ECE 2570 Midterm - Avery Peiffer

1. (a) Deliberative control is very slow and memory intensive. The information it uses can be outdated and executing a plan can be difficult.

(b) The middle layer is very difficult to build, since it must reconcile the time scales, different representations, and contradictory commands of the reactive and planning layers.

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$$R i_1(t) + L_1 \dot{i}_1(t) + v(t) = u_1(t) \qquad x_1 = i_1$$

$$L_2 \dot{i}_2(t) + v(t) = u_2(t) \qquad x_2 = i_2 \qquad y = v$$

$$i_1(t) + i_2(t) = C\dot{v}(t) \qquad x_3 = v$$

$$\dot{x} = f(x,u) \qquad y = g(x,u)$$

$$\dot{x}_1 = \dot{i}_1 = \frac{u_1 - v - Ri_1}{L_1} = \frac{u_1(t) - x_3 - Rx_1}{L_1} \qquad y = v = x_3$$

$$\dot{x}_2 = \dot{i}_2 = \frac{u_2 - v}{L_2} = \frac{u_2(t) - x_3}{L_2}$$

$$\dot{x}_3 = \dot{v} = \frac{i_1 + i_2}{C} = \frac{x_1 + x_2}{C}$$

Check:

$$\dot{x}_1 = -\frac{R}{L_1}x_1 - \frac{x_3}{L_1} + \frac{u_1}{L_1} \quad \checkmark$$

$$\dot{x}_2 = -\frac{x_3}{L_2} + \frac{u_2}{L_2}$$

$$\dot{x}_3 = \frac{x_1}{C} + \frac{x_2}{C}$$

$$\dot{x} = \underbrace{\begin{bmatrix} -\frac{R}{L_1} & 0 & -\frac{1}{L_1} \\ 0 & 0 & -\frac{1}{L_2} \\ \frac{1}{C} & \frac{1}{C} & 0 \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \underbrace{\begin{bmatrix} \frac{1}{L_1} & 0 \\ 0 & \frac{1}{L_2} \\ 0 & 0 \end{bmatrix}}_{B} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$y = \underbrace{\begin{bmatrix} 0 & 0 & 1 \end{bmatrix}}_{C} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 & 0 \end{bmatrix}}_{D} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

3. $G(s) = \dfrac{Y(s)}{R(s)} = \dfrac{3}{s^2+4s+3}$ $\quad y(0) = 1 \quad \dot{y}(0) = 0$

$r(t) = 1(t)$

Zero state response:

$$y_{zs}(s) = \frac{3}{s^2+4s+3} \cdot \frac{1}{s} = \frac{3}{(s+3)(s+1)(s)}$$

$$\frac{3}{(s+3)(s+1)(s)} = \frac{A}{s+3} + \frac{B}{s+1} + \frac{C}{s}$$

$$3 = A(s+1)(s) + B(s+3)(s) + C(s+3)(s+1)$$

$s = 0 \rightarrow 3 = C(3)(1) \rightarrow C = 1$

$s = -1 \rightarrow 3 = B(2)(-1) \rightarrow B = -\frac{3}{2}$

$s = -3 \rightarrow 3 = A(-2)(-3) \rightarrow A = \frac{1}{2}$

$$= \frac{1}{2} \cdot \frac{1}{s+3} + \frac{-3}{2} \cdot \frac{1}{s+1} + 1 \cdot \frac{1}{s}$$

$$y_{zs}(t) = \frac{1}{2}e^{-3t} - \frac{3}{2}e^{-t} + 1$$

Zero input response: Redo LT including initial condition

$y(s)[s^2+4s+3] = R(s)(3)$

$s^2y(s) + 4sy(s) + 3y(s) = 3R(s)$

$\downarrow$ ILT

$$\frac{d^2y(t)}{dt^2} + \frac{4dy(t)}{dt} + 3y(t) = 3r(t)$$

$\downarrow$ LT using init. cond.

