

Name: _____ Section: _____
MAA 2302 – Ordinary Differential Equations I
September 25, 2015
Test 1 – Version B

1. (20 points) Find the solution of the initial value problem:

$$\begin{aligned}ty' + (t + 1)y &= t, & t > 0 \\ y(\ln 2) &= 2\end{aligned}$$

2. (20 points) Classify each of the following differential equations as

(i) linear

(iii) Bernoulli

(ii) homogeneous

(iv) separable

and solve one of them:

(a) $x \frac{dy}{dx} + y = \frac{1}{y^2}, \quad x > 0$

(b) $x \frac{dy}{dx} = y + 2xe^{y/x}, \quad x > 0$

3. (15 points) Show that the differential equation

$$2x + y^2 + 2xyy' = 0$$

is exact and find the solution that satisfies the initial condition $y(2) = 10$.

4. (20 points) A skydiver weighing (with equipment included) 200 lbs falls vertically downward from a very high altitude with initial velocity $v(0) = 0$ and opens the parachute after 30 seconds of free fall. Assume that the force of air resistance, which is directed opposite to the velocity, is of magnitude $3|v|$ when the parachute is closed, and is of magnitude $20v^2$ when the parachute is open.

(a) Find the differential equation for the velocity $v(t)$ before the parachute opens.

(b) Find the differential equation for the velocity for $t > 30$ seconds.

(c) State the conditions that you need to use to determine the values of the constants in the solutions of the differential equations in parts (a) and (b), but do NOT solve the equations.

(d) Find the limiting velocity.

5. (24 points) Short questions (unrelated).

(a) Find all values of r for which $y = x^r$ is a solution of $x^2 y'' - 2y = 0$.

(b) Construct a first order linear differential equation such that all its solutions are asymptotic to the curve $y = t^2 - 1$.

(c) Determine (without solving the problem) an interval in which the solution of the initial value problem

$$\begin{aligned}(4 - t^2)y' + 2ty &= 3t^2 \\ y(1) &= 2\end{aligned}$$

is certain to exist.