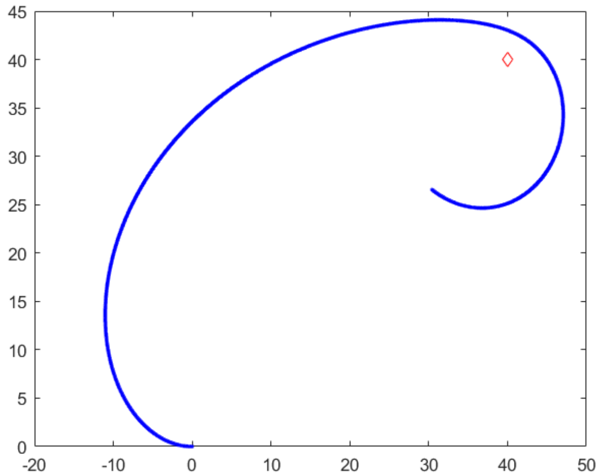
ECE 2570 - Homework 4

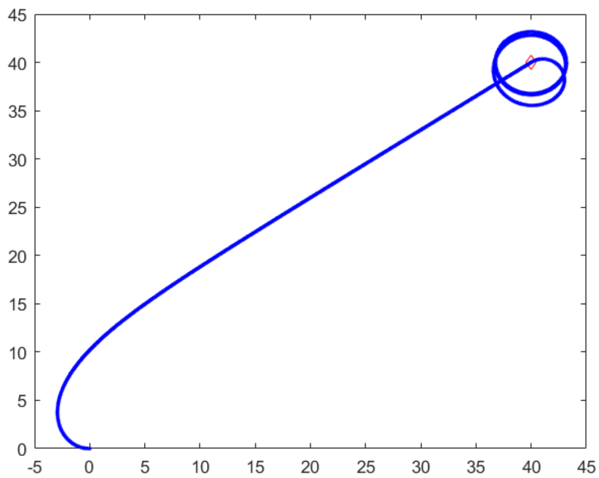
Avery Peiffer

1. Plots below:

K_p = 0.5



K_p = 2



2. $\dot{x} = \begin{bmatrix} 3 & -2 \\ 4 & -3 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u$

$y = \begin{bmatrix} 1 & 1 \end{bmatrix} x$

$x(0) = \begin{bmatrix} 3 \\ 3 \end{bmatrix} \qquad u(t) = 3 \cdot 1(t)$

$$y(s) = C(sI-A)^{-1}x(0) + C(sI-A)^{-1}B\hat{u}(s)$$

$$sI - A = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 3 & -2 \\ 4 & -3 \end{bmatrix} = \begin{bmatrix} s-3 & 2 \\ -4 & s+3 \end{bmatrix}$$

$$(sI-A)^{-1} = \begin{bmatrix} \frac{s+3}{s^2-1} & \frac{-2}{s^2-1} \\ \frac{4}{s^2-1} & \frac{s-3}{s^2-1} \end{bmatrix}$$

$$y(s) = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} \frac{s+3}{s^2-1} & \frac{-2}{s^2-1} \\ \frac{4}{s^2-1} & \frac{s-3}{s^2-1} \end{bmatrix} \begin{bmatrix} 3 \\ 3 \end{bmatrix} + \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} \frac{s+3}{s^2-1} & \frac{-2}{s^2-1} \\ \frac{4}{s^2-1} & \frac{s-3}{s^2-1} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \frac{3}{s}$$

$$= \begin{bmatrix} \frac{s+7}{s^2-1} & \frac{s-5}{s^2-1} \end{bmatrix} \begin{bmatrix} 3 \\ 3 \end{bmatrix} + \begin{bmatrix} \frac{s+7}{s^2-1} & \frac{s-5}{s^2-1} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \frac{3}{s}$$

$$= \left[\frac{6s+6}{s^2-1} \right] + \left[\frac{2s+2}{s^2-1} \right] \frac{3}{s}$$

$$= \left[\frac{6s+6}{s^2-1} \right] + \left[\frac{6s+6}{s(s^2-1)} \right]$$

$$= \left[\frac{6}{s-1} \right] + \left[\frac{6}{s(s-1)} \right]$$

$$y(t) = \mathcal{L}^{-1}\{y(s)\} = \mathcal{L}^{-1}\left\{ \frac{6}{s-1} + \frac{6}{s(s-1)} \right\}$$

$$= 6e^t - 6(t) - 6e^t$$

PF expansion

$$\boxed{y(t) = 6\left(2e^t - 1(t)\right)}$$

$$\frac{6}{s(s-1)} = \frac{A}{s} + \frac{B}{(s-1)}$$

$$6 = A(s-1) + Bs$$

$$s = 1 \longrightarrow 6 = B$$

$$s = 0 \longrightarrow 6 = -A$$

$$A = -6$$

$$= \frac{-6}{s} + \frac{6}{s-1}$$

$$\mathcal{L}^{-1}\left\{ \frac{-6}{s} + \frac{6}{s-1} \right\} = -6(t) + 6e^{t}$$