Progress Report On Orientation Determination With Maximum Likelihood Methods

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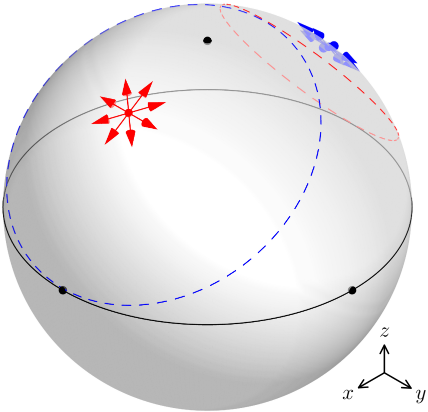
Microscope Geometry



1.1 and 0.71 NA

8 Poisson-distributed intensity measurements

Goal: Estimate the orientation of the dipole.

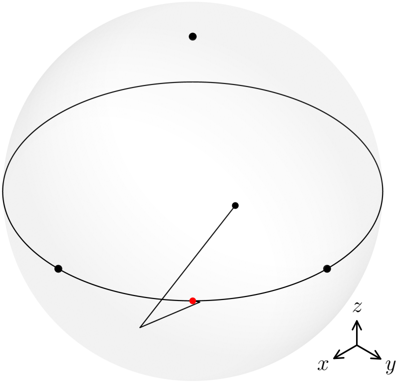
Fisher Scoring Algorithm

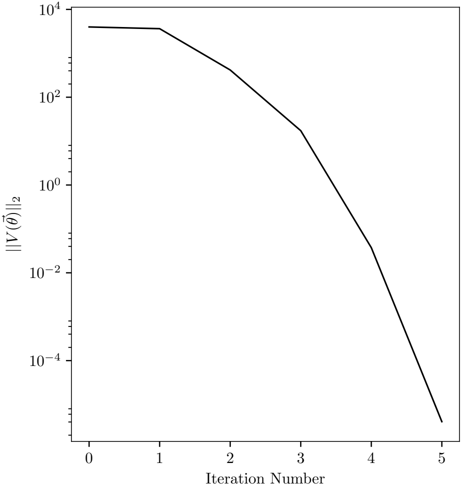
$$\begin{aligned} \vec{\theta} = \{\Theta, \Phi\} &\rightarrow \text{Parameters} \\ \vec{X} &\rightarrow \text{Data (8 intensity measurements)} \\ \mathbf{F}(\vec{\theta}) &\rightarrow \text{Fisher information matrix} \\ \vec{V}(\vec{\theta}, \vec{X}) = \frac{\partial}{\partial \vec{\theta}} \log L(\vec{\theta}, \vec{X}) &\rightarrow \text{Score} \end{aligned}$$

$$\vec{\theta}_{i+1} = \vec{\theta}_i + \mathbf{F}^{-1}(\vec{\theta}) \vec{V}(\vec{\theta}, \vec{X})$$

Fisher Scoring Run 1

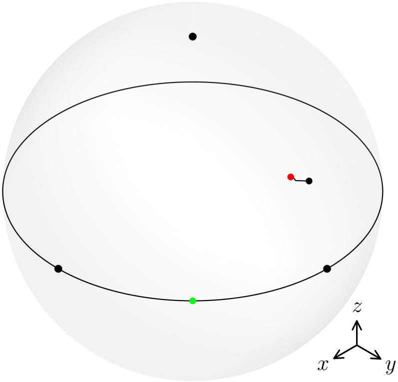
True orientation: $\Theta = \pi/2, \Phi = \pi/4$
Starting orientation: $\Theta = \pi/3, \Phi = \pi/3$

