Lab 3 Report
Lead-Lag Compensation

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1 Introduction

Real world systems can be modelled using frequency domain representation and block diagrams. Modelling systems allow for analysis of the systems behaviour. In this lab, the aim is to perform experiments to improve the Kitti-Copter system's performance and achieve the desired specifications that were not able to be met in the last iteration by using a lead-lag circuit. A lead lag circuit can change the phase, gain and also shift the poles such that the response fits the specifications. Below is the block diagram showing the modelled Kitti-Copter from the previous iteration.

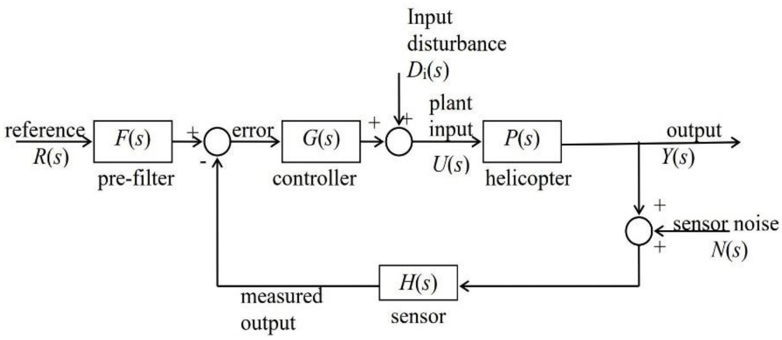

Fig 1: Block Diagram of Kitti-Copter system

Briefly, a reference is input to the system which is subtracted by the feedback. The proportional controller scales the error term. A more complex controller can change phase, response time and gain. The copter system processes the error and sends a value as output and feedback.

The aim of this lab is to improve the tracking of the input by decreasing settling time, overshoot and improve robustness to disturbances such as noise.

The controller is changed such that it provides phase correction, gain correction and proportional correction rather than simply proportional control.

2 System Modelling and Identification (Lab 2)

The copter system is unchanged from the prior experiment therefore the system model remains the same.

2.1 Modelled System

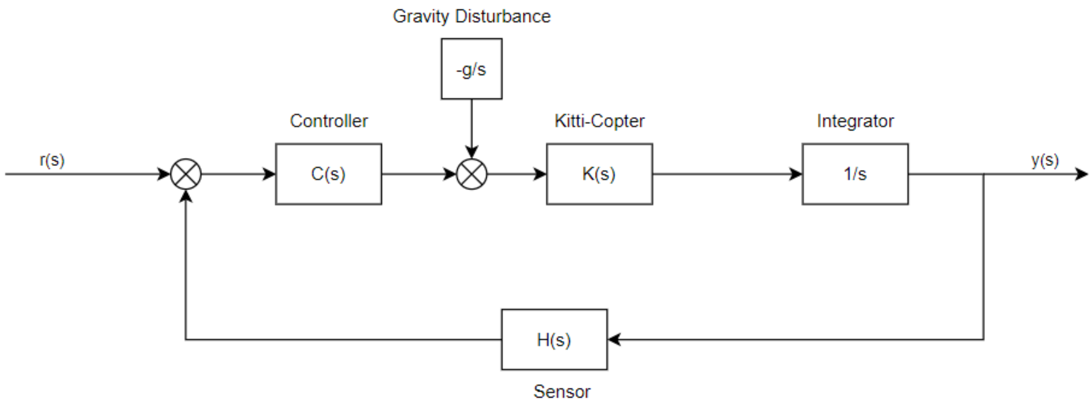

Fig 3: Block Diagram of the system

Lab 2 Findings	Values
Differential Equation modelling the system	$m\frac{d^2y}{dt^2} = -b\frac{dy}{dt} - mg + F_t(t)$
K sensor constant	0.65
Gain of copter (A)	22.096
Time constant (τ)	12.2
Model	$\frac{22.096}{12.2s+1}$

2.2 Method

1. The region where the design specifications were met was established.
2. The location of the zero and pole compensators were found.
3. The resistor and capacitor values were found.
4. The circuit was design and soldered on to the Veroboard.
5. The circuit was connected to the ADC and DAC and results were taken.
6. The data was plotted and analysed in this report.

