# Medical Physics 386:

# The Physics of Medical Imaging 1

# X-ray Imaging Notes

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# 1    Introduction

There are many different imaging modalities in the radiology department— mammography, computed radiography, digital radiography, fluoroscopy, computed tomography (CT), magnetic resonance imaging (MRI), positron-emission tomography (PET), single-photon emission tomography (SPECT), ultrasound, and more. Why are there so many modalities? Because no single modality is optimal for the wide variety of imaging tasks that arise when we try to diagnose disease. Each imaging task has different imaging requirements, so no single modality can be used for all tasks.

A major goal of an imaging medical physicist is to choose and optimize the imaging modality that maximizes the diagnostic content of the image while minimizing cost. This is not a simple task—diagnostic content depends on each specific task, and cost depends on the equipment price, the number of patients that can be imaged per unit time, the radiologists interpretation time, the cost of the exam to the patient, the risks to the patient, the consequences of a false negative exam, the cost of a false positive exam, and more.

One way to understand a medical physicist's task is to examine the relationship between cost, image quality, and diagnostic accuracy. If we assume that there is a monotonic relationship between image quality and diagnostic accuracy (Figure XXX), and if we assume that there is a similar relationship between image quality and cost (Figure XXX), then

In this part of the course, you will learn how to quantify image quality and how imaging affect image quality. First, we will review the mathematics required and discuss common image quality metrics. Second, we will discuss x-ray imaging physics and learn how to optimize and x-ray imaging system using the previously developed metrics. Finally, we will discuss the interpretation of the x-ray images and how our interpretation fits into the broader context.

These notes draw heavily from...

Our main reference will be [1].

# 2 X-Ray Image Quality

In this section we will introduce the basics of x-ray imaging and develop three tools that we can use to quantify image quality—contrast, resolution, and noise. We will focus on x-ray imaging in these notes, but these tools are useful for analyzing all imaging modalities.

## 2.1 X-ray Imaging Basics

We can create an x-ray imaging system by assembling an x-ray **source**, an **object**, and a **detector**. Figure 1 shows a simple example of an x-ray imaging system—we place a constant x-ray fluence $\phi_0$ incident on an object with two attenuation coefficients $\mu_1$ and $\mu_2$. X-rays are attenuated as they pass through the object so
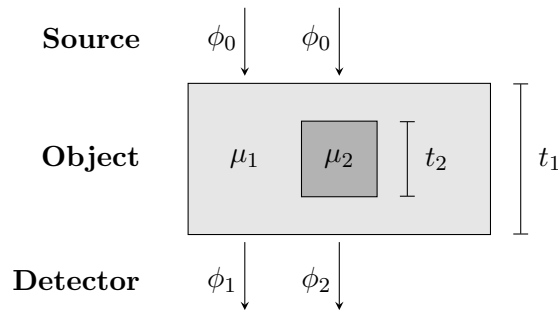


Figure 1: Simple x-ray imaging schematic.

the fluences that exit the object are related to the input fluence by

$$\phi_1 = \phi_0 e^{-\mu_1 t_1} \tag{1}$$

$$\phi_2 = \phi_0 e^{-\mu_1(t_1 - t_2) - \mu_2 t_2}. \tag{2}$$

We measure the x-rays that exit the object with a detector and create an image.

We will examine more realistic sources and detectors in later sections, but the simple example in Figure 1 is sufficient for us to model image quality in x-ray imaging systems.

## 2.2 Rose Model

Suppose that we'd like to detect the presence or absence of the small object with attenuation $\mu_2$ in Figure 1 with our imaging system. How well can we perform this task? What conditions do we need to meet to confidently say that the object is present or absent? How should we design our imaging system to meet these conditions? The Rose model supplies answers to these questions and gives us a solid framework for understanding image quality.

The signal is the difference in intensity between the object and its background:

$$\Delta S = S_1 - S_2 = A\phi_1 - A\phi_2 = A\Delta\phi.$$

where $A$ is the cross sectional area of the object, $\phi$ is the x-ray fluence with units of photons per unit area and $\Delta\phi = \phi_1 - \phi_2$.

