This activity is for attendance but does not count toward your grade otherwise.

Name: ____________________

1. Find $\lim\limits_{x\to 0} x^2 + x + 2$.

2. Find $\lim\limits_{t\to 1} \dfrac{t^2 + t - 2}{t^2 - 1}$.

3. Let $g(x)$ be a function such that $x^2 \le g(x) \le x^3$ for $x \in [1, \infty)$ and $x^3 \le g(x) \le x^2$ for $x \in (-\infty, 1]$. Find $\lim_{x\to 0} g(x)$.

4. Find $\lim\limits_{\alpha\to 3^+} \dfrac{\alpha^2 - 2\alpha + 2}{\alpha^3 - 5\alpha^2 + 8\alpha - 6}$.

5. Suppose $f(x)$ is a continuous function such that $\lim\limits_{x\to a^-} f(x) \ge 0$ and $\lim\limits_{x\to a^+} f(x) \le 0$. What is $f(a)$?

6. Let $g(x) = x^2$. Find $\lim\limits_{h\to 0} \dfrac{g(x+h) - g(x)}{h}$.

7. Let $b_n = \sum_{j=1}^{n} 2^{-j}$ be the nth partial sum of the geometric series with common ratio $r = 1/2$, and let $\varepsilon = 1/1000$. Find the smallest natural number N such that $|b_n - 1| < \varepsilon$ for all $n \geq N$. (Hint: $2^{10} = 1024$).

8. Suppose $f(x)$ is a strictly increasing function with domain $(-\infty, \infty)$, and $f(3) = 6$. If $f^{-1}(5) = 2$ and $f^{-1}(7) = 9$, find the largest $\delta > 0$ such that $|x - 3| < \delta$ implies $|f(x) - 6| < 1$.

9. Consider the sequence $(a_n)_{n=1}^{\infty} = 1.1, 1.01, 1.001, 1.0001, \ldots$ whose nth term is $a_n = 1 + 10^{-n}$. Prove this sequence converges to the value 1 by finding the smallest natural number N, for any $\varepsilon > 0$ such that $|a_n - 1| < \varepsilon$ whenever $n \geq N$. Write your solution in terms of ε. (Hint: you will need to use a logarithm and the least integer function $\lceil x \rceil$).

10. Prove the function $f(x) = x^2$ is continuous at the point $(1, 1)$ using the ε-δ definition of limit. That is, given an $\varepsilon > 0$, find a $\delta > 0$ in terms of ε such that $|x^2 - 1| < \varepsilon$ whenever $|x - 1| < \delta$. Your answer should be an equation for δ in terms of ε. (Hint: $|x - 1| < \delta$ means $-\delta < x - 1 < \delta$. Use the difference of squares on $|x^2 - 1|$. Assume $\delta < 1$, so that $\delta^2 < \delta$).