Name: ____________________

MAP 2302 – Differential Equations I

October 30, 2015

Test 2 Makeup

1. (25 points) Find the solutions of the nonhomogeneous differential equation

$$y'' + 2y' + 5y = 15t$$

that satisfies the initial conditions

$$y(0) = 0 \quad \text{and} \quad y'(0) = 0.$$

2. (20 points) Consider the nonhomogeneous differential equation

$$y''' - 4y'' + 4y' = g(x)$$

(a) Find a fundamental set of solutions of the associated homogeneous equation.

(b) Determine the <u>form</u> of a particular solution of the nonhomogenous equation given above for each of the following choices of $g(x)$. Do <u>not</u> determine the constants.

(i) $g(x) = 2x^2 + 1$

(ii) $g(x) = xe^{2x}$

(iii) $g(x) = e^x + \cos x$

3. (20 points) We seek a particular solution of the differential equation

$$y'' - 2y' + y = \frac{e^t}{t^2}$$

by the method of variation of parameters. Noethat $y_1(t) = e^t$ and $y_2(t) = te^t$ are linearly independent solutions of the associated homogeneous equation. We seek a particular solution in the form $y_p = u_1 y_1 + u_2 y_2$.

(a) Write down the two equations that u_1' and u_2' must satisfy.

(b) Solve the equations in part (a) for u_1' and u_2'.

(c) Find $y_p(t)$.

4. (20 points)

(a) Find all complex roots of the algebraic equation

$$z^4 + 64 = 0$$

(b) Using part (a), find a fundamental set of solutions of $y^{(4)}(t) + 64y(t) = 0$, where $y^{(4)}$ denotes the 4th derivative.

(c) Find by inspection a particular solution of the nonhomogeneous equation

$$y^{(4)}(t) + 64y(t) = 32$$

5. (15 points)

(a) Show that the set of functions

$$f(t) = t^2 + 9, \quad g(t) = 3t, \quad h(t) = (t+3)^2$$

is linearly dependent on any interval.

(b) Abel's theorem states that, if y_1 and y_2 are solutions of $y'' + p(t)y' + q(t)y = 0$, where p and q are continuous functions on an open interval I, then their Wronskian is equal to $W[y_1, y_2] = Ce^{-\int p(t)\,dt}$, where C is a constant. Using Abel's formula, determine the Wronskian of two solutions of $(b(t)y'(t))' + q(t)y(t) = 0$, where $b(t)$ is differentiable and $b(t) > 0$.