigid transformations (con't)

Unified treatment using homogeneous coordinates:

$$\begin{bmatrix} {}^{B}P \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & {}^{B}O_{A} \\ \mathbf{0}^{T} & 1 \end{bmatrix} \begin{bmatrix} {}^{B}_{A}R & 0 \\ \mathbf{0}^{T} & 1 \end{bmatrix} \begin{bmatrix} {}^{A}P \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} {}^{B}_{A}R & {}^{B}O_{A} \\ \mathbf{0}^{T} & 1 \end{bmatrix} \begin{bmatrix} {}^{A}P \\ 1 \end{bmatrix}$$

Rigid transformations (con't)

And even better:

$$\begin{bmatrix} {}^{B}P \\ 1 \end{bmatrix} = \begin{bmatrix} {}^{B}_{A}R & {}^{B}O_{A} \\ \mathbf{0}^{T} & 1 \end{bmatrix} \begin{bmatrix} {}^{A}P \\ 1 \end{bmatrix} = {}^{B}_{A}T \begin{bmatrix} {}^{A}P \\ 1 \end{bmatrix}$$

Invertible!

so

$$\begin{bmatrix} {}^{A}P \\ 1 \end{bmatrix} = {}^{A}_{B}T \begin{bmatrix} {}^{B}P \\ 1 \end{bmatrix} = \left({}^{B}_{A}T\right)^{-1} \begin{bmatrix} {}^{B}P \\ 1 \end{bmatrix}$$

Translation and rotation

From frame A to B:
Non-homogeneous ("regular) coordinates

$${}^{B}p = {}^{B}_{A}R \; {}^{A}p + {}^{B}_{A}t$$

3x1 translation vector

3x3 rotation matrix

Translation and rotation

Homogeneous coordinates:

$${}^{B}\vec{p} = {}^{B}_{A}T \; {}^{A}\vec{p}$$

$${}^{B}\vec{p} = \begin{pmatrix} \left(\quad {}^{B}_{A}R \quad \right) & {}^{B}_{A}\vec{t} \\ 0 \quad 0 \quad 0 & 1 \end{pmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Homogenous coordinates allows us to write coordinate transforms as a single matrix!

From World to Camera

Rotation from world to camera frame

Translation from world to camera frame

$$ {}^{C}\vec{p} = {}^{C}_{W}R \; {}^{W}\vec{p} + {}^{C}_{W}\vec{t} $$

Point now in camera frame

Point in world frame

Non-homogeneous coordinates

From World to Camera

$$ \begin{pmatrix} \\ {}^{C}\vec{p} \\ \\ \\ \end{pmatrix} = \begin{pmatrix} - & - & - & | \\ - & {}^{C}_{W}R & - & {}^{C}_{W}\vec{t} \\ - & - & - & | \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \\ {}^{W}\vec{p} \\ \\ \\ \end{pmatrix} $$

Homogeneous coordinates

From world to camera is the extrinsic parameter matrix (4x4)
(sometimes 3x4 if using for next step in projection - not worrying about inversion)

Quiz

How many degrees of freedom are there in the 3x4 extrinsic parameter matrix?

a) 12
b) 6
c) 9
d) 3

From World to Camera

$$\begin{pmatrix} \\ {}^{C}\vec{p} \\ \\ \\ \end{pmatrix} = \begin{pmatrix} - & - & - & | \\ - & {}^{C}_{W}R & - & {}^{C}_{W}\vec{t} \\ - & - & - & | \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \\ {}^{W}\vec{p} \\ \\ \\ \end{pmatrix}$$

Homogeneous coordinates

From world to camera is the ***extrinsic*** *parameter matrix (4x4)*
(sometimes 3x4 if using for next step in projection - not worrying about inversion)