Introduction to Diagnostic x-ray Imaging
(Part II)

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Outline

- Brief historical survey
- Linear-system model
- Focal spot and magnification
- Scatter
- Image detector

Importance of Image Detectors

- An important part of the imaging chain
- A field of its own

Image Detectors

- Ideal detectors
- Analog detectors
- Digital detectors

Ideal Imaging Detector

- Detect every photon incident upon it
- Provide error-free information on location, energy, and time of arrival of incident photons
- Accurate to arbitrarily fast photon arrival

Harmful Effect of x-ray: the Need to Optimize its Use in Imaging

- Tradeoff between the harmful effect of radiation dose and the benefit of a potential diagnosis
- Minimize dose for an expected diagnostic benefit
- Dose increase may be justified for greater expected diagnostic benefit

Important Concepts for Image Detector

- Quantum efficiency (QE)
  - The fraction of photons that contribute to the image
- "Detector noise"
  - Additional statistical fluctuations originated within the detector, which degrade the image
- Detective quantum efficiency (DQE)
- Noise equivalent quanta (NEQ)

Detective Quantum Efficiency (DQE)

- The fraction of incident photons that would have to be detected without additional noise to yield the same signal-to-noise ratio as is actually observed by the detector
  - Barrett & Swindell , P194

Detective Quantum Efficiency (DQE)

- The fraction of incident photons that would have to be detected without additional noise to yield **the same signal-to-noise ratio** as is actually observed by the detector

$$DQE = \left[\frac{SNR(out)}{SNR(in)}\right]^2$$

Noise equivalent quanta (NEQ)

- The equivalent number of input quanta per unit area required by an ideal imaging system to give the same SNR achieved by an actual system
  - Barrett & Myers, P866

Noise equivalent quanta (NEQ)

- The equivalent number of input quanta per unit area required by an ideal imaging system to give the same SNR achieved by an actual system

Signal-to-Noise Ratio (SNR)

- Signal:
  - contrast of low-contrast object
- Noise:
  - from finite number of x-ray photons

DQE and NEQ

$$DQE = \left[\frac{SNR(out)}{SNR(in)} \right]^2 \qquad DQE = \frac{NEQ}{\Phi}$$

- **Expressed in spatial frequency domain**

Use of DQE and NEQ

- **DQE is a relative quantity**
  - **x-ray fluence is not part of the expression**
- $DQE \leq QE \leq 1$
- **NEQ can be used for comparison of different detectors (absolute quantity)**

Image Detectors

- Ideal detectors
- Analog detectors
- Digital detectors

Analog Image Detectors

- Film
- Screen-film combination
- Intensifying screen/TV (fluoroscopy)
- Paper (Xerox)

