Name: _____ MAP 2302 – Ordinary Differential Equations 1
October 2, 2015
Week 6 Worksheet

1. Find the solution of the initial value problem $3y'' + 5y' - 2y = 0$, $y(0) = 5$, $y'(0) = -3$.
   (a) Find the characteristic equation.
   (b) Find the roots of the characteristic equation.
   (c) Write the general solution as a linear combination of two exponentials.
   (d) Find the derivative of the general solution.
   (e) Plug in the initial conditions to solve for the constants.

2. Determine the values of β, if any, for which all solutions of the differential equation
$$y'' - (2\beta - 10)y' + (\beta^2 - 10\beta + 16)y = 0$$
   (i) tend to zero as $t \to \infty$ and (ii) are unbounded as $t \to \infty$ when not identically zero.
   (a) Solve the characteristic equation in terms of α.
   (b) Find the interval when both solutions are negative to answer part (i).
   (c) Find the interval when both solutions are positive to answer part (ii).

3. Find the simplified Wronskian of the functions $f(t) = -2e^t \sinh t$ and $g(t) = 3e^t \cosh t$.

4. If the Wronskian of $t^2 - 1$ and g is $2t^2 - 2$, for $t > 0$, then find $g(t)$.

5. Without solving, determine the longest interval in which the following initial value problem is guaranteed to have a unique, twice differentiable solution:

$$(t^2 + t - 6)y'' + (t - 2)y' - (t + 3)y = t - 2, \quad y(3) = 0, \quad y'(3) = 1$$

6. Without solving the equation, for $t \neq 0$, find the Wronskian of two independent solutions of

$$t^2y'' - t(t - 2)y' + (t - 7)y = 0$$