Medical Physics 386:

The Physics of Medical Imaging 1

X-ray Imaging Notes

(Last updated: March 18, 2018)

1 Introduction

There are many different imaging modalities in the radiology department— mammography, computed radiography, digital radiography, fluoroscopy, computed tomography (CT), magnetic resonance imaging (MRI), positron-emission tomography (PET), single-photon emission tomography (SPECT), ultrasound, and more. Why are there so many modalities? Because no single modality is optimal for the wide variety of imaging tasks that arise when we try to diagnose disease. Each imaging task has different imaging requirements, so no single modality can be used for all tasks.

A major goal of an imaging medical physicist is to choose and optimize the imaging modality that maximizes the diagnostic content of the image while minimizing cost. This is not a simple task—diagnostic content depends on each specific task, and cost depends on the equipment price, the number of patients that can be imaged per unit time, the radiologists interpretation time, the cost of the exam to the patient, the risks to the patient, the consequences of a false negative exam, the cost of a false positive exam, and more.

One way to understand a medical physicist's task is to examine the relationship between cost, image quality, and diagnostic accuracy. If we assume that there is a monotonic relationship between image quality and diagnostic accuracy (Figure XXX), and if we assume that there is a similar relationship between image quality and cost (Figure XXX), then

In this part of the course, you will learn how to quantify image quality and how it is affected by various acquisition parameters. First, we will review the mathematics required and discuss common image quality metrics. Second, we will discuss x-ray imaging physics and learn how to optimize x-ray imaging systems using the previously developed metrics. Finally, we will discuss the interpretation of the x-ray images and how our interpretation fits into the broader context.

These notes draw heavily from...

Our main reference will be [**?**].

2 X-ray Image Quality

In this section we will introduce the basics of x-ray imaging and develop three tools that we can use to quantify image quality—contrast, resolution, and noise. We will focus on x-ray imaging in these notes, but these tools are useful for analyzing all imaging modalities.

2.1 X-ray Imaging Basics

We can create an x-ray imaging system by assembling an x-ray **source**, an **object**, and a **detector**. Figure 1 shows a simple example of an x-ray imaging system—we place an x-ray fluence $\boldsymbol{\phi}_0$ incident on an object with two attenuation coefficients μ_1 and μ_2. Note that we use bold symbols to indicate random variables. X-rays

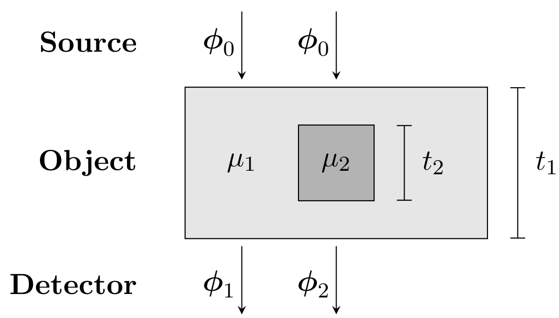

Figure 1: Simplified x-ray imaging schematic. A collimated beam of x-rays with fluence $\boldsymbol{\phi}_o$ is incident on an object with two attenuation coefficients. A detector measures the output fluences $\boldsymbol{\phi}_{1,2}$.

are attenuated as they pass through the object so the exit fluences are related to the input fluence by

$$\boldsymbol{\phi}_1 = \boldsymbol{\phi}_0 e^{-\mu_1 t_1} \tag{1}$$

$$\boldsymbol{\phi}_2 = \boldsymbol{\phi}_0 e^{-\mu_1 (t_1 - t_2) - \mu_2 t_2}. \tag{2}$$

When we place a detector in the path of the exit beam we create an image of the object.

We will examine more realistic sources and detectors in later sections, but the simple example in Figure 1 is sufficient for us to model image quality in x-ray imaging systems.

2.2 Rose Model

Suppose that we'd like to detect the presence or absence of the small object with attenuation μ_2 in Figure 1 with our imaging system. How well can we perform this task? What conditions do we need to meet to confidently say that the object is present or absent? How should we design our imaging system to meet these conditions? The Rose model supplies answers to these questions and gives us a solid framework for understanding image quality.

First, we define the **signal** S as the mean number of photons blocked by the object

$$S \equiv A(\mathrm{E}\{\boldsymbol{\phi}_1\} - \mathrm{E}\{\boldsymbol{\phi}_2\}) = A\Delta\phi \tag{3}$$

where A is the cross sectional area of the object, $\mathrm{E}\{\cdot\}$ denotes the expectation value, ϕ is the x-ray fluence in units of photons per unit area, and $\Delta\phi \equiv \mathrm{E}\{\boldsymbol{\phi}_1\} - \mathrm{E}\{\boldsymbol{\phi}_2\}$. This may seem like a peculiar way to define the signal—shouldn't the signal be the measured intensity difference between areas with and without the object? The reason for our definition is that it captures the role of object size in detectability. Intuitively, larger objects are easier to detect, so our definition of signal should reflect this.