Film Structure

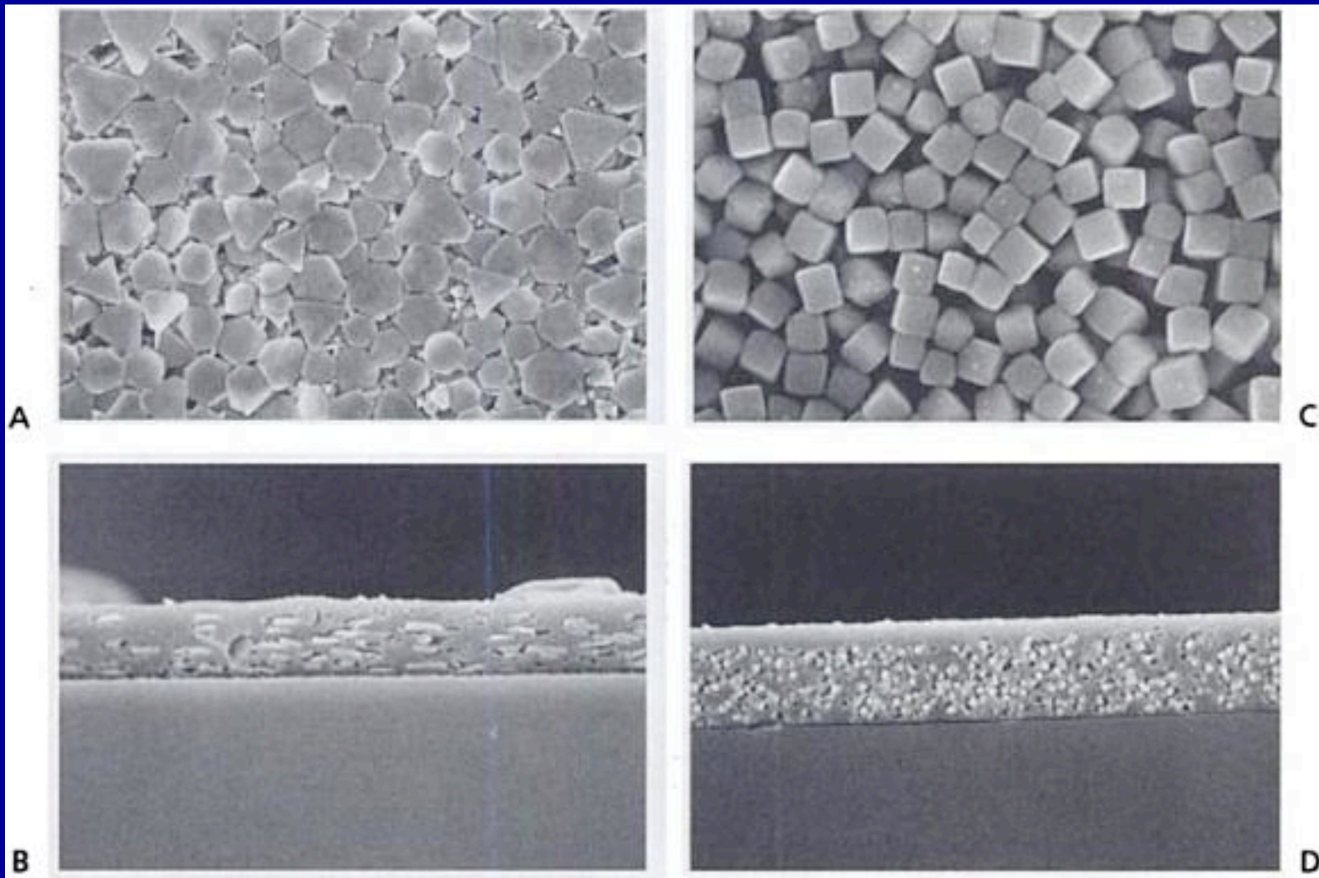
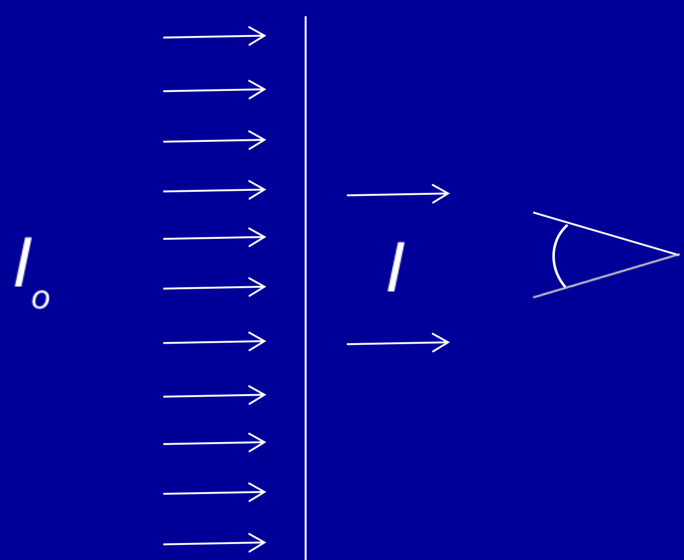


FIGURE 6-13. Scanning electron micrographs (SEM) are shown. **A:** A top-down SEM image of the emulsion layer of T grain emulsion is shown. **B:** A cross section of the T grain emulsion film is illustrated, showing the grains in the gelatin layer, supported by the polymer film base below. **C:** Cubic grain film emulsion is shown in a top-down SEM view. **D:** A cross sectional SEM image of the cubic grain film is illustrated. SEM photographs courtesy of Drs. Bernard Apple and John Sabol.