The noise is the square-root of the variance in background over areas the size of the object. Assuming the noise follows Poisson statistics, where the variance means the mean value

$$\text{Noise} = \sqrt{\sigma^2} = \sqrt{\frac{A\phi_1 + A\phi_2}{2}} = \sqrt{A\bar{\phi}} \tag{3}$$

Therefore, the signal-to-noise ratio is,

$$\text{SNR} = \frac{A\Delta\phi}{\sqrt{A\bar{\phi}}} \tag{4}$$

$$\text{SNR} = C\sqrt{A\bar{\phi}} \tag{5}$$

where $C$ is the radiation contrast:

$$C = \frac{\Delta\phi}{\bar{\phi}} \tag{6}$$

This is the SNR for an ideal detector where we've assumed that

- complete absorption of incident quanta

- no added noise

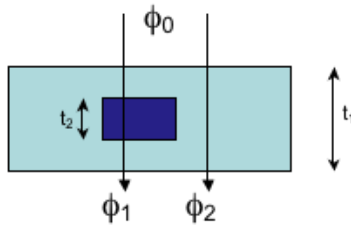- no loss of spatial resolution (i.e., no blurring)
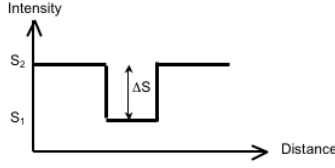


Figure 2: Test

Figure 3: Test

# 3 X-ray Imaging

## 3.1 X-ray Imaging Physics

### 3.1.1 Sources

X-ray tubes. kVp, mA, etc. Focal spot size

### 3.1.2 Interactions

Review of interactions in x-ray energy regime Scatter, grids

### 3.1.3 Detectors

The role of the x-ray detector is to convert the x-ray image into a visible image, and it should do so without altering it. Ideally, the x-ray detector should absorb all incident x-ray quanta.

The fraction of incident x-ray photons that interact in the detector, $\eta$, is called the quantum detection efficiency and is given by

$$\eta = 1 - e^{-\mu t}, \tag{7}$$

where $\mu$ is the linear attenuation coefficient of the x-ray detector and $t$ is its thickness.

We desire to have $\mu t$ to be as large as possible. Therefore, the x-ray detector should be thick and made of a material with high atomic number and high physical density.

SNR for a detector taking into account the quantum detection efficiency is

$$\text{SNR} = C\sqrt{\eta A \bar{\phi}}. \tag{8}$$

*Types of X-ray Detectors*

The two major classes of x-ray detectors are analog and digital.

