

## Lectures 3 and 4: Robot Control Architectures; Time Responses of Dynamical Systems; PID Control

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### Outline

- Homework 2 (due Feb. 9)
- Robot control architectures
  - Deliberative control
  - Reactive control
  - "Hybrid" control
  - Behavior-based control
- Time responses of dynamical systems
  - First-order systems
  - Second-order systems
  - Time response specifications of design
  - Frequency responses
- PID control

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### Robot control architectures

- Why does a robot need a control architecture?
  - How would you put multiple feedback controllers together?
  - What if you need more than feedback control?
  - How would you decide what is needed, which part of the control system to use in a given situation and for how long, and what priority to assign to it?

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*Might have multiple subtasks*

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## Robot control architectures

- Why does a robot need a control architecture?
- What is control architecture?
  - A robot *control architecture* provides guiding principles and constraints for organizing a robot's control system (its brain)
  - Robot control can take place in hardware and in software, but the more complex the controller, the more likely it is to be implemented in software

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## Robot control architectures

- Why does a robot need a control architecture?
- What is control architecture?
  - A robot *control architecture* provides guiding principles and constraints for organizing a robot's control system (its brain)
  - Robot control can take place in hardware and in software, but the more complex the controller, the more likely it is to be implemented in software
  - Robot controllers can be implemented in various languages: *C/C++*, *Python*, *C#*, *Matlab*, *Java*, *Assembly*, *Hardware Description Languages* (HDLs), *LISP*, *industrial robot languages*, *BASIC*, *Pascal*, etc.

**Note:** There is *no "best" language*; as robotics grows and matures, there are more and more specialized programming languages and tools.

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## Robot control architectures

- Why does a robot need a control architecture?
- What is control architecture?
- Types of control architectures
  - Deliberative control
  - Reactive control
  - "Hybrid" control
  - Behavior-based control

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## Deliberative control

- Definition
  - *Deliberation* refers to thinking hard (“thoughtfulness in decision and action”)
    - Deliberative control grew out of early AI (e.g. Shakey)
    - Deliberative control looks into the future, and it works on a *long time-scale*

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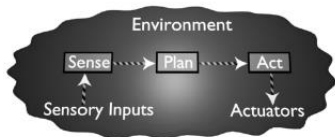
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## Deliberative control

- Definition
  - *Deliberation* refers to thinking hard
  - Deliberative control involves three steps: SPA
    - *Sensing (S)*
    - *Planning (P)*, the process of determining possible outcomes of actions and searching for the best sequence of actions to achieve a goal
    - *Acting (A)*



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## Deliberative control

- Definition
  - Drawbacks
    - *Time-scale*: it can be very slow (due to large state space)
    - *Space*: it can be very memory-intensive (state space representation requires significant storage)
    - *Information*: sometimes information is outdated
    - *Execution*: executing a plan can be difficult
- Note:** Since the 1980s, purely deliberative architectures are no longer used for the majority of physical robots

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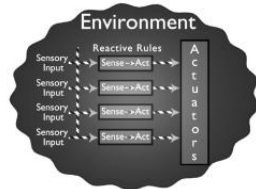
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## Reactive control

- Definition
  - **Reactive control** is control that tightly couples sensing and acting
    - It does not plan ahead (does not use any internal representations of the environment)
    - It is very fast
    - It is the most common control method in robotics



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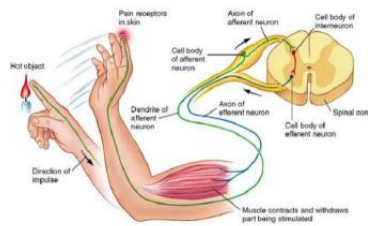
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## Reactive control

- Definition
  - **Reactive control** is control that tightly couples sensing and acting
  - Reactive control is similar to neural reflexes



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spinal  
cord

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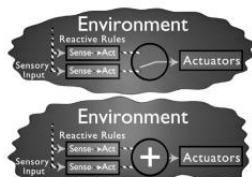
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## Reactive control

- Definition
- Action selection
  - **Action selection** is the process of deciding among multiple possible actions or behaviors
  - Two basic types
    - **Arbitration**: select one candidate
    - **Fusion**: combine multiple candidate actions into a single action



