ECE 2570 - Homework 2

Avery Peiffer

1. Chapter 10 - 10 min
   Chapter 11 - 15 min
   Chapter 12 - 5 min
   Chapter 13 - 8 min
   Chapter 14 - 20 min
   Chapter 15 - 10 min
   Chapter 16 - 15 min
   Chapter 17 - 10 min
   Chapter 18 - 12 min

2. $u_1 = F_1$ $\quad x_1 = 2x$ $\quad y_1 = x$

   $u_2 = F_2$ $\quad x_2 = \dot{x}$ $\quad y_2 = y$

   $x_3 = 3y$ $\quad y_3 = \theta$

   $x_4 = \dot{y}$

   $x_5 = 4\theta$

   $x_6 = \dot{\theta}$

$\dot{x}_1 = 2\dot{x} = 2x_2$

$\dot{x}_2 = \ddot{x} = \frac{F_1}{m}\cos\theta - \frac{F_2}{m}\sin\theta - c\frac{\dot{x}}{m} = \frac{u_1}{m}\cos\left(\frac{x_5}{4}\right) - \frac{u_2}{m}\sin\left(\frac{x_5}{4}\right) - \frac{c}{m}x_2$

$\dot{x}_3 = 3\dot{y} = 3x_4$

$\dot{x}_4 = \ddot{y} = \frac{1}{m}F_1\sin\theta + \frac{1}{m}F_2\cos\theta - g - \frac{c}{m}\dot{y} = \frac{u_1}{m}\sin\left(\frac{x_5}{4}\right) + \frac{u_2}{m}\cos\left(\frac{x_5}{4}\right) - g - \frac{c}{m}x_4$

$\dot{x}_5 = 4\dot{\theta} = 4x_6$

$\dot{x}_6 = \ddot{\theta} = \frac{r}{j}F_1 = \frac{r}{j}u_1$

$y_1 = x = \frac{x_1}{2}$

$y_2 = y = \frac{x_3}{3}$

$y_3 = \theta = \frac{x_5}{4}$

$$\dot{x} = \begin{bmatrix} 2x_2 \\ \frac{u_1}{m}\cos\left(\frac{x_5}{4}\right) - \frac{u_2}{m}\sin\left(\frac{x_5}{4}\right) - \frac{c}{m}x_2 \\ 3x_4 \\ \frac{u_1}{m}\sin\left(\frac{x_5}{4}\right) + \frac{u_2}{m}\cos\left(\frac{x_5}{4}\right) - g - \frac{c}{m}x_4 \\ 4x_6 \\ \frac{r}{j}u_1 \end{bmatrix}$$

$$Y = \begin{bmatrix} x_1/2 \\ x_3/3 \\ x_5/4 \end{bmatrix}$$

3. $$J\frac{d^2\varphi}{dt^2} - \frac{Dv_0}{b}\frac{d\delta}{dt} = mgh\sin\varphi + \frac{mv_0^2h}{b}\delta$$

(a) $u = \delta \quad x_1 = \varphi \qquad y = \varphi$

$$x_2 = \dot{\varphi} - \frac{Dv_0}{bJ}\delta$$

$$\dot{x}_1 = \dot{\varphi} = x_2 + \frac{Dv_0}{bJ}\delta = x_2 + \frac{Dv_0}{bJ}u$$

$$\dot{x}_2 = \ddot{\varphi} - \frac{Dv_0}{bJ}\dot{\delta} = \frac{mgh}{J}\sin\varphi + \frac{mv_0^2h}{bJ}\delta = \frac{mgh}{J}\sin x_1 + \frac{mv_0^2h}{bJ}u$$

$$y = \varphi = x_1$$

$$\dot{x} = \begin{bmatrix} x_2 + \frac{Dv_0}{bJ}u \\ \frac{mgh}{J}\sin x_1 + \frac{mv_0^2h}{bJ}u \end{bmatrix} \qquad Y = [x_1]$$

(b) For small $\varphi \longrightarrow \sin\varphi \approx \varphi$

$$\dot{x} = \begin{bmatrix} x_2 + \frac{Dv_0}{bJ}u \\ \frac{mgh}{J}x_1 + \frac{mv_0^2h}{bJ}u \end{bmatrix} \qquad Y = [x_1]$$

$$x = \begin{bmatrix} 0 & 1 \\ \frac{mgh}{J} & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \frac{Dv_0}{bJ} \\ \frac{mv_0^2h}{bJ} \end{bmatrix}u$$

$$Y = \begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix}u$$