Next, we consider the **noise** N that corrupts our signal. Note that in signal-to-noise ratio discussions the word "noise" usually refers to the standard deviation of a random variable. We will use this meaning. In general the word "noise" refers to any random or unwanted signals.

We define the noise as the standard deviation of the number of photons detected in an area the size of the object when the object is absent.

$$N \equiv \sqrt{\mathrm{Var}\{A\boldsymbol{\phi}_1\}}. \tag{4}$$

$A\boldsymbol{\phi}_1$ is a Poisson-distributed random variable, so its variance is identical to its mean

$$N = \sqrt{\mathrm{E}\{A\boldsymbol{\phi}_1\}} \tag{5}$$

$$N = \sqrt{A\mathrm{E}\{\boldsymbol{\phi}_1\}}. \tag{6}$$

Finally, the signal-to-noise ratio is given by

$$\mathrm{SNR} \equiv \frac{S}{N} = \frac{A\Delta\phi}{\sqrt{AE\{\boldsymbol{\phi}_1\}}} \tag{7}$$

where C is the radiation contrast:

$$C = \frac{\Delta\phi}{\bar{\phi}} \tag{8}$$

This is the SNR for an ideal detector where we've assumed that there is

- complete absorption of incident quanta
- no added noise
- no loss of spatial resolution (i.e., no blurring)

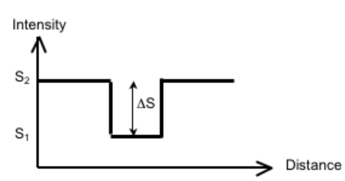

Figure 2: Test

3 X-ray Imaging

3.1 X-ray Imaging Physics

3.1.1 Sources

X-ray tubes. kVp, mA, etc. Focal spot size

3.1.2 Interactions

Review of interactions in x-ray energy regime Scatter, grids

3.1.3 Detectors

An ideal x-ray detector converts an x-ray image into a visible image without altering it. In this section we will review the major types of x-ray detectors and examine how they fail to be ideal.

The first task of any x-ray detector is to absorb all incident x-rays—a detector should not let any x-rays pass through. The fraction of incident x-rays that interact in the detector, η, is called the quantum detection efficiency. The quantum detection efficiency is related to the linear attenuation coefficient of the x-ray detector, μ, and the thickness of the detector t by

$$\eta = 1 - e^{-\mu t}. \tag{9}$$

An ideal detector will stop all x-rays, so we would like μt to be as large as possible. Therefore, an x-ray detector should be thick and made of a material with high atomic number and high physical density. If the quantum detection efficiency is less than one, then the detector's SNR will be reduced by $\sqrt{\eta}$ and Eq. XXX becomes

$$\mathrm{SNR} = C\sqrt{\eta A \bar{\phi}}. \tag{10}$$

Recall from the Rose model that we need to an SNR of approximately 5 to detect a signal on a background. This means that reducing the quantum efficiency will require us to increase the incident x-ray fluence to maintain a fixed SNR. Increasing the photon fluence will increase the dose to the patient, so clearly quantum efficiency is an important consideration for all detectors.

Analog Detectors

Analog images are acquired and displayed on film. Film consists of a plastic base coated with a gelatin binder that contains light-sensitive silver halide crystals. Figures 3 and 4 show schematic and scanning electron micrograph views of film.

When a photon hits a (transparent) silver halide crystal, a photochemical reaction occurs that creates two (opaque) metallic silver atoms. Under typical conditions the number of metallic silver atoms is far too small for the film to become visibly opaque to human eyes, but the film now contains a latent image in the form of silver atoms within silver halide grains. To make the image visible, we need to "amplify" the number of metallic silver atoms in each grain using a *developer solution*, then remove the unexposed silver halide crystals using a *fixing solution*. The result is a *photographic negative* that is dark (lots of metallic silver) in regions that have been exposed and light in regions that have not been exposed. In x-ray imaging the photographic negative is used directly, but in photographic imaging you need to invert the film brightness by projecting light through the negative, imaging the result on film, then repeating the development process. See [**?**, **?**] for a more detailed discussion of the film development process.

For an x-ray film with a thin coat of silver halide grains, the grain size determines the spatial resolution. Film grains are approximately 0.2-2 μm in diameter which gives us a sense of the smallest resolvable feature. For comparison a typical 2018 scientific CCD has a pixel width of 6 μm and an iPhone X display has a pixel width of 55 μm. Film has unmatched spatial resolution even in 2018.