$[s^2y(s) - \dot{y}(0) - sy(0)] + 4[sy(s) - y(0)] + 3y(s) = 6r(s)$ $\quad r(s) = 0$

$s^2y(s) - 0 - s(1) + 4sy(s) - 4 + 3y(s) = 0$

$y(s)[s^2+4s+3] = s+4$

$y(s) = \dfrac{s+4}{s^2+4s+3}$ $\quad \dfrac{s+4}{(s+3)(s+1)} = \dfrac{A}{s+3} + \dfrac{B}{s+1}$

$s+4 = A(s+1) + B(s+3)$

$s = -1 \rightarrow 3 = B(2) \rightarrow B = \frac{3}{2}$

$s = -3 \rightarrow 1 = A(-2) \rightarrow A = -\frac{1}{2}$

$$= -\frac{1}{2} \cdot \frac{1}{s+3} + \frac{3}{2} \cdot \frac{1}{s+1}$$

$$y_{zI}(t) = -\frac{1}{2}e^{-3t} + \frac{3}{2}e^{-t}$$

$$y(t) = y_{zs}(t) + y_{zI}(t) = \frac{1}{2}e^{-3t} - \frac{3}{2}e^{-t} + 1 - \frac{1}{2}e^{-3t} + \frac{3}{2}e^{-t}$$

$$= 1(t) \quad \smile$$

4. $\dot{x} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \qquad y = [2 \;\; 0]\, x$

$x(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad u(t) = 1(t)$

$y(s) = C(sI-A)^{-1}x(0) + C(sI-A)^{-1}B\,u(s) \qquad u(s) = \frac{1}{s}$

$$sI - A = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} = \begin{bmatrix} s & -1 \\ 2 & s+3 \end{bmatrix}$$

$$\det(sI-A) = s(s+3) - -2 = s^2+3s+2 = (s+2)(s+1)$$

$$(sI-A)^{-1} = \begin{bmatrix} \frac{s+3}{(s+2)(s+1)} & \frac{1}{(s+2)(s+1)} \\ \frac{-2}{(s+2)(s+1)} & \frac{s}{(s+2)(s+1)} \end{bmatrix}$$

$\xrightarrow{\text{check}} \begin{bmatrix} s & -1 \\ 2 & s+3 \end{bmatrix}\begin{bmatrix} \cdot & \cdot \\ \cdot & \cdot \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ ✓

$$\frac{s(s+3)}{(s+2)(s+1)} + \frac{2}{(s+2)(s+1)} = \frac{s^2+3s+2}{(s+2)(s+1)} = 1$$

$$\frac{s}{(s+2)(s+1)} - \frac{s}{(s+2)(s+1)} = 0$$

$$C(sI-A)^{-1} = [2 \;\; 0]\begin{bmatrix} \frac{s+3}{(s+2)(s+1)} & \frac{1}{(s+2)(s+1)} \\ \frac{-2}{(s+2)(s+1)} & \frac{s}{(s+2)(s+1)} \end{bmatrix} = \begin{bmatrix} \frac{2(s+3)}{(s+2)(s+1)} & \frac{2}{(s+2)(s+1)} \end{bmatrix}$$

$$y(s) = \begin{bmatrix} \frac{2(s+3)}{(s+2)(s+1)} & \frac{2}{(s+2)(s+1)} \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} \frac{2(s+3)}{(s+2)(s+1)} & \frac{2}{(s+2)(s+1)} \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}\frac{1}{s}$$

$$= \frac{2(s+3)+2}{(s+2)(s+1)} + \frac{2}{s(s+2)(s+1)}$$

$$\frac{2s+8}{(s+2)(s+1)} + \frac{2}{s(s+2)(s+1)}$$

$$\frac{2s+8}{(s+2)(s+1)} = \frac{A}{(s+2)} + \frac{B}{s+1}$$

$2s+8 = A(s+1) + B(s+2)$

$s=-1 \rightarrow 6 = B$

$s=-2 \rightarrow 4 = -A, \; A=-4$

$$= \frac{-4}{s+2} + \frac{6}{s+1}$$

$$\frac{2}{s(s+2)(s+1)} = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+1} = A(s+1)(s+2) + B(s)(s+1) + C(s)(s+2)$$

$$s=-1 \rightarrow 2 = C(-1)(1), \quad C=-2$$

$$s=0 \rightarrow 2 = A(2) \rightarrow A=1$$

$$s=-2 \rightarrow 2 = B(-2)(-1) \rightarrow B=1$$

$$= \frac{1}{s} + \frac{1}{s+2} - \frac{2}{s+1}$$

Overall: $\frac{1}{s} + \frac{1}{s+2} - \frac{2}{s+1} - \frac{4}{s+2} + \frac{6}{s+1} = \frac{1}{s} - \frac{3}{s+2} + \frac{4}{s+1} = y(s)$

