ECE 2570 - Homework 6 Avery Peiffer

1. I spent about 45 minutes reading the supplemental document.

2. $$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \end{bmatrix} u$$

$$y = \begin{bmatrix} 2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

(a) $$O = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix}$$

$$CA = \begin{bmatrix} 2 & 0 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} 4 & 6 \end{bmatrix}$$

$$O = \begin{bmatrix} 2 & 0 \\ 4 & 6 \end{bmatrix}$$, which has full rank so the system is observable

(b) $\omega_n = 5$
$\zeta = 0.8$

Desirable char eq: $\lambda^2 + 2\zeta\omega_n\lambda + \omega_n^2 = 0$

$$\lambda^2 + 2(0.8)(5)\lambda + (5)^2 = 0$$

$$\lambda^2 + 8\lambda + 25 = 0$$

Get eigenvalues of $A - LC$

$$A - LC = \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix} - \begin{bmatrix} L_1 \\ L_2 \end{bmatrix} \begin{bmatrix} 2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix} - \begin{bmatrix} 2L_1 & 0 \\ 2L_2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2-2L_1 & 3 \\ -1-2L_2 & 4 \end{bmatrix}$$

$$\text{eig}(A-LC) = \det(\lambda I - [A-LC])$$

$$= \begin{bmatrix} \lambda-2+2L_1 & -3 \\ 1+2L_2 & \lambda-4 \end{bmatrix}$$

$$= (\lambda-2+2L_1)(\lambda-4) - (-3)(1+2L_2)$$

$$= \lambda^2 - 4\lambda - 2\lambda + 8 + 2L_1\lambda - 8L_1 - [-3-6L_2]$$

$$= \lambda^2 - 6\lambda + 2L_1\lambda + 8 - 8L_1 + 3 + 6L_2$$

$$= \lambda^2 + (2L_1-6)\lambda + (-8L_1+6L_2+11)$$

$$\lambda^2 + (2L_1-6)\lambda + (-8L_1+6L_2+11) = \lambda^2 + 8\lambda + 25$$

$$2L_1 - 6 = 8 \qquad -8L_1 + 6L_2 + 11 = 25$$

$$L_1 = 14 \qquad -8(14) + 6L_2 = 14$$

$$-112 + 6L_2 = 14$$

$$6L_2 = 126$$

$$L_2 = 21$$