Introduction to Diagnostic x-ray Imaging (Part I)

Yulei Jiang

Department of Radiology, The University of Chicago

Outline

- Brief historical survey
- Linear-system model
- Focal spot and magnification
- Scatter
- Image detector

A Brief History of Diagnostic x-ray Imaging

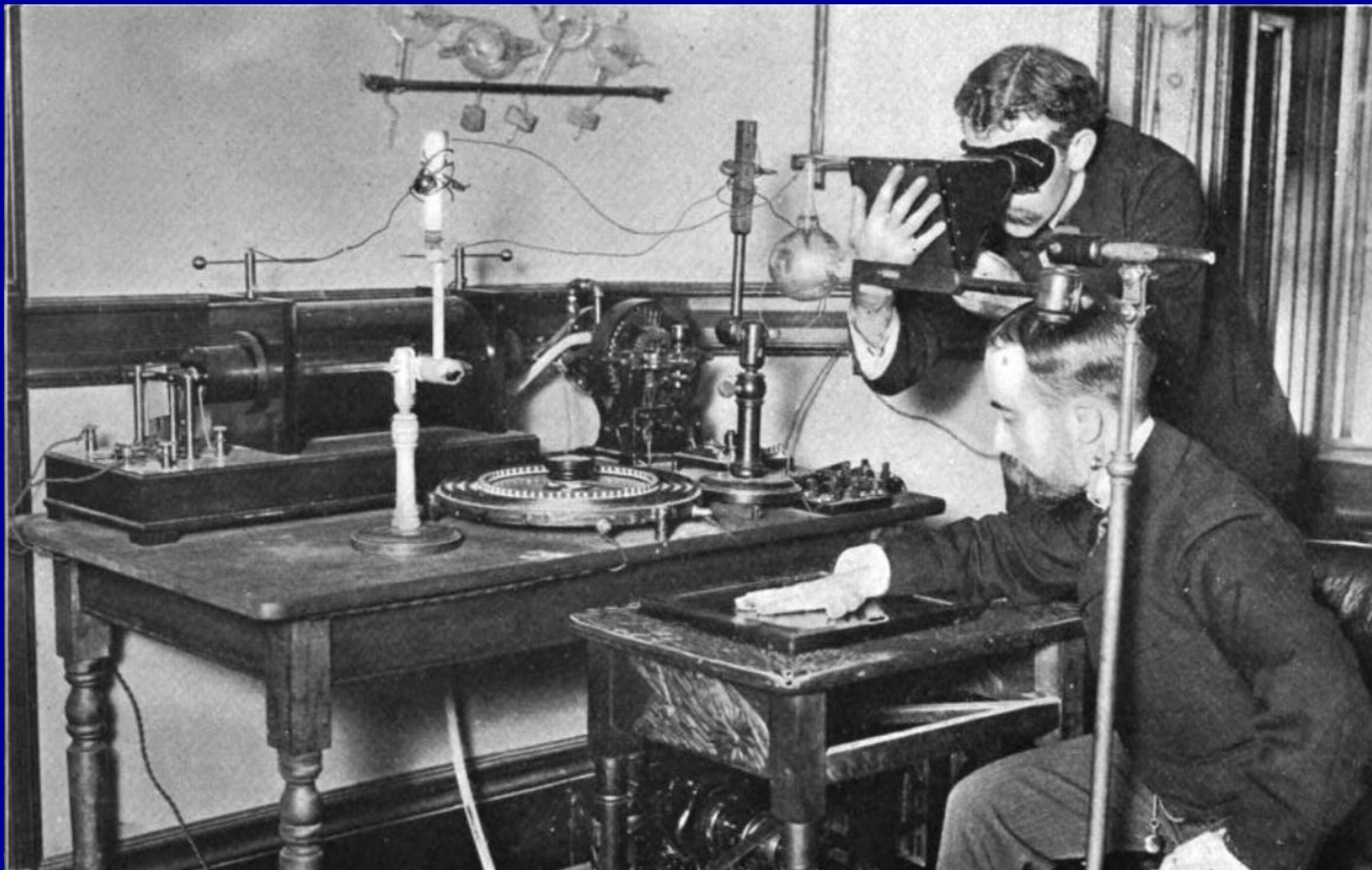
- 1895: German physicist Wilhelm Conrad Roentgen discovered x-ray, and produced the first x-ray picture of the human body—his wife's hand.

The First Medical Image

Source: Wikipedia

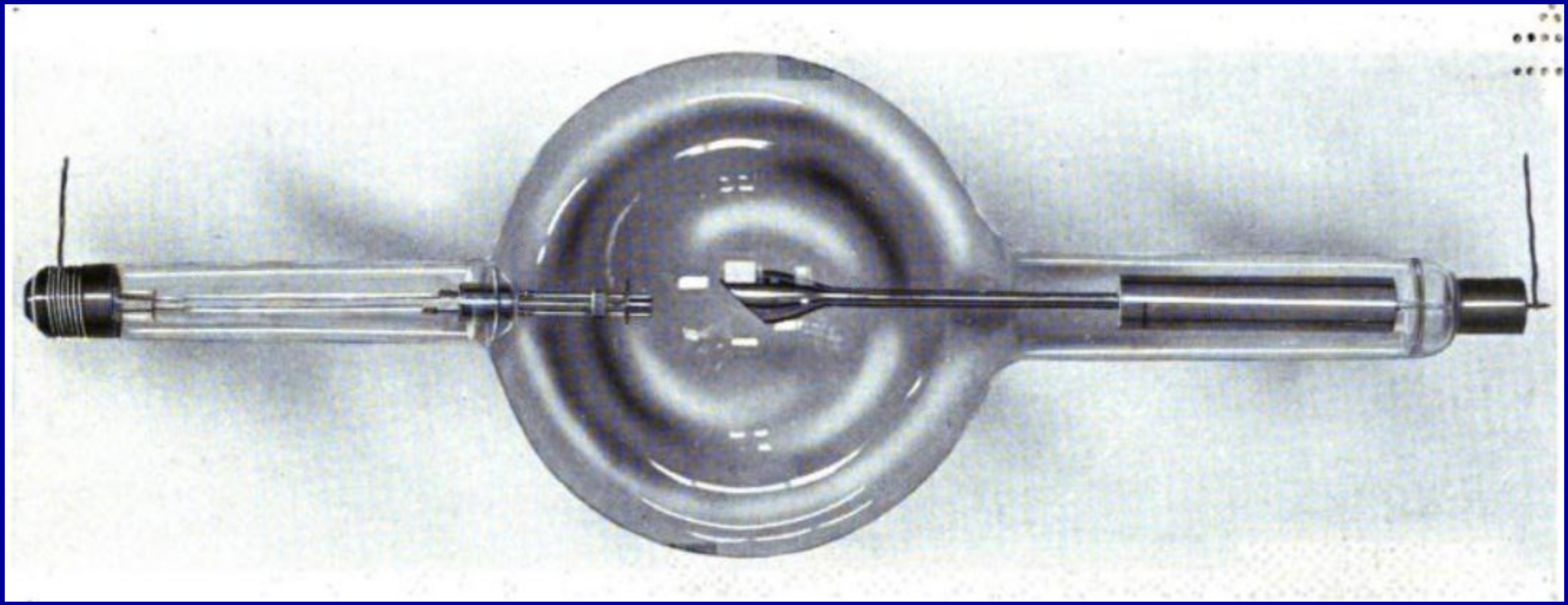
Early 20th Century

- 1900: widespread use of chest x-ray made early detection of tuberculosis a reality.
- 1906: x-ray contrast medium use began.
- 1912: Marie Curie published theory of radioactivity; investigation of x-ray radiation for patient therapy (e.g. treatment of cancer).
- 1913: William Coolidge invented the hot cathode x-ray tube



