

16 – CLASSIFICATION

CS 1656

Introduction to Data Science

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MAKING PREDICTIONS

Making predictions

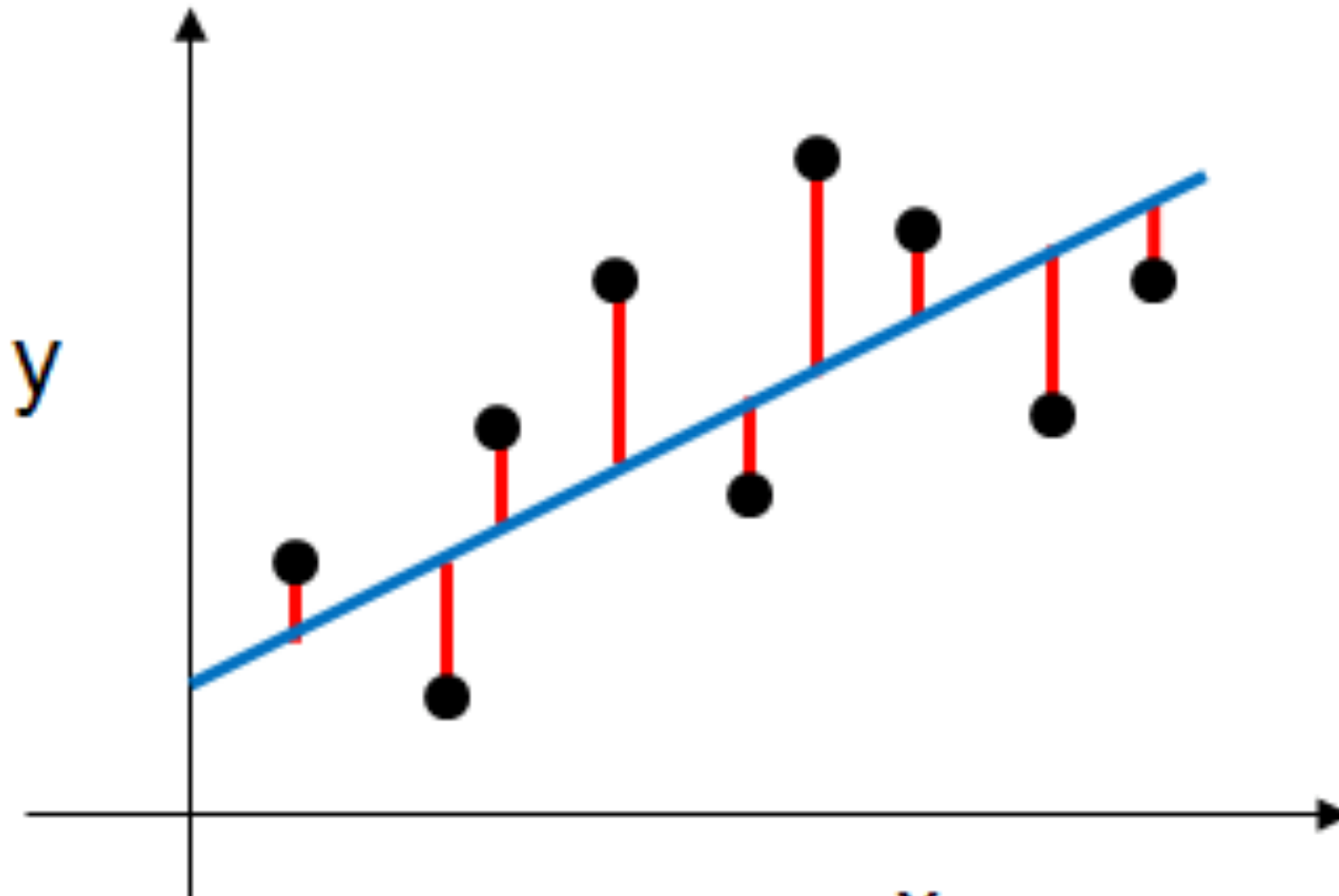
- Also referred to as **Learning**
- **Unsupervised Learning:**
 - Clustering
- **Supervised Learning**
 - Classification/
 - Difference is the existence of ``**training data**”
 - Training data is accompanied by **labels**
 - New data will be classified (i.e., assigned labels) based on training data

Supervised Learning

- Classification
 - Predict labels for categorical data
 - Classify data (=construct a model) based on training data
- Numerical Prediction
 - Predict unknown or missing values
- **MANY APPLICATIONS**
 - Textbook example: **predict credit-worthiness**
 - Medical diagnosis (In the news: Personalized Medicine Initiative)
 - Fraud detection (not frog protection)
 - ...