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## Reactive control

- Definition
- Action selection
  - Action selection is the process of deciding among multiple possible actions or behaviors
  - Two basic types
  - **Multitasking**: reactive systems must be able to support parallelism, the ability to monitor and execute multiple rules at once

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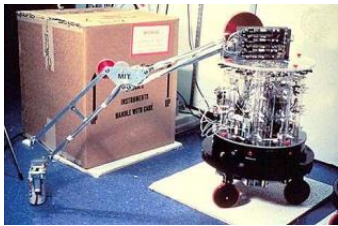
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## Reactive control

- Definition
- Action selection
- Subsumption architecture (SA)
  - SA is the best known architecture for reactive control (introduced by Prof. Rodney Brooks at MIT in 1985)



Herbert the robot

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## Reactive control

- Definition
- Action selection
- Subsumption architecture (SA)
  - SA is the best known architecture for reactive control
  - The **basic idea** is to build systems incrementally, from the simple parts to the more complex, using the already existing components as much as possible in the newly added stuff
    - This is called **bottom-up** design
    - This idea mimics our models of **evolutionary biology**

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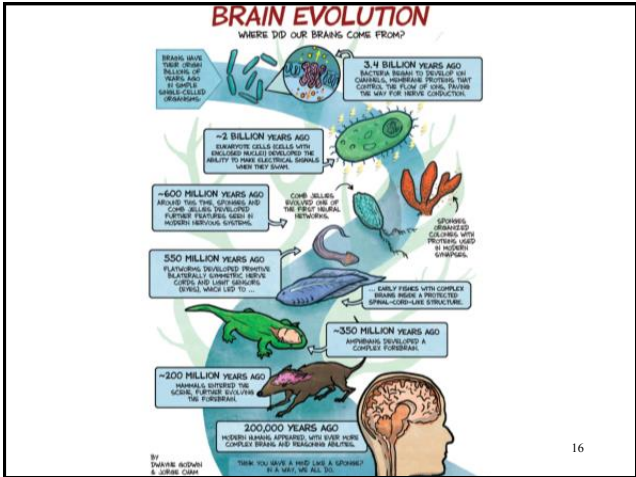
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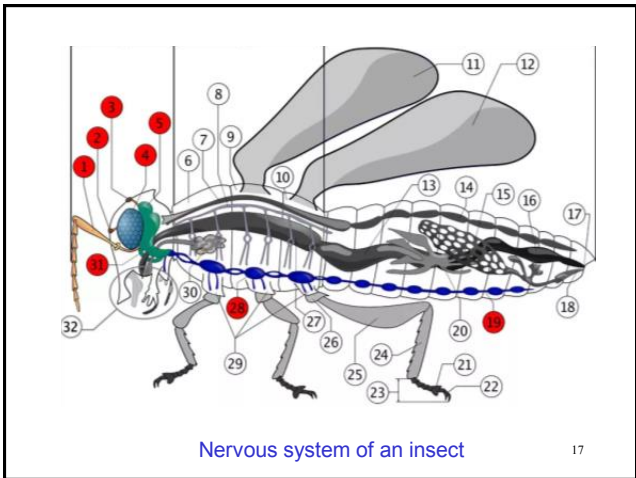
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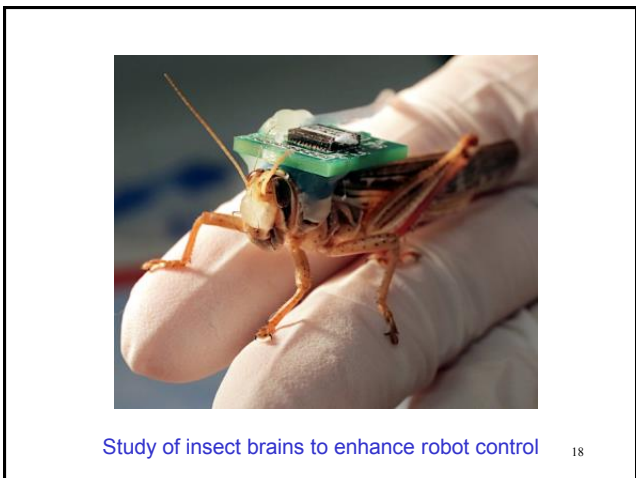
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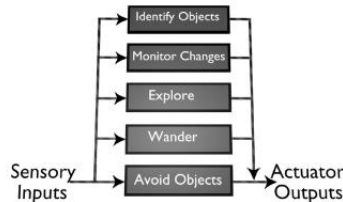
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## Reactive control