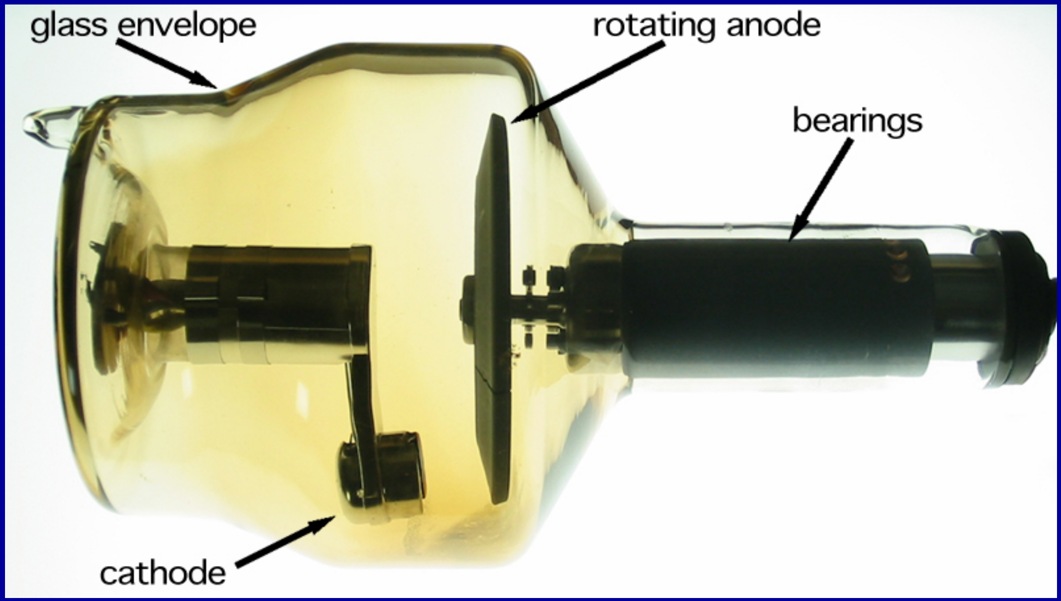
Source: Wikipedia

Coolidge Hot Cathode x-ray Tube



Source: Wikipedia

Rotating Anode x-ray Tube

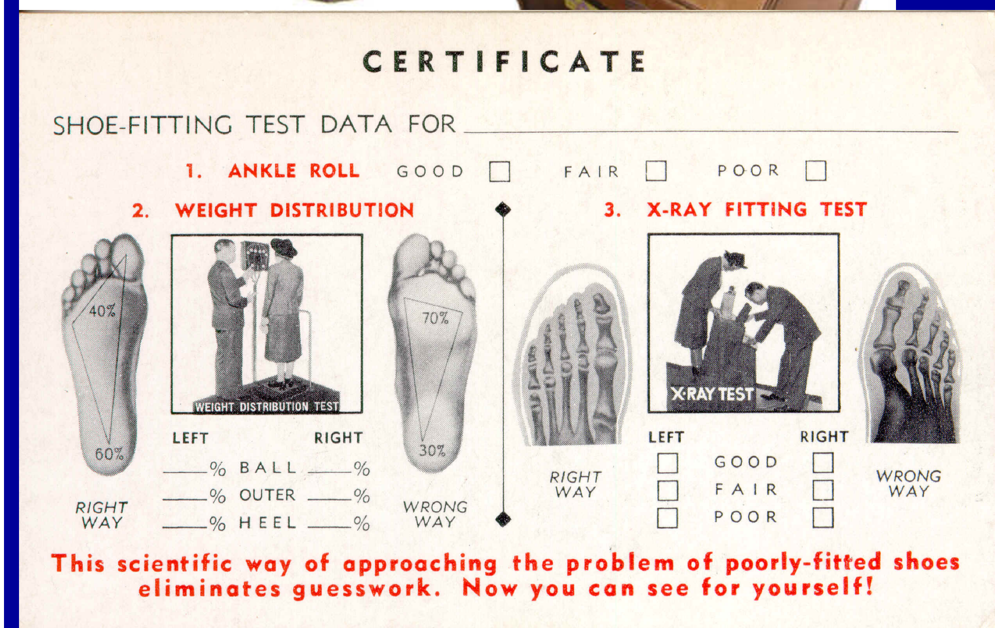


Source: Wikipedia

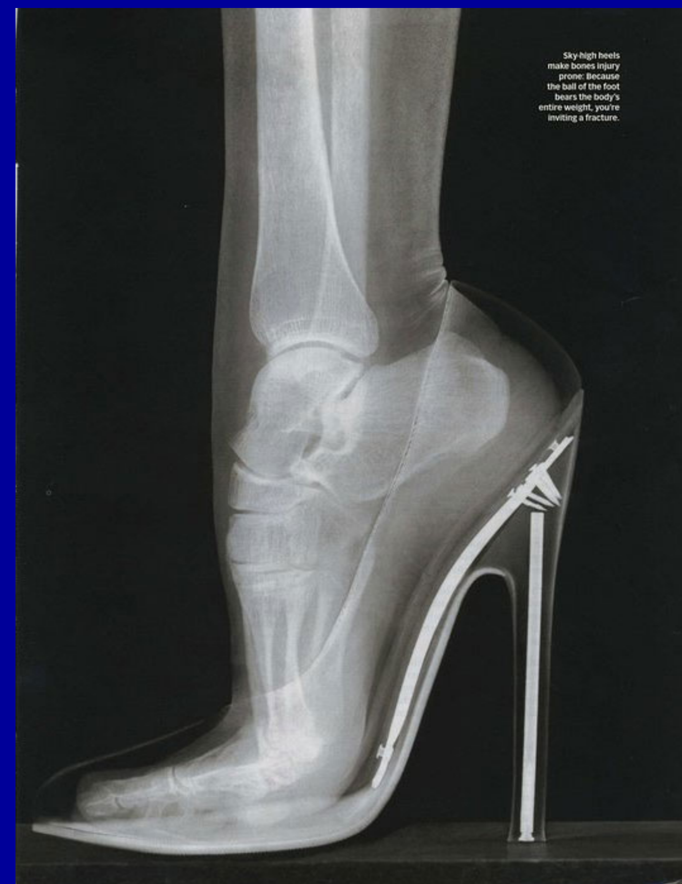
Middle and Late 20th Century

- 1950: Nuclear medicine imaging begins.
- 1955: image intensifier TV allowed dynamic x-ray imaging of beating heart and blood vessels.
- 1960: Ultrasound imaging began.
- 1970: x-ray mammography began.

Shoe-fitting Fluoroscopy



Source: Oak Ridge Associated University



Late 20th Century

- 1972: Godfrey Hounsfield and Allan Cormack invented CT.
- 1978: Digital radiography began.
- 1980: Paul Lauterbur and others developed MRI.
- 1985: University of California scientists developed PET.

Common x-ray Procedures

- Chest x-ray (TB, pneumonia, cancer, etc.)
- Orthopedic evaluations (bone fracture, bone age of children)
- Mammography (breast cancer)
- Abdomen (contrast fluoroscopy)
- Verification of surgical markers (radiation therapy verification)
- Dental examination
- Chiropractic examinations

Medical Imaging as Applied Science

- Imaging theory
- Practical aspects
- Physicists, radiologists, technologists, and engineers

Outline

- Brief historical survey
- Linear-system model
- Focal spot and magnification
- Scatter
- Image detector