Only a few percent of incident x-rays interact as they pass through film. Because quantum efficiency is so important for medical imaging, most medical x-ray film uses a photographic emulsion coating on both sides of the plastic base. We could increase the thickness of the emulsion layers to improve quantum efficiency further, but this would reduce spatial resolution and make developing the film more difficult.

We can improve the quantum efficiency of film by placing a *fluorescence screen* directly on the film. Fluorescence screens have a high attenuation coefficient and contain an x-ray phosphor that converts incident x-rays into many visible (plus infrared and ultraviolet) photons. These visible photons expose the film and the film can be developed normally. Unfortunately, the visible photons do not travel straight to the film along their generating x-ray's path—they spread out before they reach the film. This means that using a fluorescence screen improves quantum efficiency at the expense of spatial resolution. This type of detector is often called an *indirect analog* detector because it indirectly detects the incident x-rays.

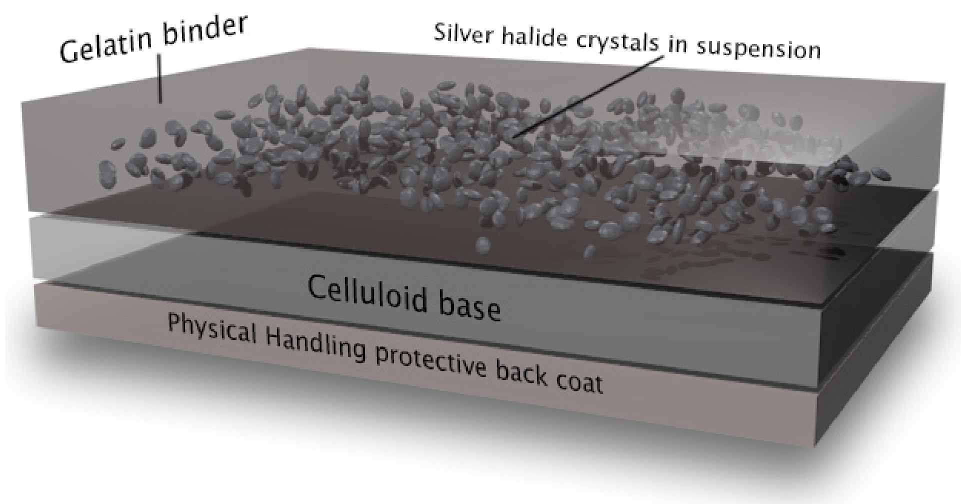

Figure 3: Schematic view of photographic film. [**?**]

The input to a film detector is the x-ray exposure measured in coulombs per kilogram (C/kg) or röntgen (R). The output of a film detector is usually measured with a unitless property called optical density (OD). If a thin beam of light with intensity I_0 hits the film and the transmitted beam has intensity I, then the optical density of the film at that point is defined as

$$\mathrm{OD} = -\ln\left(\frac{I}{I_0}\right). \tag{11}$$

A high OD means that the film is opaque which corresponds to a high x-ray exposure. The relationship between x-ray exposure (input) and optical density (output) is often called the *characteristic* curve, the *Hurter-Driffield*

