UNIVERSITY OF PITTSBURGH

Department of Electrical and Computer Engineering

ECE 2521 Analysis of Stochastic Processes (Fall 2014)

Practice Midterm

Problem 1 : A digital data transmission system selects one of the digits 1, 2, or 3 to transmit through a channel with prior probabilities 0.2, 0.4, 0.4, respectively. The conditional probabilities of receiving the digits 1, 2, or 3 given that a 1, 2, or 3 is transmitted (sent) are given in the following table.

Prob(k received | j sent)

		Received, k		
		1	2	3
Sent, j	1	0.95	0.02	0.03
	2	0.015	0.98	0.005
	3	0.02	0.05	0.93

An error is defined as the reception of any digit other than the one sent. Determine the following.

(a) Sketch a sequential tree diagram for the problem indicating the probabilities along each branch.

(b) What is the probability of receiving digit 2?

(c) Given that digit 2 was received, what is the probability that digit 2 was transmitted?

(d) Given that digit 2 was received, what is the probability that digit 1 was transmitted?

(e) How reliable is the channel. Calculate the overall probability of receiving digits correctly.

Problem 2 : A radioactive source emits an average of 0.5 beta particles per second. The medium between the source and collector absorbs a beta particle with probability of 0.2. Beta particles that are not absorbed reach the collector and are counted. The emission of beta particles from the source and absorption of beta particles by the medium are independent of each other. Let q be the number of beta particles that reach the collector in a $T = 5$ s measurement time interval.

(a) What is the probability that exactly 2 beta particles are emitted by the source in the measurement time?

(b) What is the probability that 2 or more beta particles are emitted by the source in the measurement time?

(c) What is the mean and standard deviation of the number of beta particles emitted in the measurement time?

(d) Given that exactly 6 particles are emitted, what is the conditional probability that exactly 2 particles reach the collector?

(e) Now we generalize the question asked in part (d). Given that exactly n particles were emitted, what is the conditional probability that exactly q particles reach the collector, where $0 \le q \le n$.

(f) Use the total probability theorem to determine the total probability that exactly q particles reach the collector. Remember that in order for this to happen, at least q particles must have been emitted. You should be able to sum the resulting series.

Problem 3: A toddler pulls three volumes of an encyclopedia from a bookshelf, and after being scolded, places them back in random order.

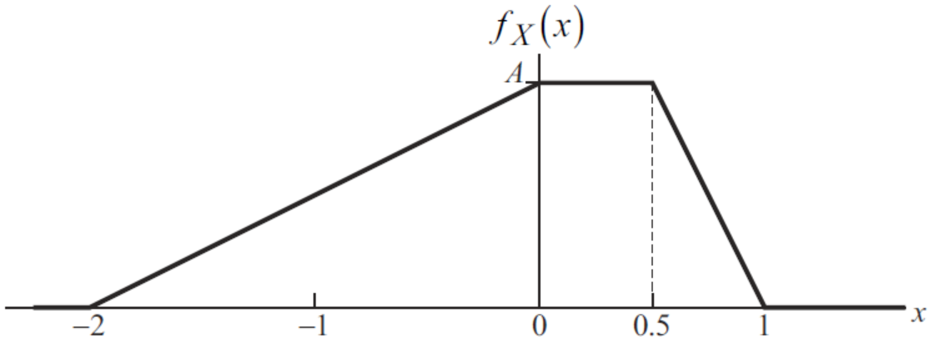
(a) What is the probability that the books are in the correct order?

(b) Two of the volumes are bound in hardcover and the remaining one has softcover. If the toddler randomly pulls two volumes from the shelf, what is the probability that both have hardcover?

(c) Suppose the toddler continues to play with the books and damages M of the books where M is random and ranges from 0 to 3 with each number being equally likely. It costs \$20 to replace a hardcover volume and \$10 to replace the softcover volume. Find the probability mass function for the cost of replacing the damaged books.

(d) Find the mean and standard deviation of the cost.

Problem 4: Random variable X is described by the density function as shown below.



a) Determine the mean $\mu_X = E[X]$ of X.
b) Determine the variance $\sigma_X^2 = E[(X - \mu_X)^2]$ of X.
c) Determine the skewness $S_X = E\left[\left(\frac{X-\mu_X}{\sigma_X}\right)^3\right]$ of X. Skewness is a pure dimensionless number which attempts to describe the leaning of the pdf. The skewness is zero if the density function is symmetric about its mean value, is positive if the shape leans towards the right, or is negative if the shape leans towards the left.
d) Determine the kurtosis $K_X = E\left[\left(\frac{X-\mu_X}{\sigma_X}\right)^4\right] - 3$ of X. Kurtosis is a dimensionless quantity which measures the relative flatness ($K_X < 0$) or peakedness ($K_X > 0$) of a density about its mean.

Problem 5: Consider the random variable X with the following probability density function

$$f_X(x) = \begin{cases} 1 - x/2 & \text{if } 0 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$$

A hard limiter produces a new random variable Y defined by

$$Y = \begin{cases} X & \text{if } X \le 1 \\ 1 & \text{if } X > 1 \end{cases}$$

1) What is the cumulative distribution (CDF) $F_X(x)$?
2) What is Prob($Y = 1$)?
3) What is $F_Y(y)$?
4) What is the pdf $f_Y(y)$?