ECE 2570 - Homework 5 Avery Peiffer

2. $$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega_0^2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

closed loop poles at $-2\omega_0$

Desirable char eq: $(\lambda - (-2\omega_0))^2 = 0$

$$(\lambda + 2\omega_0)^2 = \lambda^2 + 4\omega_0 \lambda + 4\omega_0^2 = 0$$

Char eq: $\det(\lambda I - (A - BK)) = 0$

$$\det\left(\begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \left(\begin{bmatrix} 0 & 1 \\ -\omega_0^2 & 0 \end{bmatrix} - \underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} k_1 & k_2 \end{bmatrix}}_{\begin{bmatrix} 0 & 0 \\ k_1 & k_2 \end{bmatrix}} \right) \right) = 0$$

$$\det\left(\begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -\omega_0^2 - k_1 & -k_2 \end{bmatrix} \right) = 0$$

$$\det\left(\begin{bmatrix} \lambda & -1 \\ \omega_0^2 + k_1 & \lambda + k_2 \end{bmatrix} \right) = 0$$

$$\lambda^2 + k_2 \lambda + \omega_0^2 + k_1 = 0$$

$$\lambda^2 + k_2 \lambda + \omega_0^2 + k_1 = \lambda^2 + 4\omega_0 \lambda + 4\omega_0^2$$

$k_2 \lambda = 4\omega_0 \lambda$ $\qquad$ $k_1 + \omega_0^2 = 4\omega_0^2$

$k_2 = 4\omega_0$ $\qquad$ $k_1 = 3\omega_0^2$

$$\boxed{\begin{aligned} k_1 &= 3\omega_0^2 \\ k_2 &= 4\omega_0 \end{aligned}}$$

3. $\dot{x}_1 = -2x_1 + u$

$\dot{x}_2 = -3x_2 + kx_1$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ k & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$$

Controllability Matrix: $C = [B \quad AB \quad \ldots \quad A^{n-1}B]$

$$AB = \begin{bmatrix} -2 & 0 \\ k & -3 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ k \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & -2 \\ 0 & k \end{bmatrix}$$

$\det(C) = k$, meaning that the system is controllable whenever k is nonzero