3 Controller Design

The chosen controller is a proportional controller with a lead/lag circuit which produces an output that is proportional to the error term, corrects the gain and phase as well. Lead-lad circuits aid in minimizing settling time, overshoot and improving tracking.

3.1 Design Calculations

The root locus is a plot that shows how the closed loop poles change as the gain of the controller increases. When K > 0.0037 the system becomes oscillatory.

- The open loop poles are located at s = - 0.082 and s = 0.
- The centroid $\frac{-0.082-0}{2-0} = -0.041$ is the location where the poles start to become oscillatory.
- The time constant at the centroid is $\frac{1}{0.041} = 24.39\ (s)$.
- The specifications require a minimum improvement of 80% of the settling time. $0.8 \times 24.39 = 19.51(s)$
- Therefore, the new centroid point is $\frac{1}{19.51}$ = -0.051.
- The damping ratio is given by:

$$y_p = A\left(1 - e^{\frac{-\pi z}{\sqrt{1-z^2}}}\right)$$

$$1.05 = \left(1 - e^{\frac{-\pi z}{\sqrt{1-z^2}}}\right)$$

$$z = 0.697$$

- The angle in which the new pole must lie in is:

$$\cos\theta = 0.697$$

$$\theta = 45$$

- The pole must lie between 0 degrees and 45 degrees to meet overshoot specification.
- Choose 30 degrees are first iteration with -0.1 (to the left of -0.051) being the position of the centroid.
- The frequency is restricted:

$$-0.1\tan 30 = -0.057j$$

- The angle to the pole from the pole at s = 0 is -150 degrees
- The angle to the pole from the pole at s = -0.081 is:

$$\tan^{-1}\frac{0.057}{0.1-0.081} - 180 = -108.4\ degrees$$

- The sum is -256.6 degrees which is less than -180 to meet the angle criterion.
- Lead/Lag compensation is required.
- The sum of the zero, pole and current pole angles are:

$$-258.4 - \theta_p + \theta_z = -180$$

$$-\theta_p + \theta_z = 78.4$$

- The compensator zero and pole must add an extra 78.4 degrees to achieve the specifications.
- The minimum s axis position of the zero:

$$\tan^{-1}\frac{0.057}{s} = 78.4\ degrees$$

$$s = -0.0117 - 0.1$$

$$(The\ initial\ calculation\ was\ taken\ relative\ to\ the\ pole\ chosen\ of - 0.1\ )$$

$$s = -0.1117$$

- The zero must be between -0.1 and -0.113. Choose -0.105 as the location of the zero:

$$\theta_z = \tan^{-1}\frac{0.057}{0.005}$$

$$\theta_z = 84.98$$

- The angle of the compensator pole must be -6.58 degrees.
- The location of the pole is:

$$\tan^{-1}\frac{0.057}{s} = 6.58\,degrees$$
$$s = -0.386 - 0.1$$
$$(The\ initial\ calculation\ was\ taken\ relative\ to\ the\ the\ pole\ chosen\ of - 0.1)$$
$$s = -0.486$$

