Supplemental Note for ECE 1675/2570

Classical PID Control

Based on

Goodwin, Graebe, Salgado, Prentice Hall 2000

This note examines a particular control structure that has become almost universally used in industrial control. It is based on a particular fixed structure controller family, the so-called PID controller family. These controllers have proven to be robust and extremely beneficial in the control of many important applications.

PID stands for:

P (*Proportional*)

I (*Integral*)

D (*Derivative*)

Historical Note

Early feedback control devices implicitly or explicitly used the ideas of proportional, integral, and derivative action in their structures. However, it was probably not until Minorsky's work on ship steering* published in 1922, that rigorous theoretical consideration was given to PID control.

This was the first mathematical treatment of the type of controller that is now used to control almost all industrial processes.

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* Minorsky (1922) "Directional stability of automatically steered bodies," *J. Am. Soc. Naval Eng.*, 34, p.284.

The Current Situation

Despite the abundance of sophisticated tools, including advanced controllers, the Proportional, Integral, Derivative (PID controller) is still the most widely used in modern industry, controlling more that 95% of closed-loop industrial processes*

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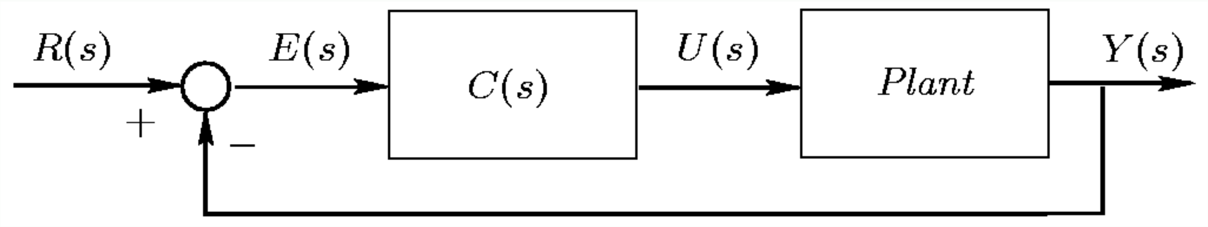
* Åström K.J. & Hägglund T.H. 1995, "New tuning methods for PID controllers," *Proc. 3rd European Control Conference*, p.2456-62; and

Yamamoto & Hashimoto 1991, "Present status and future needs: The view from Japanese industry," Chemical Process Control, *CPCIV, Proc. 4th International Conference on Chemical Process Control*, Texas, p.1-28.

PID Structure

Consider the simple SISO control loop shown in the following figure:



The *standard form* PID are:

Proportional only: $C_P(s) = K_p$

Proportional plus Integral: $C_{PI}(s) = K_p \left(1 + \frac{1}{T_r s}\right)$

Proportional plus derivative: $C_{PD}(s) = K_p \left(1 + \frac{T_d s}{\tau_D s + 1}\right)$

Proportional, integral and derivative: $C_{PID}(s) = K_p \left(1 + \frac{1}{T_r s} + \frac{T_d s}{\tau_D s + 1}\right)$

Tuning of PID Controllers

Because of their widespread use in practice, we present below several methods for tuning PID controllers. Actually these methods are quite old and date back to the 1950's. Nonetheless, they remain in widespread use today.

In particular, we will study

- *Ziegler-Nichols Oscillation Method*
- *Ziegler-Nichols Reaction Curve Method*
- *Cohen-Coon Reaction Curve Method*

(1) Ziegler-Nichols (Z-N) Oscillation Method

This procedure is only valid for open loop stable plants and it is carried out through the following steps

- Set the true plant under proportional control, with a very small gain.
- Increase the gain until the loop starts oscillating. Note that linear oscillation is required and that it should be detected at the controller output.

- Record the controller critical gain $K_p = K_c$ and the oscillation period of the controller output, P_c.
- Adjust the controller parameters according to *next slide*; there is some controversy regarding the PID parameterization for which the Z-N method was developed, but the version described here is applicable to the parameterization of standard form PID.