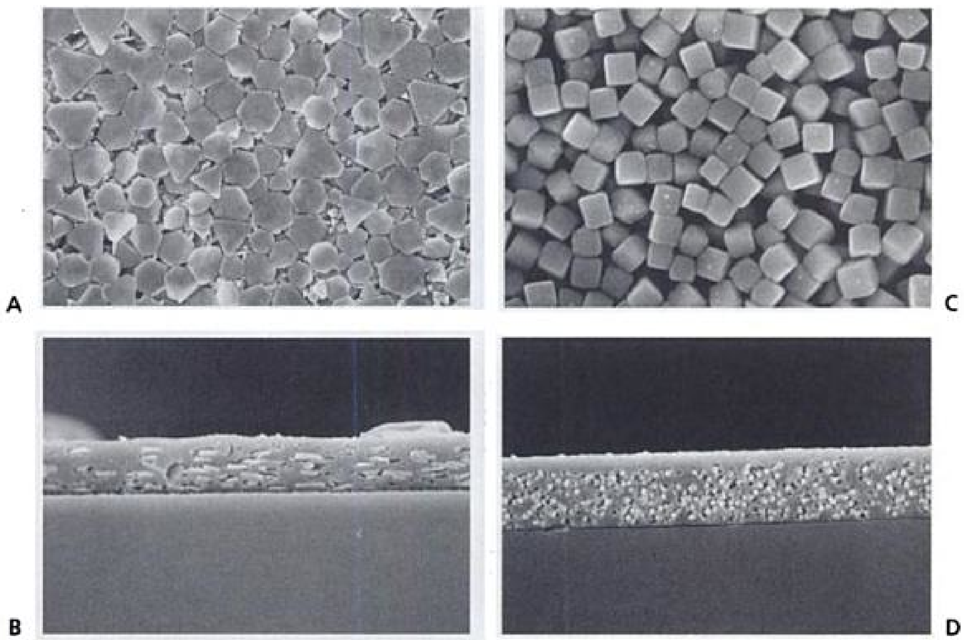

Figure 4: Scanning electron micrographs (SEM) of photographic film. **A**: A top-down SEM image of the emulsion layer of T grain emulsion. **B**: A cross section of the T grain emulsion film showing the grains in the gelatin layer supported by the polymer film base below. **C**: Cubic grain film emulsion in a top-down SEM view. **D**: Cross section of the cubic grain film. SEM courtesy of Drs. Bernard Apple and John Sabol. Citation needed.

curve, or the D—$\log E$ curve. An example of a characteristic curve is shown in Fig. 5. Note the often confused units on the characteristic curve—the horizontal axis uses a logarithmic exposure scale and the vertical axis uses a linear scale for optical density which is itself a logarithm of the input-output intensity ratio (see Eq. 11).

Let's examine the features of the characteristic curve in Fig. 5. When the film has not been exposed the optical density is not zero—even unexposed film is not completely transparent due to reflections, impurities, and thermal effects. As we start to expose the film, we begin to create metallic silver atoms. A single metallic silver atom in a grain is not enough to create a latent image though; about 4 metallic silver atoms are required in a single grain to ensure that it will be developed. This means that for low exposures the optical density will increase slowly—this is evident in the "toe" of the characteristic curve. At high exposures most of the grains in the film already have many metallic silver atoms, so increasing the exposure will not increase the optical density further—this is evident in the "shoulder" of the characteristic curve. Finally, at intermediate exposures the optical density increases linearly with the log exposure. The slope of characteristic curve is often called the *speed* of the film because it summarizes how quickly a fixed exposure rate will increase the optical density—this terminology is borrowed from photographers who will use a "fast" film in situations where they require short exposure times e.g. sports photography. See [**?**] chapter X for a complete derivation of film's characteristic curve.

The characteristic curve also depends on the film processing conditions which can vary significantly. The proportions of chemicals in the developer solution and the temperature will affect the shape of the characteristic curve and the speed point. Care is taken to keep the chemistry and the temperature of the developer as constant as possible.

Film's characteristic curve also depends on the exposure rate. The metallic silver atoms in each grain are

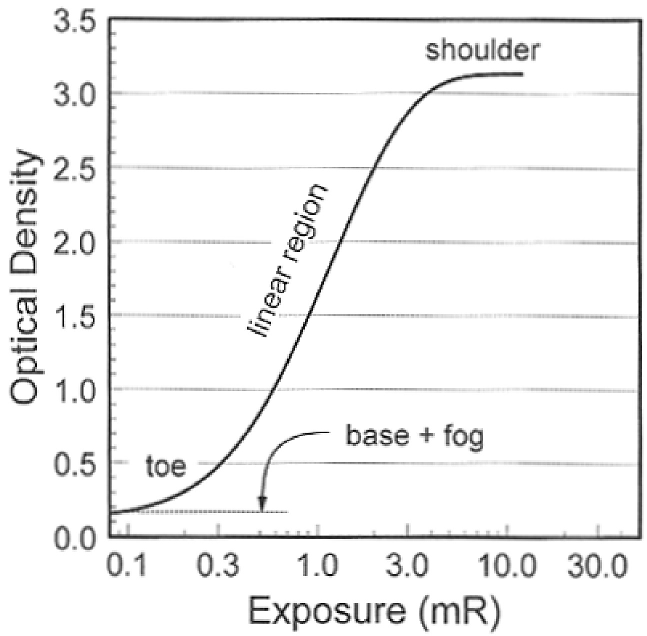

Figure 5: A characteristic curve for film. Notice the logarithmic scale on the horizontal axis and the linear scale on the vertical axis.

unstable and will re-ionize back into the silver halide crystal over time. If the exposure rate is very low, then these re-ionization events can significantly reduce the number of developable grains which will reduce the OD in the final image. In other words, the characteristic curve will shift to the right for low exposure rates. This property of film is often referred to as *reciprocity law failure* or the *Schwarzschild effect* after Schwarzchild noticed the issue while imaging dim stars.

