Recitation Review
MAP 2302 – Ordinary Differential Equations I
April 22, 2016

1. Solve the following DE:

$$\frac{dy}{dt} + \frac{1}{t}y = \frac{\sin t}{t^2 y}, \qquad t > 0$$

2. (§2.2 #36) Solve the following DE:

$$(x^2 + 3xy + y^2)dx - x^2 dy = 0$$

3. A 100 gallon tank initially contains 50 gallons of salt water at a concentration of s_0 pounds of salt per gallon. The water is emptied from the tank at a rate of q gallons per minute. Salt water at a concentration of s_r pounds of salt per gallon enters the tank at a rate of r gallons per minute. The top of the tank is open, and, if filled beyond capacity, the excess will spill out.

   (a) Find an equation $v(t)$ for the volume of the salt water in the tank at time t that is valid before the tank becomes either full or empty.

   (b) Let $Q(t)$ denote the pounds of salt in the tank at time t. Write an initial value problem for $Q(t)$ that is valid before the tank becomes either full or empty.

   (c) If $r = 10$ and $q = 5$, at what time t will the tank become full?

   (d) If $r = 10$ and $q = 5$, write a differential equation that $Q_f(t)$ satisfies *after* the tank becomes full. (The subscript f denotes that the tank is full).

   (e) Without solving for $Q(t)$, what is the initial value for $Q_f(t)$, if $r = 10$ and $q = 5$?

   (f) If $r = 10$ and $q = 5$, what is the concentration in the tank after a long amount of time?

4. (§3.8 #12) A spring-mass system has a spring constant of 3 N/m. A mass of 2 kg is attached to the spring, and the motion takes place in a viscous fluid that offers a resistance numerically equal to the magnitude of the instantaneous velocity. If the system is driven by an external force of $3\cos(3t) - 2\sin(3t)$ N, determine the stead state response. Express your answer in the form $R\cos(\omega t - \delta)$.

5. (§2.6 #27) Solve the DE by making it exact:

$$1 + \left(\frac{x}{y} - \sin y\right) y' = 0$$

6. (§4.4 #4) Use variation of parameters to solve the DE:

$$y''' + y' = \sec t, \qquad -\frac{\pi}{2} < t < \frac{\pi}{2}$$

7. Solve the following IVP using the Laplace transform:

$$y'' + y = \delta(t - \pi)\cos t + g(t), \qquad y(0) = 1, \qquad y'(0) = 1, \qquad g(t) = \begin{cases} 0, & t < 7 \\ 1 & 7 \le t < 8 \\ t, & 8 \le t \end{cases}$$

8. Find the Laplace transform of the convolution:

$$\mathcal{L}\{f * g\}, \quad f(t) = \begin{cases} t, & 0 \le t < \pi \\ t - \pi, & \pi \le t < 2\pi \\ \cdots & \\ t - n\pi, & n\pi \le t < (n+1)\pi \\ \cdots & \end{cases}, \quad g(t) = |\sin t|$$