Calculus I (MAC 2311)
Practice Skills Test 3 Version A
Friday, April 10, 2015
Time: 30 minutes

Write your answer for each question on the **Answer line**. Only the answer on the line will be graded. Neatly do the work to support your answer in the blank space provided. Each question is worth 1 point. Note: Use of any calculator will be considered as academic dishonesty.

Name: Key

Section and NID/PID: ____________________

1. Find the critical point(s) for the function $f(x) = e^{2x}$. (Write DNE, if none exists)

$f'(x) = 2e^{2x} \neq 0$

Answer: DNE

2. Let f(x) be a function satisfying all the conditions of Rolle's Theorem. Find the values of c which satisfies the conclusion for Rolle's Theorem for the function $f(x) = \sin x$ in $[0, \pi]$.(Write DNE, if none exists)

$f'(x) = \cos x = 0$

$\Rightarrow x = \pi/2$

Answer: $\pi/2$

3. Let f(x) be a function satisfying all the conditions of Mean Value Theorem (MVT). Find all number c which satisfies the conclusion for MVT for the function $f(x) = x^3$ in the interval [-1,1]. (Write DNE, if none exists)

$f'(x) = 3x^2 = \frac{(-1)^3 - (1)^3}{-1-1} = 1$

$\Rightarrow x^2 = 1/3 \Rightarrow x = \pm \frac{\sqrt{3}}{3}$

Answer: $-\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}$

4. Let f be continuous and differentiable on $(-\infty, \infty)$ and f(1)=10, $f'(x) \geq 2$ for all x. What is the minimum value of f(4)?

By MVT, $f(4) - f(1) = f'(c)(4-1) \geq 2(4-1) = 6$

$\Rightarrow f(4) \geq f(1) + 6$

$\Rightarrow f(4) \geq 16$

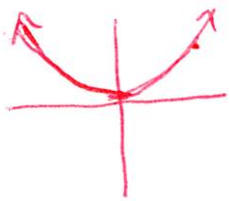
Answer: 16

5. Find the interval in which the function $f(x) = \ln x$ is increasing. $x > 0$

ln x is increasing function
(or $\frac{1}{x} > 0$ when $x>0$)

Answer: $(0, \infty)$

6. Find the **absolute maxima of the function** $f(x) = x^2$. (Write DNE, if none exists)



no max

Answer: DNE

7. Find the interval in which the function $f(x) = (x-2)^3 (0 \le x \le 5)$ is concave upwards.

$f'(x) = 3(x-2)^2$
$f''(x) = 6(x-2)$

Answer: $(2, 5)$

0 − 2 + 5

8. Find the inflection point(s) of the function $f(x) = \frac{1}{x-3}, x \neq 3$. (Write DNE, if none exists)

$f'(x) = -(x-3)^{-2}$ f not continuous at $x=3$, so
$f''(x) = 2(x-3)^{-3}$
neg: $(-\infty, 3)$ pos: $(3, \infty)$

Answer: DNE

9. Evaluate: $\lim_{x\to \cancel{0^+}\ \infty} \frac{\ln x}{x}$.(Hint: Use L'Hospital rule)

$= \lim_{x\to\infty} \frac{\frac{1}{x}}{1} = 0$

Answer: 0

10. Evaluate the value of the limit $\lim_{x\to 0} \frac{5^x - 4^x}{3^x - 2^x}$.(Hint: Use L'Hospital rule)

$= \lim_{x\to 0} \frac{(\ln 5) 5^x - (\ln 4) 4^x}{(\ln 3) 3^x - (\ln 2) 2^x}$

Answer: $\ln(5/4) / \ln(3/2)$

$= \frac{\ln(5/4)}{\ln(3/2)}$

The End.

Calculus I (MAC 2311)
Skills Test 3 Version B
Friday, April 10, 2015
Time: 30 minutes

Write your answer for each question on the **Answer line**. Only the answer on the line will be graded. Neatly do the work to support your answer in the blank space provided. Each question is worth 1 point. Note: Use of any calculator will be considered as academic dishonesty.

Name: Key

Section and NID/PID: ______

1. Find the critical point(s) for the function $f(x) = e^{2x}$. (Write DNE, if none exists)

$f'(x) = 2e^{2x} \neq 0$

Answer: DNE

2. Let f(x) be a function satisfying all the conditions of Rolle's Theorem. Find the values of c which satisfies the conclusion for Rolle's Theorem for the function $f(x) = 3\sin x$ in $[0, \pi]$.(Write DNE, if none exists)

$f'(x) = 3\cos x = 0$
$\Rightarrow \cos x = 0$
$\Rightarrow x = \pi/2$

Answer: $\pi/2$

3. Let f(x) be a function satisfying all the conditions of Mean Value Theorem (MVT). Find all number c which satisfies the conclusion for MVT for the function $f(x) = 4x^3$ in the interval [-1,1]. (Write DNE, if none exists)

$f'(x) = 12x^2 = \frac{4(-1)^3 - 4(1)^3}{-1-1} = 4$
$\Rightarrow x^2 = 1/3$
$\Rightarrow x = \pm\sqrt{3}/3$

Answer: $-\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}$

4. Let f be continuous and differentiable on $(-\infty, \infty)$ and f(1)=1, $f'(x) \geq 3$ for all x. What is the minimum value of f(4)?