Ziegler-Nichols tuning using the oscillation method

	$\mathbf{K_p}$	$\mathbf{T_r}$	$\mathbf{T_d}$
P	$0.50K_c$		
PI	$0.45K_c$	$\frac{P_c}{1.2}$	
PID	$0.60K_c$	$0.5P_c$	$\frac{P_c}{8}$

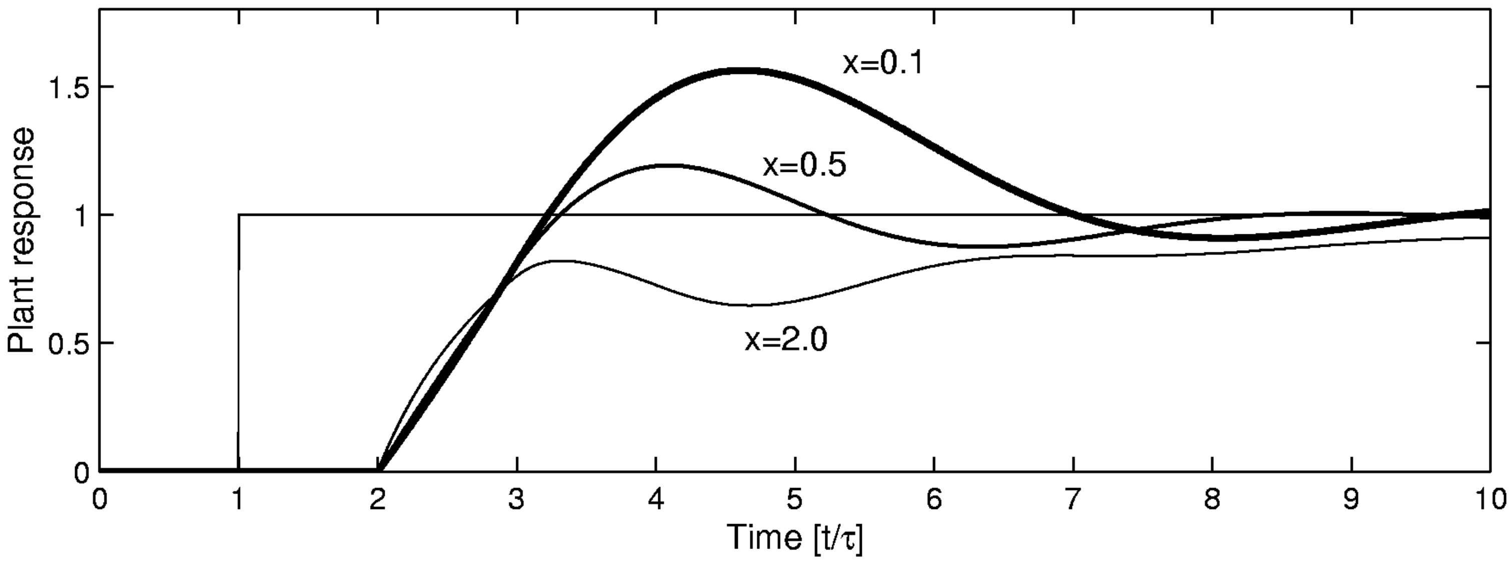
General System

If we consider a general plant of the form:

$$G_0(s) = \frac{K_0 e^{-s\tau}}{v_0 s + 1}; \quad v_0 > 0$$

then one can obtain the PID settings via Ziegler-Nichols tuning for different values of τ and v_0. The next plot shows the resultant closed loop step responses as a function of the ratio $x \stackrel{\Delta}{=} \frac{\tau}{v_0}$.

PI Z-N tuned (oscillation method) control loop for different values of the ratio $x \overset{\Delta}{=} \frac{\tau}{v_0}$



Numerical Example

Consider a plant with a model given by

$$G_o(s) = \frac{1}{(s+1)^3}$$

Find the parameters of a PID controller using the Z-N oscillation method. Obtain a graph of the response to a unit step input reference and to a unit step input disturbance.