$\downarrow$ ILT

$$y(t) = 1(t) - 3e^{-2t} + 4e^{-t}$$

5. $\dot{x} = Ax$ Check eigenvals: $\det(\lambda I - A)$

$$A_1 = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix} \quad A_2 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \quad A_3 = \begin{bmatrix} -1 & -2 \\ -2 & -1 \end{bmatrix}$$

$$\det(\lambda I - A_1) = \det\left(\begin{bmatrix} \lambda & -1 \\ +1 & \lambda+2 \end{bmatrix}\right) = \lambda(\lambda+2) - (-1) = \lambda^2 + 2\lambda + 1 = 0$$

$$(\lambda+1)(\lambda+1) = 0$$

$$\lambda_{1,2} = -1$$

A_1 is asymptotically stable (all neg. real parts)

$$\det(\lambda I - A_2) = \det\left(\begin{bmatrix} \lambda & 1 \\ -1 & \lambda \end{bmatrix}\right) = \lambda^2 - (-1) = \lambda^2 + 1 = 0$$

$$\lambda_{1,2} = \pm j$$

A_2 is marginally stable (distinct, real parts are 0)

$$\det(\lambda I - A_3) = \det\left(\begin{bmatrix} \lambda+1 & 2 \\ 2 & \lambda+1 \end{bmatrix}\right) = (\lambda+1)^2 - 4 = 0$$

$$\lambda^2 + 2\lambda + 1 - 4 = 0$$

$$\lambda^2 + 2\lambda - 3 = 0$$

$$(\lambda+3)(\lambda-1) = 0$$

$$\lambda_1 = -3$$

$\lambda_2 = 1 \longrightarrow$ Unstable (positive real part)

6. $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ w_0^2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$

Char eq. $\det(\lambda I - (A - BK)) = 0$ $\qquad \begin{bmatrix} 0 & 0 \\ k_1 & k_2 \end{bmatrix}$

$$A - BK = \begin{bmatrix} 0 & 1 \\ w_0^2 & 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} k_1 & k_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ w_0^2 - k_1 & -k_2 \end{bmatrix}$$

$$\lambda I - (A - BK) = \begin{bmatrix} \lambda & -1 \\ k_1 - w_0^2 & k_2 + \lambda \end{bmatrix}$$

$$\det(\lambda I - (A - BK)) = \lambda(k_2 + \lambda) - (-1)(k_1 - w_0^2)$$

$$= k_2 \lambda + \lambda^2 + k_1 + w_0^2$$

Desirable char eq.

$$s^2 + 2(0.7)(2w_0)s + 4w_0^2 = 0$$

$$s^2 + 2.8s + 4w_0^2 = 0$$

$$s = \frac{-2.8 \pm \sqrt{2.8^2 + (-4)(1)(4w_0^2)}}{2} = \frac{-2.8 \pm \sqrt{7.84 - 16w_0^2}}{2}$$

$$\lambda^2 + 2.8\lambda + 4w_0^2 = \lambda^2 + k_2\lambda + k_1 + w_0^2$$

$$2.8\lambda = k_2 \lambda \rightarrow k_2 = 2.8$$

$$4w_0^2 = k_1 + w_0^2 \rightarrow k_1 = 3w_0^2$$

7. $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u$

$C = \begin{bmatrix} B & AB \end{bmatrix}$

(a) $AB = \begin{bmatrix} -2 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ -1 \end{bmatrix}$

$C = \begin{bmatrix} 1 & -2 \\ 1 & -1 \end{bmatrix}$ $\quad \det(C) = (-1) + (+2) = 1 \rightarrow$ System is controllable

(b) $u = -\begin{bmatrix} k_1 & k_2 \end{bmatrix} x + r$ $\quad$ System is controllable

$0 = -k_1 x_1 - k_2 x_2 + r$ $\quad$ as long as $k_1 x_1 \neq -k_2 x_2 + r$