- Compensator Gain Calculation:

$$|KQ(s)| = 1$$
$$(0.65)K\left|\frac{-0.486(0.057j -\ 0.01 + 0.105)}{-0.11(0.057j - 0.1 + 0.486)}\right|\left|\frac{22.096}{(12.2(0.057j - 0.1) + 1)(0.057j - 0.1)}\right| = 1$$

$$K = 0.0086$$

3.2 MATLAB Simulations

Root Locus

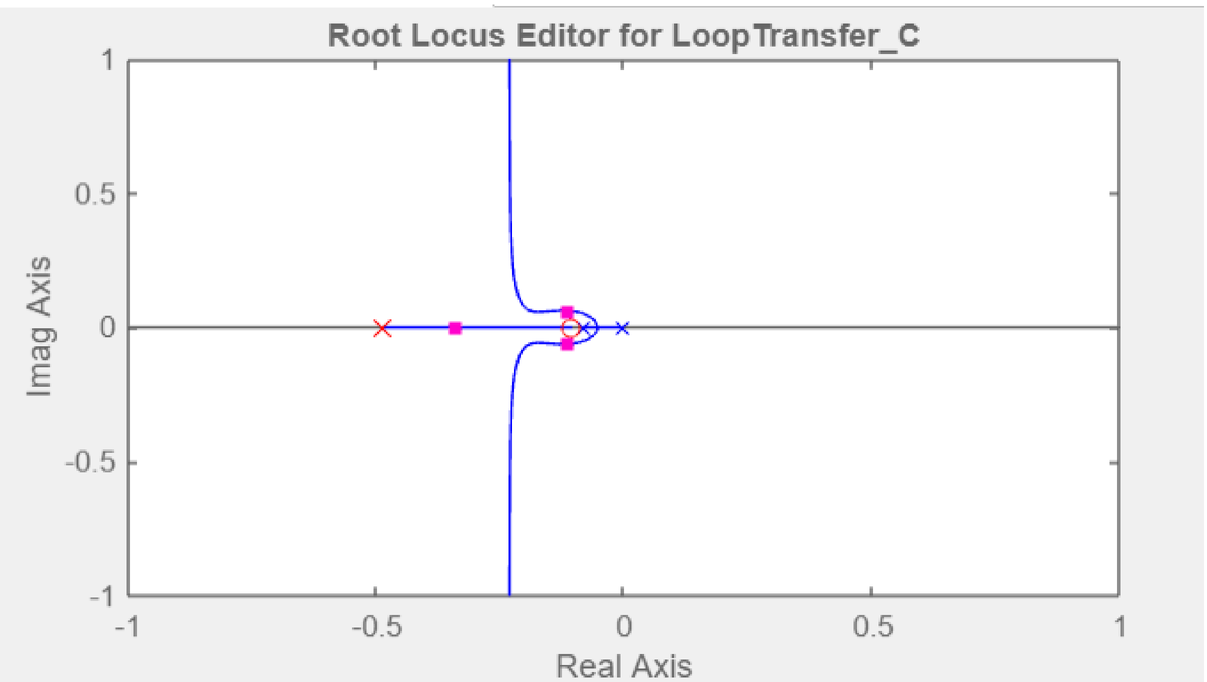

Fig 4: Plot showing Root Locus with the compensation poles and zeros.

The addition of the compensation pole and zero causes the centroid to shift to the left. Both poles move to the left of their initial positions hence causing the system to respond faster since response time is inversely proportional to frequency. At very high gains, the non-dominant pole begins to influence the response therefore gain must be kept low enough to neglect it.

