ECE 2570 - Homework 7

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1. The Robotics Primer, Chapter 19 : 15 min

Sliding mode control supplement: 25 min

2. $J\ddot{\theta} = u \rightarrow J = 1$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$x_1 = \theta \qquad x_2 = \dot{\theta}$$

(a) $g(x) = x_1 + a x_2 \quad , \quad a > 0$

$$\frac{dg}{dx} = \begin{bmatrix} \partial g / \partial x_1 \\ \partial g / \partial x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ a \end{bmatrix}$$

(b)

$$u = \begin{cases} -\frac{1}{a} x_2 - \frac{1}{a} \eta \ , & g(x) \geq 0 \\ -\frac{1}{a} x_2 + \frac{1}{a} \eta \ , & g(x) < 0 \end{cases}$$

$$\eta > 0$$

$$\dot{x} = \begin{cases} f_1(x) \ , & g(x) \geq 0 \\ f_2(x) \ , & g(x) < 0 \end{cases}$$

State space equation:

$$\dot{x}_1 = x_2 \qquad \dot{x} = \begin{bmatrix} x_2 \\ u \end{bmatrix}$$
$$\dot{x}_2 = u$$

Since f_1 is the one that corresponds to ≥ 0

$$f_1 = \begin{bmatrix} x_2 \\ -\frac{1}{a} x_2 - \frac{1}{a} \eta \end{bmatrix} \qquad f_2 = \begin{bmatrix} x_2 \\ -\frac{1}{a} x_2 + \frac{1}{a} \eta \end{bmatrix}$$

$$\dot{x} = \begin{cases} \begin{bmatrix} x_2 \\ -\frac{1}{a}x_2 - \frac{1}{a}\eta \end{bmatrix}, & g(x) \geq 0 \\ \begin{bmatrix} x_2 \\ -\frac{1}{a}x_2 + \frac{1}{a}\eta \end{bmatrix}, & g(x) < 0 \end{cases}$$

$$L_{f_1} g = \left(\frac{\partial g}{\partial x}\right)^T f_1 = \begin{bmatrix} 1 & a \end{bmatrix} \begin{bmatrix} x_2 \\ -\frac{1}{a}x_2 - \frac{1}{a}\eta \end{bmatrix} = x_2 - x_2 - \eta = -\eta$$

$$L_{f_2} g = \left(\frac{\partial g}{\partial x}\right)^T f_2 = \begin{bmatrix} 1 & a \end{bmatrix} \begin{bmatrix} x_2 \\ -\frac{1}{a}x_2 + \frac{1}{a}\eta \end{bmatrix} = x_2 - x_2 + \eta = +\eta$$

Satisfies equations, so sliding mode exists

(c) $$\sigma_1 = \frac{L_{f_2} g}{L_{f_2} g - L_{f_1} g} = \frac{\eta}{\eta - (-\eta)} = \frac{1}{2}$$

$$\sigma_2 = 1 - \sigma_1 = \frac{1}{2}$$

$$\dot{x} = \frac{1}{2}\begin{bmatrix} x_2 \\ -\frac{1}{a}x_2 - \frac{1}{a}\eta \end{bmatrix} + \frac{1}{2}\begin{bmatrix} x_2 \\ -\frac{1}{a}x_2 + \frac{1}{a}\eta \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2}x_2 + \frac{1}{2}x_2 \\ -\frac{1}{2a}x_2 \cancel{- \frac{1}{2a}\eta} - \frac{1}{2a}x_2 \cancel{+ \frac{1}{2a}\eta} \end{bmatrix}$$

$$= \begin{bmatrix} x_2 \\ -\frac{1}{a} x_2 \end{bmatrix}$$

(d) A matrix: $\begin{bmatrix} 0 & 1 \\ 0 & -\frac{1}{a} \end{bmatrix}$

$$\lambda I - A = \begin{bmatrix} \lambda & -1 \\ 0 & \lambda + \frac{1}{a} \end{bmatrix}$$

$$\lambda(\lambda + \tfrac{1}{a}) = 0$$

$\lambda = 0$, $\lambda = -\frac{1}{a}$

Marginally stable → it might converge to 0 as $t \to \infty$, but this depends on the initial condition