LINEAR REGRESSION

Trying to fit a line through data points

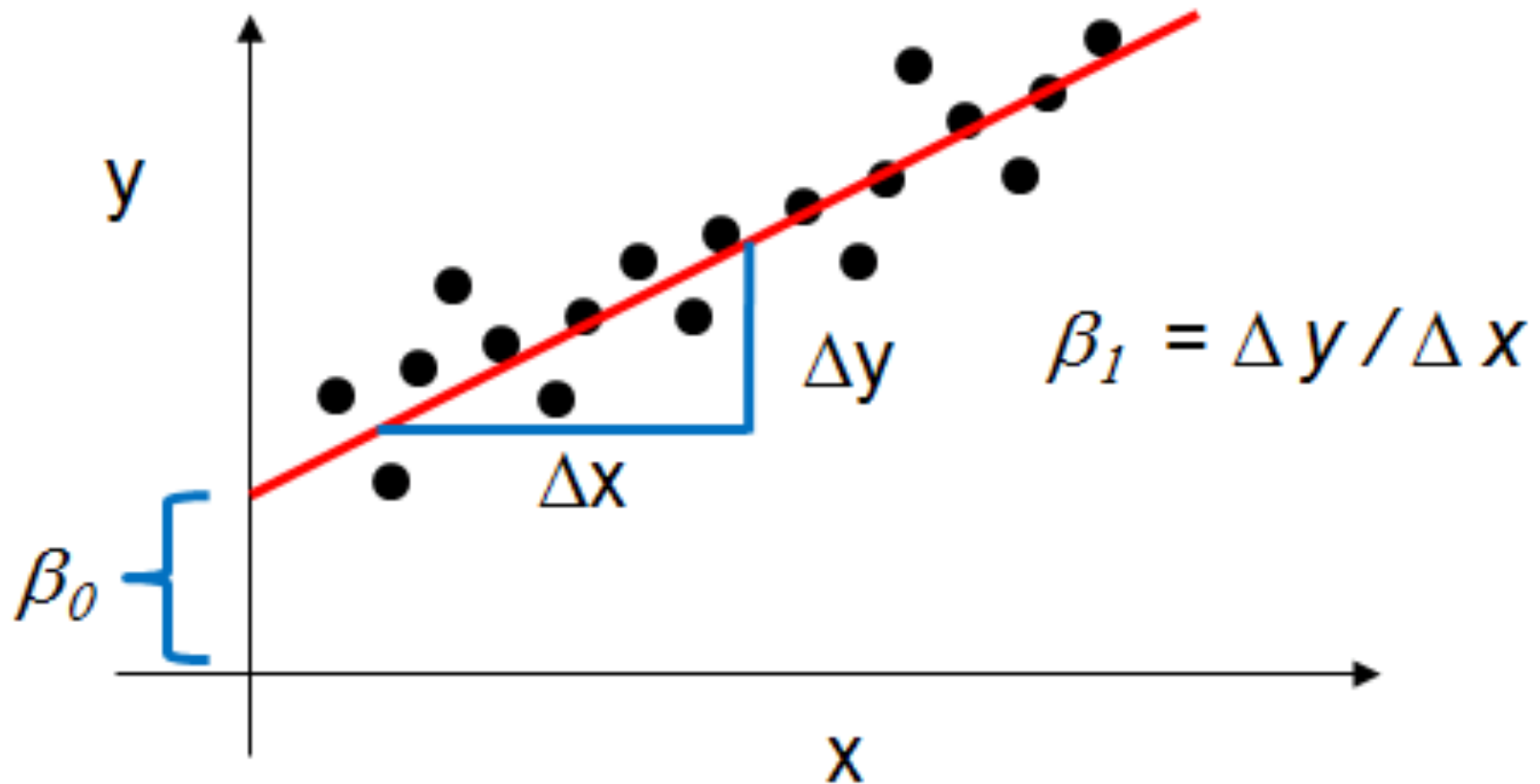


Source: <http://www.dataschool.io/linear-regression-in-python/>

What would a line look like?