System Response

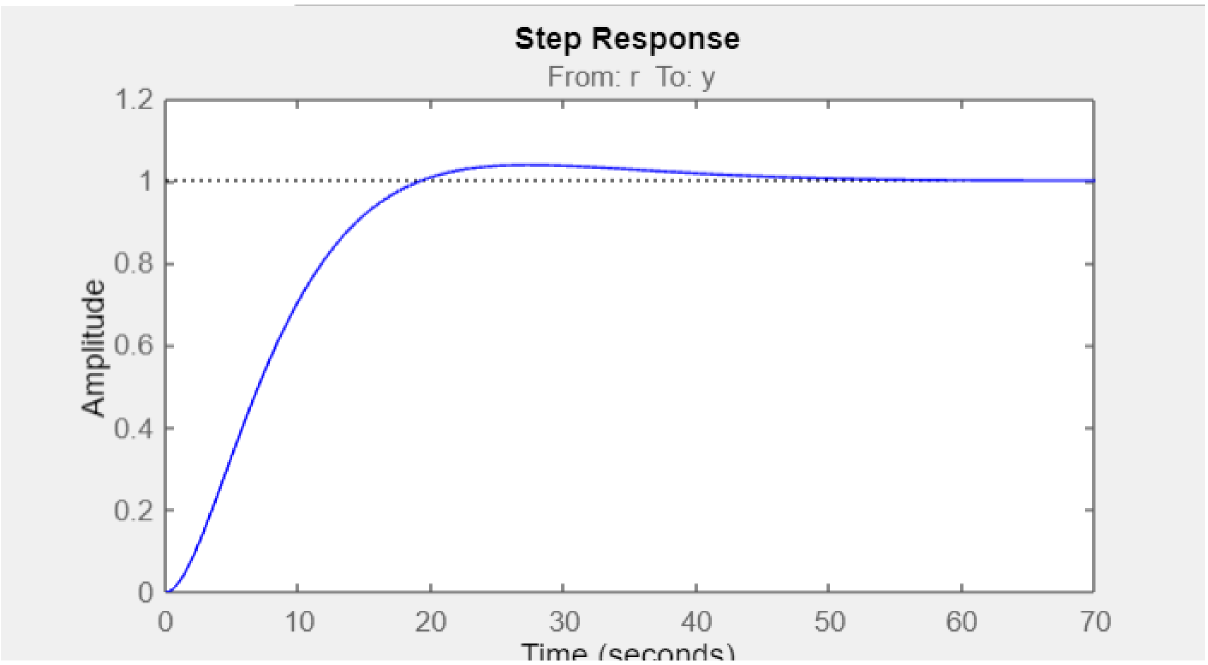

Fig 5: The step response to the compensated system

The simulated response meets the settling time, overshoot, and tracking requirements. The settling time is 35% better than the uncompensated system. The overshoot is restricted to 5% and the system tracks with 100% accuracy.

4 Circuit Design

4.1 Component calculation

$$\frac{0.105(9.52s+1)}{0.486(2.06s+1)} = \frac{R2}{R1} \times \frac{R1C1s+1}{R2C2s+1}$$

$$R1C1 = 9.52$$

$$R2C2 = 2.06$$

$$\frac{R2}{R1} = 0.216$$

Let R1 = 10K

R2 = 2.16K therefore choose 2.2K

Therefore C1 =952μF therefore choose 1000μF and C2 = 936.46μF. therefore choose 1000μF.

$$\frac{R4}{R3} = K$$

$$\frac{R4}{R3}(prop\ control\ resistor\ ratio) = 0.02(0.43)\ (0.02\ from\ proportional\ control)$$

