Update On 3D Orientation Reconstruction

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Forward Model

$$g_i = \int_{\mathbb{S}^2} d\hat{\mathbf{r}}\; h_i(\hat{\mathbf{r}}) f_i(\hat{\mathbf{r}})$$

- $g_i \rightarrow$ intensity measurement
- $h_i \rightarrow$ point response function
- $f_i \rightarrow$ orientation distribution function

Fourier Transforms

$$\text{Fourier Transform} \rightarrow F(\nu) = \int_{\mathbb{R}} dx\ f(x) e^{-2\pi i x \nu}$$

$$\text{Spherical Fourier Transform} \rightarrow F_l^m = \int_{\mathbb{S}^2} d\hat{\mathbf{r}}\ f(\hat{\mathbf{r}}) \overline{Y_l^m(\hat{\mathbf{r}})}$$

Forward Model

$$g_i = \int_{\mathbb{S}^2} d\hat{\mathbf{r}}\ h_i(\hat{\mathbf{r}}) f_i(\hat{\mathbf{r}})$$

Simplifies to:

$$g_i = \mathbf{H}^T \mathbf{F}$$

- $\mathbf{H} \rightarrow$ is a vector of the Fourier coefficients of the point response function
- $\mathbf{F} \rightarrow$ is a vector of the Fourier coefficients of the orientation distribution function

Multiple measurements:

$$\mathbf{g} = \Psi \mathbf{F}$$

No Prior Reconstruction

$$\mathbf{F} = \Psi^{+}\mathbf{g}$$

If Ψ is full column rank then we can recover $\mathbf{F}$—the spherical harmonic coefficients passed by the system.

$\mathbf{F}$ is a representation of the true object projected onto the spherical harmonic components passed by the system.

Isotropic Excitation - Single Pixel z View

$$h(\theta, \phi) = \sin^2\theta =$$

$$\frac{4\sqrt{\pi}}{3} Y_0^0$$

- - -

- - $-4\frac{\sqrt{5\pi}}{15} Y_1^0$ - -

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Isotropic Excitation - Single Pixel x View

$$h(\theta, \phi) = 1 - \sin^2\theta\cos^2\phi =$$

$$\frac{4\sqrt{\pi}}{3}Y_0^0$$

- - -

$$-\frac{\sqrt{30\pi}}{15}Y_1^{-2} \quad \text{-} \quad +2\frac{\sqrt{5\pi}}{15}Y_1^0 \quad \text{-} \quad -\frac{\sqrt{30\pi}}{15}Y_1^2$$

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Isotropic Excitation - Single Pixel y View

$$h(\theta, \phi) = 1 - \sin^2\theta \sin^2\phi =$$

$$\begin{array}{ccccccccc}
 & & & & \frac{4\sqrt{\pi}}{3}Y_0^0 & & & & \\
 & & & \text{-} & \text{-} & \text{-} & & & \\
 & & \frac{\sqrt{30\pi}}{15}Y_1^{-2} & \text{-} & +2\frac{\sqrt{5\pi}}{15}Y_1^0 & \text{-} & \frac{\sqrt{30\pi}}{15}Y_1^2 & & \\
 & \text{-} & \text{-} & \text{-} & \text{-} & \text{-} & \text{-} & \text{-} & \\
\text{-} & \text{-} & \text{-} & \text{-} & \text{-} & \text{-} & \text{-} & \text{-} & \text{-}
\end{array}$$

Isotropic Excitation - Single Pixel (Θ, Φ) View

$$h(\theta, \phi) = 1 - (\sin\Theta\cos\Phi\sin\theta\cos\phi + \sin\Theta\sin\Phi\sin\theta\sin\phi + \cos\Theta\cos\theta)^2 =$$

$\frac{4\sqrt{\pi}}{3} Y_0^0$

-   -

$+2\frac{\sqrt{5\pi}}{15}(3\sin^2\Theta - 2)Y_1^0$ $\quad -\frac{2\sqrt{30\pi}}{15}\sin\Theta\cos\Theta e^{-i\phi}Y_1^1$ $\quad -\frac{\sqrt{30\pi}}{15}\sin^2\Theta e^{-2i\phi}Y_1^2$