Analog images are acquired and displayed on film. Film has a non-linear response to x-ray exposure. As we shall see, this presents many interesting phenomena. There are two types of analog detectors. Film Roentgen
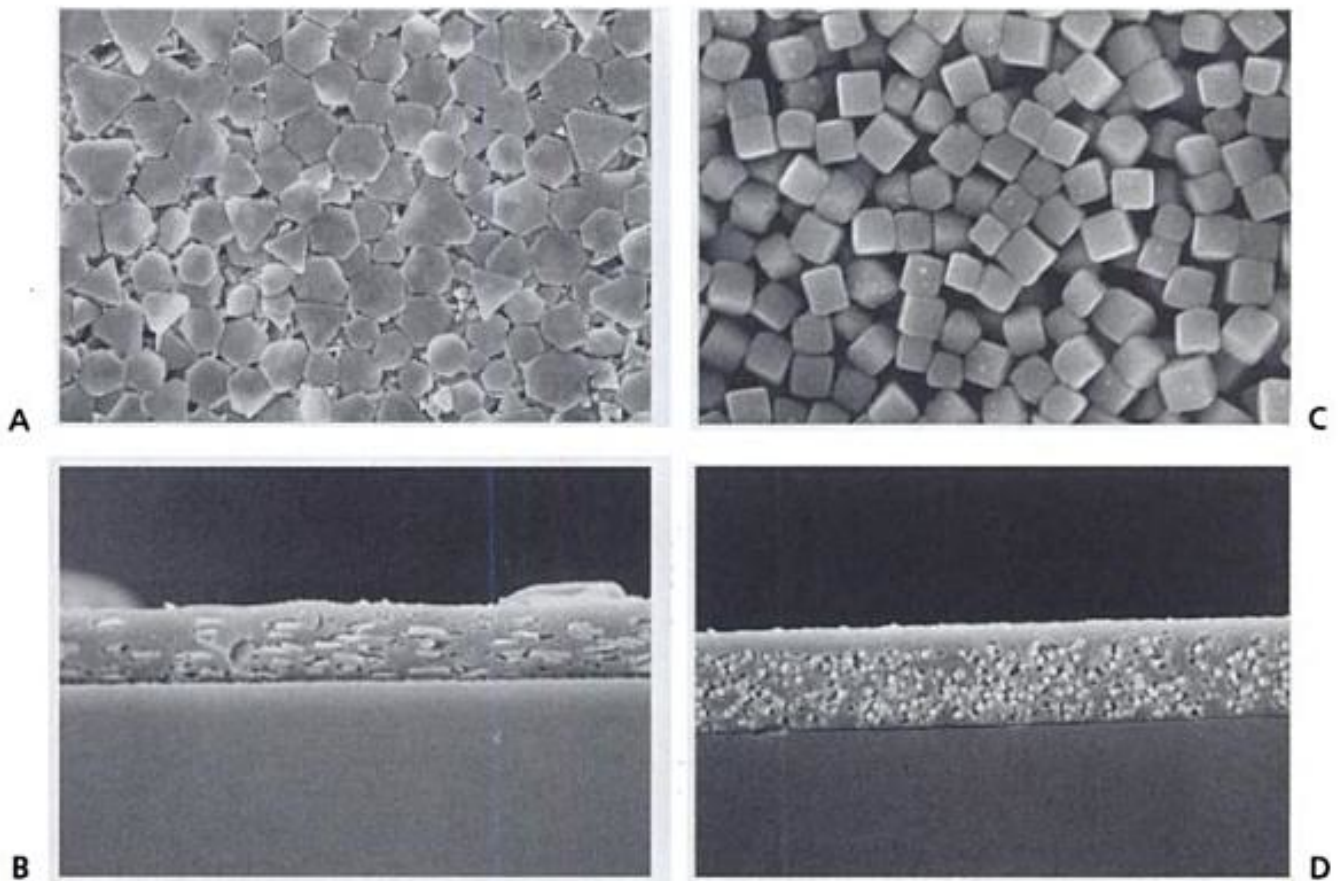
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**FIGURE 6-13.** Scanning electron micrographs (SEM) are shown. **A:** A top-down SEM image of the emulsion layer of T grain emulsion is shown. **B:** A cross section of the T grain emulsion film is illustrated, showing the grains in the gelatin layer, supported by the polymer film base below. **C:** Cubic grain film emulsion is shown in a top-down SEM view. **D:** A cross sectional SEM image of the cubic grain film is illustrated. SEM photographs courtesy of Drs. Bernard Apple and John Sabol.

Figure 4: Test

when making the first x-ray image used film. Film is made of silver bromide, and only approximately 3% of incident x-rays interact in the film. See additional material on how film works.

Screen film systems

To reduce the x-ray dose to the patient, films were used with florescence screens, which contain an x-ray phosphor. The x-ray phosphor converts the x-rays into visible light (plus infrared and ultraviolet), which exposes the film (see figure below). When the film is developed, the x-ray image appears. The film is used to both record and display the image. As we shall see, image quality can be compromised for screen-film systems compared to digital systems.

