Name: Key

MAP 2302 – Ordinary Differential Equations I

January 29, 2016

Quiz 2

Solve the following IVP:

$$y' + \frac{2}{t}y = \frac{\cos t}{t^2}, \quad y(\pi) = 0, \quad t > 0$$

($p(t)$, $g(t)$; notice abs. unnecessary)

(a) Find the integrating factor $\mu(t)$.

$$\mu(t) = e^{\int p(t)\,dt} = e^{\int \frac{2}{t}\,dt} = e^{2\ln|t|} = t^2$$

(b) Find the general solution $y(t)$. (This solution should depend on an arbitrary constant).

$$t^2 y' + 2ty = \cos t \Rightarrow$$

$$\frac{d}{dt}(t^2 y) = \cos t \Rightarrow$$

$$t^2 y = \int \cos t\,dt = \sin t + C$$

$$y = \frac{\sin t}{t^2} + \frac{C}{t^2}$$

(c) Find the solution that satisfies the initial value $y(\pi) = 0$.

$$0 = \frac{\sin \pi}{\pi^2} + \frac{C}{\pi^2} = \frac{C}{\pi^2} \Rightarrow C = 0$$

$$\therefore \quad y = \frac{\sin t}{t^2}$$

(d) What is the domain of the solution from part (c)?

$(0, \infty)$ or $t > 0$

Remark: The domain of the original problem was $t > 0$, and we put no new restrictions on t.