EEE4119F Lander Project Report

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I. Introduction

A. Background

Following a nuclear war on Earth, a group of astronauts ventured to determine whether a planet could be a suitable home for the remaining humans. The rocket due to land requires a landing system for the crew to carry out their mission.

B. System Parameters

The lander was approximated to be a rectangular body with a mass of 4000kg, a L1 = 6m and L2 = 8m. There are two thrusters at the base of the rocket with the force limited to F ¡ Fmax. A position sensor was fixed to the rocket's centre of mass and measured position with respect to the ground landing position. The centre of mass of the lander was determined through simulation as it did not coincide with the geometric centre. There was an angle sensor which measured the angle of the rocket with respect to the horizontal axis where counterclockwise is positive.

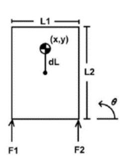

Fig. 1: Abstracted model of the rocket

C. Objectives

The aim of this project is to model the rocket and planet by determining the centre of mass position and the gravity of the planet through simulation. A controller is required to manoeuvre the landing. The success of the lander system was dependent on controlling the rocket in three scenarios.

- Scenario 1: The rocket must land after starting upright with zero velocity.
- Scenario 2: The rocket must land after starting with an initial angle of ±0.5 rad.
- Scenario 3: The rocket must land after starting upside down with an initial angular velocity of ±0.1 rad.

The rocket starts at a height greater than 900m above the landing position with zero x offset. Upon a successful landing, the base of the lander had to reach the ground (y = 0) with a speed less than 2m/s and an angle less than $10°$.

II. System Modelling

A. Modelling the motion of the system

The table below details, chronologically, the parameters required to model the system.

System Mechanics		
Parameter	**Value**	**Rational**
Generalized Variables	q = [x; y; θ]	The rockets motion is restricted to these three variables
Frames	Inertial and Body	Two frames required for referencing all the scenarios
Lander Position	$[x; y; 0]$	The sensor reads position from the COM WRT to the ground
Lander Velocity	diff($[x; y; 0]$)	The derivative of position is velocity

TABLE I: Table showing the variables required to model the mechanics of the system

Generalized Forces		
Parameter	**Value**	**Rational**
Force Vectors	[0; Fi; 0]	Forces act in the y direction and are scaled by Rot matrices when θ changes
Force Position Vectors	$[\pm\frac{l_1}{2}, -(dL+\frac{l_2}{2}), 0]$	(x, y) is the (0, 0) reference point in the body.
Partial Force Derivative	diff(rF_i, q_j)	A partial derivative was computed for each generalized variable

TABLE II: Table showing the parameters for modelling the force inputs

Generalized Forces cont.		
Parameter	**Value**	**Rational**
Q_i vector	$\sum_{i=1}^{2}$ transpose$(F_{i_0}) \cdot \frac{\partial dr_i}{\partial q_i}$	Each force was multiplied by each partial derivative
Force Position Vectors	$[\pm\frac{l_1}{2}, -(dL + \frac{l_2}{2}), 0]$	(x, y) is the (0, 0) reference point in the body.
Q matrix	$\begin{bmatrix} -sin(\theta) \cdot (F1 + F2) \\ cos(\theta) \cdot (F1 + F2) \\ -\frac{l1 \cdot (F1 - F2)}{2} \end{bmatrix}$	Concatenating the Q vectors

TABLE III: Table showing the variables required to model the mechanics of the system

Energy and Manipulator Equation		
Parameter	**Value**	**Rational**
Kinetic Energy	$\frac{1}{2}m\text{Lander}_v^T\text{Lander}_v + \frac{1}{2}\omega^T I_{zz}\omega$	The sum of linear energy and rotational energy
Potential Energy	mgy	Potential energy is determined by the y position.
Mass Matrix	$\begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & I_{zz} \end{bmatrix}$	Computed by taking the second partial derivative of the Kinetic energy
Gravity Matrix	[0, gm, 0]	Partial derivative of the Potential energy
Coriolis Matrix	$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$	The matrix of the velocity product terms
Manipulator Equation	$M * ddq + C + transpose(G) = Q$	Equation that defines the motion of the system
EOM	$\begin{bmatrix} \frac{-\sin(\theta) \cdot (F1+F2)}{m} \\ \frac{\cos(\theta) \cdot (F1-F2) - gm}{m} \\ \frac{-(F1-F2) \cdot l1}{2I_{zz}} \end{bmatrix}$	Returns the solution of the manipulator equation.

