ECE 2570 Midterm Cheat Sheet - Avery Peiffer | LAPLACE TABLE

1. Control architectures - conceptual

- Deliberative Control - long time scale
  SPA - Sensing, Planning (determining poss outcomes of actions and searching for best sequence of actions to achieve a goal), Acting
  (−) Very slow, very memory intensive, info can be outdated, executing a plan can be difficult
- Reactive control - tightly couples sensing and acting; no planning ahead (int. representations of environment), very fast → most common in robotics
  Action selection - deciding among mult possible actions/behaviors
  - Arbitration: select one candidate
  - Fusion: combine mult. candidates into one

  Multitasking: must be able to support parallelism
  Subsumption architecture: bottom-up design
  - Modular, with hierarchy among modules
  - Higher layers "subsume" lower ones

  (−) No state, no internal rep. of world, no memory, no learning
- Hybrid control - comb of deliberative and reactive
  Reactive layer, planning layer, middle layer connecting
  Middle reconciles time scales, diff. reps, and contradictory commands
  (−) Middle layer is difficult to build, M layer specialized to specific problem, R and D layers can work to detriment of each other
- Behavior-based control - "behaviors" as modules for control
  Behaviors: (closest to reactive control)
  - achieve/maintain particular goals,
  - time-extended, not instantaneous,
  - take inputs from sensors and other behavs,
  - more complex than actions

  Behaviors typically executed in parallel
  Networks of behaviors store state and construct world models
  Operate on compatible time scales
  Key properties: react in time, use reps to generate behavior, use uniform structure in system
  Emergent behavior: Structured behavior that is apparent from observer's POV but not robot

→ Note on emergent behavior
  Reactive, behavior-based use parallel rules which interact with each other and env (most em behav)
  Deliberative systems are sequential and thus require env structure for em behav
  Hybrid follows delib. model, minimizing em behav

2. Find state-space equation
$\dot{x} = f(x,u) \quad y = g(x,u)$ — Don't use mats for nonlinear
Linearize (for small angles)
$\sin x \approx x$
$\cos x \approx 1 \quad e^x \approx 1 + x$ — Make A, B, C, D matrices
$\dot{x} = Ax + Bu$
$y = Cx + Du$

3. Find response of single-input, single-output system
Transfer Function $= Y(s)/R(s) \rightarrow$ LT
Zero state response: $\underset{ZS}{y(s)} = \text{TF} \cdot \underbrace{\text{LT(input)}}_{\text{Do PF expansion}}$

Zero input response: Redo LT including initial condition, set input to 0
$\rightarrow \mathcal{L}\left[\frac{d^n f}{dt^n}\right] = s^n F(s) - s^{n-1} f(0^-) \ldots - f^{n-1}(0^-)$
$\mathcal{L}\left[\frac{d^2 y}{dt^2}\right] = s^2 y(s) - s y(0^-) - \dot{y}(0^-)$
$\mathcal{L}\left[\frac{dy}{dt}\right] = s y(s) - y(0)$
Use superposition to combine y_{ZS} and y_{ZI}

4. Solve state-space equation (find y(t) by using matrices)
$y(s) = C(sI - A)^{-1} x(0) + C(sI-A)^{-1} B u(s)$
$x(0)$ is initial condition for x
$u(s)$ is LT of $u(t)$
Do ILT to get $y(t)$

5. Stability: given A matrix, find out if system is stable
Marginally stable: A has distinct eigenvals, all eigenvals have 0 or neg. real parts
Asymptotically stable: all eigenvals of A have neg. real parts
Unstable: ∃ one eigenvalue has pos. real part
$\det(\lambda I - A)$ to get eigenvals
asymp stable ⇒ marginally stable, but not other way round
asymp. ⇒ BIBO

6. Pole placement (find K)
Desirable char eq: $(\lambda - \blacksquare_1)(\lambda - \blacksquare_2) = 0$
Char eq: $\det(\lambda I - (A - BK)) = 0$
where $K = [K_1 \ K_2]$

7. Controllability

Controllability matrix: $C = [B \ \ AB \ \ldots \ A^{n-1}B]$ → Full rank (linearly independent)

$Det(C) \neq 0$ means it's controllable

Laplace Transform Table

Time Domain	Frequency Domain
$\delta(t)$	1
A (step)	$\frac{A}{s}$
t (ramp)	$\frac{1}{s^2}$
t^2	$\frac{2}{s^3}$
$t^n, n>0$	$\frac{n!}{s^{n+1}}$
e^{-at}	$\frac{1}{s+a}$
$e^{-at}\sin(\omega t)$	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{-at}\cos(\omega t)$	$\frac{s+a}{(s+a)^2+\omega^2}$

ILT to get impulse response

Nominal second order form:

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Solve state space equation example

$$\dot{x} = \begin{bmatrix} 3 & -2 \\ 4 & -3 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u$$

$$y = [1 \ \ 1]\, x \qquad x(0) = \begin{bmatrix} 3 \\ 3 \end{bmatrix} \qquad u(t) = 3\,1(t)$$

$$y(s) = C(sI-A)^{-1}x(0) + C(sI-A)^{-1}Bu(s)$$

$$sI - A = \begin{bmatrix} s-3 & 2 \\ -4 & s+3 \end{bmatrix}$$

$$(sI-A)^{-1} = \begin{bmatrix} \frac{s+3}{s^2-1} & \frac{-2}{s^2-1} \\ \frac{4}{s^2-1} & \frac{s-3}{s^2-1} \end{bmatrix}$$

$$u(s) = \frac{3}{s}$$

Inverse Proc. (2×2)
- Switch elems in diag.
- Reverse signs of off diag
- Divide by det.

PF expansion example:

$$\frac{6}{s(s-1)} = \frac{A}{s} + \frac{B}{s-1}$$

$$6 = A(s-1) + B(s)$$

set $s=1$, $s=0$ to find $A + B$

Response of single-input, single-output example

$$\frac{d^2y(t)}{dt^2} + 4\frac{dy(t)}{dt} + 3y(t) = 6r(t)$$

$$y(0) = 3 \quad \dot{y}(0) = 0 \quad r(t) = 1,\ t \geq 0$$

LT: $s^2Y(s) + 4sY(s) + 3Y(s) = 6R(s)$

$$\frac{Y(s)}{R(s)} = \frac{6}{s^2+4s+3}$$

LT of input $(r = 1(t))$

Zero state response:

$$Y_{zs}(s) = \frac{6}{s^2+4s+3} \cdot \frac{1}{s}$$

$$= \frac{6}{(s+3)(s+1)(s)} \rightarrow \text{PF expansion}$$

$$= \frac{2}{s} + \frac{-3}{s+1} + \frac{1}{s+3} \rightarrow \text{ILT}$$

$$y_{zs}(t) = 2 - 3e^{-t} + e^{-3t}$$

Zero input response: Redo LT including initial condition (use differential theorem)

$$[s^2y(s) - \dot{y}(0) - sy(0)] + 4[sy(s) - y(0)] + 3y(s) = 6r(s)$$

→ set $r(s)$ to 0

$$y(s) = \frac{3s+12}{s^2+4s+3} \rightarrow \text{PF expansion}$$

$$= \frac{-1.5}{s+3} + \frac{4.5}{s+1} \xrightarrow{\text{ILT}} y_{zI}(t) = -1.5e^{-3t} + 4.5e^{-t}$$

$$y(t) = y_{zs}(t) + y_{zI}(t) = 2 + \frac{3e^{-t}}{2} + -\frac{e^{-3t}}{2}$$