Solution

Applying the procedure we find:

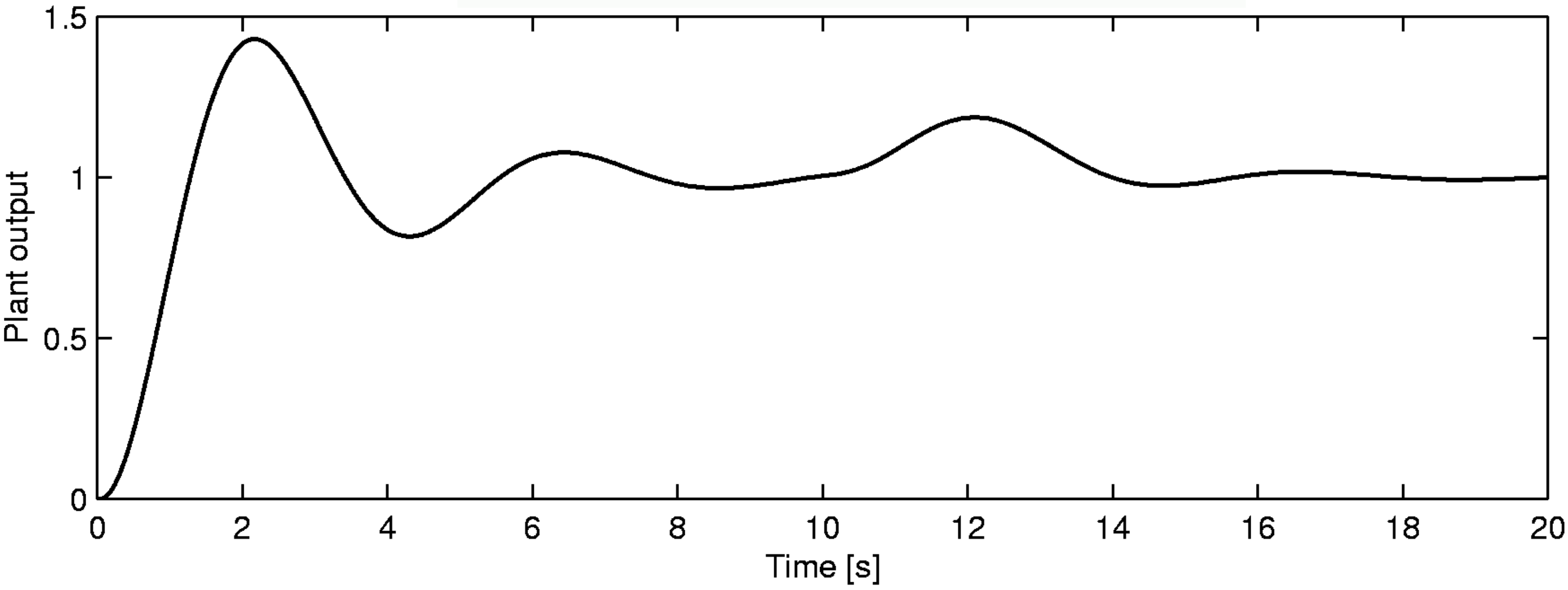
$$K_c = 8 \ \text{ and } \ \omega_c = \sqrt{3}.$$

Hence, from the table in Page 10, we have

$$K_p = 0.6 * K_c = 4.8; \qquad T_r = 0.5 * P_c \approx 1.81; \qquad T_d = 0.125 * P_c \approx 0.45$$

The closed loop response to a unit step in the reference at $t = 0$ and a unit step disturbance at $t = 10$ are shown in the next figure.

Response to step reference and step input disturbance



(2) Reaction Curve Based Methods

A linearized quantitative version of a simple plant can be obtained with an open loop experiment, using the following procedure:

1. With the plant in open loop, take the plant manually to a normal operating point. Say that the plant output settles at $y(t) = y_0$ for a constant plant input $u(t) = u_0$.
2. At an initial time, t_0, apply a step change to the plant input, from u_0 to u_∞ (*this should be in the range of* 10 *to* 20% *of full scale*).

3. Record the plant output until it settles to the new operating point. Assume you obtain the curve shown on the next slide. This curve is known as the *process reaction curve*.