TABLE IV: Table showing the energy of the system and the final equation of the system's motion

B. Modelling Gravitational Acceleration

The forces to the thrusters were set to zero and the rocket was initialized to mode 1, which resulted in the rocket free falling. The second derivative of y represents the acceleration of the system. Newton's Second Law $F_{\text{net}} = Ma$ was used to compute the value for gravitational acceleration. Since it was indicated that the position sensors have noise, multiple tests were taken and averaged out.

Test	**Force (N)**	**Acceleration (m/s^2)**
1	0	-8.574
2	1000	-8.574
3	10000	-8.575

TABLE V: Table computing various tests in determining gravitational acceleration

C. Modelling Moment of Inertia

Moment of inertia determines a body's behaviour under rotation; therefore, the equation $T = I_z\alpha$ was used to determine I_z. The right thruster was set to zero, and the left thruster was given a 1000 N force, which caused the rocket to rotate. The angular acceleration was observed from the $\ddot{\theta}$ output to be $-0.048\,\text{rad/s}^2$. The torque was found using F multiplied by half of the base as a perpendicular distance ($3\,\text{m} \Rightarrow T = 3000\,\text{Nm}$). Only one test was done for determining I_z since there was no specified noise on the sensor. I_z was found to be $62500\,\text{kg}\cdot\text{m}^2$.

D. Determining the distance between the geometric centre and the COM, dL

The link between the moment of inertia about the COM and the geometric centre is the distance d_L, which can be found using the parallel axis theorem, by $I_{\text{COM}} = I_{\text{GC}} + M \cdot d_L^2$. Since the body's density function was unknown, it was a reasonable assumption that the body was homogeneous and the moment of inertia about the geometric centre was given by $I_{\text{GC}} = \frac{1}{12}M(l_1^2 + l_2^2)$. After solving for I_{COM} and I_{GC}, dL came out to be 2.7m.

III. Control Scheme

A. State Space

State space is a better method for handling MIMO systems when compared to transfer function representation. The following equations define a systems state space:

$$\dot{x}(t) = Ax(t) + Bu(t)$$

$$y(t) = Cx(t) + Du(t)$$

Essential to the control of the system is the controller gain. Pole Placement and LQR are methods which generate gain matrices to complete the standard state space model. Pole Placement is higher level which allows for direct control given specifications however it is more suited to systems with fewer parameters. LQR on the other hand uses a cost function to reduce the error to varying degrees depending on the cost parameters given. Due to the multi-variable nature of this task, the LQR method was chosen for gain selection.

A reference gain is also used to normalize the input for control however in this system the reference is **0** therefore a gain serves no purpose.

B. Linearization around a Stable point

Non-linear systems are difficult to control however through the use of linearization, non-linear systems can be approximated to linear systems. This is accomplished using feedback linearization (dynamic) or static linearization. Static linearization initially linearizes a system to a final stable state which the controller aims to reach. Dynamic linearization continuously re-linearizes the system using a non-linear transfer function.

The rocket's final state, [dx dy $d\theta$ x y θ] = [0 0 0 0 0 0] is considered a stable operating point therefore it was chosen as the linearization point. The stable control input is required such that the rocket hovers above the landing point before the engine switches off therefore the linearized control input is $\frac{mg}{2}$ per thruster.

