Name: Key Section: ___

MAP 2302 – Ordinary Differential Equations I

March 18, 2016

Quiz 6

1. A mass weighing 1 lb stretches a spring 6 in. The mass is pushed upward, contracting the spring a distance of 4 ft, and then set in motion with a downward velocity of 32 ft/s. Assume there is no damping.

Convert to feet

(a) Find the position function $u(t)$.

$mg = 1$ lb, $L = \frac{1}{2}$ ft, $u(0) = -4$, $u'(0) = 32$, $\gamma = 0$

$\Rightarrow m = \frac{1}{32}$ slugs, $k = 2$

$mu'' + \gamma u' + ku = 0 \Rightarrow \frac{1}{32}u'' + 2u = 0$

$\Rightarrow u'' + 64u = 0$

$r^2 + 64 = 0 \Rightarrow r = \pm 8i \Rightarrow u = A\cos(8t) + B\sin(8t)$

I.C.

$u(0) = -4 \Rightarrow A\cos(8\cdot 0) + B\sin(8\cdot 0) = -4 \Rightarrow A = -4$

$u'(0) = 32 \Rightarrow -8A\sin(8\cdot 0) + 8B\cos(8\cdot 0) \Rightarrow 8B = 32$

$\Rightarrow B = 4$

So, $\boxed{u = -4\cos(8t) + 4\sin(8t)}$

(b) Write the position function in the form $u(t) = R\cos(\omega_0 t - \delta)$. Identify the frequency, amplitude, and phase of the motion.
(Hint: If $u(t) = A\cos(\omega_0 t) + B\sin(\omega_0 t)$, then $R = \sqrt{A^2 + B^2}$ and $\tan\delta = B/A$).

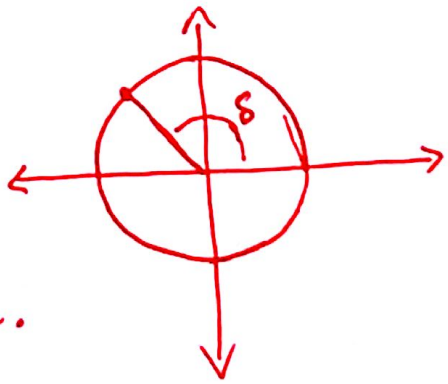
Freq. := $\boxed{\omega_0 = 8}$ (Period is $\frac{2\pi}{8} = \frac{\pi}{4}$)

Amp. := $R = \sqrt{(-4)^2 + (4)^2} = \sqrt{32} = \boxed{4\sqrt{2} = R}$

$\tan\delta = \frac{4}{-4} = -1$

Since $A < 0$ and $B > 0$,

this is in quadrant II,



so $\boxed{\delta = \frac{3\pi}{4}}$ is the phase.