- If we have y and x , then:
- Simple: $y = b * x$
- Simple+: $y = b_0 + b_1 * x$
- Simple++: $y = b_0 + b_1 * x + e$
 - Where e captures the error
- Goal: Minimize error

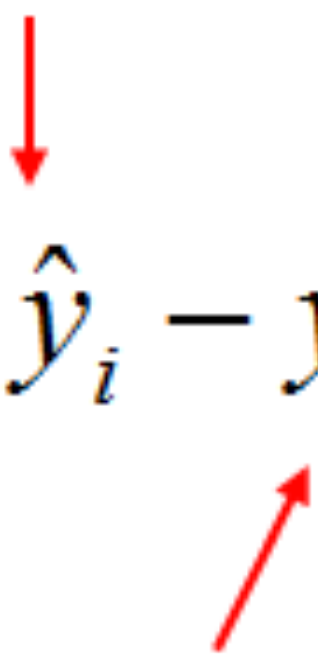
What that means



Source: <http://www.dataschool.io/linear-regression-in-python/>

How to compute errors

Model Prediction


$$SS_{residuals} = \sum_{i=1}^N (\hat{y}_i - y_i)^2$$

Observed Result

Source: <http://www.dataschool.io/linear-regression-in-python/>

How to minimize error

$$\hat{\beta}_1 = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^n (x_i - \bar{x})^2},$$
$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x},$$

Diagram illustrating the formulas for the least squares regression coefficients $\hat{\beta}_1$ and $\hat{\beta}_0$. The first formula shows the slope $\hat{\beta}_1$ as the ratio of the covariance of x and y to the variance of x . Red arrows point from the labels "avg(x)" and "avg(y)" to the terms \bar{x} and \bar{y} respectively in the formulas.

Source: An Introduction to Statistical Learning with Applications in R

MAKING PREDICTIONS

Understanding Question / Q1

- **Question 1:**
- Given the table in the handout, which attribute can be used to accurately predict the LOAN_OK attribute?
- **Possible Answers:**
 - Age
 - Credit_Rating
 - Sex
 - None of the above

Understanding Question / Q2

- **Question 2:**
- Given the updated table in the handout, can only one attribute still be used to accurately predict the LOAN_OK attribute?
- **Possible Answers:**
 - Yes
 - No

Understanding Question / Q3

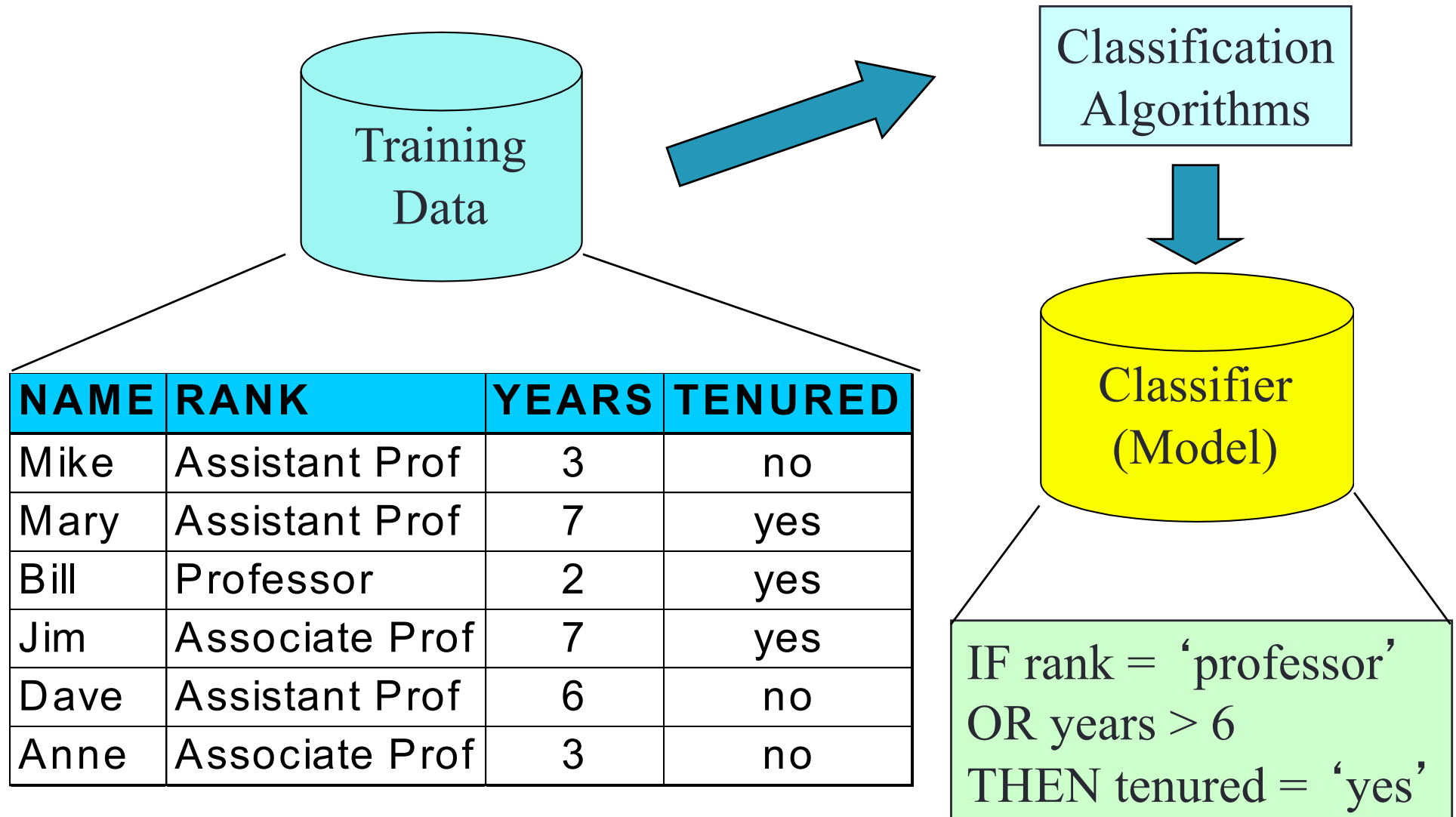
- **Question 3:**
- If you answered no to the previous question, given the updated table in the handout, which additional attribute needs to be used to accurately predict the LOAN_OK attribute?
- **Possible Answers:**
 - Age
 - Credit_Rating
 - Sex