$$R3 = 100000\Omega$$

$$\frac{R4}{100000} = 0.43$$

$R4 = 43K\Omega$ Choose 39KΩ

4.2 Circuit Schematic

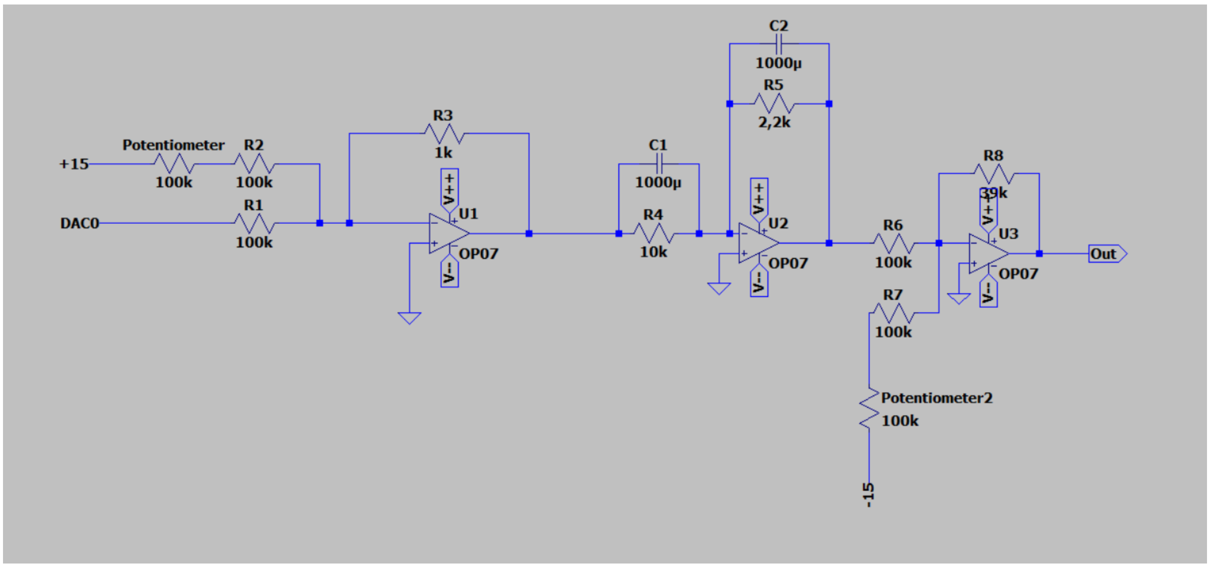

Fig 6: Diagram showing schematic of controller.

Potentiometer 2 serves as the offset to gravity (2.5V) which is manually adjusted. R1 and R2 are equal to provide unity gain for the convergence of the voltage from the potentiometer 2 and the DAC. The voltage from the potentiometer 1 is the set point. The input voltage is negative since it goes into the inverting terminal and reduces the need for an inverting op amp. The output of the DAC is the feedback from the copter system in the computer. Potentiometer 2 is the level shifter which balances the 2.5V offset to the ADC. The feedback resistor (Rf) in conjunction with R1 and R2 provides the gain as the controller would provide. The lead circuit connects to the output of the controller circuit (the first op amp).

4.3 Final Board

The wire coming out of the right most potentiometer represents -15Vand the left most potentiometer represents +15V. Current flows up and down if there are no drill bit scraped holes. The 100k resistor that is floating is the connection to DAC 0. The output of the Op Amp (pin 6) goes into ADC 1.

The back of the board has drilled holes done by a drill bit to prevent the op amps pins being connected to each other.