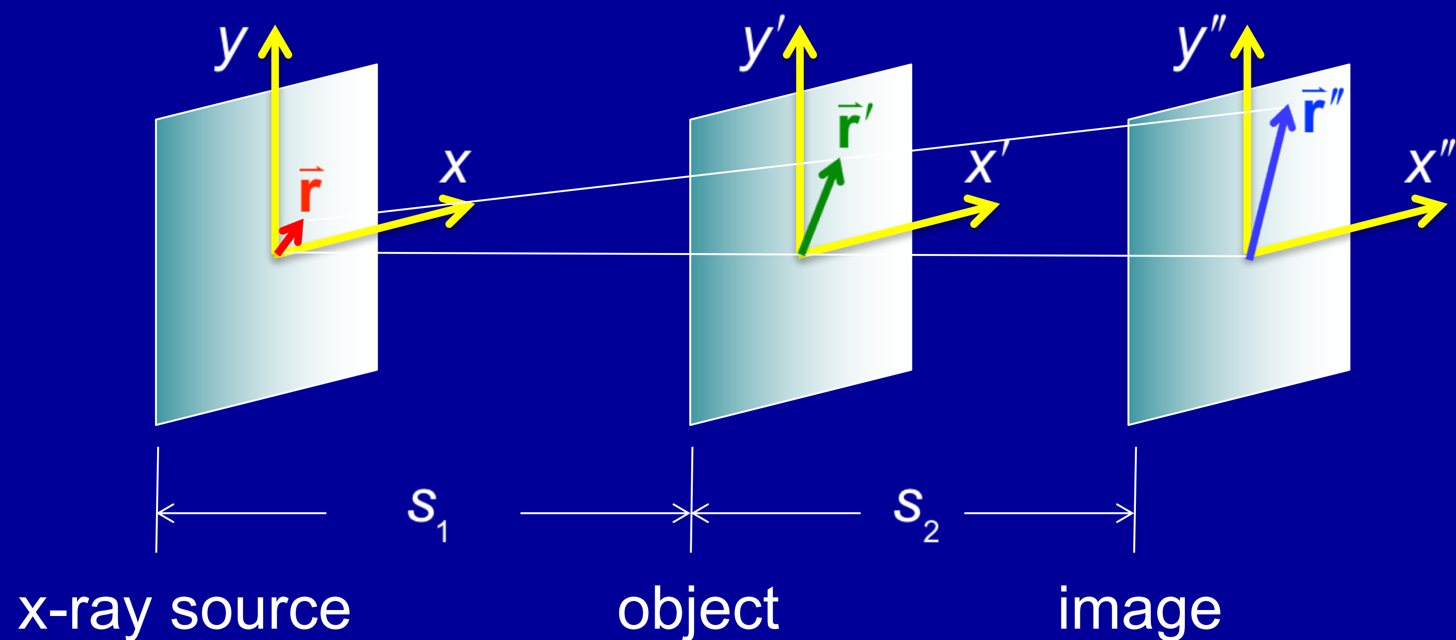
Linear-System Model of Projection (Planar) x-ray Imaging

- Based on first principle (geometry, physics)
- Math not difficult but somewhat tedious
- Idealized in several ways
- Provides a way of thinking of the imaging system
- Was not necessarily the source of system development

Summary of Objective

- Describe the image as a convolution of the object with the x-ray source

Simplified Representation of an x-ray System



Idealizations

- 3D source → 2D planar source
- Anisotropic source → isotropic source
- 3D object → 2D planar object

Geometry: From the Image Plane Perspective of the Source Plane

- **A patch of the image plane expressed as solid angle to a patch in the source plane:**

$$\mathrm{d}\Omega = \frac{\mathrm{d}^2 r'' \cos\theta}{R^2}$$

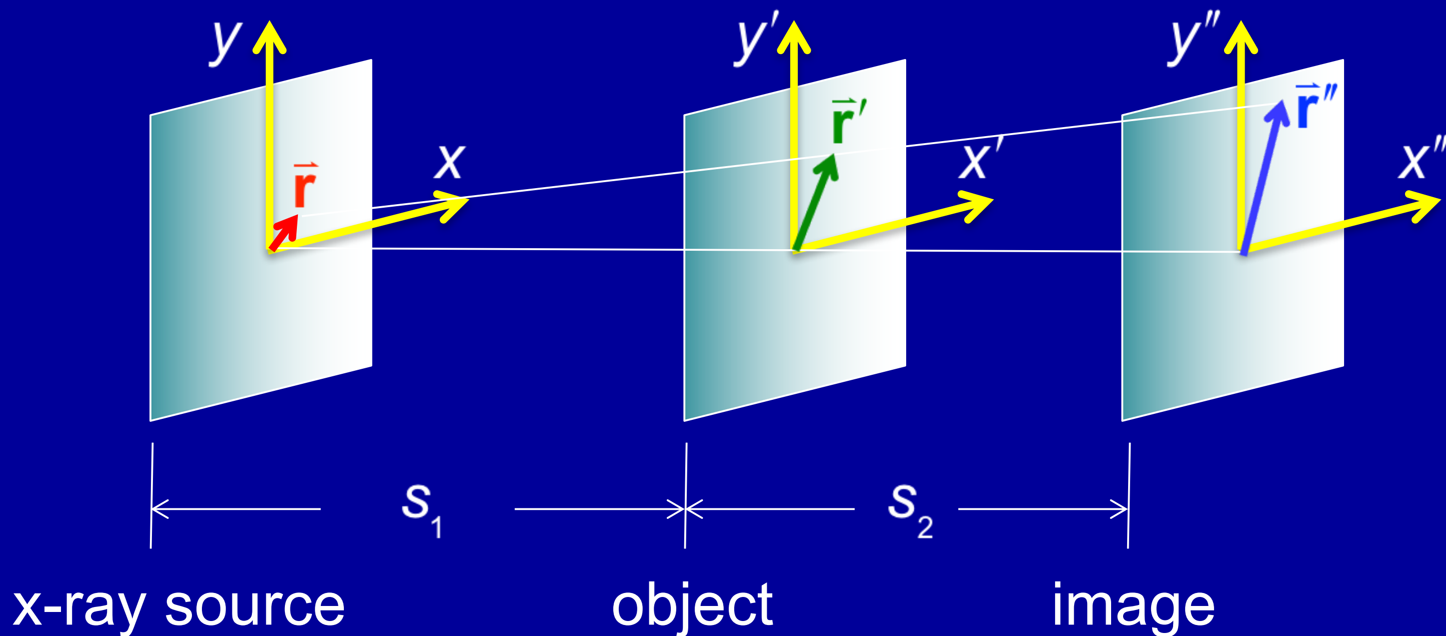
- **x-ray photon density at the patch in the image plane from the patch in the source plane:**

$$f(\vec{\mathbf{r}})\ \mathrm{d}^2 r\ \frac{\mathrm{d}\Omega}{4\pi} = f(\vec{\mathbf{r}}) \frac{\cos^3\theta}{4\pi\left(s_1 + s_2\right)^2}\ \mathrm{d}^2 r\ \mathrm{d}^2 r''$$

- **Total x-ray photon density at the patch in the image plane:**

$$h(\vec{\mathbf{r}}'')\ \mathrm{d}^2 r'' = \frac{T\ \mathrm{d}^2 r''}{4\pi\left(s_1 + s_2\right)^2} \int_{source} \mathrm{d}^2 r\ \cos^3\theta\ f(\vec{\mathbf{r}})\ g(\vec{\mathbf{r}}')$$

Simplified Representation of an x-ray System



General Description

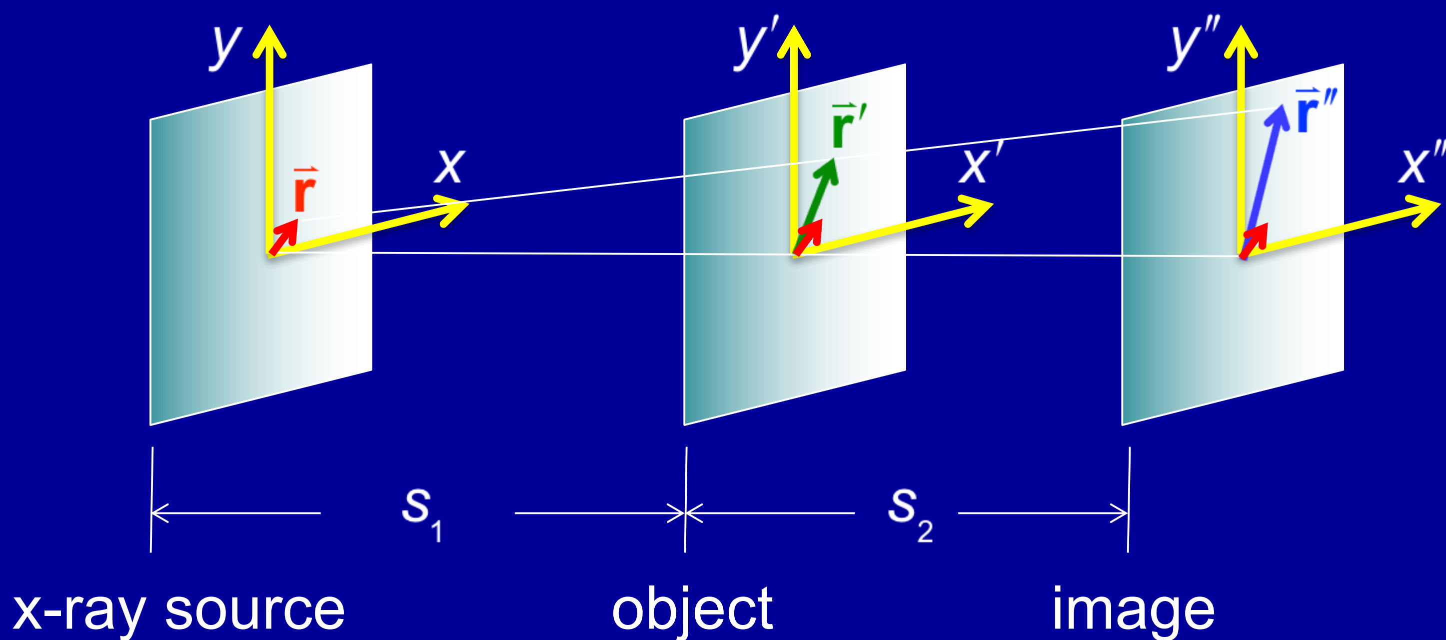
$$h\left(\vec{\mathbf{r}}''\right)\ \mathrm{d}^2 r'' = \frac{T\ \mathrm{d}^2 r''}{4\pi\left(s_1 + s_2\right)^2} \int_{source} \mathrm{d}^2 r\ \cos^3\theta\ f\left(\vec{\mathbf{r}}\right)\ g\left(\vec{\mathbf{r}}'\right)$$

