

**Homework 2, ECE 1675/2570 Robotic Control, Spring 2022**  
**(Due Wednesday Feb. 9)**

**Problem 1.** Please read Chapters 10-18 of the textbook by Mataric (The Robotics Primer) and submit a table summarizing the approximate amount of time that you spent in reading each chapter.

**Problem 2.** Please derive the nonlinear state-space equation for the Harrier “jump jet” system described on Slide 32 of Lecture 2. Consider the following definitions of input, state, and output variables:

$$\begin{aligned} u_1 &= F_1, u_2 = F_2; \\ x_1 &= 2x, x_2 = \dot{x}, x_3 = 3y, x_4 = \dot{y}, x_5 = 4\theta, x_6 = \dot{\theta}; \\ y_1 &= x, y_2 = y, y_3 = \theta. \end{aligned}$$

**Problem 3.** Consider the motion dynamics of a bicycle (see Fig. 1), which can be described by

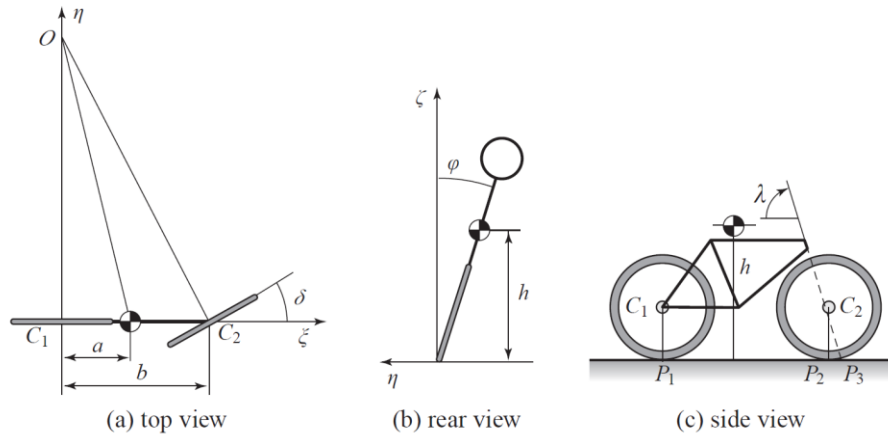
$$J \frac{d^2 \varphi}{dt^2} - \frac{D v_0}{b} \frac{d\delta}{dt} = mgh \sin \varphi + \frac{m v_0^2 h}{b} \delta$$

where  $\delta$  is the steering angle,  $\varphi$  the tilt angle,  $J$  the moment of inertia of the bicycle body with respect to the  $\zeta$ -axis,  $v_0$  the velocity of the bicycle at the rear wheel,  $m$  the total mass of the system,  $g$  the gravitational acceleration,  $D = mah$ , and  $a$ ,  $b$ , and  $h$  are explained in Fig. 1.

(a) Please derive the nonlinear state-space equation for the bicycle motion with the following definitions of input, state, and output variables:

$$u = \delta; \quad x_1 = \varphi, \quad x_2 = \dot{\varphi} - \frac{D v_0}{b J} \delta; \quad y = \varphi.$$

(b) Please linearize the equation obtained in (a) for small  $\varphi$ .



**Figure 1.** Schematic views of a bicycle.