Topics: Normal distribution, Functions of Random Variables

1. The time required for servicing transmissions is normally distributed with μ = 45 minutes and σ = 8 minutes. The service manager plans to have work begin on the transmission of a customer's car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?

   A. 0.3875
   B. 0.2676
   C. 0.5
   D. 0.6987

ANS;

Since work being 10 mins after the car is dropped, the time left to complete work is 50 mins.
Probability that Service Manager cannot meet his commitment = P(X>50) = 1-Pr(x<=50)
X=the time taken to complete work,
Standard normal variable Z= (X- μ) /σ =(x-45)/8
P(X<=50) =P (Z<= (50-45)/8) =PR(Z<=0.625) =0.73237 =73.237% (Z-TABLE)
Probability that services manager will not meet his commitment is
100-73.237 =26.763% =0.2676
So, the answer is **B. 0.2676**

2. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean μ = 38 and Standard deviation σ=6. For each statement below, please specify True/False. If false, briefly explain why.
   A. More employees at the processing center are older than 44 than between 38 and 44.
   B. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

ANS;

μ = 38 & σ=6

A)

FALSE; As 84% employees are having age less than 44.

B)

TRUE; As about 9% of employees comes under age 30 and also out of 400 if we consider 9%it will be 36 persons.

3. If $X_1 \sim N(\mu, \sigma^2)$ and $X_2 \sim N(\mu, \sigma^2)$ are *iid* normal random variables, then what is the difference between 2 X_1 and $X_1 + X_2$? Discuss both their distributions and parameters.

ANS;

As we know that if $X \sim N(\mu1, \sigma1^2)$, and $Y \sim N(\mu2, \sigma2^2)$ are two independent random variables then $X + Y \sim N(\mu1 + \mu2, \sigma1^2 + \sigma2^2)$, and $X - Y \sim N(\mu1 - \mu2, \sigma1^2 + \sigma2^2)$.
Similarly, if Z = ax + by,
where X and Y are as defined above, i.e., Z is linear combination of X and Y,
then $Z \sim N(a\mu1 + b\mu2, a^2\sigma1^2 + b^2\sigma2^2)$.
Therefore,
$2X1 \sim N(2u, 4\sigma^2)$ &
$X1+X2 \sim N(\mu + \mu, \sigma^2 + \sigma^2) \sim N(2u, 2\sigma^2)$
$2X1-(X1+X2) = N(4\mu, 6\sigma^2)$

4. Let $X \sim N(100, 20^2)$. Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.

    A. 90.5, 105.9
    B. 80.2, 119.8
    C. 22, 78
    D. 48.5, 151.5
    E. 90.1, 109.9

ANS;
Since we need to find out the values of a and b, which are symmetric about the mean, such that the probability of random variable taking a value between them is 0.99, we have to work out in reverse order.
The Probability of getting value between a and b should be 0.99.
So, the Probability of going wrong, or the Probability outside the a & b area is 0.01 (i.e., 1-0.99).
The Probability towards left from a = -0.005 (i.e., 0.01/2).
The Probability towards right from b = +0.005 (i.e., 0.01/2).
So, since we have the probabilities of a and b, we need to calculate X, the random variable at a and b which has got these probabilities.
By finding the Standard Normal Variable Z (Z Value), we can calculate the X values.
$Z = (X - \mu) / \sigma$
For Probability 0.005 the Z Value is -2.57 (from Z Table).
$Z * \sigma + \mu = X$
$Z(-0.005) * 20 + 100 = -(-2.57) * 20 + 100 = 151.4$
$Z(+0.005) * 20 + 100 = (-2.57) * 20 + 100 = 48.6$
So, **option D** is correct.

5. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions $Profit_1 \sim N(5, 3^2)$ and $Profit_2 \sim N(7, 4^2)$ respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
    A. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
    B. Specify the 5^{th} percentile of profit (in Rupees) for the company

C. Which of the two divisions has a larger probability of making a loss in a given year?

ANS;

In [1]:
```
import pandas as pd
import numpy as np
from scipy import stats
from scipy.stats import norm
```
executed in 1.24s, finished 20:18:08 2021-09-22

In [2]:
```
# Mean profits from two different divisions of a company
Mean = 5+7
print('Mean Profit is Rs', Mean*45,'Million')
```
executed in 11ms, finished 20:18:08 2021-09-22

```
Mean Profit is Rs 540 Million
```

In [3]:
```
# Variance of profits from two different divisions of a company
SD = np.sqrt((9)+(16))
print('Standard Deviation is Rs',SD*45,'Million')
```
executed in 10ms, finished 20:18:08 2021-09-22

```
Standard Deviation is Rs 225.0 Million
```

A

#95% probability for the annual profit of the company

In [4]:
```
print('Raange is Rs',(stats.norm.interval(0.95,540,225)),'in Million')
```
executed in 25ms, finished 20:18:08 2021-09-22

```
Raange is Rs (99.00810347848784, 980.9918965215122) in Million
```

B

#Specify the 5th percentile of profit (in Rupees) for the company

In [5]:
```
#Formula  X= μ + Zσ; and from z-table => 5% = -1.645
X = 540+(-1.645)*(225)
print('5th percentile of profit (in Million Rupees) is',np.round(X,))
```
executed in 11ms, finished 20:18:08 2021-09-22

```
5th percentile of profit (in Million Rupees) is 170.0
```

C

#Which of the two divisions has a larger probability of making a loss in a given year?

In [6]:
```
# Probability of Division 1 making a loss P(X<0)
stats.norm.cdf(0,5,3)
```
executed in 29ms, finished 20:18:08 2021-09-22

Out[6]:
```
0.0477903522728147
```

In [7]:
```
# Probability of Division 2 making a loss P(X<0)
stats.norm.cdf(0,7,4)
```
executed in 12ms, finished 20:18:08 2021-09-22

Out[7]:
```
0.040059156863817086
```

Inference: Probability of Division 1 making a loss in a given year is more than Division 2.