The output of the film image is measured in film optical density (OD). It is a measure of light transmission

through the film.

$$OD = -\ln\left(\frac{I}{I_0}\right).$$ (9)

Higher OD means lower transmission and a darker film, which corresponds to a higher x-ray exposure (see characteristic curve to the left). The characteristic is very dependent on the film processing conditions, which can be quite variable. The exact proportion of chemicals and the exact temperature of the developer will affect the shape of the characteristic curve and the speed point. Care is taken to keep the chemistry and the temperature of the developer as constant as possible.
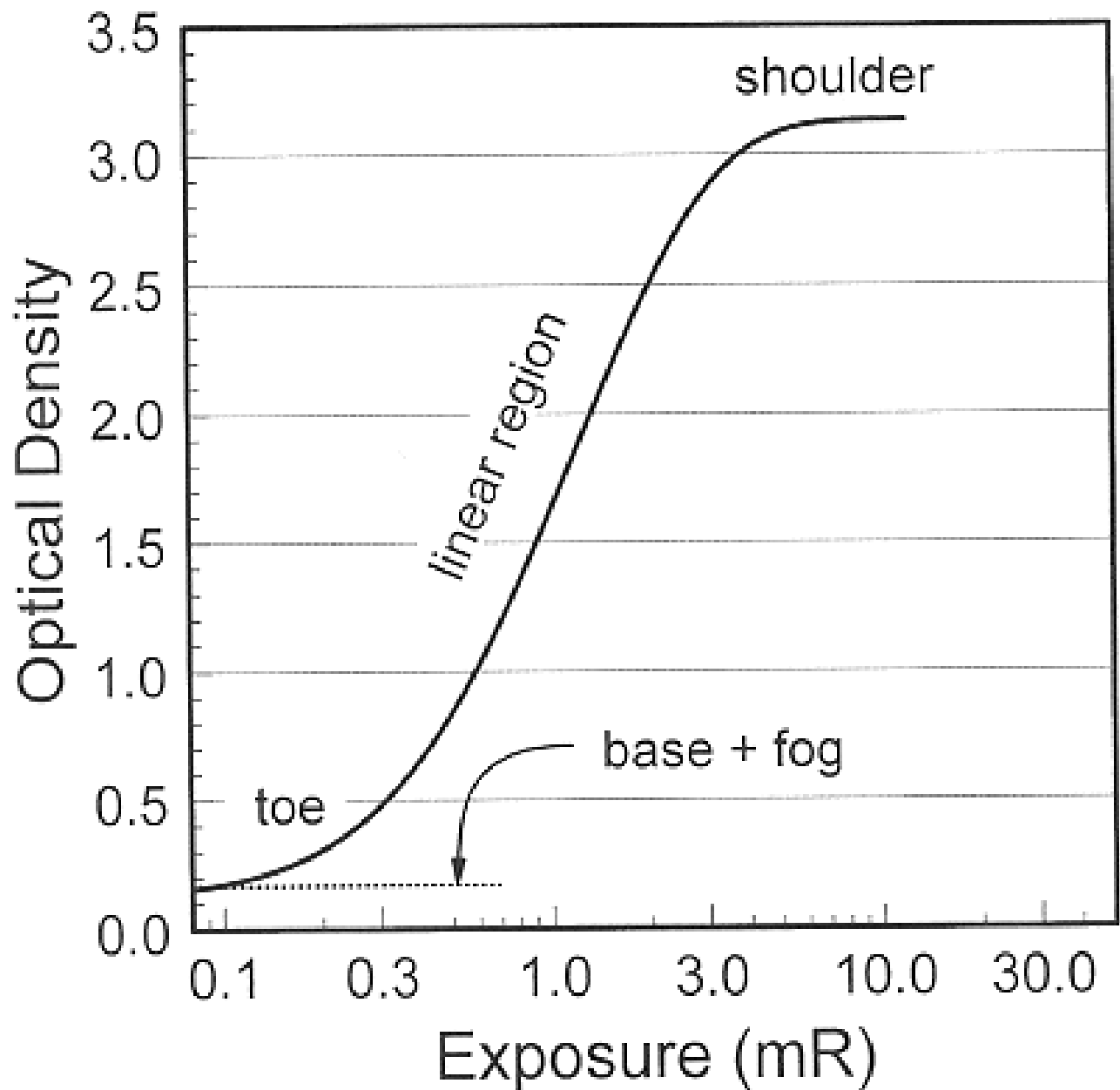