CLASSIFICATION

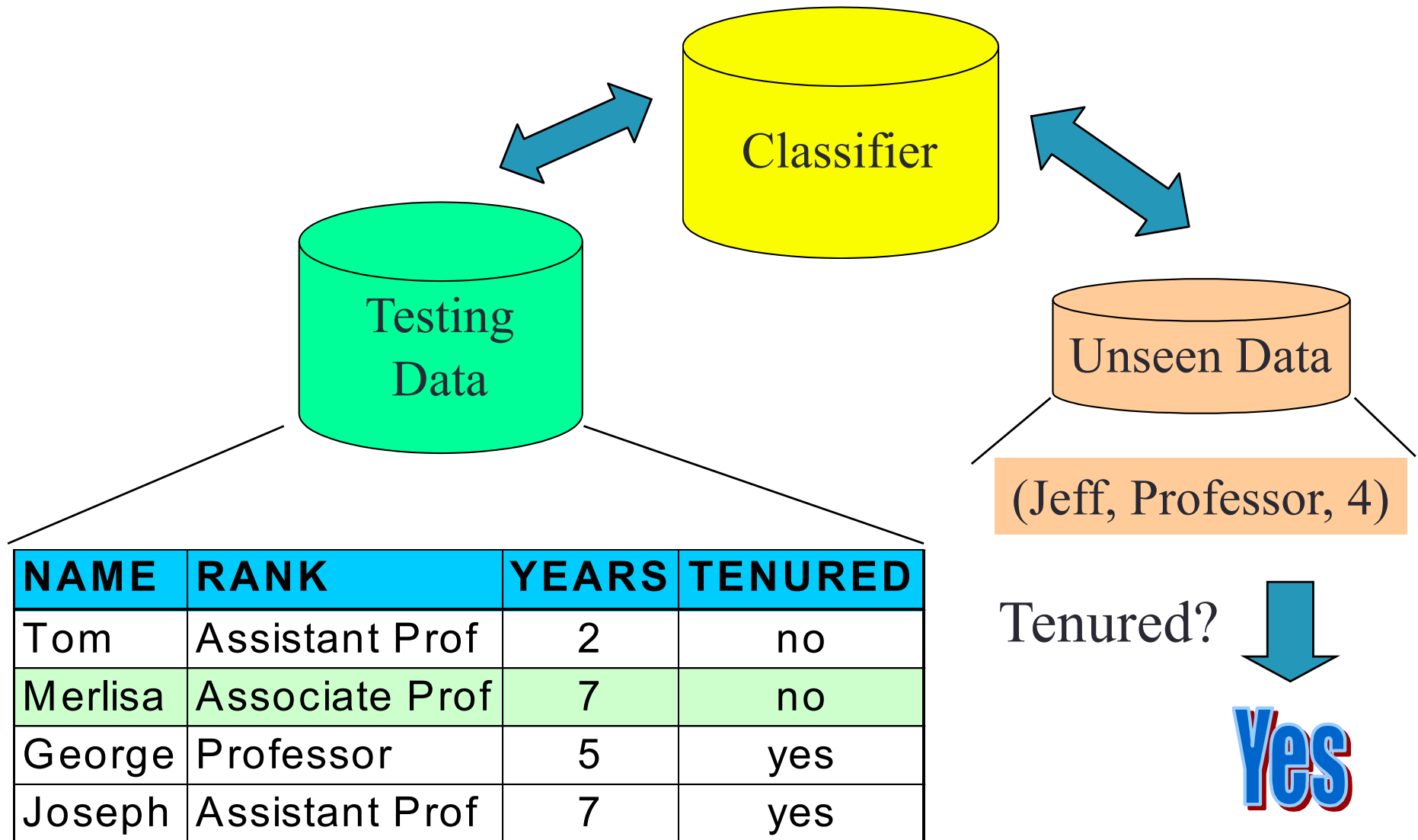
Classification—A Two-Step Process

- **Model construction:** describing a set of predetermined classes
 - Each tuple/sample is assumed to belong to a predefined class, as determined by the **class label attribute**
 - The set of tuples used for model construction is **training set**
 - The model is represented as classification rules, decision trees, or mathematical formulae
- **Model usage:** for classifying future or unknown objects
 - **Estimate accuracy** of the model