- Definition
- Action selection
- Subsumption architecture (SA)
  - SA is the best known architecture for reactive control
  - The basic idea
  - SA is modular, with a hierarchy among the modules, which are suggestively called layers



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## Reactive control

- Definition
- Action selection
- Subsumption architecture (SA)
  - SA is the best known architecture for reactive control
  - The basic idea
  - SA is modular, with a hierarchy among the modules, which are suggestively called layers
    - Each layer performs some task (e.g. avoiding objects)
    - Each layer is largely independent of other layers (allowing each to be designed and debugged separately)

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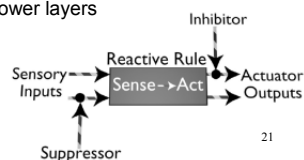
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## Reactive control

- Definition
- Action selection
- Subsumption architecture (SA)
  - SA is the best known architecture for reactive control
  - The basic idea
  - SA is modular, with a hierarchy among the modules, which are suggestively called layers
    - Each layer performs some task
    - Each layer is largely independent of other layers
    - **Subsumption:** higher layers can, under certain conditions, "subsume" aspects of lower layers

The subsumption can occur by either suppressing the inputs or inhibiting the outputs



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## Reactive control

- Definition
- Action selection
- Subsumption architecture (SA)
  - SA is the best known architecture for reactive control
  - The basic idea
  - SA is modular, with a hierarchy among the modules, which are suggestively called layers
    - Each layer performs some task
    - Each layer is largely independent of other layers
    - Subsumption
  - “*The world is its own best model*,” so no internal model is used
  - No sequencing of tasks between layers is used; they are all running in parallel all the time

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## Reactive control

- Definition
- Action selection
- Subsumption architecture (SA)
- Drawbacks of reactive control
  - No (or minimal) state
  - No internal representations of the world
  - No memory
  - No (or minimal) learning

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## “Hybrid” control

- Definition
  - “*Hybrid*” control involves the combination of reactive and deliberative control within a single robot control system (it roughly means to think and act independently and concurrently)
  - This concept is different from the concept of hybrid control used in controls literature

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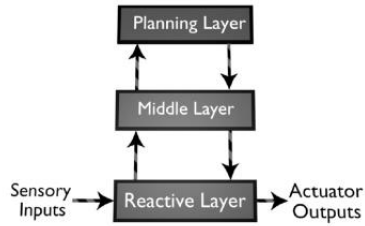
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## "Hybrid" control

- Definition
- Three-layer architecture
  - Three layers: a *reactive layer*, a *planning layer*, and a *middle layer* linking the two



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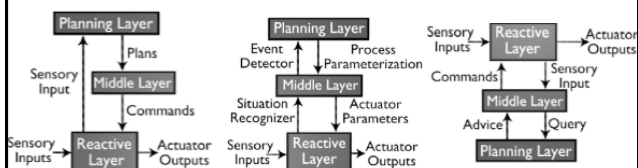
## "Hybrid" control

- Definition
- Three-layer architecture
  - Three layers: a *reactive layer*, a *planning layer*, and a *middle layer* linking the two
  - The "magic middle"
    - compensates for the limitations of the other two
    - reconciles their disparate time-scales
    - reconciles their different representations
    - reconciles contradictory commands

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## "Hybrid" control

- Definition
- Three-layer architecture
  - Three layers: a *reactive layer*, a *planning layer*, and a *middle layer* linking the two
  - The "magic middle"
  - Various ways of managing layer interaction



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## "Hybrid" control

- Definition
- Three-layer architecture
- Drawbacks of "hybrid" control
  - The middle layer is difficult to design and build
  - The middle layer is specialized to a specific problem/robot
  - Sometimes the reactive and deliberative layers work to the detriment of each other

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## Behavior-based control

- Definition
  - *Behavior-based control* involves the use of "behaviors" as modules for control
  - About *behaviors*
    - Behaviors achieve and/or maintain particular goals
    - Behaviors are time-extended, not instantaneous
    - Behaviors can take inputs from sensors and also from other behaviors, and can send outputs to effectors and to other behaviors—we can create network of behaviors
    - Behaviors are more complex than actions