-   -   -   -

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- Need >=4 single pixel measurements to satisfy full rank condition.
- Choose orientations so that the spherical harmonic coefficients are measured as independently as possible. I expect a tetrahedron pattern is optimal, but I haven't shown this.

diSPIM - polarized illuminatation from z - detect from x - ϕ_p parameter

$$h(\theta,\phi) = \sin^2\theta\cos^2(\phi-\phi_\mathrm{p})\cdot 2[A + B(\cos^2\theta + \sin^2\theta\sin^2\phi)] =$$

$$\begin{array}{ccccccccc}
 & & & & H_0^0Y_0^0 & & & & \\
 & & & - & - & - & & & \\
 & & +\overline{H_2^2}Y_2^{-2} & - & +H_2^0Y_2^0 & - & +H_2^2Y_2^2 & & \\
 & - & - & - & - & - & - & - & \\
+\overline{H_4^4}Y_4^{-4} & - & +\overline{H_4^2}Y_4^{-2} & - & +H_4^0Y_4^0 & - & +H_4^2Y_4^2 & - & +H_4^4Y_4^4
\end{array}$$

$$H_0^0 = \frac{4\sqrt{\pi}}{15}(5A + 2B\sin^2\phi_\mathrm{p})$$

$$H_2^0 = \frac{-4\sqrt{5\pi}}{105}(7A + 4B\sin^2\phi_\mathrm{p})$$

$$H_2^2 = \frac{-2\sqrt{30\pi}}{105}(7iA\sin(2\phi_\mathrm{p}) - 7A\cos(2\phi_\mathrm{p}) + 4iB\sin(2\phi_\mathrm{p}) - 4B\cos(2\phi_\mathrm{p}))$$

$$H_4^0 = \frac{-4\sqrt{\pi}105}{B}\cos(2\phi_\mathrm{p})$$

$$H_4^2 = \frac{2\sqrt{10\pi}}{105}B(1 + e^{-2i\phi_\mathrm{p}})$$

$$H_4^4 = \frac{-2\sqrt{70\pi}}{105}Be^{-2i\phi_\mathrm{p}}$$

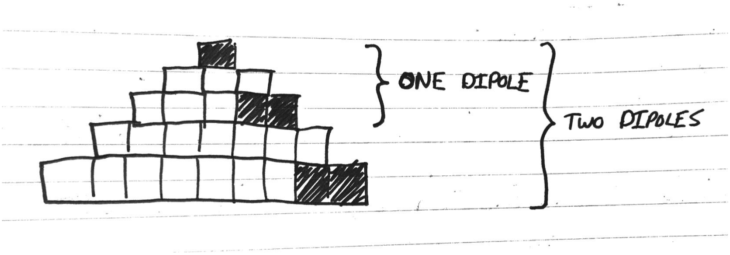
diSPIM - polarized illuminatation from x - detect from z - ϕ_{p} parameter

$$h(\theta,\phi) = (\sin\theta\sin\phi\sin\phi_{\mathrm{p}} - \cos\theta\cos\phi_{\mathrm{p}})^2 \cdot 2(A + B\sin^2\theta) =$$

$$\begin{array}{ccccccccc}
 & & & & H_0^0 Y_0^0 & & & & \\
 & & & - & - & - & & & \\
 & & +\overline{H_2^2}Y_2^{-2} & +\overline{H_2^1}Y_2^{-1} & +H_2^0Y_2^0 & +H_2^2Y_2^1 & +H_2^2Y_2^2 & & \\
 & - & - & - & - & - & - & - & \\
- & - & +\overline{H_4^2}Y_4^{-2} & +\overline{H_4^1}Y_4^{-1} & +H_4^0Y_4^0 & +H_4^1Y_4^1 & +H_4^2Y_4^2 & - & -
\end{array}$$