By MVT, $f(4) - f(1) = f'(c)(4-1) \geq 3(4-1) = 9$
$\Rightarrow f(4) \geq f(1) + 9$
$\Rightarrow f(4) \geq 10$

Answer: 10

5. Find the interval in which the function $f(x) = \ln(3x)$ is increasing.

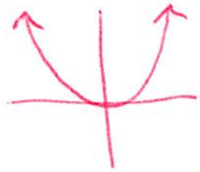
$x > 0$

$\ln(3x)$ is always increasing

(or $\frac{1}{x} > 0$ when $x > 0$)

Answer: $(0, \infty)$

6. Find the **absolute maxima of the function** $f(x) = 5x^2$. (Write DNE, if none exists)



There is none

Answer: DNE

7. Find the interval in which the function $f(x) = (x-2)^3 (0 \le x \le 3)$ is concave upwards.

$f'(x) = 3(x-2)^2$

$f''(x) = 6(x-2)$

Answer: $(2, 3)$

0 − 2 + 3

8. Find the inflection point(s) of the function $f(x) = \frac{1}{x-11}, x \neq 11$. (Write DNE, if none exists)

$f'(x) = -(x-11)^{-2}$

But f is not continuous at $x = 11$, so

$f''(x) = 2(x-11)^{-3}$

Answer: DNE

neg: $(-\infty, 11)$ pos: $(11, \infty)$

9. Evaluate: $\lim_{x \to \cancel{0^+}\ \infty} \frac{9 \ln x}{x}$.(Hint: Use L'Hospital rule)

$$= \lim_{x \to \infty} \frac{9 \frac{1}{x}}{1} = 0$$

Answer: 0

10. Evaluate the value of the limit $\lim_{x \to 0} \frac{15^x - 14^x}{13^x - 11^x}$.(Hint: Use L'Hospital rule)

$$= \lim_{x \to 0} \frac{(\ln 15) 15^x - (\ln 14) 14^x}{(\ln 13) 13^x - (\ln 11) 11^x}$$

Answer: $\ln(15/14) / \ln(13/11)$

$$= \frac{\ln(15/14)}{\ln(13/11)}$$

The End.

Calculus I (MAC 2311)
Skills Test 3 Version C
Friday, April 10, 2015
Time: 30 minutes

Write your answer for each question on the **Answer line**. Only the answer on the line will be graded. Neatly do the work to support your answer in the blank space provided. Each question is worth 1 point. Note: Use of any calculator will be considered as academic dishonesty.

Name: Key

Section and NID/PID: ____________________

1. Find the critical point(s) for the function $f(x) = e^{6x}$. (Write DNE, if none exists)

$f'(x) = 6e^{6x} \neq 0$

Answer: DNE

2. Let f(x) be a function satisfying all the conditions of Rolle's Theorem. Find the values of c which ~~satisfies~~ satisfy the conclusion for Rolle's Theorem for the function $f(x) = 5\sin x$ in $[0, \pi]$.(Write DNE, if none exists)

$f'(x) = 5\cos x = 0$

$\Rightarrow \cos x = 0$

$\Rightarrow x = \frac{\pi}{2}$

Answer: $\frac{\pi}{2}$

3. Let f(x) be a function satisfying all the conditions of Mean Value Theorem (MVT). Find all numbers c which satisfies the conclusion for MVT for the function $f(x) = 6x^3$ in the interval [-1,1]. (Write DNE, if none exists)

$f'(x) = 18x^2 = \frac{6(-1)^3 - 6(1)^3}{-1 - 1} = 6$

$\Rightarrow x^2 = \frac{1}{3} \Rightarrow x = \pm\frac{\sqrt{3}}{3}$

Answer: $-\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}$

4. Let f be continuous and differentiable on $(-\infty, \infty)$ and f(3)=4, $f'(x) \geq 3$ for all x. What is the minimum value of f(9)?

By MVT, $f(9) - f(3) = f'(c)(9-3) = 6f'(c) \geq 18$

$\Rightarrow f(9) \geq f(3) + 18$

$\Rightarrow f(9) \geq 22$

Answer: 22

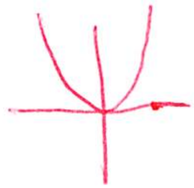
5. Find the interval in which the function $f(x) = \ln(8x)$ is increasing. $x > 0$ (domain)

always increasing

(or $\frac{1}{x} > 0$ when $x > 0$)

Answer: $(0, \infty)$

6. Find the **absolute maxima of the function** $f(x) = 12x^2$. (Write DNE, if none exists)



no max

Answer: DNE

7. Find the interval in which the function $f(x) = (x-4)^3 (0 \leq x \leq 5)$ is concave upwards.

$f'(x) = 3(x-4)^2$

$f''(x) = 6(x-4)$

0 $-$ 4 $+$ 5

Answer: $(4, 5)$

8. Find the inflection point(s) of the function $f(x) = \frac{1}{x-10}, x \neq 10$. (Write DNE, if none exists)

$f'(x) = -(x-10)^{-2}$ f is not continuous at 10, so

$f''(x) = 2(x-10)^{-3}$

negative: $(-\infty, 10)$

positive: $(10, \infty)$

Answer: DNE

9. Evaluate: $\lim_{x\to \cancel{0+}\,\infty} \frac{7\ln x}{x}$.(Hint: Use L'Hospital rule)

$= \lim_{x\to\infty} \frac{7\frac{1}{x}}{1} = 0$

Answer: 0

10. Evaluate the value of the limit $\lim_{x\to 0} \frac{8^x - 7^x}{5^x - 3^x}$.(Hint: Use L'Hospital rule)

$= \lim_{x\to 0} \frac{(\ln 8) 8^x - (\ln 7) 7^x}{(\ln 5) 5^x - (\ln 3) 3^x}$

$= \frac{\ln(8/7)}{\ln(5/3)}$

Answer: $\ln(8/7) / \ln(5/3)$

The End.