 - The known label of test sample is compared with the classified result from the model
 - **Accuracy** rate is the percentage of test set samples that are correctly classified by the model
 - **Test set** is independent of training set (otherwise overfitting)
 - If the accuracy is acceptable, use the model to **classify new data**
- Note: If *the test set* is used to select models, it is called **validation (test) set**

Process (1): Model Construction



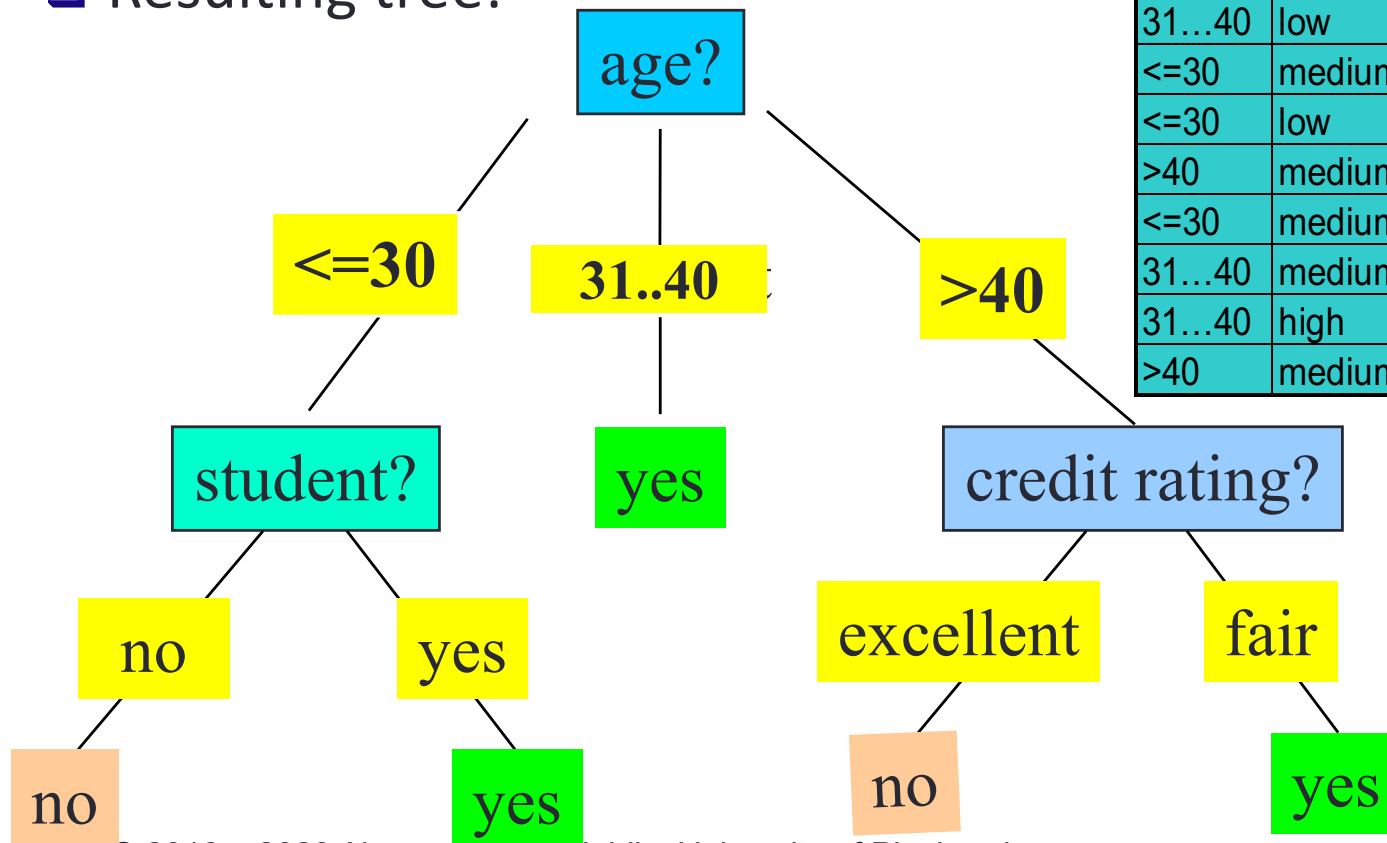
Process (2): Using the Model in Prediction



