Homework 7, ECE 1675/2570 Robotic Control, Spring 2022
(Due Wednesday March 30)

Problem 1. Please read (i) Chapter 19 of the textbook by Mataric (The Robotics Primer) and (ii) the supplemental material on sliding mode control and write down the approximate amount of time that you spent in reading each document.

Problem 2. Consider a single-joint robot whose dynamics is described by $J\ddot{\theta} = u$, where θ is the joint angle, $J = 1$ is the moment of inertia set to 1, and u is the control input (torque). The system's state-space description is as follows:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix}u \qquad \dot{x}_1 = x_2 \quad \dot{x}_2 = u$$

$$y = [1 \;\; 0]\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

where the state variables are defined as $x_1 = \theta, x_2 = \dot{\theta}$.

(a) We choose the sliding surface as $g(\mathbf{x}) = x_1 + ax_2, a > 0$. Please calculate the gradient $\partial g / \partial \mathbf{x}$.

(b) If we use the control law: $u = \begin{cases} -\frac{1}{a}x_2 - \frac{1}{a}\eta, & g(\mathbf{x}) \geq 0 \\ -\frac{1}{a}x_2 + \frac{1}{a}\eta, & g(\mathbf{x}) < 0 \end{cases}$

where η is a positive constant, then the controlled system becomes a hybrid automaton in the form of

$$\dot{\mathbf{x}} = \begin{cases} \mathbf{f}_1(\mathbf{x}), & g(\mathbf{x}) \geq 0 \\ \mathbf{f}_2(\mathbf{x}), & g(\mathbf{x}) < 0. \end{cases}$$

Please determine $\mathbf{f}_1, \mathbf{f}_2, L_{\mathbf{f}_1} g, L_{\mathbf{f}_2} g$.

(c) Please determine the induced mode or sliding mode $\dot{\mathbf{x}} = \sigma_1 \mathbf{f}_1 + \sigma_2 \mathbf{f}_2$ on the sliding surface.

(d) In the induced mode, what is the value of $\theta(t)$ as $t \to \infty$?