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## Behavior-based control

- Definition
- Connections with the other control architectures
  - Behavior-based control is closer to reactive control than to hybrid control, and farthest from deliberative control
  - Behavior based systems have reactive components, just as hybrid systems do, but they do not have traditional deliberative components

**Note:** Reactive control is too inflexible (incapable of representation or learning); deliberative control is too slow and cumbersome; and hybrid systems require complex interaction among components

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## Behavior-based control

- Definition
- Connections with the other control architectures
- Principles of design
  - Behaviors are typically executed in parallel
  - Networks of behaviors are used to store state and construct world models/representations
  - Behaviors operate on compatible time-scales

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## Behavior-based control

- Definition
- Connections with the other control architectures
- Principles of good design
- Key properties
  - The ability to react in real-time
  - The ability to use representations to generate (not only reactive) behavior
  - The ability to use a uniform structure and representation throughout the system (with no intermediate layers)

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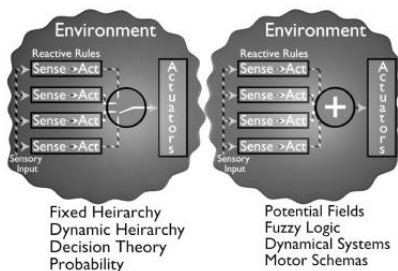
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## Behavior-based control

- Definition
- Connections with the other control architectures
- Principles of good design
- Key properties
- Behavior coordination (or action selection)



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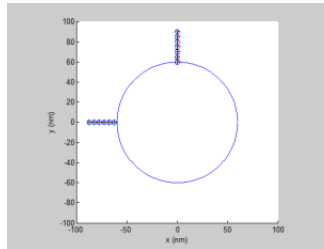
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## Behavior-based control

- Definition
- Connections with the other control architectures
- Principles of good design
- Key properties
- Behavior coordination

### Emergent behavior

- Definition: *Emergent behavior* is structured (patterned, meaningful) behavior that is apparent from the observer's viewpoint, but not from the controller's /robot's view point



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## Behavior-based control

- Definition
- Connections with the other control architectures
- Principles of good design
- Key properties
- Behavior coordination

### Emergent behavior

- Definition
- Architectures and emergence
  - *Reactive* and *behavior-based* systems employ parallel rules and behaviors, respectively, which interact with each other and the environment, thus providing the perfect foundation for exploiting emergent behavior by design
  - *Deliberative* systems are sequential (with no parallel interactions between the components) and thus would require environment structure to have any behavior emerge over time
  - *Hybrid* systems follow the deliberative model in attempting to produce a coherent, uniform output of the system, minimizing interactions and thus minimizing emergence

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## Time responses of first-order systems

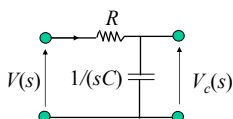
### First-order systems

$$G(s) = \frac{C(s)}{R(s)} = \frac{b_0}{s + a_0} = \frac{K}{\tau s + 1}$$

dc gain

Time constant

– Examples:



$$G(s) = \frac{V_c(s)}{V(s)} = \frac{1/(Cs)}{R + 1/(Cs)} = \frac{1}{RCs + 1}$$

Question: What does this circuit often used for?

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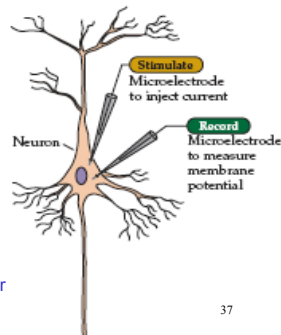
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## Time responses of first-order systems

### • First-order systems

#### – Examples

- Cruise control model
- Leaky water tank model
- Eye movement control model



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## Time responses of first-order systems

### • First-order systems

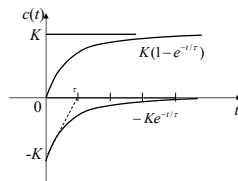
$$G(s) = \frac{C(s)}{R(s)} = \frac{K}{\tau s + 1}$$

### • Step response

$$R(s) = 1/s,$$

$$C(s) = \frac{1}{s} \frac{K}{\tau s + 1} = \frac{K}{s} - \frac{K}{s + 1/\tau},$$

$$c(t) = K(1 - e^{-t/\tau}), \quad t > 0$$



The limit of  $c(t)$  as  $t$  goes to infinity is called the **final value**, or **steady-state value** of the response.