Let: $\mathrm{C} = \dfrac{T}{4\pi\left(s_1 + s_2\right)^2}$

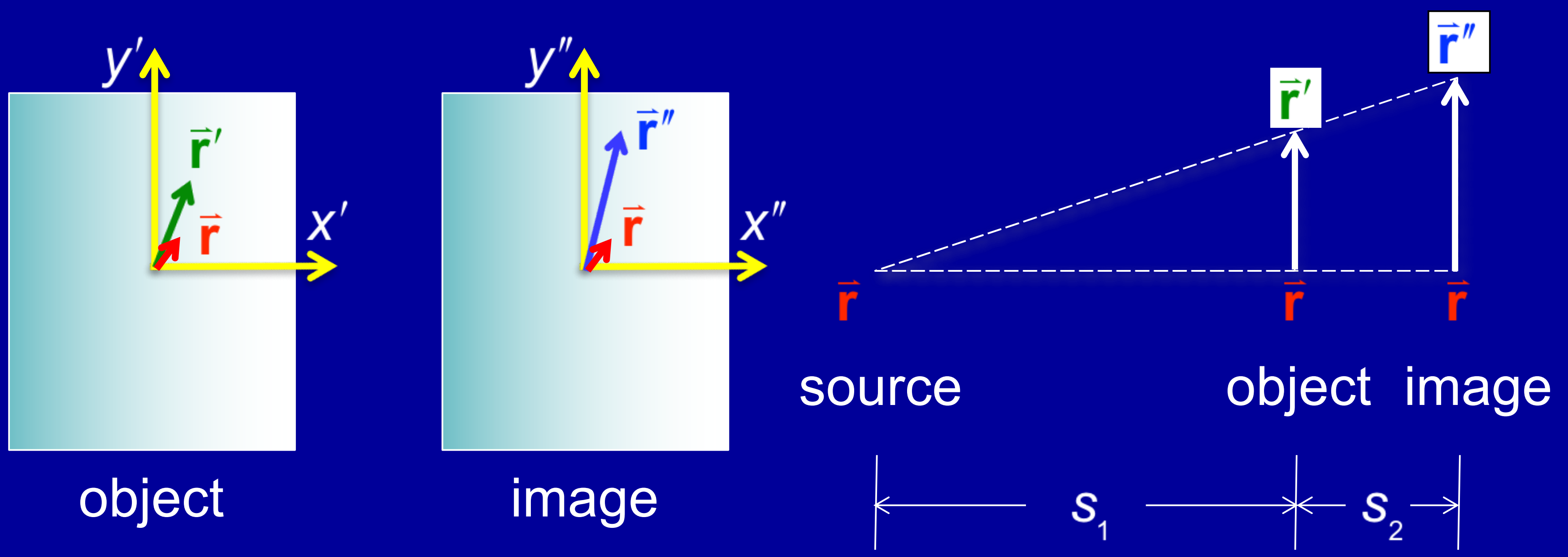
and consider source far from the imaging plane:

$$h\left(\vec{\mathbf{r}}''\right) \approx \mathrm{C} \int_{source} d^2 r\ f_{source}\left(\vec{\mathbf{r}}\right)\ g_{object}\left(\vec{\mathbf{r}}'\right)$$

Geometry: Determine the Object-Plane Vector by Line of Sight



Geometry: The Object Plane Determination by the Line of Sight



Geometry: Line-of-Sight Determination of Object-Plane Vector

$$\frac{\vec{\mathbf{r}}' - \vec{\mathbf{r}}}{s_1} = \frac{\vec{\mathbf{r}}'' - \vec{\mathbf{r}}}{s_1 + s_2} \Rightarrow \vec{\mathbf{r}}' = \frac{s_1 \vec{\mathbf{r}}'' + s_2 \vec{\mathbf{r}}}{s_1 + s_2} = a\vec{\mathbf{r}}'' + b\vec{\mathbf{r}}$$

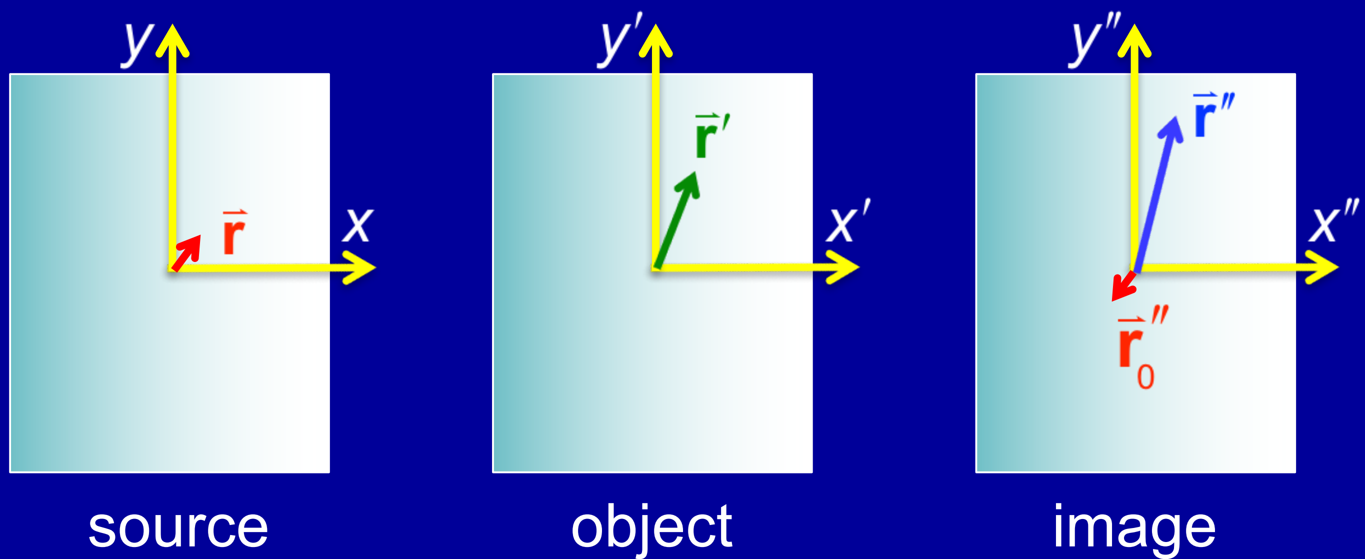
Define: $a = \dfrac{s_1}{s_1 + s_2} \quad b = \dfrac{s_2}{s_1 + s_2} = 1 - a$