Figure 5: Test

An important property of screen-film systems is known as reciprocity law failure. If a certain exposure is required to reach a certain film OD, then if the same exposure is given but over a longer period of time, the film will have a lower OD. This is a basic property of film that is exposed using light. Film requires 3-10 light quanta to produce approximately 3-10 silver atoms in order for the grain to be developed grain (How Film Works handout). If the flux of light is too low, then after receiving fewer than the threshold number of light quanta, the silver atoms, being unstable can ionize before additional silver atoms are produced.

DIGITAL

Images are acquired in digital form and are usually displayed electronically, although they can be printed on film. Thus, the acquisition of the image is separate from the display of the image and each component can be optimized separately. Digital systems are usually respond linearly to x-ray exposure. The output of a digital image is in pixel value.

3. Indirect digital detectors Phosphor (often CsI) converts x rays to light and it is coupled to a CCD camera or a thin-film transistor (TFT) wafer. The latent image is then digitized and transferred to a computer for processing and display

4. Direct digital detectors These use materials (e.g., Se, a-Si, CdZnTe) that produce electron-hole pairs that can be collected directly. The an electric field is applied across the width of the detector and the electron-hole pairs follow the field lines that are perpendicular to the surface of the detector.

5. Photon counting detectors. It is possible for certain types of direct digital detectors to act as photon counting systems, where each individual x-ray interaction is count directly. Non-photon counting detectors integrate quanta (photons or electrons) over the total exposure time of the image acquisition.

6. Computed radiography (CR) [as oppose to digital radiography (DR)] systems These store a latent image as electrons trapped in the bulk of the phosphor (usually BaFCl). The electrons are subsequently read out by scanning the phosphor with a laser beam. The laser light stimulates the trapped electrons back into the conduction band where they can return back to the valence band with the emission of light. This light is collect, usually using a PMT. Wavelength of the laser readout is lower than the light that is emitted by the phosphor.

5. Image Quality Properties of X-ray Detectors

a. Contrast

The contrast in the image incident on the detector is given (ignoring any scattered radiation) is given by the radiation contrast, Eq. [4]. We want to know the contrast in the image called the radiographic contrast. To determine this, we need to know how x-ray exposure incident on the detector is converted to a visible image. This relationship between the input exposure and the output image is given by the characteristic curve.

b. Characteristic Curve

Gives the relationship between the detector output and the exposure to the detector. For a digital detector the characteristic curve is linear. That is, , where PV is the pixel value in the image and G is the slope of the characteristic curve. Further:

$$\frac{\Delta E}{\bar{E}} = \frac{\Delta \text{PV}}{\bar{\text{PV}}} = C. \tag{10}$$

Therefore, for a digital detector, the radiographic contrast is equal to the radiation contrast and this is true for all exposure values. It is independent of the slope of the characteristic curve.

For screen-film systems, the characteristic curve is called the H&D curve, named after Hurter and Driffield, and it is a plot OD versus log relative exposure to the screen.

For screen-film systems, the response is non-linear and there is a toe, a shoulder region, and a linear region in between. Base+fog is the minimum OD due to the transparency of the film base and any darkening of the film due to thermal effects. The net OD is the gross OD minus the base+fog level.

Characteristic curve for a screen-film system depends on the properties of the screen-film system and the film processing (developer) conditions.

For a screen-film system, the radiographic contrast is given by difference in optical density, D. It depends on the radiation contrast and the slope of the H&D curve, called gamma (G). Radiographic contrast in a screen-film image is given by:

$$G = \frac{\Delta D}{\Delta(\ln E)} \tag{11}$$

$$\Delta D = G(\log_{10} e)\Delta(\ln E) = G(\log_{10} e)\frac{\Delta E}{\bar{E}} = CG \log_{10} e, \tag{12}$$

since $\phi = kE$, then $\frac{\Delta E}{\bar{E}} = \frac{\Delta \phi}{\phi} = C$ = radiation contrast.