The parameter  $\tau$  is called **time constant**; we may consider an exponential term to be zero after **four** time constants.

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## Time responses of first-order systems

### • First-order systems

$$G(s) = \frac{C(s)}{R(s)} = \frac{K}{\tau s + 1}$$

### • Step response

$$R(s) = 1/s,$$

$$C(s) = \frac{1}{s} \frac{K}{\tau s + 1} = \frac{K}{s} - \frac{K}{s + 1/\tau}$$

$$c(t) = K - Ke^{-t/\tau}, \quad t > 0$$

Forced response or steady-state response

Natural response or transient response

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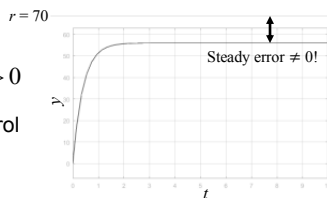
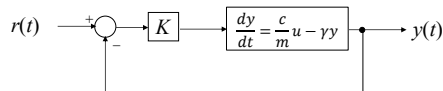
## Time responses of first-order systems

- First-order systems

### Step response

$$c(t) = K(1 - e^{-t/\tau}), \quad t > 0$$

- Example: using P control in cruise control



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## Time responses of first-order systems

- First-order systems
- Step response

$$G(s) = \frac{C(s)}{R(s)} = \frac{K}{\tau s + 1}$$

### System dc gain

- The system dc gain is the steady-state gain to a constant input for the case the output has a final value, and it is equal to the system transfer function evaluated at  $s = 0$  (why?)

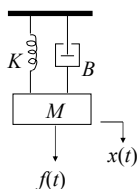
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## Time responses of second-order systems

### Second-order systems

$$G(s) = \frac{C(s)}{R(s)} = \frac{b_0}{s^2 + a_1 s + a_0} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

- Examples:



$$M \frac{d^2 x}{dt^2} = f(t) - B \frac{dx}{dt} - Kx$$

$$G(s) = \frac{X(s)}{F(s)} = \frac{1}{Ms^2 + Bs + K}$$

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## Time responses of second-order systems

- Second-order systems
  - Examples:

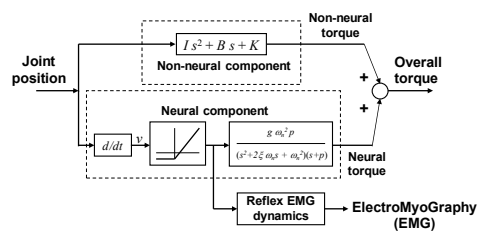
An example from  
Dr. Ruiping Xia's  
research project  
(I am a collaborator)



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## Time responses of second-order systems

- Second-order systems
  - Examples:



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## Time responses of second-order systems

- Second-order systems  $G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$

### • Step response

- Case 1:  $\zeta < 1$  (underdamped), including  $\zeta = 0$  (undamped)

$$c(t) = 1 - \frac{1}{\beta} e^{-\zeta\omega_n t} \sin(\beta\omega_n t + \theta), \quad \text{where } \beta = \sqrt{1 - \zeta^2} \text{ and } \theta = \tan^{-1}(\beta / \zeta)$$

- Case 2:  $\zeta > 1$  (overdamped)

$$c(t) = 1 + k_1 e^{-t/\tau_1} + k_2 e^{-t/\tau_2}, \quad \text{where } \tau_{1,2} = 1 / (\zeta\omega_n \pm \omega_n \sqrt{\zeta^2 - 1})$$

- Case 3:  $\zeta = 1$  (critically damped)

$$c(t) = 1 + k_1 e^{-t/\tau} + k_2 t e^{-t/\tau}, \quad \text{where } \tau = 1 / \omega_n$$

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## Time responses of second-order systems

- Second-order systems

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

- Step response

Case 1:  $\zeta < 1$  (underdamped)

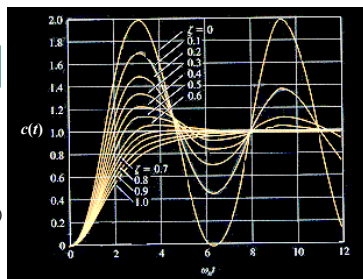
$$c(t) = 1 - \frac{1}{\beta} e^{-\zeta\omega_n t} \sin(\beta\omega_n t + \theta)$$