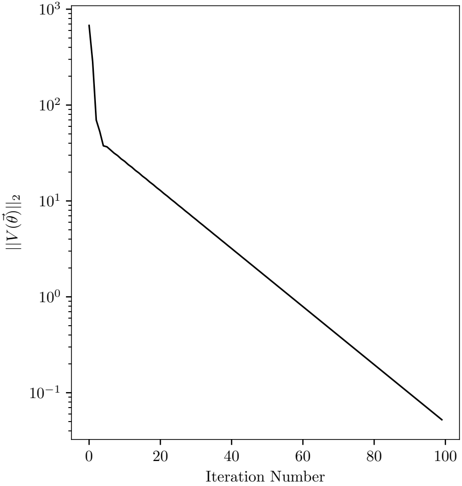




Fisher Scoring Run 2

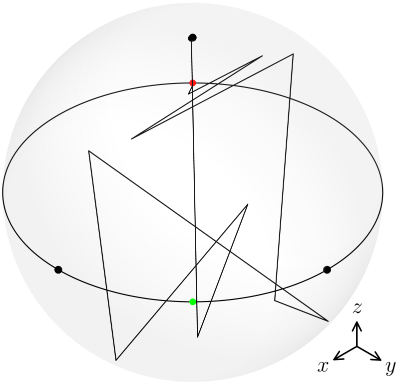
True orientation: $\Theta = \pi/2, \Phi = \pi/4$
Starting orientation: $\Theta = \pi/3, \Phi = \pi/2$

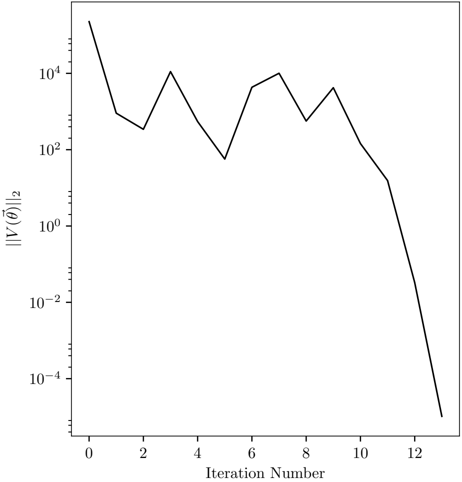




Fisher Scoring Run 3

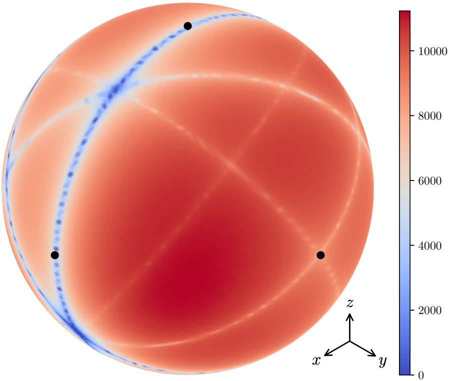
True orientation: $\Theta = \pi/2, \Phi = \pi/4$
Starting orientation: $\Theta = 0, \Phi = 0$





Log-likelihood as a function of the estimate orientation

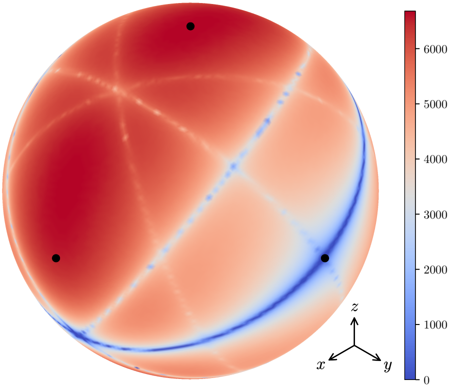
True orientation: $\Theta = \pi/2, \Phi = \pi/4$



Gradient descent isn't going to cut it.
I need to be careful with derivatives on curved spaces.

Log-likelihood as a function of the estimate orientation

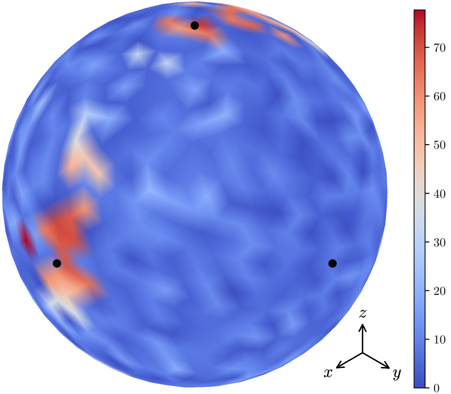
True orientation: $\Theta = 0, \Phi = 0$



Brute Force Optimization

Choose number of points (100).

Plot error in degrees (great circle arc) between truth and estimate as a function of truth.



Particle Swarm Optimization

Choose starting number of points (25).

Choose max number of function evaluations (100).

Plot error in degrees (great circle arc) between truth and estimate as a function of truth.