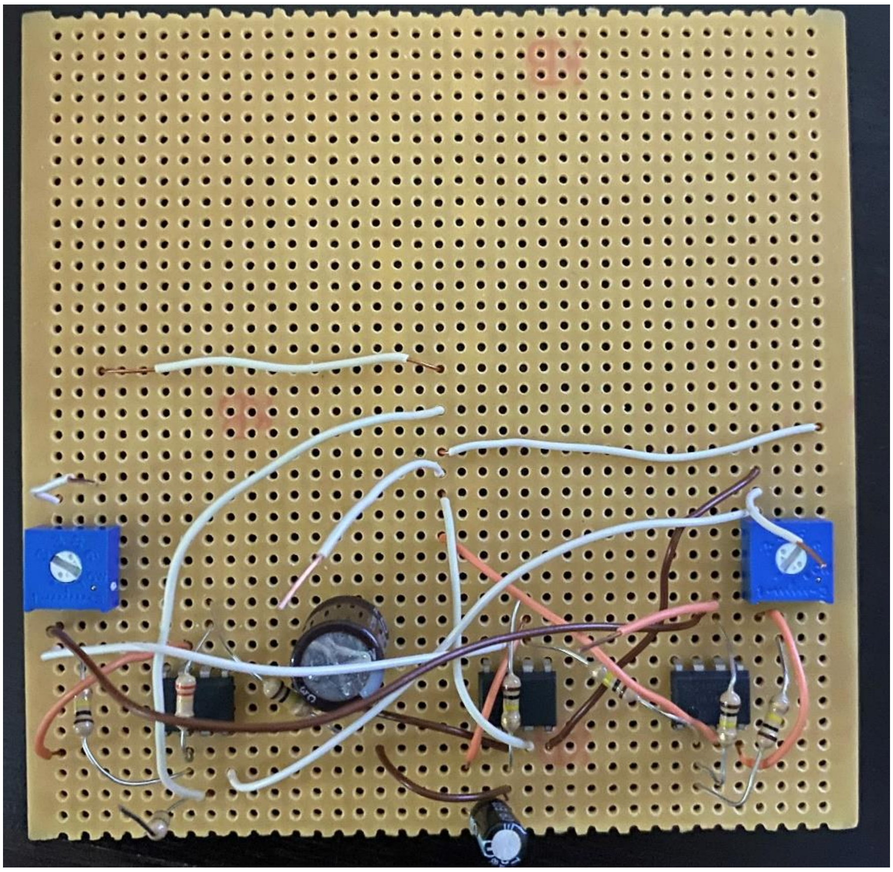

Fig 7: Top of Vero board

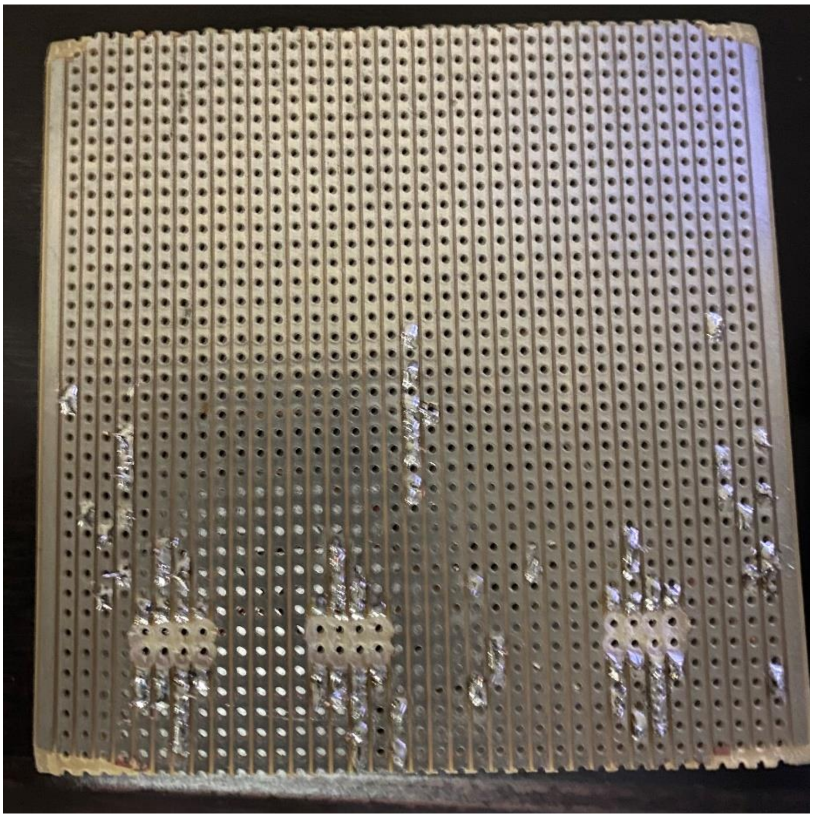

Fig 7: Back of Veroboard showing solder and drillings

5 Controller Testing

5.1 Results from Lab Testing

The tests were conducted several times, and an average result was calculated and plotted.

