Name: Key Section: ___

MAP 2302 – Ordinary Differential Equations I

April 8, 2016

Quiz 7

1. Consider the following IVP:

$$y^{(4)} - y = 0, \qquad y(0) = 1, \quad y'(0) = 0, \quad y''(0) = 1, \quad y'''(0) = 0$$

(a) Find the Laplace transform $Y(s) = \mathcal{L}\{y(t)\}$.

$$\mathcal{L}\{y^{(4)} - y\} = \mathcal{L}\{0\} = 0$$

$$\Rightarrow \mathcal{L}\{y^{(4)}\} - \mathcal{L}\{y\} = 0$$

$$\Rightarrow (s^4Y - s^3y(0) - s^2y'(0) - s\,y''(0) - y'''(0)) - Y = 0$$

$$\Rightarrow s^4Y - s^3 - s - Y = 0$$

$$\Rightarrow (s^4 - 1)Y = s^3 + s$$

$$\Rightarrow Y = \frac{s^3 + s}{s^4 - 1} = \frac{s(s^2+1)}{(s^2+1)(s^2-1)} = \boxed{\frac{s}{s^2-1}}$$

(b) Find the continuous solution $y(t) = \mathcal{L}^{-1}\{Y(s)\}$ using the inverse Laplace transform.

$$\mathcal{L}^{-1}\left\{\frac{s}{s^2-1}\right\} = \cosh(t)$$