Decision Tree Induction: An Example

- Training data set: Buys_computer
- The data set follows an example of Quinlan's ID3 (Playing Tennis)
- Resulting tree:

age	income	student	credit_rating	buys_computer
<=30	high	no	fair	no
<=30	high	no	excellent	no
31...40	high	no	fair	yes
>40	medium	no	fair	yes
>40	low	yes	fair	yes
>40	low	yes	excellent	no
31...40	low	yes	excellent	yes
<=30	medium	no	fair	no
<=30	low	yes	fair	yes
>40	medium	yes	fair	yes
<=30	medium	yes	excellent	yes
31...40	medium	no	excellent	yes
31...40	high	yes	fair	yes
>40	medium	no	excellent	no



Algorithm for Decision Tree Induction

- **Basic algorithm (a greedy algorithm)**
 - Tree is constructed in a **top-down recursive divide-and-conquer manner**
 - At start, all the training examples are at the root
 - Attributes are categorical (if continuous-valued, they are discretized in advance)
 - Examples are partitioned recursively based on selected attributes
 - Test attributes are selected on the basis of a heuristic or statistical measure (e.g., **information gain**)
- **Conditions for stopping partitioning:**
 - All samples for a given node belong to the same class
 - There are no remaining attributes for further partitioning – **majority voting** is employed for classifying the leaf
 - There are no samples left

Attribute Selection Measure: Information Gain (ID3/C4.5)

- Select the attribute with the highest information gain
- Let p_i be the probability that an arbitrary tuple in D belongs to class C_i , estimated by $|C_{i,D}|/|D|$
- **Expected information** (entropy) needed to classify a tuple in D :

$$Info(D) = -\sum_{i=1}^m p_i \log_2(p_i)$$

- **Information** needed (after using A to split D into v partitions) to classify D :

$$Info_A(D) = \sum_{j=1}^v \frac{|D_j|}{|D|} \times Info(D_j)$$

- **Information gained** by branching on attribute A

$$Gain(A) = Info(D) - Info_A(D)$$

Attribute Selection: Information Gain

■ Class P: buys_computer = “yes”

■ Class N: buys_computer = “no”

$$Info(D) = I(9,5) = -\frac{9}{14} \log_2\left(\frac{9}{14}\right) - \frac{5}{14} \log_2\left(\frac{5}{14}\right) = 0.940$$

age	p_i	n_i	$I(p_i, n_i)$
≤ 30	2	3	0.971
31...40	4	0	0
> 40	3	2	0.971

age	income	student	credit_rating	buys_computer
≤ 30	high	no	fair	no
≤ 30	high	no	excellent	no
31...40	high	no	fair	yes
> 40	medium	no	fair	yes
> 40	low	yes	fair	yes
> 40	low	yes	excellent	no
31...40	low	yes	excellent	yes
≤ 30	medium	no	fair	no
≤ 30	low	yes	fair	yes
> 40	medium	yes	fair	yes
≤ 30	medium	yes	excellent	yes
31...40	medium	no	excellent	yes
31...40	high	yes	fair	yes
> 40	medium	no	excellent	no

$$Info_{age}(D) = \frac{5}{14} I(2,3) + \frac{4}{14} I(4,0) + \frac{5}{14} I(3,2) = 0.694$$

