§ 2.2 # 3. Solve $y' + y^2 \sin x = 0$

$$\frac{dy}{dx} = -y^2 \sin x \Rightarrow \quad \text{(separate)}$$

$$-\frac{dy}{y^2} = (\sin x)\, dx \Rightarrow \quad \text{(integrate)}$$

$$\frac{1}{y} = (-\cos x) + C \Rightarrow \quad \text{(solve for } y\text{)}$$

$$\boxed{y = \frac{1}{C - \cos x}}$$

Note: This was the quiz problem.

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§ 2.2 # 5. Solve $y' = (\cos^2 x)(\cos^2 2y)$

$$\frac{dy}{\cos^2 2y} = (\cos^2 x)\, dx \Rightarrow \quad \text{(trig. identities)}$$

$$\frac{1}{2}\int 2 \sec^2(2y)\, dy = \int \frac{1 + \cos(2x)}{2}\, dx \Rightarrow \quad \text{(integrate)}$$

$$\frac{\tan(2y)}{2} = \frac{x}{2} + \frac{\sin 2x}{4} + C \Rightarrow$$

$$\boxed{2\tan(2y) = 2x + \sin(2x) + C}$$

This uses:

$$\frac{d}{dx}\tan x = \sec^2 x \quad \text{and} \quad \cos^2 x = \frac{1 + \cos(2x)}{2}$$

§ 2.2 # 7. Solve $\dfrac{dy}{dx} = \dfrac{x - e^{-x}}{y + e^{y}}$

$(y+e^{y})\,dy = (x - e^{-x})\,dx \Rightarrow$ (integrate)

$$\boxed{\frac{y^2}{2} + e^{y} = \frac{x^2}{2} + e^{-x} + C} \quad (y+e^{y} \neq 0)$$

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§ 2.2 # 11. Solve $x\,d\cancel{x} + ye^{-x}dy = 0$, $y(0) = 1$

$x\,dx = -ye^{-x}dy \Rightarrow$ (separate)

$xe^{x}\,dx = -y\,dy \Rightarrow$ (integrate)

$(x-1)e^{x} + c = -\dfrac{y^2}{2}$

Now, solve for c:

$(0-1)e^{0} + c = -\dfrac{1}{2} \Rightarrow c = \dfrac{1}{2}$

Now, solve for y:

$y^2 = (2-2x)e^{x} - 1 \Rightarrow$ Since $y(0)=1$, take positive root

$$\boxed{y = \sqrt{(2-2x)e^{x} - 1}}$$

(valid on open interval between the two solutions of $(2-2x)e^{x} = 1$)

§2.2 # 13. Solve $y' = \dfrac{2x}{y + x^2 y}$, $y(0) = -2$

$$\frac{dy}{dx} = \frac{2x}{y(1+x^2)} \Rightarrow \text{(separate)}$$

$$y\,dy = \frac{2x}{1+x^2}\,dx \Rightarrow \text{(integrate)}$$

note: abs. not needed b/c $1+x^2 > 0 \ \forall x \in \mathbb{R}$

$$\frac{y^2}{2} = \ln(1+x^2) + C$$

solve for C: $\dfrac{(-2)^2}{2} = \ln(1+0^2) + C \Rightarrow$

$$2 = C$$

solve for y:

Since $y(0)=-2$, take negative root.

$$y^2 = \ln[(1+x^2)^2] + 4 \Rightarrow$$

$$y = -\sqrt{\ln[(1+x^2)^2] + 4}$$

(valid for all real #'s)

§2.2 # 17. Solve $y' = \dfrac{3x^2 - e^x}{2y-5}$, $y(0) = 1$

$$(2y-5)\,dy = (3x^2 - e^x)\,dx \Rightarrow \text{(Integrate)}$$

$$y^2 - 5y = x^3 - e^x + C$$

solve for C: $1^2 - 5 \cdot 1 = 0^3 - e^0 + C \Rightarrow$

$$-4 = -1 + C \Rightarrow C = -3$$

solve for y: $y^2 - 5y - x^3 + e^x + 3 = 0 \Rightarrow$

$$\boxed{y = \frac{5 - \sqrt{25 - 4(e^x + 3 - x^3)}}{2}}$$

(Valid between the the two roots of $25 = 4(e^x + 3 - x^3)$)

§ 2.2 # 23. Solve $y' = 2y^2 + xy^2$, $y(0) = 1$ & find minimum

$$\frac{dy}{dx} = y^2(2+x) \Rightarrow \quad \text{(separate)}$$

$$\frac{dy}{y^2} = (2+x)\,dx \Rightarrow \quad \text{(integrate)}$$

$$-\frac{1}{y} = 2x + \frac{x^2}{2} + C$$

Solve for C: $-\frac{1}{1} = 2\cdot 0 + \frac{0^2}{2} + C \Rightarrow C = -1$

Solve for y:

$$y = \frac{1}{1 - 2x - \frac{x^2}{2}}$$

Max of $1 - 2x - \frac{x^2}{2}$ is the same as min of y

For quadratic $ax^2 + bx + c$, max/min occurs at $\frac{-b}{2a}$,

so minimum of y occurs at $x = -2$.

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§ 2.2 # 33. Solve $\frac{dy}{dx} = \frac{4y - 3x}{2x - y}$

Homogeneous: substitute $y = vx$, $y' = v'x + v$

$$v'x + v = \frac{4vx - 3x}{2x - vx} = \left(\frac{4v-3}{2-v}\right)\frac{x}{x} = \frac{4v-3}{2-v} \Rightarrow$$

(partial fractions) $$v'x = \frac{4v-3}{2-v} - \frac{v(2-v)}{(2-v)} = \frac{v^2 + 2v - 3}{(2-v)} = \frac{(v+3)(v-1)}{2-v}$$

$$\Rightarrow \frac{2-v}{(v+3)(v-1)}\,dv = \frac{dx}{x} \Rightarrow \int \frac{-5/4}{v+3} + \frac{1/4}{v-1}\,dv = \ln|x|$$

$$\Rightarrow \ln|v+3|^{-5/4} + \ln|v-1|^{1/4} = \ln|x| + C \Rightarrow$$

$$\boxed{\left|\frac{y}{x} + 3\right|^{-5/4} \cdot \left|\frac{y}{x} - 1\right|^{1/4} = C \cdot |x|}$$

(can simplify further if desired)