Since the characteristic curve is not linear, the exposure to the detector is very important. The image can be properly exposed, but also under or over exposed, where the radiographic contrast will be low (because G is low). For digital system, this is not a problem (at least in terms of contrast) as illustrated below.

Effect of characteristic curve shape. Top is for screen-film, which have a non-linear response. Bottom is for a digital system with a linear response. This figure only illustrates the effect on radiographic contrast and not noise nor SNR.

c. Speed

Speed is defined as the reciprocal of the exposure required to reach a net OD of 1.0. The speed point is considered the exposure to give a properly exposed image. A fast system has high speed and slow system has low speed. Screen-film systems have an optimum exposure that must be used in order to produce a useful image.

Factors Affecting Speed 1. X-ray absorption by the screen - phosphor type (atomic number, k-edge energy) - thickness and packing density - x-ray energy - crystal size 2. Conversion Efficiency of Screen (fraction of x-ray energy converted in optical energy) - physical properties of phosphor - optical properties of screen - concentration of activator atoms - x-ray energy 3. Film Sensitivity - silver content - sensitizers - film gain size, structure, etc. 4. Matching of light emission of the screen to the spectral sensitivity of the film 5. Film processing

d. Latitude

For screen-film systems, since the curve is non-linear, the system has limited latitude. Latitude refers to the range in exposure that will produce density within the accepted range for diagnostic radiology (usually considered to be 0.25 to 2.0). Latitude does not apply to digital systems. For screen-film systems, there is a tradeoff in latitude and contrast. Generally speaking, systems with high contrast (large G) have limited latitude and vice versa.

For the image on the right, System A has higher speed and wider latitude than System B. System B has higher contrast, but limited latitude.

Wide latitude is important for imaging tasks where there are large difference in tissue types. For example a chest image requires that image display lung tissue (mostly air) and ribs (bone). Wide latitude is required to image both of these simultaneous with good contrast. With a digital detector, since the response to x-ray exposure is linear, the display of the image can be manipulated so that bone can be displayed properly and then lung tissue; or image processing can be used so that both are imaged optimally in a single image.

e. Resolution In a phosphor screen, x rays are converted to optical photons that must travel through the bulk of the screen to escape. For screens that are composed of crystals of phosphor in a binder material (turbid screen), the light is scattered multiple times as illustrated below and light can be absorbed in the screen. The light at the output of the screen is spread over a finite area, reducing spatial resolution. The scattering of light in the screen increases spatial resolution because it preferentially reduces light photons that travel a long distance. Recall that resolution can be characterized by the point spread function (psf) and the modulation transfer function (MTF). The image below gives a qualitative depiction of how the scattering of light broadens the psf and thus reduces the high frequency components of the MTF.

The spatial resolution is reduced (more spread of light) as the thickness of the screen increases. The further the distance the light needs to travel to exit the screen, the broader the psf will be. In many instances a film is sandwiched between two thinner screens rather than be used with a single thick screen. This can improve the resolution compared to using a single thick screen. It is important that the screen and film be in close contact, as any space (poor contact) will increase the area over which the light has spread.

In CsI phosphor, the crystals of CsI form long needle shaped structures. These needles act like an optical fiber reducing the lateral spread of light improving the resolution compared to turbid screens of equal thickness.

For direct digital detectors, the spatial resolution can be very high. An electric field can be placed across

the photoconductor forcing the electrons to travel in direction perpendicular to the surface of the detector greatly reducing the lateral spread of the electrons.

f. X-ray Quantum Noise

The signal in a screen-film system, the signal is radiographic contrast, as given in Eq. [9]. The noise in as screen-film image is $\sigma_D$, and it is related to the noise in the x-ray image incident on the detector, $\sigma_E$.

