Update On 3D Orientation Reconstruction

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Spheroid Distribution

$\kappa = 0.1$ $\kappa = 0.25$ $\kappa = 0.5$ $\kappa = 1$ $\kappa = 2$ $\kappa = 4$ $\kappa = 10$

$$f(\vec{r}; \mathbf{A}) = \frac{r^T \mathbf{A}^{-1} r}{|\mathbf{A}|}$$

$$f(\vec{r}; \vec{r}', \kappa) = \frac{\kappa}{S(\kappa)\sqrt{1 + (\vec{r} \cdot \vec{r}')^2 \left(\frac{1}{\kappa^2} - 1\right)}}$$

$$S(\kappa) = 2\pi \left(1 + \frac{\kappa^2}{2\sqrt{1-\kappa^2}} \log\left[\frac{1+\sqrt{1-\kappa^2}}{1-\sqrt{1-\kappa^2}}\right]\right)$$

Forward Model: Single diSPIM Frame

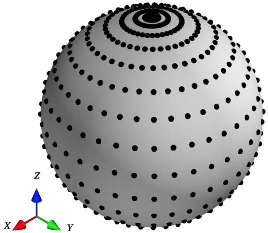
$$\text{Measured Intensity: } I = \int_{\mathbb{S}^2} d\vec{r}\; I^s(\vec{r}) f(\vec{r}; \vec{r}', \kappa)$$

$$\text{Single Fluorophore: } I^s(\vec{r}) = 2[A + B(1 - \cos^2\phi \sin^2\theta)] \times \sin^2\theta \cos^2(\phi - \phi_{\text{pol}})$$

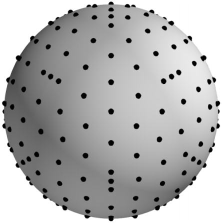
$$\text{Spheroid Distribution: } f(\vec{r}; \vec{r}', \kappa) = \frac{\kappa}{S(\kappa)\sqrt{1 + (\vec{r} \cdot \vec{r}')^2 \left(\frac{1}{\kappa^2} - 1\right)}}$$

- No luck integrating in closed form ;(

Integrating Faster With Spherical Quadrature



(A) Gaussian product grid with $N = 800$.



(B) Lebedev grid with $N = 266$.

Beentjes, 2012

- Instead of evaluating integrals one after the other and using Gaussian quadrature (left), evaluate the integral all at once using Lebedev quadrature (right)
- How many grid points do I need to use to accurately integrate? To within 1%? Perfectly? $\rightarrow$

Harmonic Analysis On The Sphere

$$\text{Measured Intensity: } I = \int_{\mathbb{S}^2} d\vec{r}\; I^s(\vec{r}) f(\vec{r})$$

- Rewrite $I^s(\vec{r})$ and $f(\vec{r})$ in terms of orthogonal spherical harmonics
- Expensive integral becomes a cheap dot product of coefficients
- With measurements under different polarization settings we get $\vec{g} = \mathcal{H}\vec{f}$ where $\vec{g}$ is a vector of intensity measurements, $\vec{f}$ is a vector of spherical harmonic coefficients of the fluorophore distribution, and the rows of $\mathcal{H}$ are the spherical harmonic coefficients of $I^s(\vec{r})$ for each measurement
- Null space of $\mathcal{H}$ corresponds to symmetries/degeneracies

Harmonic Analysis

- Harmonic analysis on $\mathbb{R}^n \to n$-D Fourier Transform
- Harmonic analysis on $\mathbb{S}^1$ (circle) $\to$ Fourier Series
- Harmonic analysis on $\mathbb{S}^2$ (sphere) $\to$ 2D Fourier Series

Spherical Harmonics

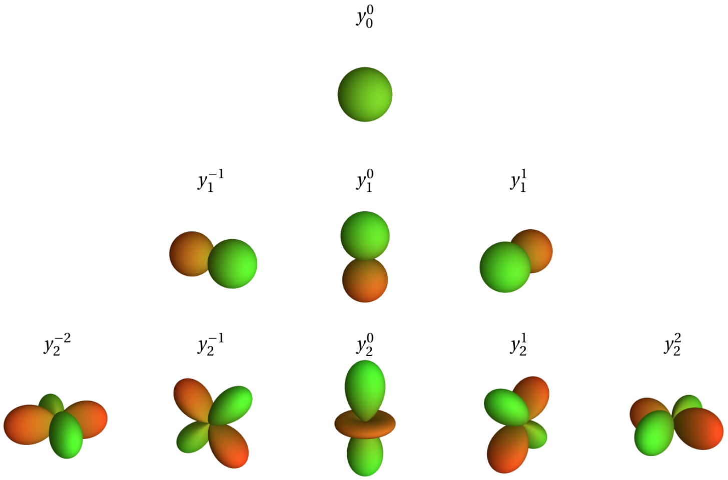


Figure B.1: Plots of the real-valued spherical harmonic basis functions. Green indicates positive values and red indicates negative values.

Jarosz, 2008

Single View diSPIM Frame

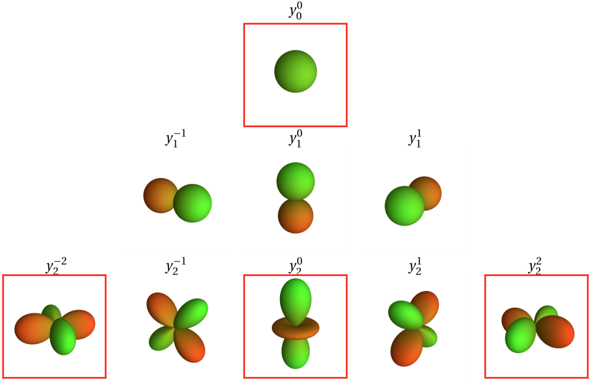


Figure B.1: Plots of the real-valued spherical harmonic basis functions. Green indicates positive values and red indicates negative values.

Jarosz, 2008

- Non-zero coefficients for single view diSPIM frame shown in red boxes

Early Conclusions + Upcoming Work

- Odd bands aren't antipodally symmetric so they aren't present in the sample.
- Zero coefficients on $y_2^{\pm 1}$. Corresponds to degeneracy? Investigating.
- Adding polarizers on the detection side with give us access to 4th order band. Still investigating degeneracy.
- A single fluorophore has non-zero components in all even bands (sort of like FT of $\delta(x)$ is 1). We can only measure the first few bands which constrains the accuracy of our estimates.
- Now we're estimating spherical harmonic coefficients instead of parameters of a hypothesized distribution. That's all of the info we can access.
- Upcoming—reconstruction in terms of matrix inversion!