ECE 1675/2570: Robot Control

Midterm Exam **(7 problems, 30 points)**

Problem 1 [2 points]. (a) Why is the deliberative control architecture no longer used for the majority of physical robots? [1 point]
(b) What is the main drawback of the "hybrid" control architecture? [1 point]

Problem 2 [5 points]**.** Consider a system represented by the following differential equations:

$$Ri_1(t) + L_1\frac{di_1(t)}{dt} + v(t) = u_1(t)$$
$$L_2\frac{di_2(t)}{dt} + v(t) = u_2(t)$$
$$i_1(t) + i_2(t) = C\frac{dv(t)}{dt}$$

where R, L_1, L_2, and C are given constants, and u_1(t) and u_2(t) are inputs. Let the state variables be defined as $x_1 = i_1, x_2 = i_2, x_3 = v$. Please obtain a state-space description of the system where the output is $y = v$ and determine the **A**, **B**, **C**, **D** matrices.

Problem 3 [5 points]**.** Consider a system represented by the transfer function

$$G(s) = \frac{Y(s)}{R(s)} = \frac{3}{s^2 + 4s + 3}$$

where the initial conditions are $y(0) = 1,\ \dot{y}(0) = 0$, and the input $r(t) = 1(t)$. Please calculate the response $y(t)$ for $t \geq 0$.

Problem 4 [5 points]**.** Consider an LTI system: $\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}u$ and $y = [2\ \ 0]\mathbf{x}$. Given $\mathbf{x}(0) = [1, 1]'$ and $u(t) = 1(t)$, please compute $y(t)$.

Problem 5 [3 points]. Given the linear time-invariant system $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x}$, determine whether the following **A** matrices correspond to a system that is marginally stable, asymptotically stable, or unstable:

$$\mathbf{A}_1 = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}, \mathbf{A}_2 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \mathbf{A}_3 = \begin{bmatrix} -1 & -2 \\ -2 & -1 \end{bmatrix}.$$

Problem 6 [5 points]**.** Consider a robot whose state-space description is given by

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \omega_0^2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u.$$

Please find the state-feedback control law so that the closed-loop system has a natural frequency of $2\omega_0$ and damping ratio of 0.7.

Problem 7 [5 points]**.** Consider a robot system represented by

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u.$$

(a) Determine the controllability of this system. [2 points]
(b) Now consider state-feedback control $u = -[K_1 \quad K_2]\mathbf{x} + r$**.** Determine all possible values of K_1 and K_2 with which the closed-loop system is controllable. [3 points]