Elimination of the Object-Plane Vector

$$h\left(\vec{\mathbf{r}}''\right) \approx C \int_{source} d^2r \; f_{source}\left(\vec{\mathbf{r}}\right) \; g_{object}\left(\vec{\mathbf{r}}'\right)$$

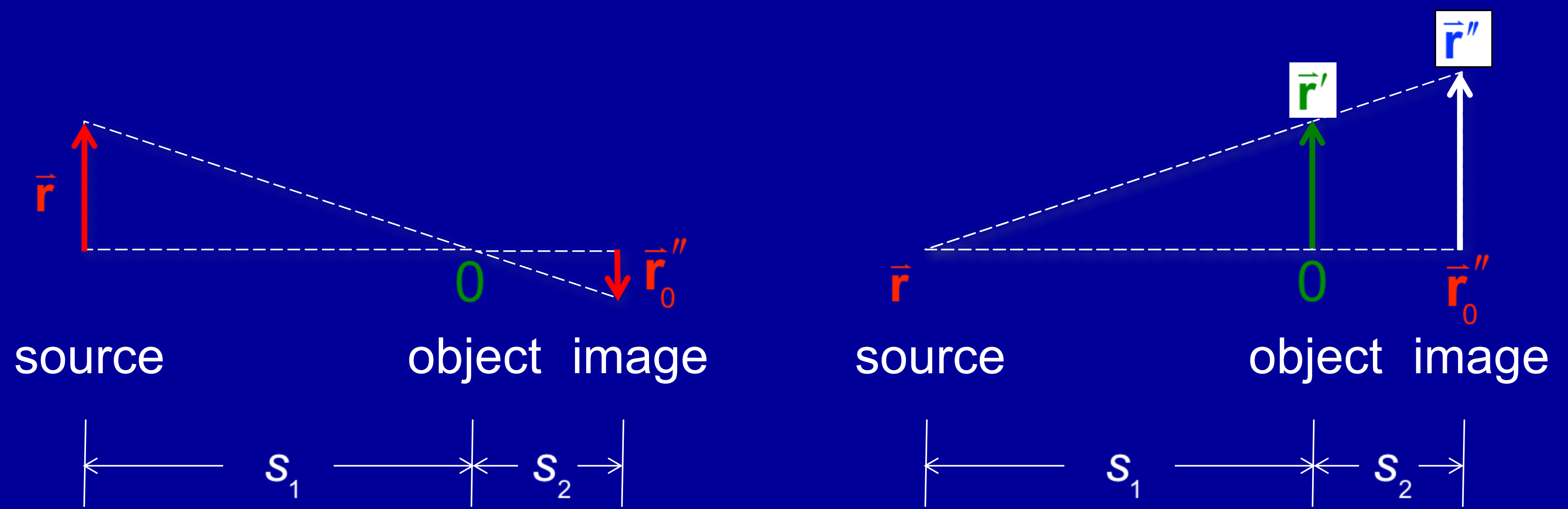
$$h\left(\vec{\mathbf{r}}''\right) \; d^2r'' \approx C \; d^2r'' \int_{source} d^2r \; f_{source}\left(\vec{\mathbf{r}}\right) g_{object}\left(a\vec{\mathbf{r}}'' + b\vec{\mathbf{r}}\right)$$

Geometry: Project the Source through the Origin of the Object Plane to the Image Plane



$$\vec{\mathbf{r}}_0'' = -\frac{b}{a}\vec{\mathbf{r}}$$

Geometry: Projection of the Source through the Object Plane to the Image Plane



Geometry: Evaluation of Source and Object at Image-Plane Coordinates

$$f_{image}\left(\vec{\mathbf{r}}_0''\right) = f_{source}\left(\vec{\mathbf{r}}\right) = f_{source}\left(-\frac{a}{b}\vec{\mathbf{r}}_0''\right)$$

$$g_{image}\left(\vec{\mathbf{r}}_0''\right) = g_{object}\left(a\vec{\mathbf{r}}_0''\right)$$

$$g_{object}\left(a\vec{\mathbf{r}}'' + b\vec{\mathbf{r}}\right) = g_{object}\left(a\vec{\mathbf{r}}'' - a\vec{\mathbf{r}}_0''\right) = g_{image}\left(\vec{\mathbf{r}}'' - \vec{\mathbf{r}}_0''\right)$$

Important Results

$$h\left(\vec{\mathbf{r}}''\right)\; d^2r'' \;\approx\; C\, d^2r'' \int_{source} d^2r\; f_{source}\left(\vec{\mathbf{r}}\right) g_{object}\left(a\vec{\mathbf{r}}'' + b\vec{\mathbf{r}}\right)$$

$$h\left(\vec{\mathbf{r}}''\right) = \left(\frac{a}{b}\right)^2 C \int_{image} d^2r_0''\; f_{image}\left(\vec{\mathbf{r}}_0''\right) g_{image}\left(\vec{\mathbf{r}}'' - \vec{\mathbf{r}}_0''\right)$$

$$= \left(a/b\right)^2 C\, f_{image}\left(\vec{\mathbf{r}}''\right) \otimes g_{image}\left(\vec{\mathbf{r}}''\right)$$

$$\mathbf{H}_{f,\,image}\left(\vec{\rho}''\right) = \left(a/b\right)^2 C\; \mathbf{F}_{f,\,image}\left(\vec{\rho}''\right) \mathbf{G}_{f,\,image}\left(\vec{\rho}''\right)$$

$$= \left(C/a^2\right)\; \mathbf{F}_{f,\,source}\left(-b\vec{\rho}''/a\right) \mathbf{G}_{f,\,object}\left(\vec{\rho}''/a\right)$$

Review of Assumptions

- Source far away from the imaging plane
- Isotropy of x-ray source
- 2D planar x-ray source
- 2D planar object

Have Not Considered

- Scatter
- Image detector
- Patient motion

Remarks

- x-ray source is “in” every image
- x-ray source is an important part of the imaging chain
- Linear system analysis becomes possible
- Based on system-geometry consideration alone

Several Forms of the Convolution Equation

$$\mathbf{H}_{f,\,image}\left(\vec{\rho}''\right) = \left(C/a^2\right)\ \mathbf{F}_{f,\,source}\left(-b\vec{\rho}''/a\right)\mathbf{G}_{f,\,object}\left(\vec{\rho}''/a\right)$$

$$\mathbf{H}_{f,\,image}\left(a\vec{\rho}'\right) = \left(C/a^2\right)\ \mathbf{F}_{f,\,source}\left(-b\vec{\rho}'\right)\mathbf{G}_{f,\,object}\left(\vec{\rho}'\right)$$

$$\mathbf{H}_{f,\,image}\left(-a\vec{\rho}/b\right) = \left(C/a^2\right)\mathbf{G}_{f,\,object}\left(-\vec{\rho}/b\right)\mathbf{F}_{f,\,source}\left(\vec{\rho}\right)$$

Outline

- Brief historical survey
- Linear-system model
- Focal spot and magnification
- Scatter
- Image detector