Case 2:  $\zeta > 1$  (overdamped)

$$c(t) = 1 + k_1 e^{-t/\tau_1} + k_2 e^{-t/\tau_2}$$

Case 3:  $\zeta = 1$  (critically damped)

$$c(t) = 1 + k_1 e^{-t/\tau} + k_2 t e^{-t/\tau}$$



## Time responses of second-order systems

- Second-order systems

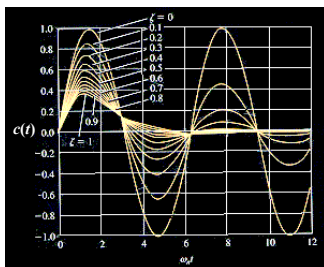
$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

- Step response

- Case 1
- Case 2
- Case 3

Initial condition  
and impulse response

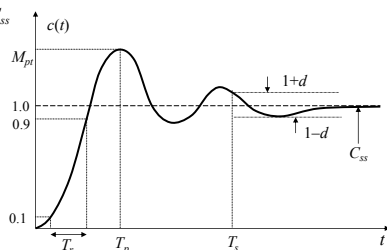
The initial condition excitation of higher-order systems **cannot** be modeled as simply as that of the first-order system; however, the impulse response of any system **does** give an indication of the nature of the initial-condition response, and thus the transient response



## Time response specifications in design

- Some parameters

- Rise time,  $T_r$
- Peak value of the step response,  $M_{pt}$ ; time to reach it,  $T_p$  (how to calculate  $T_r$ ?)
- Steady state value,  $C_{ss}$
- Percent overshoot,  $\frac{M_{pt} - C_{ss}}{C_{ss}} \times 100$
- Settling time,  $T_s$  (how to calculate  $T_r$ ?)





## Time response specifications in design

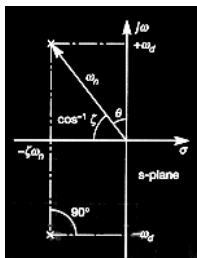
- Some parameters

### Time response and pole locations

- The settling time is inversely related to the real part of the poles (the speed of response is increased by moving the poles to the left in the  $s$ -plane)

$$T_s = k\tau = \frac{k}{\zeta\omega_n}$$

- Decreasing the angle  $\cos^{-1}\zeta$  (increasing  $\zeta$ ) reduces the percent overshoot



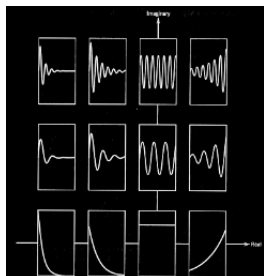
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## Time response specifications in design

- Some parameters

### Time response and pole locations

- The settling time is inversely related to the real part of the poles (the speed of response is increased by moving the poles to the left in the  $s$ -plane)
- Decreasing the angle  $\cos^{-1}\zeta$  (increasing  $\zeta$ ) reduces the percent overshoot



This picture shows how changing pole locations in the  $s$ -plane affects responses

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## Frequency response of systems

- Frequency response: steady-state response of systems to sinusoidal inputs

$$r(t) = A \cos \omega_1 t, \quad R(s) = \frac{As}{s^2 + \omega_1^2},$$

$$C(s) = G(s)R(s) = \frac{k_1}{s - j\omega_1} + \frac{k_2}{s + j\omega_1} + C_s(s)$$

$$\lim_{t \rightarrow \infty} c_s(t) = 0$$

$$k_1 = \frac{1}{2} AG(j\omega_1), \quad k_2 = \frac{1}{2} AG(-j\omega_1), \quad G(j\omega_1) = |G(j\omega_1)| e^{j\phi(\omega_1)}$$

$$c_{ss}(t) = k_1 e^{j\omega_1 t} + k_2 e^{-j\omega_1 t} = A |G(j\omega_1)| \frac{e^{j(\omega_1 t + \phi(\omega_1))} + e^{-j(\omega_1 t + \phi(\omega_1))}}{2}$$

$$= A |G(j\omega_1)| \cos(\omega_1 t + \phi(\omega_1))$$

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## Frequency response of systems

