Calculus I (MAC 2311)
Skills Test 2 Version C
Friday, March 6, 2015
Time: 30 minutes

Write your answer for each question on the **Answer line**. Only the answer on the line will be graded. Neatly do the work to support your answer in the blank space provided. Each question is worth 1 point. Note: Use of any calculator will be considered as academic dishonesty.

Name: _____

Section and NID/PID: _____

1. Find an equation of the tangent line to the curve $y = 3x^2\sqrt{x}$ at the point $(1, 3)$.

$y = 3x^2\sqrt{x} = 3x^2 \cdot x^{1/2} = 3x^{5/2}$ → $y - 3 = \frac{15}{2}(x-1)$

$y' = \frac{5}{2}(3x^{5/2-1}) = \frac{15}{2}x^{3/2}$

$y'(1) = 15/2$

Answer: $y = (15/2)x - 9/2$

2. The position function of a particle is $s(t) = 5t^3 - 2t^2 + t + 4$, where s is measured in centimeters and t in seconds. Find the acceleration of the particle as a function of time.

Velocity is $s' = 15t^2 - 4t + 1$

acceleration is $s'' = 30t - 4$

Answer: $30t - 4$

3. If $f(x) = x^4e^x$, find $f'(x)$.

$f'(x) = \frac{d}{dx}(x^4) \cdot e^x + x^4 \frac{d}{dx}(e^x)$

$= 4x^3e^x + x^4e^x$

Answer: $x^3e^x(4+x)$

4. If $f(x) = \dfrac{1+x^2}{x-1}$, find $f'(0)$.

$f'(x) = \dfrac{\frac{d}{dx}(1+x^2) \cdot (x-1) - (1+x^2)\frac{d}{dx}(x-1)}{(x-1)^2}$

$= \dfrac{2x(x-1) - (1+x^2)}{(x-1)^2}$

$f'(0) = \dfrac{2(0)(0-1) - (1+(0)^2)}{(0-1)^2} = \dfrac{0-1}{1} = -1$

Answer: $f'(0) = -1$

5. If $y = \dfrac{1}{\sin x - 1}$, find y'.

$$y' = \frac{(\sin x - 1)\frac{d}{dx}(1) - (1)\frac{d}{dx}(\sin x - 1)}{(\sin x - 1)^2}$$

$$= \frac{0 - \cos x}{(\sin x - 1)^2}$$

Answer: $y' = -\dfrac{\cos x}{(\sin x - 1)^2}$

6. Let $g(x) = (x^4 - 3x + 2)^{100}$. Find $g'(x)$.

$$g'(x) = 100\,(x^4 - 3x + 2)^{99}(4x^3 - 3)$$

Answer: $g'(x) = 100(4x^3 - 3)(x^4 - 3x + 2)^{99}$

7. Find $\dfrac{dy}{dx}$ implicitly: $x^2 - xy + y^2 = 1$.

$$\frac{dy}{dx} \Rightarrow 2x - y - x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx}(2y - x) = y - 2x$$

Answer: $\dfrac{dy}{dx} = \dfrac{y - 2x}{2y - x}$

8. Find $f'(x)$ if $f(x) = x^2 \sin^{-1} x$.

$$f'(x) = \frac{d}{dx}(x^2)\cdot \sin^{-1} x + x^2 \cdot \frac{d}{dx}(\sin^{-1} x)$$

$$= 2x \sin^{-1} x + x^2 \frac{1}{\sqrt{1 - x^2}}$$

Answer: $2x\sin^{-1} x + \dfrac{x^2}{\sqrt{1 - x^2}} = f'(x)$

9. If $f(x) = \ln(x^4 + 2x + 1)$, find $f'(x)$.

$$f'(x) = \frac{1}{x^4 + 2x + 1}(4x^3 + 2)$$

Answer: $f'(x) = \dfrac{4x^3 + 2}{x^4 + 2x + 1}$

10. Find the points on the curve $y = 2x^3 - 12x^2 + 18x + 1$ where the tangent line is horizontal.

Set $y' = 6x^2 - 24x + 18 = 0$

$$\Rightarrow 6(x^2 - 4x + 3) = 0$$

$$\Rightarrow (x - 3)(x - 1) = 0$$

Answer: $x = 1, 3$ or $(1, 9), (3, 1)$

The End.