For a uniform exposure, we can average the square of $\Delta D = D(x, y) - \bar{D}$ over an area in the image to calculate $\sigma_D$:

$$\sigma_D^2 = \frac{1}{4XY} \int_{-X}^{X} \int_{-Y}^{Y} \Delta D^2(x, y) dx dy. \tag{13}$$

Similar equations can be written in terms of PV and exposure to the detector, $E$.

$$\sigma_{PV}^2 = \frac{1}{4XY} \int_{-X}^{X} \int_{-Y}^{Y} \Delta PV^2(x, y) dx dy \tag{14}$$

$$\sigma_E^2 = \frac{1}{4XY} \int_{-X}^{X} \int_{-Y}^{Y} \Delta E^2(x, y) dx dy \tag{15}$$

Then by Eqs [10] and [11]"

$$\Delta D = CG \log_{10} e = G(\log_{10} e) \frac{\Delta E}{\bar{E}}, \tag{16}$$

but

$$E = kN \text{ and} \tag{17}$$

$$\sigma_E^2 = k^2 \sigma_N^2 \tag{18}$$

where $N$ is the number of photons, which is $N = A\phi$, where $A$ is the cross-sectional area and $\phi$ is the fluence.

Eq. [14] becomes:

$$\sigma_D^2 = G^2(\log_{10}^2 e) \frac{k^2 \sigma_N^2}{k^2 \bar{N}^2} = G^2(\log_{10}^2 e) \frac{\sigma_N^2}{\bar{N}^2} \tag{19}$$

For Poisson statistics,

$$\sigma_N^2 = \bar{N} = A\bar{\phi} \text{and} \frac{\sigma_N^2}{\bar{N}^2} = \frac{1}{\bar{N}} = \frac{1}{A\bar{\phi}}. \tag{20}$$

Inserting Eq. [19] into [17] gives:

$$\sigma_D^2 = \frac{G^2(\log_{10}^2 e)}{A\bar{\phi}}. \tag{21}$$

For a digital system, $PV = GE$ and therefore

$$\Delta PV = G\Delta E. \tag{22}$$

Now using Eqs. [13] and [20]

$$\sigma_{PV}^2 = G^2\sigma_E^2. \tag{23}$$

Using Eqs [16], [17], [18] and [21]

$$\sigma_{PV}^2 = G^2 k^2 \sigma_N^2 = G^2 k^2 A\bar{\phi} \tag{24}$$

Note $\sigma_D^2 \propto \frac{1}{\phi}$, but $\sigma_{PV}^2 \propto \bar{\phi}$.

Note in real imaging systems, there are other noise sources. In particular, in a screen-film system there is noise due the finite size and number of the silver grains in the developed film; and in a digital system there is electronic noise from the device that captures the light (indirect detectors) or the electrons (direct detectors). These are usually small compared to quantum noise, at low spatial frequencies. More about this in the lecture on noise.

SNR (ignoring image blurring and considering only x-ray quantum noise)

For screen-film systesms, using Eqs. [10] and [19]

$$SNR_{film} = \frac{\Delta D}{\sigma_D} = \frac{GC(\log_{10} e)}{\sqrt{\frac{G^2(\log_{10}^2 e)}{A\bar{\phi}}}} = C\sqrt{A\bar{\phi}}, \tag{25}$$

which is the same as Eq. [3].

For a digital system, using Eqs. [15], [20] and [22]

$$SNR_{digital} = \frac{\Delta PV}{\sigma_{PV}} = \frac{G\Delta E}{\sqrt{G^2 k^2 A\phi}} = \frac{k\Delta N}{\sqrt{k^2 A\phi}} = \frac{A\Delta\phi}{\sqrt{A\bar{\phi}}} = C\sqrt{A\bar{\phi}}, \tag{26}$$

which is again Eq. [3].

Non-ideal Detectors

Assume the imaging system is linear or linearizable. Further assume $w_{in}(u)$ is the input stimulus where $u$ is

an independent variable and $w_{out}(u)$ is the output response of the system. If there are two inputs, which produce two outputs, that is: $w'_{out}(u) = w'_{in}(u)$ and $w''_{out}(u) = w''in(u)$, the system is said to be linear if, when both inputs are applied together, $w_{in}(u) = w'in(u)+w''in(u)$, the output is given by: $wout(u) = w'out(u)+w''out(u)$.