Focal Spot Consideration

- Simple uniform-disk focal spot
- To derive the PSF of the focal spot

Focal Spot Imaged Through a Point Object

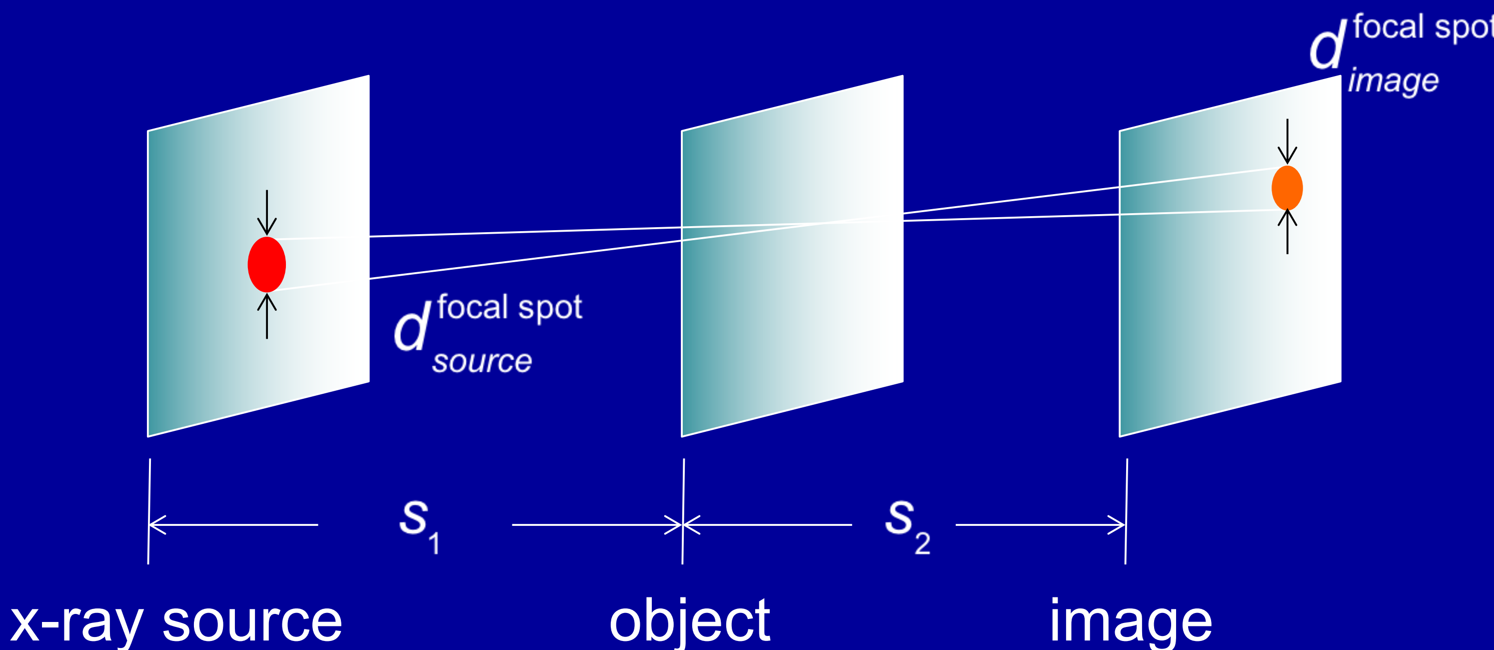
$$f_{source}\left(\vec{\mathbf{r}}\right) = f_0 \ \mathrm{circ}\left(\frac{2r}{d_{source}^{\text{focal spot}}}\right)$$

$$g_{object}\left(\vec{\mathbf{r}}'\right) = \delta\left(\vec{\mathbf{r}}' - \vec{\mathbf{r}}_1'\right)$$

$$h\left(\vec{\mathbf{r}}''\right) = \mathrm{C} \int_{source} d^2 r \ f_0 \ \mathrm{circ}\left(\frac{2r}{d_{source}^{\text{focal spot}}}\right)\delta\left(a\vec{\mathbf{r}}'' + b\vec{\mathbf{r}} - \vec{\mathbf{r}}'\right)$$

$$= \frac{\mathrm{C}}{b^2} \ f_0 \ \mathrm{circ}\left(\frac{2\left|\vec{\mathbf{r}}'' - \vec{\mathbf{r}}_1'/a\right|}{b \ d_{source}^{\text{focal spot}}/a}\right)$$

Geometric Consideration



$$d^{\text{focal spot}}_{image} = \frac{b}{a} d^{\text{focal spot}}_{source} = \frac{s_2}{s_1} d^{\text{focal spot}}_{source}$$

Remarks

- Consideration of this simple hypothetical focal spot helps to convince one that the image is a convolution of the object with the focal spot
- Note importance of spatial (and hence spatial frequency) scaling

Total System PSF

Point object

$$\mathrm{PSF}_{tot} = \left(\mathrm{C}/a^2\right)\mathrm{Fourier}^{-1}\left\{\mathbf{D}_{f,\,\mathrm{image}}\left(a\vec{\rho}'\right)\mathbf{F}_{f,\,source}\left(-b\vec{\rho}'\right)\right\}$$
$$= \mathrm{PSF}_{focal\;\;spot}\left(\vec{\mathbf{r}}'\right) \otimes \mathrm{PSF}_{detector}\left(\vec{\mathbf{r}}'\right)$$

$$\mathrm{PSF}_{focal\;\;spot}\left(\vec{\mathbf{r}}'\right) = \frac{\mathrm{C}}{a^2}\mathrm{Fourier}^{-1}\left\{\mathbf{F}_{f,\,source}\left(-b\vec{\rho}'\right)\right\} = \frac{\mathrm{C}}{a^2 b^2}\; f_{source}\left(-\frac{\vec{\mathbf{r}}'}{b}\right)$$

$$\mathrm{PSF}_{detector}\left(\vec{\mathbf{r}}'\right) = \mathrm{Fourier}^{-1}\left\{\mathbf{D}_{f,\,\mathrm{image}}\left(a\vec{\rho}'\right)\right\} = \frac{1}{a^2}\; d_{detector}\left(\frac{\vec{\mathbf{r}}'}{a}\right)$$

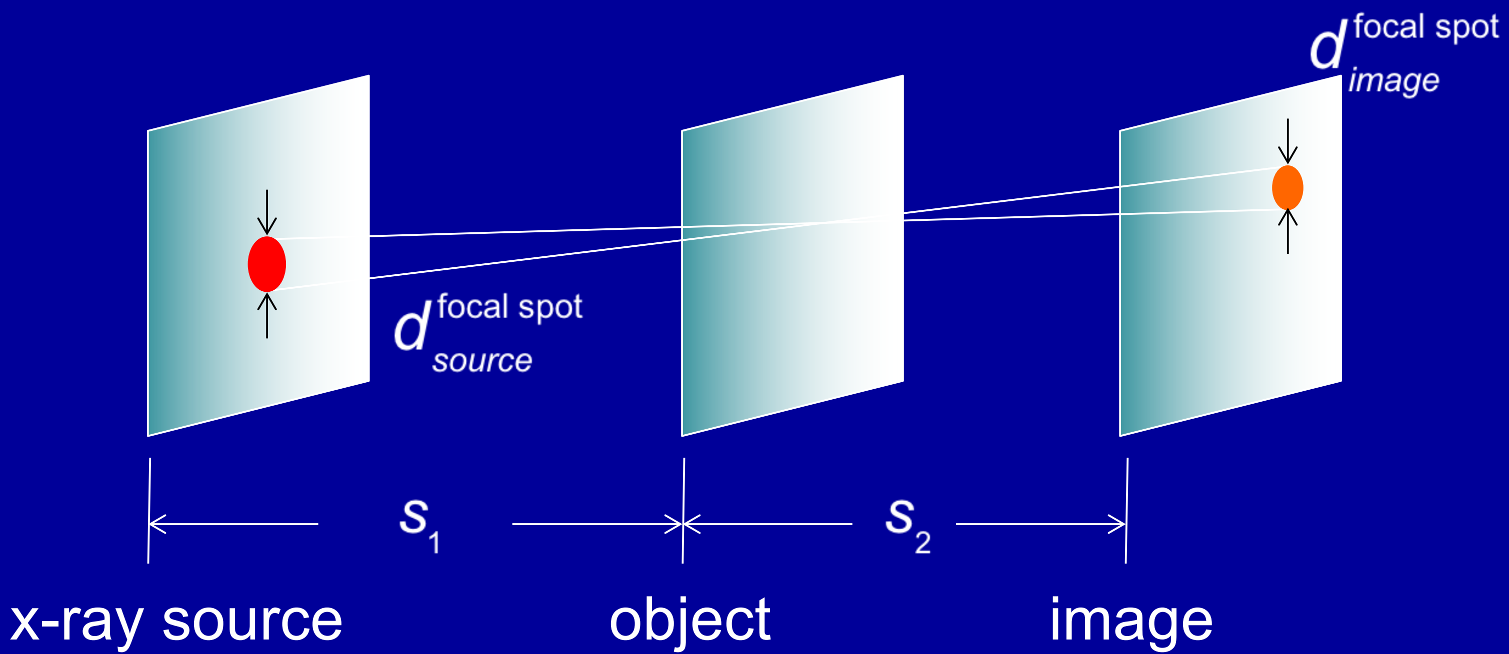
Optimal Magnification

$$\mathbf{F}_{f,\,source}\left(\vec{\rho}\right) \propto e^{-\pi\left(\frac{\rho}{\rho_{focal\ spot}}\right)^2} \qquad \mathbf{D}_{f,\,\mathrm{image}}\left(\vec{\rho}''\right) \propto e^{-\pi\left(\frac{\rho''}{\rho_{detector}}\right)^2}$$

$$\mathrm{MTF}_{tot} \propto e^{-\pi\rho'^2\left[\frac{1}{\left(m_t\,\rho_{detector}\right)^2}+\frac{\left(m_t-1\right)^2}{\left(m_t\,\rho_{focal\ spot}\right)^2}\right]}$$

$$m_t^{optimal} = 1 + \left(\frac{\rho_{focal\ spot}}{\rho_{detector}}\right)^2$$