$\frac{5}{14} I(2,3)$ means “age ≤ 30 ” has 5 out of 14 samples, with 2 yes’ es and 3 no’ s. Hence

$$Gain(age) = Info(D) - Info_{age}(D) = 0.246$$

Similarly,

$$Gain(income) = 0.029$$

$$Gain(student) = 0.151$$

$$Gain(credit_rating) = 0.048$$

Gain Ratio for Attribute Selection (C4.5)

- Information gain measure is biased towards attributes with a large number of values
- C4.5 (a successor of ID3) uses gain ratio to overcome the problem (normalization to information gain)

$$SplitInfo_A(D) = - \sum_{j=1}^v \frac{|D_j|}{|D|} \times \log_2 \left(\frac{|D_j|}{|D|} \right)$$

- $GainRatio(A) = Gain(A)/SplitInfo(A)$
- Ex. $SplitInfo_{income}(D) = -\frac{4}{14} \times \log_2 \left(\frac{4}{14} \right) - \frac{6}{14} \times \log_2 \left(\frac{6}{14} \right) - \frac{4}{14} \times \log_2 \left(\frac{4}{14} \right) = 1.557$
 - $gain_ratio(income) = 0.029/1.557 = 0.019$
- The attribute with the maximum gain ratio is selected as the splitting attribute

EVALUATION METRICS

Classifier Evaluation Metrics: Confusion Matrix

Confusion Matrix:

Actual class\Predicted class	C_1	$\neg C_1$
C_1	True Positives (TP)	False Negatives (FN)
$\neg C_1$	False Positives (FP)	True Negatives (TN)

Example of Confusion Matrix:

Actual class\Predicted class	buy_computer = yes	buy_computer = no	Total
buy_computer = yes	6954	46	7000
buy_computer = no	412	2588	3000
Total	7366	2634	10000

- Given m classes, an entry, $\mathbf{CM}_{i,j}$ in a **confusion matrix** indicates # of tuples in class i that were labeled by the classifier as class j
- May have extra rows/columns to provide totals

Classifier Evaluation Metrics:

Accuracy, Error Rate, Sensitivity and Specificity

A\P	C	¬C	
C	TP	FN	P
¬C	FP	TN	N
	P'	N'	All

- **Classifier Accuracy**, or recognition rate: percentage of test set tuples that are correctly classified

$$\text{Accuracy} = (TP + TN)/All$$

- **Error rate**: $1 - \text{accuracy}$, or
$$\text{Error rate} = (FP + FN)/All$$

- **Class Imbalance Problem:**

- One class may be *rare*, e.g. fraud, or HIV-positive
- Significant *majority of the negative class* and minority of the positive class
- **Sensitivity**: True Positive recognition rate
 - **Sensitivity** = TP/P
- **Specificity**: True Negative recognition rate
 - **Specificity** = TN/N

Classifier Evaluation Metrics: Precision and Recall

- **Precision:** exactness – what % of tuples that the classifier labeled as positive are actually positive

$$precision = \frac{TP}{TP + FP}$$

- **Recall:** completeness – what % of positive tuples did the classifier label as positive?

$$recall = \frac{TP}{TP + FN}$$

- Perfect score is 1.0
- Inverse relationship between precision & recall

Classifier Evaluation Metrics: Example

Actual Class\Predicted class	cancer = yes	cancer = no	Total	Recognition(%)
cancer = yes	90	210	300	30.00 (<i>sensitivity</i>)
cancer = no	140	9560	9700	98.56 (<i>specificity</i>)
Total	230	9770	10000	96.40 (<i>accuracy</i>)