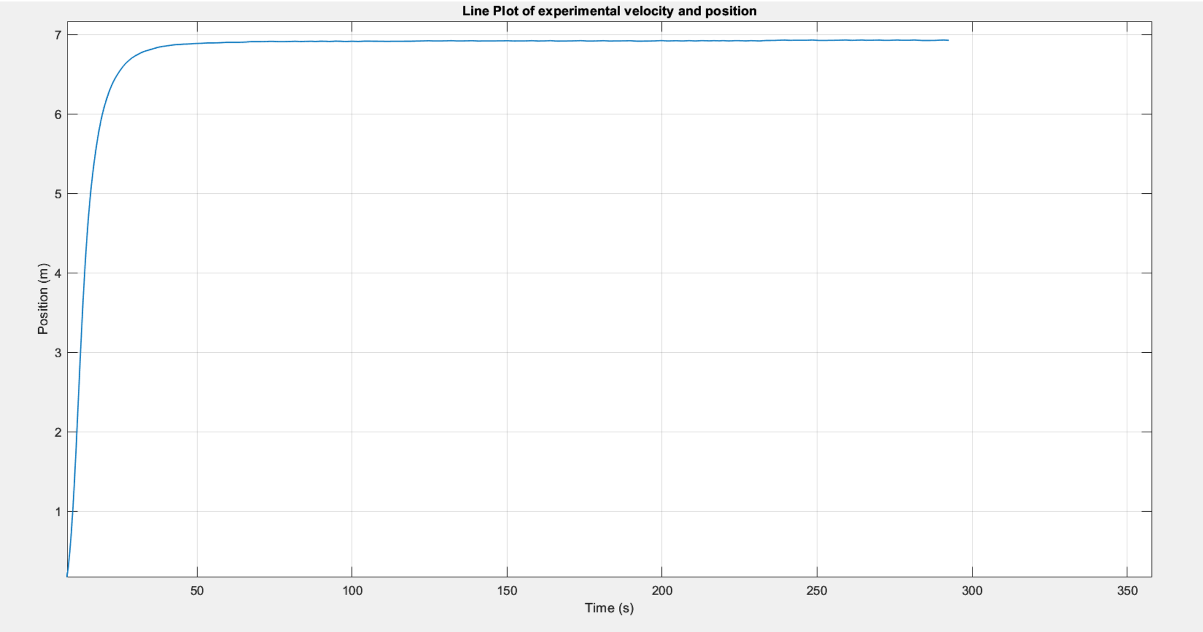

Fig 9: Plot showing response of the kitti-copter with the controller and compensator.

$$Overshoot = \frac{Max - Reference}{Referece} x\ 100$$

$$Overshoot = \frac{6.915 - 7}{7} x\ 100$$

$$Overshoot = 1.12\%$$

$$Settling\ time\ difference = \frac{Controlled\ time - Uncontrolled\ time}{Uncontrolled\ time}$$

$$Settling\ time\ difference = \frac{62 - 85}{85}$$

$$Settling\ time\ difference = 27\%\ improvement$$

- As it can be observed from the plot, there is a no observable overshoot, and the settling time has improved compared to the open loop and proportional controller only systems.
- It was possible to achieve all the specifications using a simple proportional controller. The dominant poles move left hence decreasing settling time since time is inversely proportional to frequency.
- The value for the controller and the gain did not match perfectly to calculations, this is due to resistor tolerances and compensating using a variety of test values.

- There were sources of error in the controls lab which also affected the controller performance.
- The machines in control lab are cheap and not precise as to track perfectly therefore the expected results are not achieved due to the window of parameters being so small.
- The gain calculated was smaller than the gain used in the circuit. This is due to adjusting the gain due to resistor tolerances, noise, and fluctuations.
- The controller was able to deal with robustness since fluctuations were dealt in the output.