Geometric Consideration



$$d_{image}^{focal\ spot} = \frac{b}{a} d_{source}^{focal\ spot} = \frac{s_2}{s_1} d_{source}^{focal\ spot}$$

Summary on Focal Spot and Magnification

- Large focal spot ok for contact imaging
- Small focal spot required for magnification

Outline

- Brief historical survey
- Linear-system model
- Focal spot and magnification
- Scatter
- Image detector

Linear System Analysis

$$PSF_{total}\left(\vec{\mathbf{r}}\right) = PSF_{primary}\left(\vec{\mathbf{r}}\right) + PSF_{scatter}\left(\vec{\mathbf{r}}\right)$$

Modeling of Scatter PSF

$$\mathrm{PSF}_{scatter}\left(\vec{\mathbf{r}}\right)=$$

$$\int_0^L dz\left[\Phi_0 \mathrm{e}^{-\mu(L-z)}\right]\left[\frac{r_0^2}{2}\left(1+cos^2\theta\right)\right]\left[\mathrm{n}_e A\right]\left[\frac{cos^3\theta}{\left(s+z\right)^2}\right]\left[\mathrm{e}^{-\mu z\ sec\theta}\right]$$

$$\mathrm{PSF}_{primary}\left(\vec{\mathbf{r}}\right) = \Phi_0 \mathrm{e}^{-\mu L}$$

Modeling Scatter at Gaussian

$$\mathrm{PSF}_{scatter}\left(\vec{\mathbf{r}}\right) = A_s T \, \mathrm{e}^{-\pi \beta_s^2 r^2} \qquad \mathbf{P}_{scatter}\left(\vec{\rho}\right) = \frac{A_s T}{\beta_s^2} \, \mathrm{e}^{-\pi \rho^2 / \beta_s^2}$$

$$\mathrm{PSF}_{primary}\left(\vec{\mathbf{r}}\right) = A_p T \, \mathrm{e}^{-\pi \beta_p^2 r^2} \qquad \mathbf{P}_{primary}\left(\vec{\rho}\right) = \frac{A_p T}{\beta_p^2} \, \mathrm{e}^{-\pi \rho^2 / \beta_p^2}$$

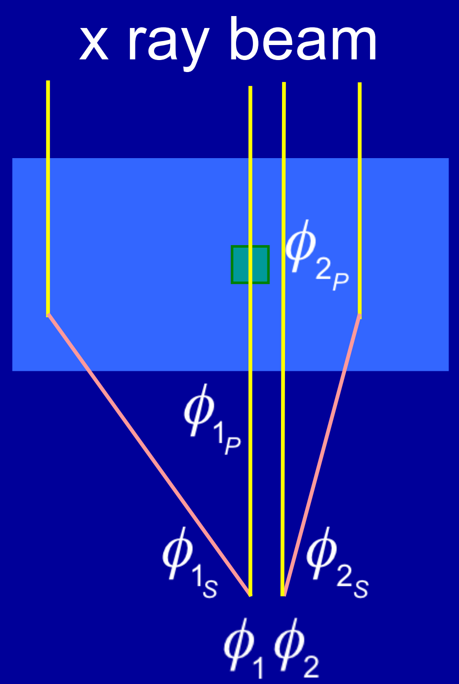
$$\mathrm{MTF}_{total} = \frac{\mathbf{P}_{primary}\left(\vec{\rho}\right) + \mathbf{P}_{scatter}\left(\vec{\rho}\right)}{\mathrm{PSF}_{primary}(0) + \mathrm{PSF}_{scatter}(0)}$$

$$= \frac{\mathrm{e}^{-\pi \rho^2 / \beta_p^2} + \mathrm{SPR}\, \mathrm{e}^{-\pi \rho^2 / \beta_s^2}}{1 + \mathrm{SPR}}$$

Important Results

$$\frac{\mathbf{P}_{total}\left(\vec{\rho}\right)}{\mathbf{P}_{primary}\left(\vec{\rho}\right)} \approx \frac{1}{1+\mathrm{SPR}} \qquad \beta_s << \vec{\rho} << \beta_p$$

Effect of Scatter on Radiation Contrast



$$C_{NS} = \frac{\Delta\phi}{\bar{\phi}} = \frac{\phi_1 - \phi_2}{(\phi_1 + \phi_2)/2} = \frac{\phi_{1_P} - \phi_{2_P}}{(\phi_{1_P} + \phi_{2_P})/2} = \frac{\phi_{1_P} - \phi_{2_P}}{P}$$

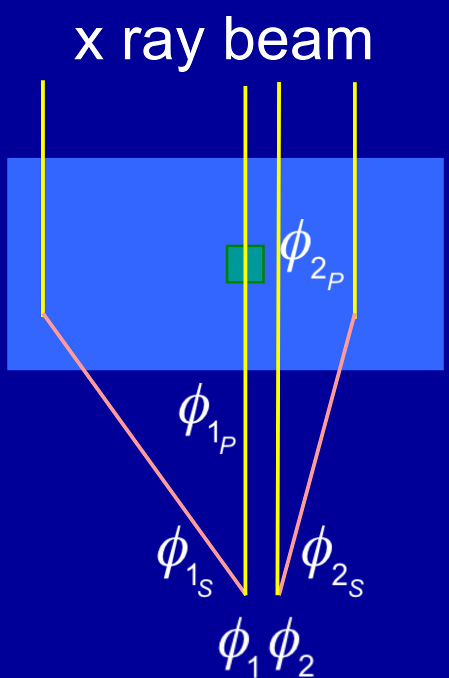
$$C_S = \frac{(\phi_{1_P} + \phi_{1_S}) - (\phi_{2_P} + \phi_{2_S})}{[(\phi_{1_P} + \phi_{1_S}) + (\phi_{2_P} + \phi_{2_S})]/2}$$

$$= \frac{(\phi_{1_P} - \phi_{2_P}) + (\phi_{1_S} - \phi_{2_S})}{[(\phi_{1_P} + \phi_{2_P}) + (\phi_{1_S} + \phi_{2_S})]/2} = \frac{\phi_{1_P} - \phi_{2_P}}{P + S}$$

$$P = \frac{(\phi_{1_P} + \phi_{2_P})}{2},\ S = \phi_S$$

$$\phi_{1_S} \approx \phi_{2_S} = \phi_S$$

Effect of Scatter on Radiation Contrast



$$C_{NS} = \frac{\Delta\phi}{\bar{\phi}} = \frac{\phi_1 - \phi_2}{\left(\phi_1 + \phi_2\right)/2} = \frac{\phi_{1_P} - \phi_{2_P}}{\left(\phi_{1_P} + \phi_{2_P}\right)/2} = \frac{\phi_{1_P} - \phi_{2_P}}{P}$$

$$C_S = \frac{\phi_{1_P} - \phi_{2_P}}{P} \frac{P}{P+S} = C_{NS} \frac{P}{P+S} = C_{NS} \left(1 - \frac{S}{P+S}\right)$$