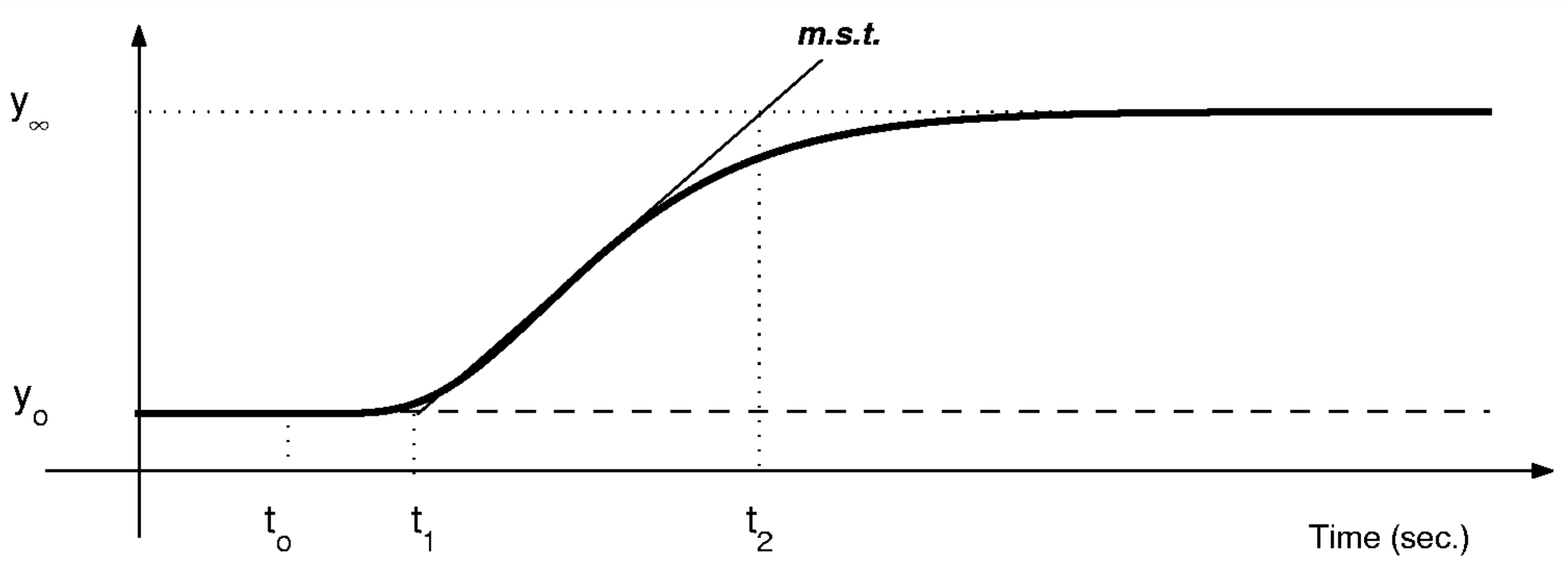
   In the figure on next page, m.s.t. stands for *maximum slope tangent*.

4. Compute the parameter model as follows

$$K_o = \frac{y_\infty - y_o}{u_\infty - u_o}; \qquad \tau_o = t_1 - t_o; \qquad \nu_o = t_2 - t_1$$

Plant step response

The suggested parameters are shown in next slide.



Ziegler-Nichols tuning using the reaction curve

	K_p	T_r	T_d
P	$\frac{\nu_o}{K_o\tau_o}$		
PI	$\frac{0.9\nu_o}{K_o\tau_o}$	$3\tau_o$	
PID	$\frac{1.2\nu_o}{K_o\tau_o}$	$2\tau_o$	$0.5\tau_o$