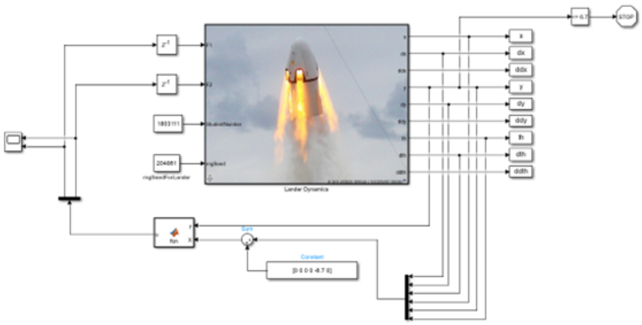

Fig. 2: Control System

The lander dynamics block modelled the rocket and its parameters for motion. An output X, the state variables, was generated by the lander dynamics block and multiplexed into a state vector. The state vector was subtracted by the set point which was the linearization point discussed previously. A slight difference in the y set point was due to the sensor having an offset of 6.7m from the base, therefore a $-6.7m$ y set point allows the base to land and the system to stop. The control input was found by using the LQR gain and multiplying it by the state vector. Two delay blocks were necessary since the control input would attempt to change the force while it was being changed resulting in an error.

A was computed by evaluating $\frac{\partial f}{\partial x}$ at the equilibrium point (x^{*}, u^{*}). Similarly, B was computed by evaluating $\frac{\partial f}{\partial u}$ at (x^{*}, u^{*}). Small angle approximation was used to remove the non-linear terms from A and B.

C. Optimizing the controller (Milestone 2)

1) Mode 1: The Q matrix (LQR) was initialized to 1000 for each of its parameters. The rocket was not able to land since the controller attempted to decrease the y velocity to zero to meet the set point value. The changes that followed are documented in the table below. Note: increases in each iteration are done on the previous iteration and the parameters were normalized by dividing by 1000 after the second iteration.

Variation	Rationale	Effect
10x (y) inc	dy was constrained, priority was shifted to reaching the y set point.	y tracking improved however lander speed is $10m \cdot s^{-1}$ at y = 0.
50x (dy) inc	final dy was too large therefore prioritize dy	The rocket landed successfully in 72s
25x (dy) inc	The landing time was too long	The rocket landed successfully in 19s

TABLE VI: Table computing significant iterations for mode 1

2) Mode 2: Due to the various stages of motion in mode 2, Gain Scheduling was chosen as the control method. Gain Scheduling allows for independent control gains to be selected depending on the stage of the journey. After running the simulation with the gain control for mode 1, a 3-stage gain was chosen to control the landing. The first stage intended to correct the θ error, the second stage intended to correct the x offset caused by the first stage and the last stage focused on landing the rocket safely. The following table details the iterations. Parameters for Q were normalized to 1 to start.

Variation	Rationale	Effect
100000x (θ) inc AND 50000x ($d\theta$) inc	Using experience from mode 1, costs were set large to start. Emphasis was placed on fixing θ before the rocket strays whilst ensuring the angular velocity didn't overshoot	The angle was correct however the rocket strayed horizontally as it did not correct quickly enough.
Several tests	Increases were done on θ until a fast enough gain was found	The angle corrected itself better after each iteration.
100x ($d\theta$) dec AND 10000x (θ) inc	The θ was increased to speed the angle correct and $d\theta$ was decreased to allow the angle to correct whilst not heavily constraining the angular velocity	The rocket corrected its angle however it strayed in the x direction
100000x (x) inc	To prevent the rocket from straying horizontally	The rocket corrected its angle and ventured back to x = 0 in 4s

TABLE VII: Table computing significant iterations for mode 2, stage 1

This stage was used for the first 50m of descent as the rocket moved vertically whilst attempting to correct θ and x.

All Q parameters were normalized to 1.

Variation	Rationale	Effect
1000000x (y) inc	In this stage, getting y to reach its set point as quickly as possible was the priority	The rocket travelled vertically however not quickly enough.
100x (y) dec	Increasing the gain on y counters dy since a large negative velocity is required to reach the setpoint	The max speed increased more than 3 times
Free-fall	To save fuel and allow the rocket to reach its maximum possible speed. The free-fall gain was found by several iterations of finding a gain value that forces the control input to be a negative number.	The max speed increased by 9 times.

TABLE VIII: Table computing significant iterations for mode 2, stage 2

The free-fall motion occurred after the angle correction until the last 250m of the journey. A small correction on the x position ensured that the rocket travelled vertically only.

