You have 30 minutes to work on this group activity before presenting.

Name: _____

1. The equation $y'' + y' - 2y = x^2$ is called a *differential equation* because it involves an unknown function y and its derivatives y' and y''. Find constants A, B, and C such that the function $y = Ax^2 + Bx + C$ satisfies the equation.

2. Evaluate $\lim_{x \to 1} \dfrac{x^{1000} - 1}{x - 1}$.

3. The *Reciprocal Rule* states $\dfrac{d}{dx}\left[\dfrac{1}{g(x)}\right] = -\dfrac{g'(x)}{[g(x)]^2}$.
   (a) Use the Quotient Rule to prove the Reciprocal Rule.
   (b) Use the Reciprocal Rule to verify that the Power Rule is valid for negative integers, meaning
   $$\frac{d}{dx}\left(x^{-n}\right) = -nx^{-n-1}$$
   for all positive integers n.

4. Let $f(x) = x^2 e^x$. The principle of mathematical induction states that, if a proposition $P(n)$ is true for $n = 1$, and $P(n)$ true implies $P(n+1)$ true, then $P(n)$ is true for every natural number n.
   (a) Find expressions for the first five derivatives of $f(x)$.
   (b) Guess a formula for the nth derivative $f^{(n)}(x)$.
   (c) Prove your formula holds for all natural numbers n using the principal of mathematical induction.

5. Find constants A and B such that the function $y = A\sin x + B\cos x$ satisfies the differential equation $y'' + y' - 2y = \sin x$.

6. A ladder 10ft long rests against a vertical wall. Let θ be the angle between the top of the ladder and the wall and let x be the distance from the bottom of the ladder to the wall. If the bottom of the ladder slides away from the wall, how fast does x change with respect to θ when $\theta = \pi/3$?

7. Let $f(x)$ and $g(x)$ be infinitely differentiable, meaning the nth derivatives $f^{(n)}(x)$ and $g^{(n)}(x)$ exist for all positive integers n, and let $F(x) = f(x)g(x)$.
   (a) Show that $F'' = f''g + 2f'g' + fg''$.
   (b) Find similar formulae for F''' and $F^{(4)}$.
   (c) Guess a formula for $F^{(n)}$.

8. For what values of a and b is the line $2x + y = b$ tangent to the parabola $y = ax^2$ when $x = 2$?

9. Let f, g, and h be differentiable functions.
   (a) Use two applications of the Product Rule to prove that $(fgh)' = f'gh + fg'h + fgh'$.
   (b) Take $f = g = h$ to show that $\frac{d}{dx}[f(x)]^3 = 3[f(x)]^2 f'(x)$ using part (a).
   (c) Use part (b) to differentiate $y = e^{3x}$.