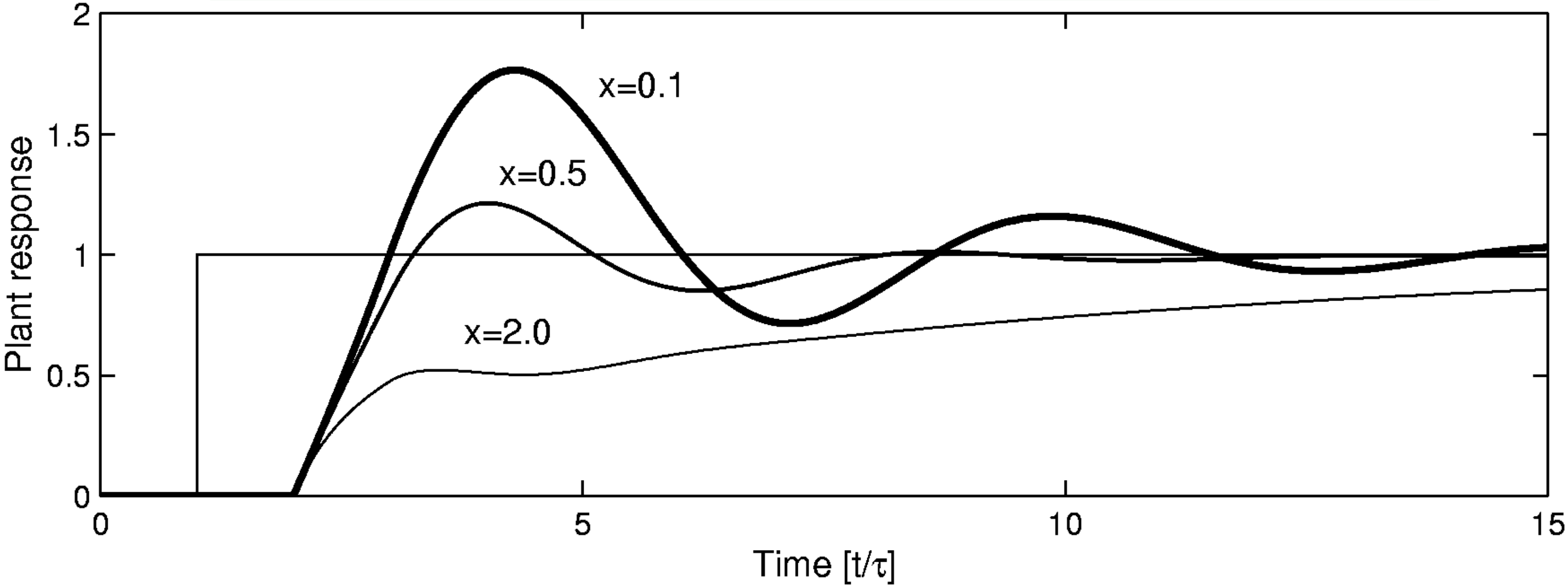
General System Revisited

Consider again the general plant:

$$G_0(s) = \frac{K_0 e^{-s\tau}}{v_0 s + 1}$$

The next slide shows the closed loop responses resulting from Ziegler-Nichols Reaction Curve tuning for different values of $x \stackrel{\Delta}{=} \frac{\tau}{v_0}$.

PI Z-N tuned (reaction curve method) control loop



Observation

We see from the previous slide that the Ziegler-Nichols reaction curve tuning method is very sensitive to the ratio of delay to time constant.

(3) Cohen-Coon Reaction Curve Method

Cohen and Coon carried out further studies to find controller settings which, based on the same model, lead to a weaker dependence on the ratio of delay to time constant.

Cohen-Coon tuning using the reaction curve

	$\mathbf{K_p}$	$\mathbf{T_r}$	$\mathbf{T_d}$
P	$\dfrac{\nu_o}{K_o\tau_o}\left[1+\dfrac{\tau_o}{3\nu_o}\right]$		
PI	$\dfrac{\nu_o}{K_o\tau_o}\left[0.9+\dfrac{\tau_o}{12\nu_o}\right]$	$\dfrac{\tau_o[30\nu_o+3\tau_o]}{9\nu_o+20\tau_o}$	
PID	$\dfrac{\nu_o}{K_o\tau_o}\left[\dfrac{4}{3}+\dfrac{\tau_o}{4\nu_o}\right]$	$\dfrac{\tau_o[32\nu_o+6\tau_o]}{13\nu_o+8\tau_o}$	$\dfrac{4\tau_o\nu_o}{11\nu_o+2\tau_o}$