Variation	Rationale	Effect
normalize (y) AND 100x (dy) inc	The priority was to decrease the speed quickly enough to reach the base at a speed less than specified.	The rocket slowed down however it hit the ground 5 times faster than required.
20x (dy) dec	The gain aims to decelerate quicker.	The rocket decelerated to zero however it hovered about the base at a height of 6m.
50x (y)	This aimed to track the position set point better.	The rocket landed successfully.

TABLE IX: Table computing significant iterations for mode 2, stage 3

Mode 2 Optimization:

- The rocket stabilized quickly however, the controller was in stage 1 for too long therefore the controller focused on the angle rather than the altitude for stage 1.
- The rocket was allowed to free fall until a safe height to start decelerating.
- An optimized period for each stage was found to ensure slow regions were restricted to small distances without creating oscillations and the fast region was maximized to speed to landing.

D. *Optimizing the controller (Milestone 3)*

The controller designed for Milestone 2 was robust enough to achieve the specifications for Milestone 3 as the rocket was able to land successfully without any changes. Changes were made however to improve the landing operation and minimize the time.

The first stage of Milestone 2 was replaced by a stage that corrected the angular velocity as well as angle offsets.

Variation	Rationale	Effect
500000x (θ) inc AND 1000000x ($d\theta$) inc	A stronger priority was placed on fixing the angular velocity since the angular velocity causes unpredictable motion	The rocket re-orientated itself however it oscillated about x.
50000x (x) inc	The rocket should align itself with the lander without oscillations.	There were smaller oscillations.
2x (x) inc AND 5x (θ) inc	This aims to reduce the fine oscillations by getting the rocket to turn around without requiring angle correction.	The rocket completed a successful turn and aligned itself with the lander.

TABLE X: Table computing significant iterations for mode 3, stage 1

The gravity change had to be compensated for as the rocket hit the base at a high speed, therefore the gains were directly increased on dy. The free fall period of the motion was unchanged as the rocket was aligned in stage one and the rocket was able to land successfully in mode 3.

IV. Results

A. *Landing Time*

Mode	Best (s)	Worst	Average
1	18.3	22.3	20.7
2	21.3	29.1	23.4
3	24.7	35.2	28.8

TABLE XI: Table showing time results for the different modes

The time taken for the lander to hit the ground in free-fall was around 14.9 seconds. In mode 1, the lander averaged 20.7 seconds due to the lander decelerating at a safe rate for a successful landing. If the lander decelerates too quickly to zero, the astronauts may experience G-LOC. In mode 2, the increase in average time is due to the oscillation and the delay in the free-fall stage. The worst case in mode 2 is at a much higher deviation to the average than in mode 1 due to the random generation of the initial angle. In mode 3, the rocket travelled a longer distance with more random, non-zero initial parameters. As a result, the average time and worst time are worse than the previous modes.

Mode	Best (m)	Worst (m)	Oscillation Avg
1	0	0	1×10^{-5}
2	1×10^{-5}	1×10^{-2}	3
3	1×10^{-3}	1×10^{-1}	10

TABLE XII: Table showing x offset results for the different modes

In mode 2, there were minimal oscillations with the first overshoot resulting in the biggest x offset however the landing spot was accurate to the cm. In mode 3, the orientation correction resulted in a large first oscillation however the motion was corrected to the nearest tenth of a meter.

V. Conclusion

Automating the execution tasks, specifically landing the rocket in this project, requires control. State space and the transfer function method provide a system to control the rocket and perform the landing. State space was used as it provides greater control over multi-variable systems hence it was chosen for this project. It was partnered with LQR optimization as it was the intuitive method for controlling several parameters. Static linearization was chosen for the project as it provided a simple and resource-effective solution.

Three distinct modes were focused on in this project, each with added complexity. Mode 1 was easily controlled with one LQR gain however modes 2 and 3 required gain scheduling which focused on the different stages in the journey. The controller was able to control all three modes achieving a quick landing and a small x offset of the landing base. The combination of state-space modeling, LQR optimization, and gain scheduling enabled precise control over the rocket's attitude, altitude, and velocity, resulting in a smooth and accurate landing.