Digital Detectors

We will briefly discuss several types of digital detectors and leave the details for Patrick's lectures. The defining feature of a digital detector is that its output is a digital signal with discrete values. Digital images are usually displayed electronically, but they can be also printed on film. Thus, the acquisition of the image is separate from the display of the image and each component can be optimized separately. The input to a digital detector is exposure, and the output is a pixel value (PV). Digital systems usually respond linearly to x-ray exposure.

Indirect digital detectors use a phosphorescent screen (often CsI) coupled to a charge-couple device (CCD) camera or a think-film transistor (TFT) water. Incident x-rays are converted to visible photons in the screen which generates a signal in the CCD. Indirect digital detectors are analogous to indirect analog detectors.

Direct digital detectors use materials that produce electron-hole pairs that can be collected directly. Selenium (Se), amorphous silicon (a-Si), and cadmium zinc telluride (CZT) are common materials for direct digital detectors. An electric field is applied across the width of the detector and the electron-hole pairs follow the field lines that are perpendicular to the surface of the detector where they are collected and read out.

Photon counting detectors can measure individual x-ray interactions. Some direct digital detectors can act as photon-counting detectors. Non-photon counting detectors integrate quanta (photons or electrons) over the total exposure time of the image acquisition.

Computed radiography (CR) detectors store a latent image as electrons trapped in a phosphor screen (usually

BaFCl). The electrons are subsequently read out by scanning the phosphor with a laser beam. The laser light stimulates the trapped electrons back into the conduction band where they can return back to the valence band with the emission of light. The emitted light is collected to create a digital image. These detectors use the same principle as optically stimulated luminescent dosimeters (OSLD) in radiation dosimetry.

3.1.4 *Comparison of Analog and Digital Detectors

Contrast

Speed

Latitude

Resolution

Noise

SNR

The contrast in the image incident on the detector is given (ignoring any scattered radiation) by the radiation contrast, Eq. [4]. We want to know the radiographic contrast — the contrast in the final image. To determine this, we need to know how the x-ray exposure incident on the detector is converted to a visible image. This relationship between the input exposure and the output image is given by the characteristic curve.

b. Characteristic Curve

Gives the relationship between the detector output and the exposure to the detector. For a digital detector the characteristic curve is linear. That is,

$$\mathrm{PV} = GE \tag{12}$$

where PV is the pixel value in the image, G is the slope of the characteristic curve and E is the incident exposure. Further:

$$\frac{\Delta E}{\bar{E}} = \frac{\Delta \mathrm{PV}}{\bar{\mathrm{PV}}} = C. \tag{13}$$

That is, for a digital detector, the radiographic contrast is equal to the radiation contrast and this is true for all exposure values. It is independent of the slope of the characteristic curve.

The characteristic curve for a screen-film system depends on the properties of the screen-film system and the film processing (developer) conditions.

For a screen-film system, the radiographic contrast is given by the difference in optical density, D. It depends on the radiation contrast and the slope of the H&D curve, called gamma (G). Radiographic contrast in a screen-film image is given by:

$$G = \frac{\Delta D}{\Delta(\ln E)} \tag{14}$$

$$\Delta D = G(\log_{10} e)\Delta(\ln E) = G(\log_{10} e)\frac{\Delta E}{\bar{E}} = CG\log_{10} e, \tag{15}$$

since $\phi = kE$, then $\frac{\Delta E}{E} = \frac{\Delta \phi}{\phi} = C =$ radiation contrast.

Since the characteristic curve is not linear, the exposure to the detector is very important. The image can be properly exposed, but also under or over exposed, where the radiographic contrast will be low (because G is low). For digital system, this is not a problem (at least in terms of contrast) as illustrated below.

Effect of characteristic curve shape. Top is for screen-film, which have a non-linear response. Bottom is for a digital system with a linear response. This figure only illustrates the effect on radiographic contrast and not noise nor SNR.

c. Speed

Speed is defined as the reciprocal of the exposure required to reach a net OD of 1.0. The speed point is considered the exposure to give a properly exposed image. A fast system has high speed and slow system has low speed. Screen-film systems have an optimum exposure that must be used in order to produce a useful image.

Factors Affecting Speed 1. X-ray absorption by the screen - phosphor type (atomic number, k-edge energy) - thickness and packing density - x-ray energy - crystal size 2. Conversion Efficiency of Screen (fraction of x-ray energy converted in optical energy) - physical properties of phosphor - optical properties of screen - concentration of activator atoms - x-ray energy 3. Film Sensitivity - silver content - sensitizers - film gain size, structure, etc. 4. Matching of light emission of the screen to the spectral sensitivity of the film 5. Film processing

d. Latitude

For screen-film systems, since the curve is non-linear, the system has limited latitude. Latitude refers to the range in exposure that will produce density within the accepted range for diagnostic radiology (usually considered to be 0.25 to 2.0). Latitude does not apply to digital systems. For screen-film systems, there is a tradeoff in latitude and contrast. Generally speaking, systems with high contrast (large G) have limited latitude and vice versa.

