ECE 2195 Assignment 1 - Avery Peiffer

Problem 1

Sample point $\langle -1, 1, 2 \rangle$

Distance $AB = \sqrt{(x_{1,1} - x_{1,2})^2 + (x_{2,1} - x_{2,2})^2 + (x_{3,1} - x_{3,2})^2}$

Obs.	1	2	3	4	5	6	t-point
1	0	3.61	3.61	2.83	3.32	2.45	3
2	3.61	0	3.74	3	3.16	1.73	3.74
3	3.61	3.74	0	1	2.45	2.24	1.41
4	2.83	3	1	0	1.73	1.41	1
5	3.32	3.16	2.45	1.73	0	2.23	1.41
6	2.45	1.73	2.24	1.41	2.23	0	2.23

K=1: 4 is the nearest neighbor, so prediction is red

K=3: 3, 4, and 5 are the 3 nearest neighbors, so the votes are [Green, Red, Red] → overall, prediction is red.

K=5: 1, 3, 4, 5, and 6 are the 5 nearest neighbors. The votes are [Green, Green, Red, Red, Green] → Overall result is green.

Problem 2

$\bar{x} = 107.33$

$\bar{y} = 118.11$

x	y	$x_i y_i$	x^2	$(y_i - \bar{y})(x_i - \bar{x})$	$(x_i - \bar{x})^2$
127	115	14605	16129	-61.174	386.909
121	128	15488	14641	135.196	186.869
94	128	12032	8836	-131.834	177.689
126	156	19656	15876	707.406	348.569
102	101	10302	10404	91.196	28.409
111	128	14208	12321	36.296	13.469
95	115	10925	9025	38.346	152.029
89	105	9345	7921	240.306	335.989
101	87	8787	10201	196.926	40.069
SUM 966	1063	115348	105354	1252.664	1670.0009

$$\hat{\beta}_1 = \frac{115348 - (118.11)(966)}{105354 - (107.33)(966)} = \frac{1253.74}{1673.22} = 0.749$$

$$\hat{\beta}_0 = 118.11 - (0.749)(107.33) = 37.72$$

$\bar{y} = 112.4$

x_i	y_i	$\hat{y}_i = 37.72 + 0.749 x_i$	$(y_i - \hat{y}_i)^2$	$(y_i - \bar{y})^2$
90	103	105.13	4.5369	88.36
106	131	117.114	192.821	345.96
105	85	116.365	983.763	750.76
115	99	123.855	617.771	179.56
113	144	122.357	468.419	998.56
SUM			2267.411	2363.2

$RSS = 2267.411$

$TSS = 2363.2$

$$R^2 = 1 - \frac{2267.411}{2363.2} = .0405$$

Problem 6

$\mu_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ $\Sigma_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $\Sigma_1^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $|\Sigma_1| = 1$

$\mu_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $\Sigma_2 = \begin{bmatrix} 2 & 2 \\ 2 & 3 \end{bmatrix}$ $\Sigma_2^{-1} = \begin{bmatrix} 3/2 & -1 \\ -1 & 1 \end{bmatrix}$ $|\Sigma_2| = 2$

$$g_1(x) = -\ln(2\pi) - \frac{1}{2}\ln|\Sigma_1| - \frac{1}{2}\begin{bmatrix} x_1 - \mu_{11} & x_2 - \mu_{12} \end{bmatrix} \Sigma_1^{-1} \begin{bmatrix} x_1 - \mu_{11} \\ x_2 - \mu_{12} \end{bmatrix}$$

$$g_2(x) = -\ln(2\pi) - \frac{1}{2}\ln|\Sigma_2| - \frac{1}{2}\begin{bmatrix} x_1 - \mu_{21} & x_2 - \mu_{22} \end{bmatrix} \Sigma_1^{-1} \begin{bmatrix} x_1 - \mu_{21} \\ x_2 - \mu_{22} \end{bmatrix}$$

Decision boundary: $g_1(x) = g_2(x)$

$$-\frac{1}{2}\ln(1) - \frac{1}{2}\begin{bmatrix} x_1+1 & x_2-1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x_1+1 \\ x_2-1 \end{bmatrix} = -\frac{1}{2}\ln(2) - \frac{1}{2}\begin{bmatrix} x_1-1 & x_2 \end{bmatrix}\begin{bmatrix} 3/2 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} x_1-1 \\ x_2 \end{bmatrix}$$

$$\cancel{-\frac{1}{2}\ln(1)} - \frac{1}{2}\begin{bmatrix} x_1+1 & x_2-1 \end{bmatrix}\begin{bmatrix} x_1+1 \\ x_2-1 \end{bmatrix} = -\frac{1}{2}\ln(2) - \frac{1}{2}\begin{bmatrix} 3/2x_1 - \frac{3}{2} - x_2 & 1 - x_1 + x_2 \end{bmatrix}\begin{bmatrix} x_1-1 \\ x_2 \end{bmatrix}$$

$$\cancel{-\frac{1}{2}}\left[(x_1+1)^2 + (x_2-1)^2\right] = \cancel{-\frac{1}{2}}\left[\ln(2) + (x_1-1)\left(\frac{3}{2}x_1 - x_2 - \frac{3}{2}\right) + (x_2)(1 - x_1 + x_2)\right]$$

$$(x_1+1)^2 + (x_2-1)^2 = \ln(2) + \frac{3}{2}x_1(x_1-1) - x_2(x_1-1) - \frac{3}{2}(x_1-1) + x_2 - x_1x_2 + x_2^2$$

$$x_1^2 + 2x_1 + 1 + \cancel{x_2^2} - 2x_2 + 1 = \ln(2) + \frac{3}{2}x_1^2 - \frac{3}{2}x_1 - x_1x_2 + x_2 - \frac{3}{2}x_1 + \frac{3}{2} + x_2 - x_1x_2 + \cancel{x_2^2}$$

$$x_1^2 + 2x_1 - 2x_2 + 2 = \ln(2) + \frac{3}{2}x_1^2 - 3x_1 - 2x_1x_2 + 2x_2 + \frac{3}{2}$$

$$\frac{1}{2}x_1^2 - 5x_1 + 4x_2 - 2x_1x_2 - \frac{1}{2} + \ln(2) = 0$$

$$= 2x_1x_2 - 4x_2$$

$$= x_2(2x_1 - 4)$$

$$\boxed{x_2 = \frac{\frac{1}{2}x_1^2 - 5x_1 + 0.193}{(2x_1 - 4)}}$$