Name: Key   Section: ___

MAP 2302 – Ordinary Differential Equations I

March 4, 2016

Quiz 5

1. Consider the following differential equation:

$$x^2y'' - 2xy' + 2y = x\cos x$$

(a) Verify that $y_1(x) = x$ and $y_2(x) = x^2$ are solutions to the associated homogeneous problem:

$$x^2y'' - 2xy' + 2y = 0$$

$$y_1 = x \qquad y_2 = x^2$$
$$y_1' = 1 \qquad y_2' = 2x$$
$$y_1'' = 0 \qquad y_2'' = 2$$

y_1: $x^2 \cdot 0 - 2x \cdot 1 + 2x = 0$ ✓

y_2: $x^2 \cdot 2 - 2x \cdot 2x + 2x^2 = 0$ ✓

(b) Find $u_1(x)$ and $u_2(x)$ with $u_1'(x) + xu_2'(x) = 0$ such that $y_p(x) = x\overset{u_1}{\cancel{c_1}}(x) + x^2\overset{u_2}{\cancel{c_2}}(x)$ solves the inhomogeneous problem (using variation of parameters).

$$\begin{array}{r|l} 2 & y_p = xu_1 + x^2u_2 \\ -2x & y_p' = u_1 + \cancel{xu_1'} + 2xu_2 + \cancel{x^2u_2'} \\ x^2 & y_p'' = u_1' + 2u_2 + 2xu_2' \end{array}$$

, take $xu_1' + x^2u_2' = 0$

$$(\overset{0}{\cancel{2x - 2x}})u_1 + (2x^2 + \overset{0}{\cancel{(-2x)(2x)}} + 2x^2)u_2 + x^2u_1' + 2x^3u_2' = x\cos x$$

Thus, $xu_1' + x^2u_2' = 0$

$x^2u_1' + 2x^3u_2' = x\cos x$ $\xrightarrow{\text{simplify}}$ $u_1' + xu_2' = 0$

$u_1' + 2xu_2' = \dfrac{\cos x}{x}$

Solve via Cramer's rule:

$$D = \begin{vmatrix} 1 & x \\ 1 & 2x \end{vmatrix} = x,$$

$$u_1' = \frac{\begin{vmatrix} 0 & x \\ \frac{\cos x}{x} & 2x \end{vmatrix}}{x} = -\frac{\cos x}{x}$$

$$u_2' = \frac{\begin{vmatrix} 1 & 0 \\ 1 & \cos x/x \end{vmatrix}}{x} = \frac{\cos x}{x^2}$$

$$\boxed{\begin{aligned} u_1 &= -\int \frac{\cos x}{x}\,dx \\ u_2 &= \int \frac{\cos x}{x^2}\,dx \end{aligned}}$$