Practice Skills Test 1 – Calculus I
Solutions

1. Find the domain of the function $f(x) = \dfrac{1}{x^2+1}$.

   Since $f(x)$ is a rational function, it is defined everywhere its denominator is not zero. Thus, it's domain is $\{x \in \mathbb{R} : x^2 + 1 \neq 0\}$. However,

   $$x^2 + 1 = 0 \iff x^2 = -1$$

   Since $x^2 = -1$ has no real solution, there is no $x \in \mathbb{R}$ such that $x^2 + 1 = 0$. Therefore, the denominator is never zero, and the domain of $f(x)$ is all real numbers. $\boxed{(-\infty, \infty)}$

2. Find the inverse function of $y = x + \dfrac{1}{x}$.

   To find the inverse function, we first reverse the roles of x and y to get the equation

   $$x = y + \frac{1}{y}$$

   and then solve for the new y. First, we eliminate the fraction by multiplying both sides of the equation by y.

   $$xy = \left(y + \frac{1}{y}\right) y = y^2 + 1$$

   Now, we have a quadratic function in y (it has a y^2 term), so we move all the terms to one side to set it equal to zero and put it in standard form. In this case, this means subtracting xy from both sides.

   $$0 = y^2 - xy + 1$$

   Now, we can use the quadratic formula with $a = 1$, $b = -x$, and $c = 1$ to solve for y in terms of x. We have

   $$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-x) \pm \sqrt{(-x)^2 - 4(1)(1)}}{2(1)} = \boxed{\frac{x \pm \sqrt{x^2 - 4}}{2}}$$

   Now, we must be careful to restrict our domain to ensure this inverse is actually a function.

3. If $f \circ g \circ h = \arctan(\sqrt{|x| + 5})$, find functions that can qualify as f, g, and h.

   We want meaningful functions, not functions like $f(x) = x$. Working from the outside inward, the first function on the right hand side is $\arctan(\cdot)$. Thus, we take $f(x) = \arctan(x)$ and set $(g \circ h)(x) = \sqrt{|x| + 5}$, and we have that $f((g \circ h)(x)) = \arctan(\sqrt{|x| + 5})$.

   Now, we can repeat this process on $g \circ h = \sqrt{|x| + 5}$. Working inward on the right, the first function is $\sqrt{\cdot}$, so, by taking $g(x) = \sqrt{x}$ and $h(x) = |x| + 5$, we have $g(h(x)) = (g \circ h)(x) = \sqrt{|x| + 5}$.

   Our solution is $\boxed{f(x) = \arctan(x),\ g(x) = \sqrt{x}, \text{ and } h(x) = |x| + 5}$. These are not the only possibilities. for instance, we could have chosen $f(x) = \arctan \sqrt{x}$, $g(x) = x + 5$, and $h(x) = |x|$.

4. Find infinite limit: $\lim\limits_{x \to \frac{\pi}{2}^-} \tan(x)$.

   As x approaches $\pi/2$ from the left, x is an angle in the first quadrant. In quadrant I, $\sin(x)$ and $\cos(x)$ are both positive, and $\lim_{x \to \pi/2^-} \sin(x) = 1$ and $\lim_{x \to \pi/2^-} \cos(x) = 0$. Thus,

$$\lim_{x\to\frac{\pi}{2}^-} \tan(x) = \lim_{x\to\frac{\pi}{2}^-} \frac{\sin(x)}{\cos(x)} = \boxed{\infty}$$

5. Determine the limit, if it exists: $\lim_{x\to 1} \frac{|x|-1}{x-1}$.

   When x is close to 1, x is positive, and, when x is positive, $|x| = x$. Thus, when x is close to one, we can replace $|x|$ with x, and we have

   $$\lim_{x\to 1} \frac{|x|-1}{x-1} = \lim_{x\to 1} \frac{x-1}{x-1} = \boxed{1}$$

   To be precise, "x is close to 1" means $|x-1| < 1$.

6. Find the limit, if it exists: $\lim_{x\to 2} |x| + 2$.

   When x is close to 2, x is positive, and, when x is positive, $|x| = x$. Thus, when x is close to two, we can replace $|x|$ with x, and we have

   $$\lim_{x\to 2} |x| + x = \lim_{x\to 2} x + x = \boxed{4}$$

   To be precise, "x is close to 2" means $|x-2| < 2$.

7. Obtain the limit, if it exists: $\lim_{u\to 2} \frac{\sqrt{5u+6}-1}{u-3}$.

   The function $f(x) = \sqrt{x}$ is continuous for $x > 0$, as are polynomials (like $5x+6$, $x-1$, and $x-3$) and quotients, provided the denominator is not zero. Since, for $u = 2$, $u - 3 = 2 - 3 = -1 \neq 0$ and $5u+6 = 5(2)+6 = 16 > 0$, our function is continuous at $u = 2$. Thus, we may simply substitute $u = 2$ to find the limit:

   $$\lim_{u\to 2} \frac{\sqrt{5u+6}-1}{u-3} = \frac{\sqrt{5(2)+6}-1}{(2)-3} = \frac{\sqrt{16}-1}{-1} = -(4-1) = \boxed{-3}$$

8. Is $f(x) = \pi x \cos\left(\frac{\pi}{x}\right)$ an even or odd or neither function?

   To test whether a function is even, odd, or neither, we substitute $-x$ for x and see if $f(-x) = f(x)$ (even), $f(-x) = -f(x)$ (odd), or neither. We have

   $$f(-x) = \pi(-x)\cos\left(\frac{\pi}{(-x)}\right) = -\pi x \cos\left(-\frac{\pi}{x}\right) = -\left[\pi x \cos\left(\frac{\pi}{x}\right)\right] = -f(x)$$

   Here, we used the fact that cosine is even to turn $\cos(-\pi/x)$ into $\cos(\pi/x)$. Since $f(-x) = -f(x)$, our function is $\boxed{\text{odd}}$.

9. A bacteria culture starts with 300 bacteria and doubles in size every half hour. How many bacteria are there after 4 hours?

   Every half an hour, the previous population is doubled. Thus, if n is the number of half hours since the start, the population will have doubled n times. This means the number 300 multiplied by n twos, or $2^n \cdot 300$. Since there are 8 half hour periods in 4 hours, the population is $2^8 \cdot 300 = 256 \cdot 300 = \boxed{76800}$.

10. Find the domain of $f(t) = \sqrt{1-3^t}$.

    Since our function have a square root, which is only defined for nonnegative numbers, what goes into the square root cannot be negative. Thus, we must satisfy the condition $1 - 3^t \geq 0$, which is the same as $1 \geq 3^t$. Since $\log_3(x)$ is an increasing function, it preserves inequalities, so we can take the base three log of both sides to get $\log_3 1 \geq \log_3 3^t$. The log of one is zero in any base, and $\log_3 3^t = t$. Thus, $0 \geq t$, i.e., our domain is $\boxed{(-\infty, 0]}$.