For the image on the right, System A has higher speed and wider latitude than System B. System B has higher contrast, but limited latitude.

Wide latitude is important for imaging tasks where there are large difference in tissue types. For example a chest image requires that image display lung tissue (mostly air) and ribs (bone). Wide latitude is required to image both of these simultaneous with good contrast. With a digital detector, since the response to x-ray exposure is linear, the display of the image can be manipulated so that bone can be displayed properly and then lung tissue; or image processing can be used so that both are imaged optimally in a single image.

e. Resolution In a phosphor screen, x rays are converted to optical photons that must travel through the bulk of the screen to escape. For screens that are composed of crystals of phosphor in a binder material (turbid screen), the light is scattered multiple times as illustrated below and light can be absorbed in the screen. The light at the output of the screen is spread over a finite area, reducing spatial resolution. The scattering of light in the screen increases spatial resolution because it preferentially reduces light photons that travel a long distance. Recall that resolution can be characterized by the point spread function (psf) and the modulation transfer function (MTF). The image below gives a qualitative depiction of how the scattering of light broadens the psf and thus reduces the high frequency components of the MTF.

The spatial resolution is reduced (more spread of light) as the thickness of the screen increases. The further the distance the light needs to travel to exit the screen, the broader the psf will be. In many instances a film is sandwiched between two thinner screens rather than be used with a single thick screen. This can improve the resolution compared to using a single thick screen. It is important that the screen and film be in close contact, as any space (poor contact) will increase the area over which the light has spread.

In CsI phosphor, the crystals of CsI form long needle shaped structures. These needles act like an optical fiber reducing the lateral spread of light improving the resolution compared to turbid screens of equal thickness.

For direct digital detectors, the spatial resolution can be very high. An electric field can be placed across the photoconductor forcing the electrons to travel in direction perpendicular to the surface of the detector greatly reducing the lateral spread of the electrons.

f. X-ray Quantum Noise

The signal in a screen-film system, the signal is radiographic contrast, as given in Eq. [9]. The noise in as screen-film image is σ_D, and it is related to the noise in the x-ray image incident on the detector, σ_E.

For a uniform exposure, we can average the square of $\Delta D = D(x,y) - \bar{D}$ over an area in the image to calculate σ_D:

$$\sigma_D^2 = \frac{1}{4XY}\int_{-X}^{X}\int_{-Y}^{Y}\Delta D^2(x,y)dxdy. \tag{16}$$

Similar equations can be written in terms of PV and exposure to the detector, E.

$$\sigma_{\mathrm{PV}}^2 = \frac{1}{4XY}\int_{-X}^{X}\int_{-Y}^{Y}\Delta\mathrm{PV}^2(x,y)dxdy \tag{17}$$

$$\sigma_{\mathrm{E}}^2 = \frac{1}{4XY}\int_{-X}^{X}\int_{-Y}^{Y}\Delta\mathrm{E}^2(x,y)dxdy \tag{18}$$

Then by Eqs [10] and [11]"

$$\Delta D = CG\log_{10}e = G(\log_{10}e)\frac{\Delta E}{\bar{E}}, \tag{19}$$

but

$$E = kN \text{ and} \tag{20}$$

$$\sigma_E^2 = k^2\sigma_N^2 \tag{21}$$

where N is the number of photons, which is $N = A\phi$, where A is the cross-sectional area and ϕ is the fluence.

Eq. [14] becomes:

$$\sigma_D^2 = G^2(\log_{10}^2 e)\frac{k^2\sigma_N^2}{k^2\bar{N}^2} = G^2(\log_{10}^2 e)\frac{\sigma_N^2}{\bar{N}^2} \tag{22}$$

For Poisson statistics,

$$\sigma_N^2 = \bar{N} = A\bar{\phi}\text{and}\frac{\sigma_N^2}{\bar{N}^2} = \frac{1}{\bar{N}} = \frac{1}{A\bar{\phi}}. \tag{23}$$

Inserting Eq. [19] into [17] gives:

$$\sigma_D^2 = \frac{G^2(\log_{10}^2 e)}{A\bar{\phi}}. \tag{24}$$

For a digital system, $\mathrm{PV} = GE$ and therefore

$$\Delta\mathrm{PV} = G\Delta E. \tag{25}$$

Now using Eqs. [13] and [20]

$$\sigma_{PV}^2 = G^2 \sigma_E^2. \tag{26}$$

Using Eqs [16], [17], [18] and [21]

$$\sigma_{\rm PV}^2 = G^2 k^2 \sigma_N^2 = G^2 k^2 A \bar{\phi} \tag{27}$$

Note $\sigma_D^2 \propto \frac{1}{\phi}$, but $\sigma_{\rm PV}^2 \propto \bar{\phi}$.