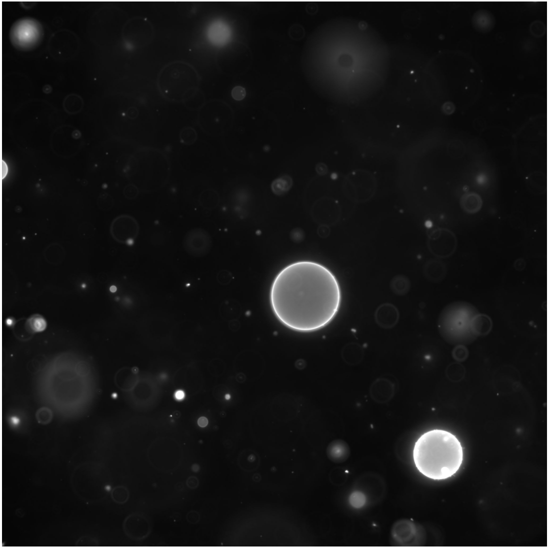
$$\begin{aligned}
H_0^0 &= \frac{4\sqrt{\pi}}{15}(5A + 2B\sin^2\phi_{\mathrm{p}}) \\
H_2^0 &= \frac{4\sqrt{5\pi}}{105}(-21A\sin^2\phi_{\mathrm{p}} + 14A - 10B\sin^2\phi_{\mathrm{p}} + 2B) \\
H_2^1 &= \frac{-2\sqrt{30\pi}i}{105}(7A + 4B)\sin(2\phi_{\mathrm{p}}) \\
H_2^2 &= \frac{-2\sqrt{30\pi}}{105}(7A + 6B)\sin^2\phi_{\mathrm{p}} \\
H_4^0 &= \frac{16\sqrt{\pi}B}{105}(3\sin^2\phi_{\mathrm{p}} - 2) \\
H_4^1 &= \frac{8\sqrt{5\pi}iB}{105}\sin(2\phi_{\mathrm{p}}) \\
H_4^2 &= \frac{4\sqrt{10\pi}B}{105}\sin^2\phi_{\mathrm{p}}
\end{aligned}$$

Single Molecule Prior Reconstruction

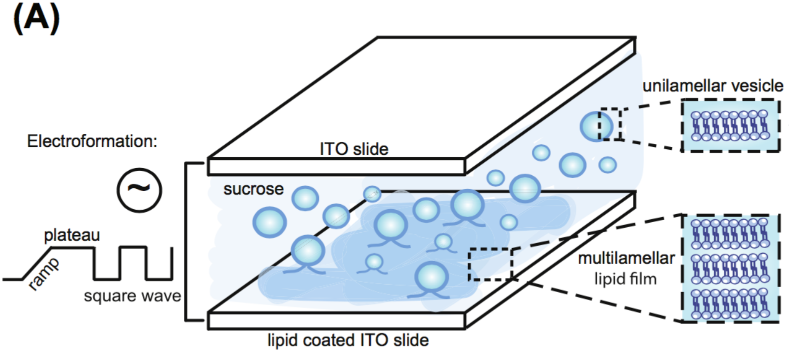


M. Vetterli, P. Marziliano and T. Blu, "Sampling signals with finite rate of innovation," in IEEE Transactions on Signal Processing, vol. 50, no. 6, pp. 1417-1428, Jun 2002.
S. Deslauriers-Gauthier and P. Marziliano, "Sampling signals with a finite rate of innovation on the sphere," in IEEE Transactions on Signal Processing, vol. 61, no. 18, pp. 4552-4561, Sept.15, 2013.

Giant Unilamallar Vesicles (GUV)
FOV $\approx 150 \times 150\ \mu$m

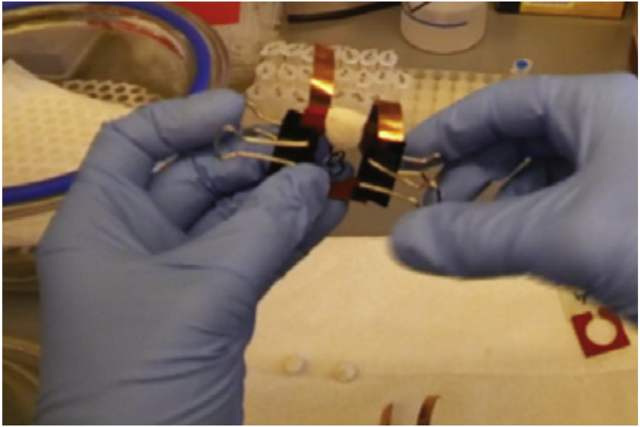


GUV Protocol



Schmid, 2015

GUV Chamber



Schmid, 2015