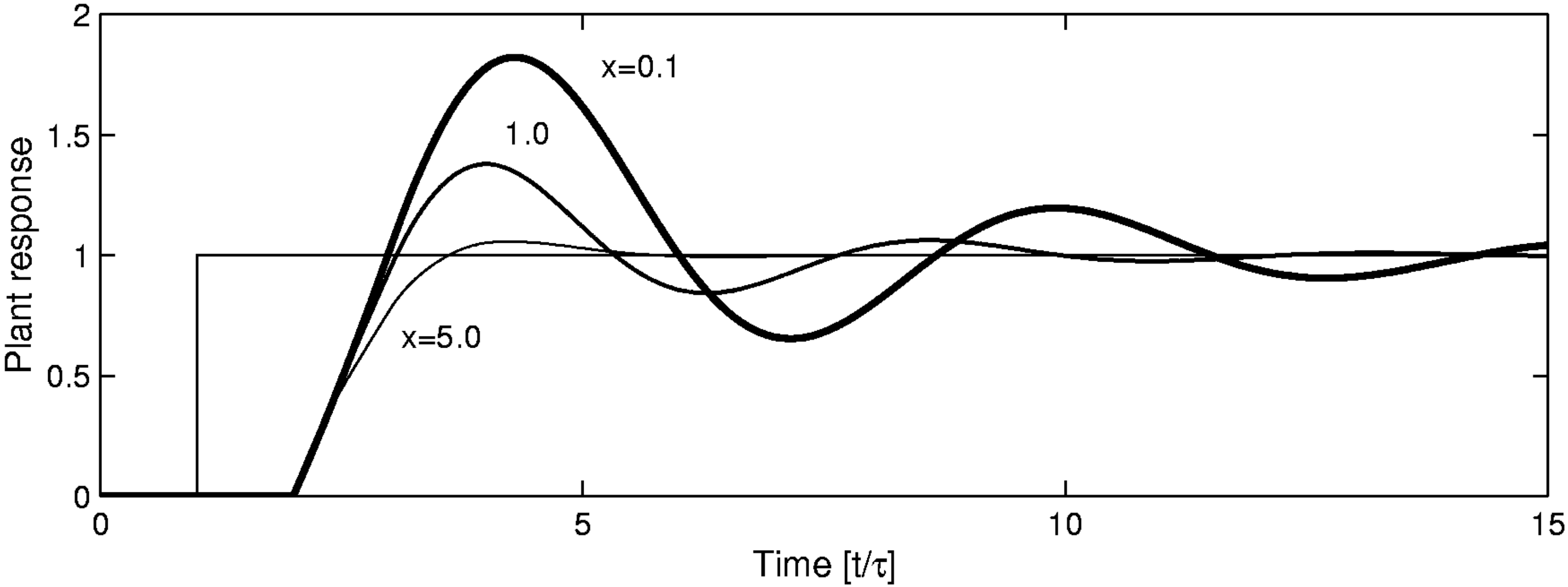
General System Revisited

Consider again the general plant:

$$G_0(s) = \frac{K_0 e^{-s\tau}}{\nu_0 s + 1}$$

The next slide shows the closed loop responses resulting from Cohen-Coon Reaction Curve tuning for different values of $x = \frac{\tau}{\nu_0}$.

PI Cohen-Coon tuned (reaction curve method) control loop



Lead-lag Compensators

Closely related to PID control is the idea of lead-lag compensation. The transfer function of these compensators is of the form:

$$C(s) = \frac{\tau_1 s + 1}{\tau_2 s + 1}$$

If $\tau_1 > \tau_2$, then this is a *lead network* and when $\tau_1 < \tau_2$, this is a *lag network*.

Summary

- PI and PID controllers are widely used in industrial control.
- From a modern perspective, a PID controller is simply a controller of (up to second order) containing an integrator. Historically, however, PID controllers were tuned in terms of their **P**, **I**, and **D** terms.
- It has been empirically found that the PID structure often has sufficient flexibility to yield excellent results in many applications.