Note in real imaging systems, there are other noise sources. In particular, in a screen-film system there is noise due the finite size and number of the silver grains in the developed film; and in a digital system there is electronic noise from the device that captures the light (indirect detectors) or the electrons (direct detectors). These are usually small compared to quantum noise, at low spatial frequencies. More about this in the lecture on noise.

SNR (ignoring image blurring and considering only x-ray quantum noise)

For screen-film systesms, using Eqs. [10] and [19]

$$\mathrm{SNR}_{\rm film} = \frac{\Delta D}{\sigma_D} = \frac{GC(\log_{10} e)}{\sqrt{\frac{G^2(\log_{10}^2 e)}{A\bar{\phi}}}} = C\sqrt{A\bar{\phi}}, \tag{28}$$

which is the same as Eq. [3].

For a digital system, using Eqs. [15], [20] and [22]

$$\mathrm{SNR}_{\rm digital} = \frac{\Delta \mathrm{PV}}{\sigma_{PV}} = \frac{G\Delta E}{\sqrt{G^2 k^2 A\phi}} = \frac{k\Delta N}{\sqrt{k^2 A\phi}} = \frac{A\Delta\phi}{\sqrt{A\phi}} = C\sqrt{A\bar{\phi}}, \tag{29}$$

which is again Eq. [3].

Non-ideal Detectors

Assume the imaging system is linear or linearizable. Further assume $w_{in}(u)$ is the input stimulus where u is an independent variable and $w_{out}(u)$ is the output response of the system. If there are two inputs, which produce two outputs, that is: $w'_{out}(u) = w'_{in}(u)$ and $w''_{out}(u) = w''in(u)$, the system is said to be linear if, when both inputs are applied together, $w_{in}(u) = w'in(u) + w''in(u)$, the output is given by: $wout(u) = w'out(u) + w''out(u)$.

For a real (non-ideal) imaging system, the input maybe localized to a location u_0, the response at the output is spread over a range of u centered on $u0$. Conversely, any point at the output will depend on input stimuli over a range of positions at the input, that is:

$$w_{out}(u) = \int_{-\infty}^{\infty} p(u, u') w_{\rm in}(u') du'$$

Let $w_{\rm in}(u) = \delta(u - u_0)$.