For a real (non-ideal) imaging system, the input maybe localized to a location $u_0$, the response at the output is spread over a range of $u$ centered on $u0$. Conversely, any point at the output will depend on input stimuli over a range of positions at the input, that is:

$$w_{out}(u) = \int_{-\infty}^{\infty} p(u, u')w_{\text{in}}(u')du'$$

Let $w_{\text{in}}(u) = \delta(u - u_0)$.

Therefore,

## 3.2 Characterizing An X-ray Imaging System

### 3.2.1 Contrast

### 3.2.2 Resolution

### 3.2.3 Noise

## 3.3 Effect of Scatter
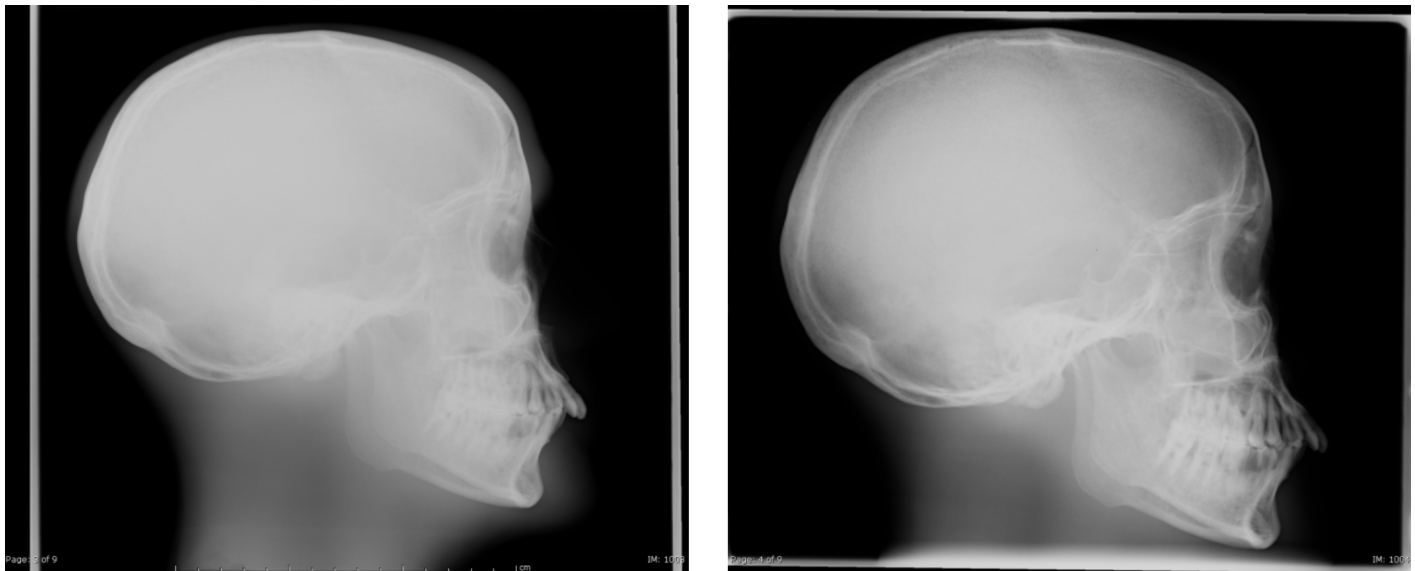


Figure 6: Test

## 3.4 Effect of Focal Spot Size

# 4    Image Interpretation

## 4.1    Societal Levels

## 4.2    Phantoms

### 4.2.1    Contrast-Detail

### 4.2.2    Resolution

# 5   Conclusion

# References

[1] Harrison H. Barrett and William Swindell. *Radiological imaging : the theory of image formation, detection, and processing.* Academic Press, New York, 1981. `https://doi.org/10.1016/c2009-0-02376-9`.