- $Precision = 90/230 = 39.13\%$ $Recall = 90/300 = 30.00\%$

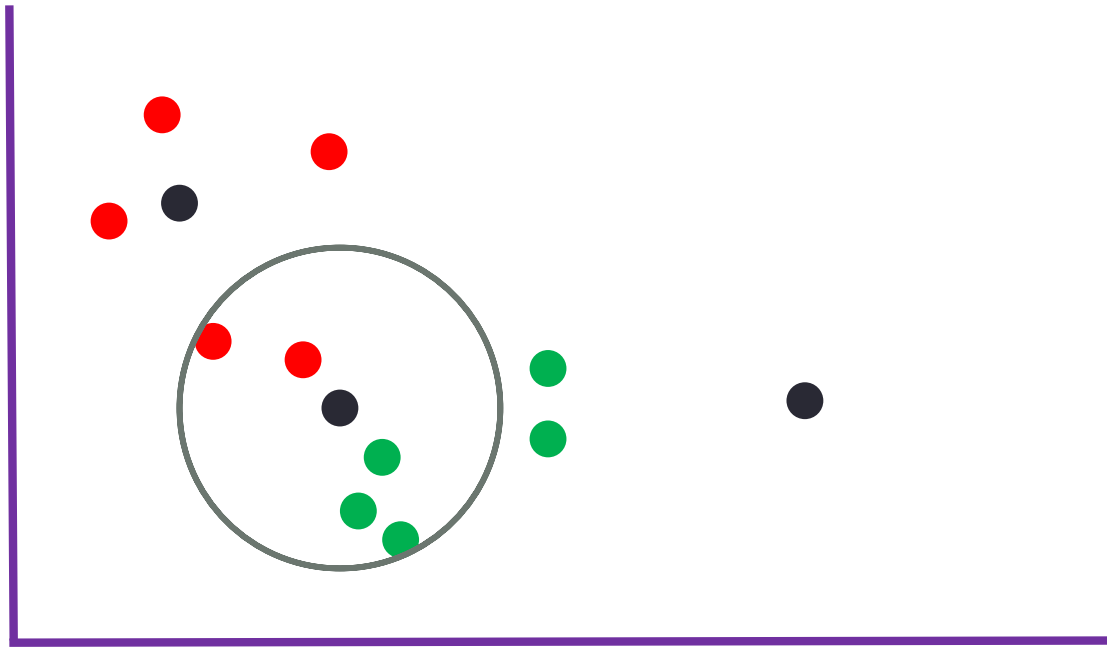
KNN

K Nearest Neighbors

kNN algorithm

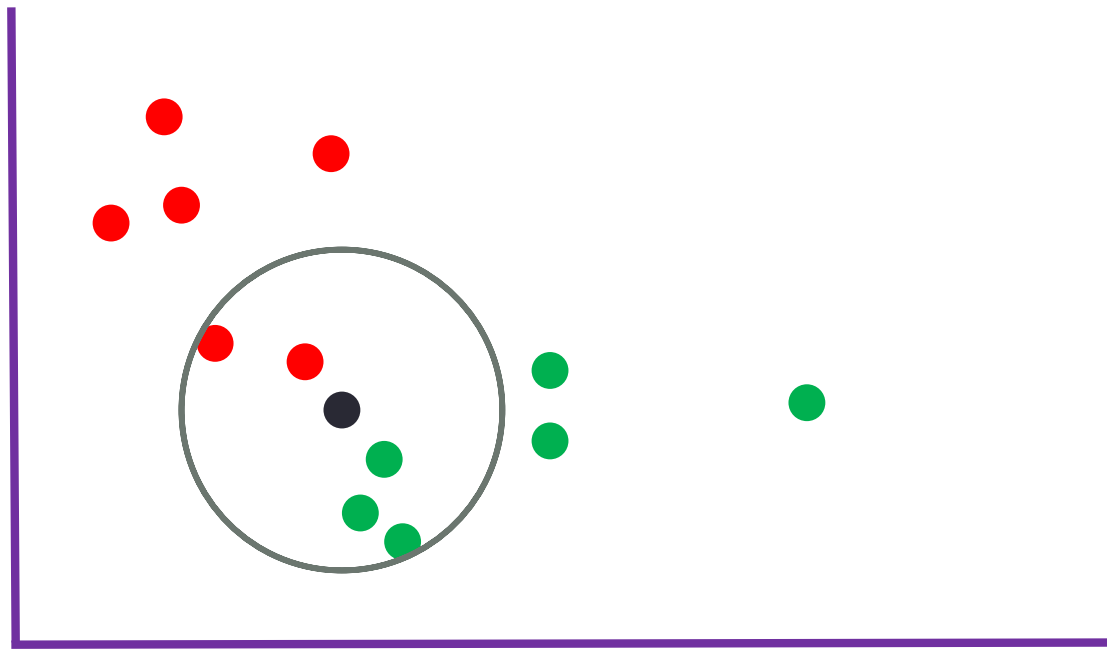
- Very simple supervised learning algorithm
- **Requirements:**
 - Value of k
 - Pre-labeled points (label = class membership)
 - Distance function
- **Algorithm:**
 - For every new point, identify k closest neighbors
 - Decide label for new point based on majority of labels from neighbors
 - K must not be multiple of the number of classes

kNN example



- Assume two classes: red and green

kNN example (2)



- Classifying last item:
 - $k=1 \Rightarrow$ red, $k=2 \Rightarrow$ tie, $k=3 \Rightarrow$ green, $k=4 \Rightarrow$ tie, $k=5 \Rightarrow$ green