§2.2 # 24: $y'(x) = \dfrac{2-e^x}{3+2y(x)} \iff (3+2y)y' = 2-e^x$

or $(3+2y)\,dy = (2-e^x)\,dx$. Integrate to get

(*) $\boxed{3y + y^2 = 2x - e^x + C}$ General solution in an implicit form.

constant

If $y(0)=0 \Rightarrow 3\cdot 0 + 0^2 = 2\cdot 0 - e^0 + C \Rightarrow C = 1$

So the solution to the Initial Value Problem

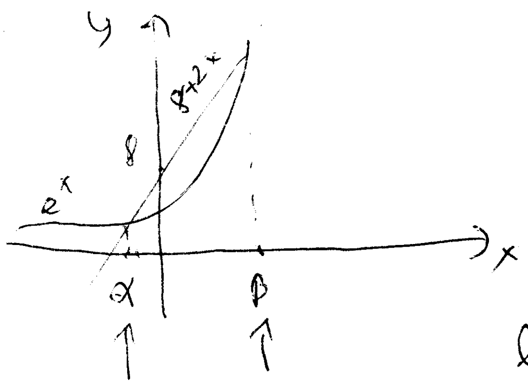
$$\begin{cases} y' = \dfrac{2-e^x}{3+2y} \\ y(0) = 0 \end{cases} \quad \text{is} \quad \underline{y^2 + 3y + e^x - 2x - 1 = 0} \quad (**)$$

Note that (**) can be solved explicitly:

$$y_{1,2} = \frac{-3 \pm \sqrt{9 - (e^x - 2x - 1)}}{2} \quad \text{but only} \quad \boxed{y = \frac{-3 + \sqrt{8+2x-e^x}}{2}}$$

satisfies $y(0) = 0$.

Domain of y: All x for which $8+2x-e^x \geq 0$. For x in $[\alpha, \beta]$



The maximum of $y(x)$ is attained at the same x for which $8+2x-e^x$ has a maximum for $\alpha \leq x \leq \beta$

Let $g(x) = 8+2x-e^x$

then $g(\alpha) = g(\beta) = 0$

and $g(x) \geq 0$ for x in $[\alpha, \beta]$

Find critical points of g: $0 = g'(x) = 2 - e^x \Rightarrow x = \ln 2$

and $g'(x) > 0$ for $x < \ln 2$
$g'(x) < 0$ for $x > \ln 2$ $\Rightarrow$ $\boxed{x = \ln 2}$ is a max.

§2.2 #31. $y' = \dfrac{x^2+xy+y^2}{x^2} \iff y' = 1 + \dfrac{y}{x} + \left(\dfrac{y}{x}\right)^2$ Note $x \neq 0$.

~~Make~~ Seek $u(x) = \dfrac{y(x)}{x} \Rightarrow y(x) = x \cdot u(x) \Rightarrow y'(x) = u(x) + x \cdot u'(x)$

and substitute to get:

$$\cancel{u} + x u' = 1 + \cancel{u} + u^2 \Rightarrow x u' = 1 + u^2 \Rightarrow \frac{u'}{1+u^2} = \frac{1}{x}$$

$$\Rightarrow \frac{du}{1+u^2} = \frac{dx}{x}.$$

Solve for $x>0$: integrate $\Rightarrow \displaystyle\int \frac{du}{1+u^2} = \int \frac{dx}{x} \Rightarrow \arctan(u) = \ln(x) + C$

$$\Rightarrow u = \tan(\ln(x) + C) \Rightarrow \boxed{y(x) = x \cdot \tan(\ln(x) + C)}$$

general solution