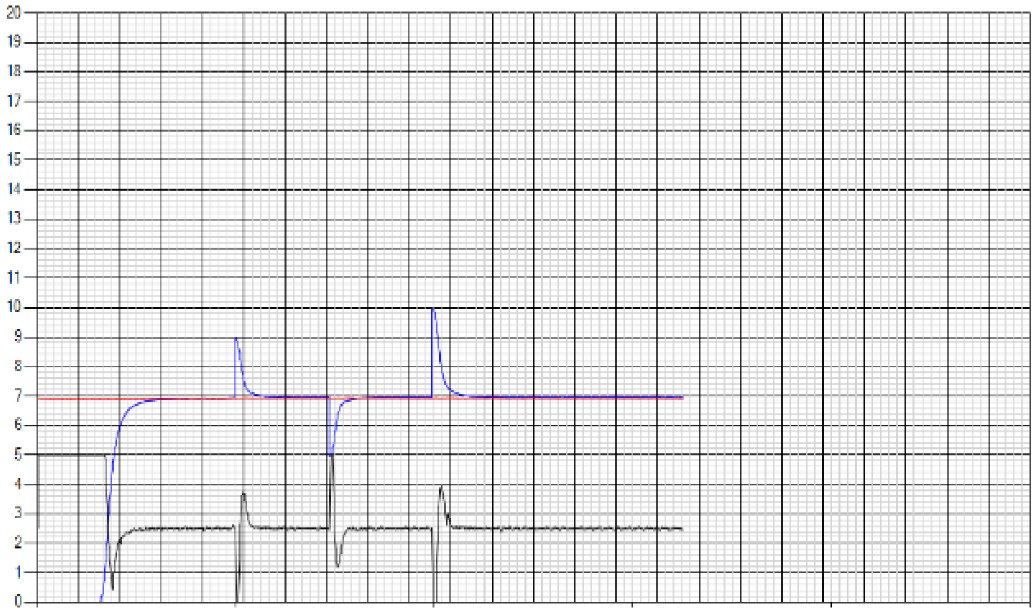

Fig 10: Plot showing robustness to disturbances.

5.2 Experiment analysis compared to calculations.

- The gain calculated was 0.0086 for the system however a gain of 0.041 achieved the specifications which is 5 times the calculated value.
- The time improvement was 27% which beat the calculated improved time percentage of 20%.
- The gain increasing meant that the error term also decreased thereby tracking closer to the set point.

5.3 Recommendations for Future Iterations

- To achieve a faster response time, chose a zero that is more left to the one chosen in this experiment. A zero that is left of the current value will result in the settling time decreasing and the increasing the gain threshold for the non-dominant pole to have an influence.
- Choose a gain and adjust the offset until the tracking is close to desired. If it's not possible to track with that gain, then pick a gain depending on the offset. I spent a large amount of time trying specific gain values that returned redundant results.
- Choose resistors in the thousands to hundred thousand range as the mega-ohm resistors are prone to noise.

- The capacitor values used were the same, it would be better to do another round of experiments where the capacitor values match the calculated values.
- Test the power in the soldered circuit after soldering every wire as shorts appear quickly.

6 Conclusion

The helicopter in the closed loop system flies off without control when the system has no controller. The controller aids in control by providing a correcting signal to the helicopter system. A proportional controller is a simple controller which improves control by providing a signal proportional to the error signal thereby improving the tracking.

In the previous laboratory, the parameters of the system tested resulted in the proportional controller being insufficient to meet all the specifications. Improving the

A controller that included a lead circuit improved the results in this experiment. The lead compensator improved the response time by 27%, overshoot by 15% and achieved 98% tracking.

The lead compensator provided an extra pole and zero which shifted the dominant poles left resulting in better settling time. Since the poles moved left, a smaller gain was required to improved settling time and hence improving overshoot since smaller gains lower the risk of overshoot.

7 References

[1] UCT EEE3094 Control Laboratory document

[2] *Control system designer* (no date) *MATLAB & Simulink Example*. Available at: https://www.mathworks.com/help/control/ug/getting-started-with-the-control-system-designer.html (Accessed: 08 October 2023).

[3] *Control system designer* (no date) *MATLAB & Simulink Example*. Available at: https://www.mathworks.com/help/control/ug/getting-started-with-the-control-system-designer.html (Accessed: 08 October 2023).