- Frequency response: steady-state response of systems to sinusoidal inputs

$$r(t) = A \cos \omega_1 t, \quad G(j\omega_1) = |G(j\omega_1)| e^{j\phi(\omega_1)}$$

$$c_{ss}(t) = A |G(j\omega_1)| \cos(\omega_1 t + \phi(\omega_1))$$

- The steady-state gain of a system for a sinusoidal input is the **magnitude** of the transfer function evaluation at  $s = j\omega_1$ , and the **phase shift** of the output sinusoid relative to the input sinusoid is the angle of  $G(j\omega_1)$

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## Frequency response of systems

- Frequency response: steady-state response of systems to sinusoidal inputs

- The steady-state gain of a system for a sinusoidal input is the **magnitude** of the transfer function evaluation at  $s = j\omega_1$ , and the **phase shift** of the output sinusoid relative to the input sinusoid is the angle of  $G(j\omega_1)$
- $G(j\omega)$  is defined as the **frequency response function**

$$G(j\omega) = |G(j\omega)| e^{j\phi(\omega)}$$

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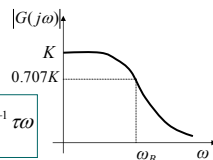
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## Frequency response of systems

- Frequency response: steady-state response of systems to sinusoidal inputs
- Frequency response of first-order systems

$$G(s) = \frac{K}{\tau s + 1}$$

$$|G(j\omega)| = \frac{K}{(1 + \tau^2 \omega^2)^{1/2}}, \quad \phi(\omega) = -\tan^{-1} \tau \omega$$



- System bandwidth**,  $\omega_B$ : The frequency at which the gain is equal to  $1/\sqrt{2}$  (approximately 0.707) times the gain at very low frequencies

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## Frequency response of systems

- Frequency response: steady-state response of systems to sinusoidal inputs
- Frequency response of first-order systems

### • Frequency response of second-order systems

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} = \frac{1}{(s/\omega_n)^2 + 2\zeta(s/\omega_n) + 1}$$

$$G(j\omega) = \frac{1}{[1 - (\omega/\omega_n)^2] + j2\zeta(\omega/\omega_n)}$$

$$|G(j\omega)| = \frac{1}{\left[1 - (\omega/\omega_n)^2\right]^2 + (2\zeta(\omega/\omega_n))^2}^{\frac{1}{2}}$$

**Question:** What will happen if  $\zeta = 0$  and  $\omega = \omega_n$ ?

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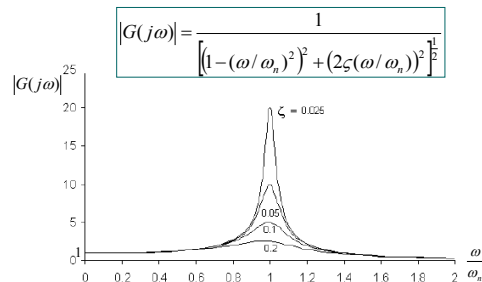
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## Frequency response of systems

- Frequency response: steady-state response of systems to sinusoidal inputs
- Frequency response of first-order systems

### • Frequency response of second-order systems



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## PID control

- Proportional control
  - In proportional control, steady-state error tends to depend inversely upon proportional gain
  - Proportional control has a tendency to make a system faster
  - Proportional control does not change the order of the system

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## PID control

- Proportional control
- Integral control
  - In integral control, steady-state error should be zero (**prerequisite**: the closed loop system has to be stable)
  - Integral control has a tendency to make a system slower and may even sacrifice stability
  - Integral control changes the order of the system

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## PID control

- Proportional control
- Integral control
- Derivative control
  - Derivative control tends to increase the stability of the system
  - Derivative control tends to reduce the overshoot and improve the transient response
  - Derivative control changes the order of the system

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## PID control

- Proportional control
- Integral control
- Derivative control

Closed-loop response	Rise time	Overshoot	Settling time	Steady-state error
$K_P$	Decrease	Increase	Small change	Decrease
$K_I$	Decrease	Increase	Increase	Eliminate
$K_D$	Small change	Decrease	Decrease	Small change

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## PID control

- Proportional control
- Integral control
- Derivative control

- Another view on PID control
  - The proportional term gives the controller output a component that is a function of the present state of the system
  - The integrator output is determined by the past state of the system
  - The differentiator is a function of the slope of its input and thus can be considered to be a predictor of the future state of the system
  - The PID controller can viewed as giving control that is a function of the past, the present, and the predicted future

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