Therefore,

3.2 Characterizing An X-ray Imaging System

3.2.1 Contrast

3.2.2 Resolution

3.2.3 Noise

3.3 Effect of Scatter

3.3.1 On Radiation Contrast

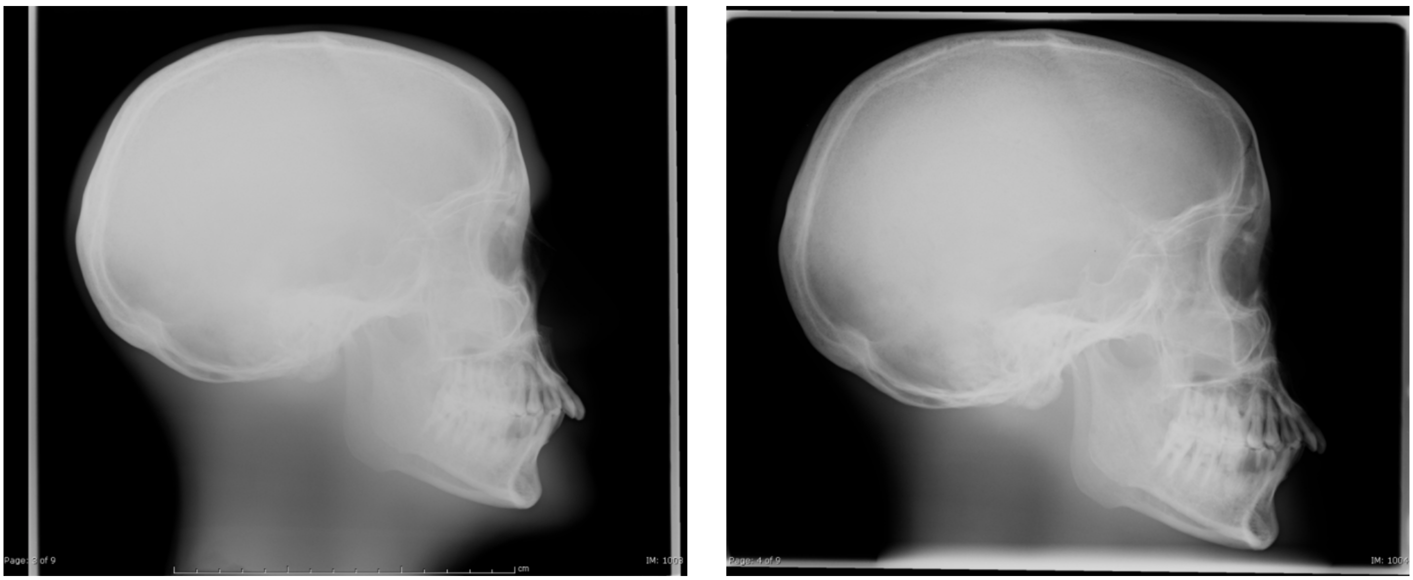

Figure 6: Left: Radiograph of the skull with contrast. Right: Radiograph of the same skull with an anti-scatter grid in place. Note the improvement in image contrast.

The derivation of expressions for contrast and SNR under the Rose model in section 2.2 neglected the effects of scattered radiation. In general, scattered radiation always works to decrease the contrast and overall quality of an image, as seen in Figure 6. This effect can be quantitatively demonstrated with a simple extension of the Rose model.

Recall our previous definition of contrast under the Rose model,

$$C = \frac{\Delta\phi}{\bar{\phi}} = \frac{\phi_1 - \phi_2}{(\phi_1 + \phi_2)/2} \tag{30}$$

Referring to Figure 7, we can define a "no scatter" contrast, C_{NS}, accounting only for contrast from the primary fluence components, ϕ_{1p} and ϕ_{2p}, That is,

$$C_{NS} = \frac{\phi_{1p} - \phi_{2p}}{(\phi_{1p} + \phi_{2p})/2} = \frac{\phi_{1p} - \phi_{2p}}{P} \tag{31}$$

where we define P as the mean primary fluence, $(\phi_{1p} + \phi_{2p})/2$.

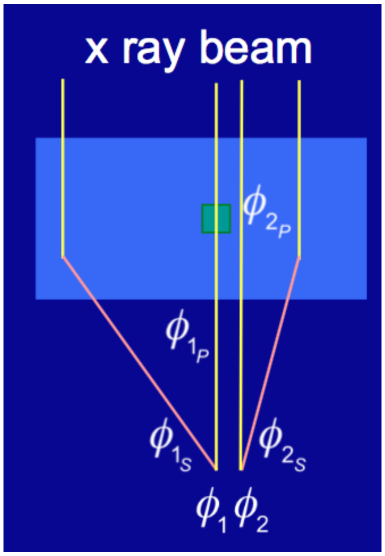

Figure 7: Rose model with scatter.

Likewise, we can define a "scatter" contrast, C_S,

$$
\begin{aligned}
C_S &= \frac{(\phi_{1p} + \phi_{1s}) - (\phi_{2p} + \phi_{2s})}{[(\phi_{1p} + \phi_{1s}) + (\phi_{2p} + \phi_{2s})]/2} && (32) \\
&= \frac{(\phi_{1p} - \phi_{2p}) + (\phi_{1s} - \phi_{2s})}{[(\phi_{1p} + \phi_{2p}) + (\phi_{1s} + \phi_{2s})]/2} && (33) \\
& && (34)
\end{aligned}
$$

If we assume that the scatter component of each fluence is roughly equal, that is $\phi_{1s} \approx \phi_{2s} = \phi_S$, and define $S = \phi_S$, then C_S can be written

$$C_S = \frac{\phi_{1p} - \phi_{2p}}{P + S} \tag{35}$$

and we see that scatter reduces contrast.

We can relate the scatter and no-scatter contrasts as follows:

$$
\begin{aligned}
C_S &= \frac{\phi_{1p} - \phi_{2p}}{P + S}\left(\frac{P}{P}\right) && (36) \\
&= C_{NS}\left(\frac{P}{P + S}\right) && (37) \\
&= C_{NS}\left(1 - \frac{S}{P + S}\right) && (38) \\
&= C_{NS}(1 - \mathrm{SF}) = C_{NS}\left(\frac{1}{1 + S/P}\right) && (39)
\end{aligned}
$$

using the scatter fraction:

$$\mathrm{SF} = \frac{S}{P + S} \tag{40}$$

and scatter-to-primary ratio, S/P.

3.4 Effect of Focal Spot Size

4 Image Interpretation

4.1 Societal Levels

4.2 Phantoms

4.2.1 Contrast-Detail

4.2.2 Resolution

5 Conclusion

References

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