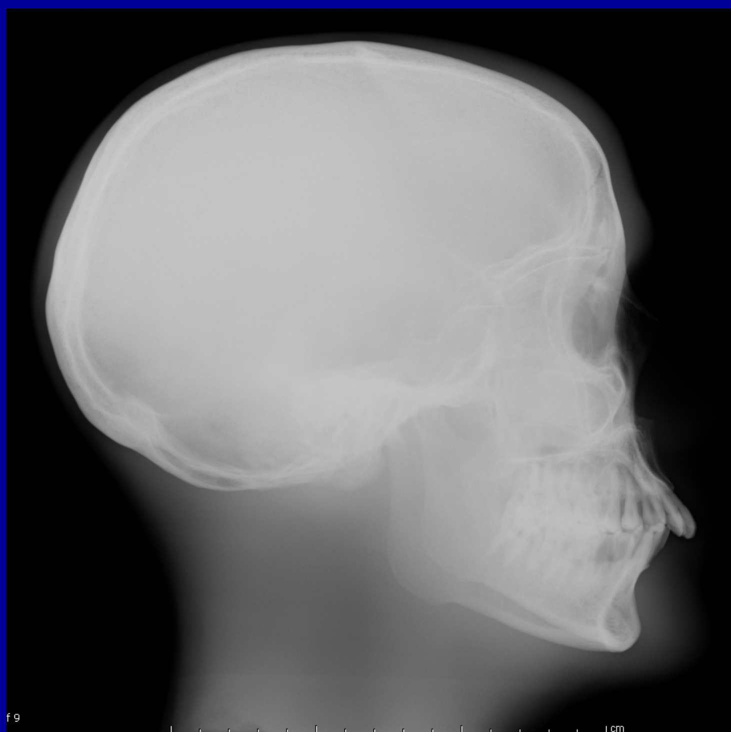
$$= C_{NS}\left(1 - SF\right) = C_{NS} \frac{1}{1 + S/P}$$

$$P = \frac{\left(\phi_{1_P} + \phi_{2_P}\right)}{2}, \quad S = \phi_S$$

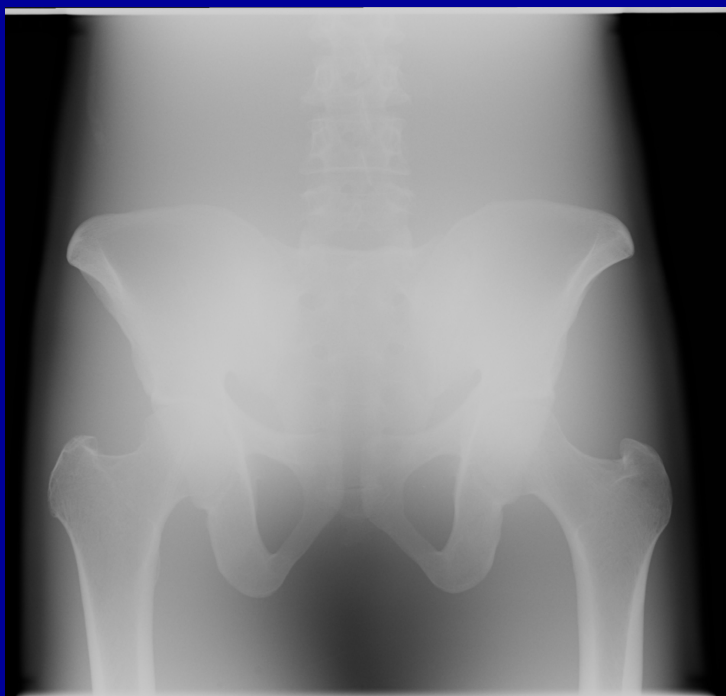
$$\phi_{1_S} \approx \phi_{2_S} = \phi_S$$

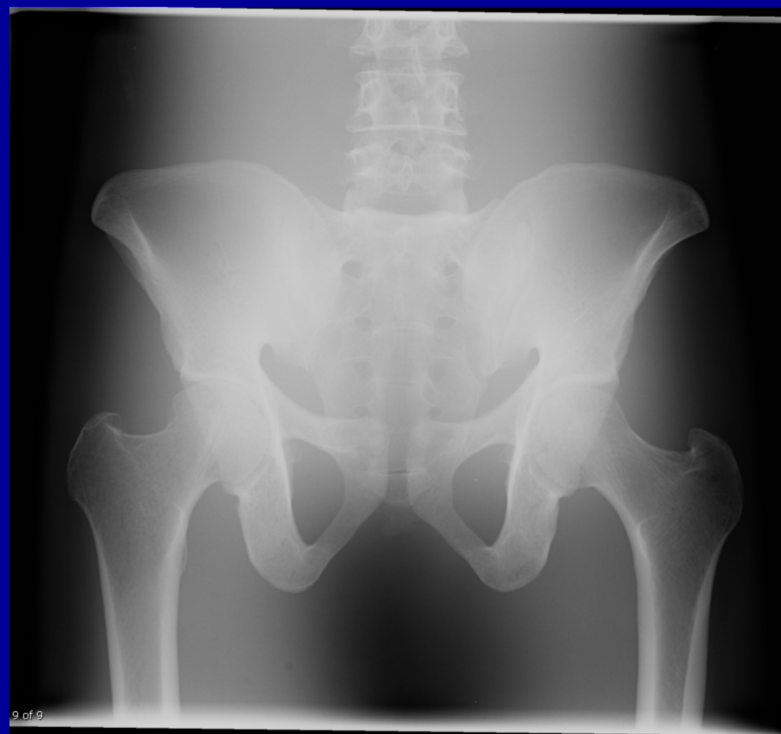
Define scatter fraction & scatter-to-primary ratio: $SF = \frac{S}{P+S} = \frac{S/P}{1+S/P}$

Example Images of Reduced Scatter



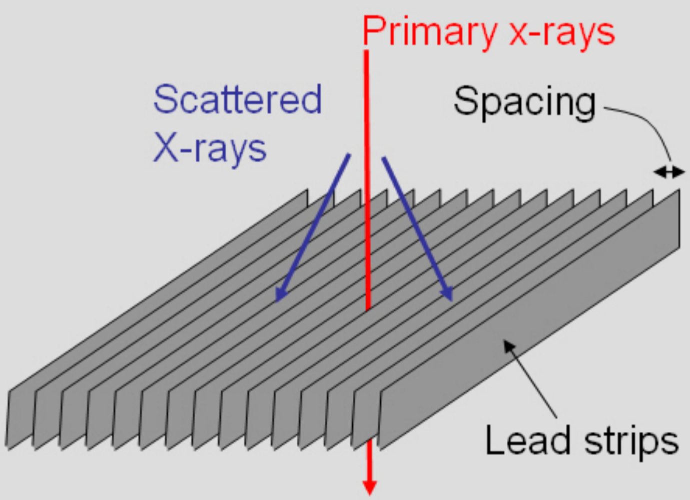
Example Images of Reduced Scatter





Anti-scatter Grip Basics

Antiscatter grid design



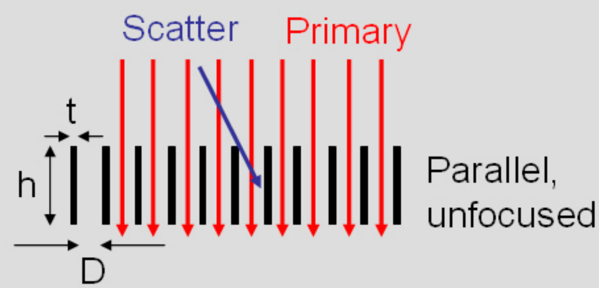
Grid characteristics:

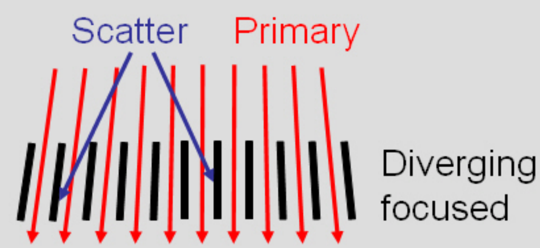
t = thickness of lead strips
h = height of lead strips
D = distance between lead strips

Grid Ratio = $\frac{h}{D}$

Grid Frequency = $\frac{1}{t + D}$

Focal range: determined by geometry of lead strips





Summary

- A general linear-system model that describes planar x-ray imaging
- Image is a convolution of the object with the focal spot
- Magnification important for this convolution
- Scatter degrades image by inserting a broad PSF into the linear-system model