Optical Density

$$D = -\log\left(\frac{I}{I_o}\right)$$



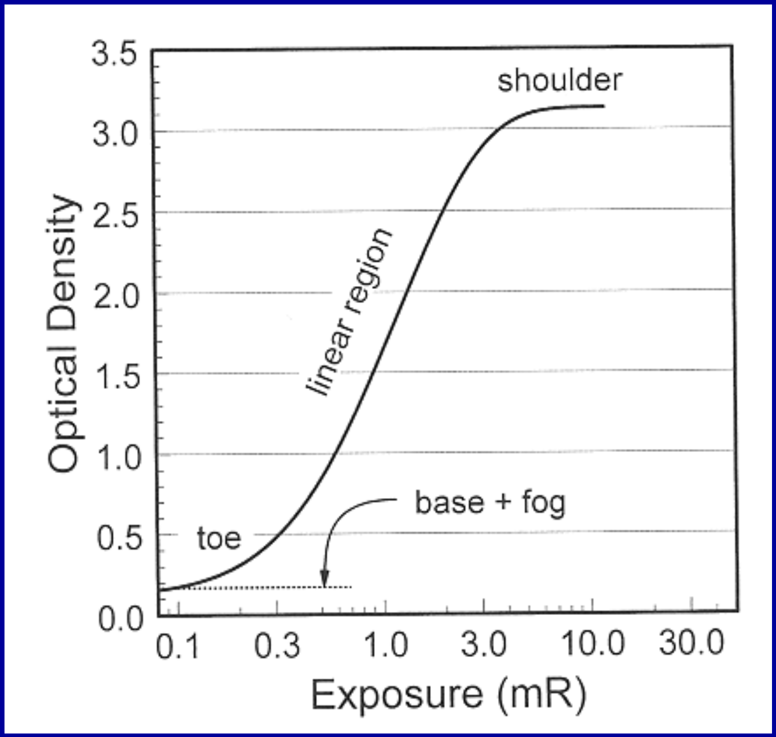
Film Density—Logarithmic Transformation

D	I/I_o
0.11	0.78
1	0.1
2	0.01
3	0.001
4	0.0001

Film Density—Logarithmic Transformation

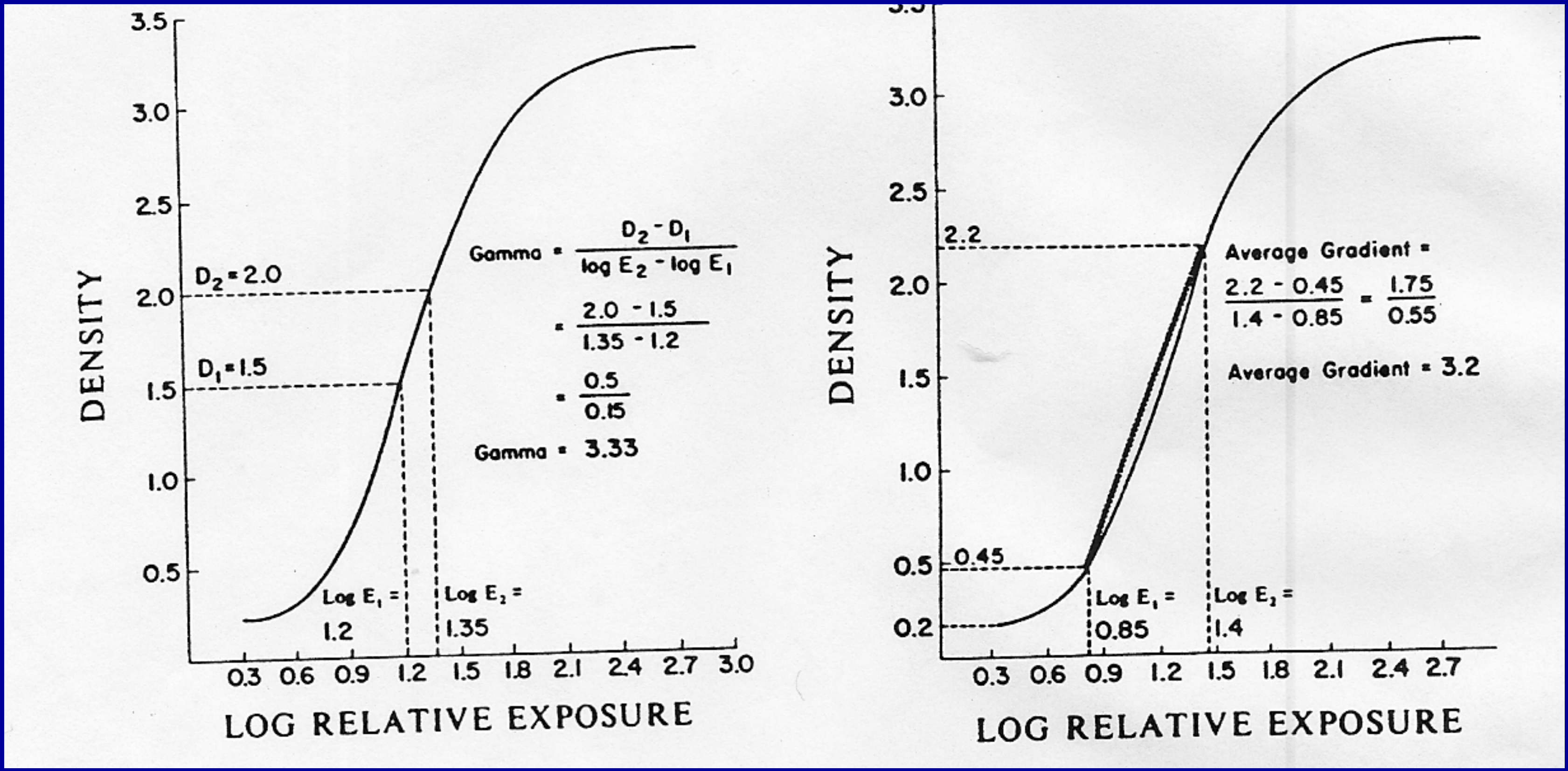
- Film is not linear
- Problem for linear-system theory
- Linear system approximately valid for low contrast

H&D Curve



3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
Optical Density
shoulder
linear region
toe
base + fog
0.1
0.3
1.0
3.0
10.0
30.0
Exposure (mR)

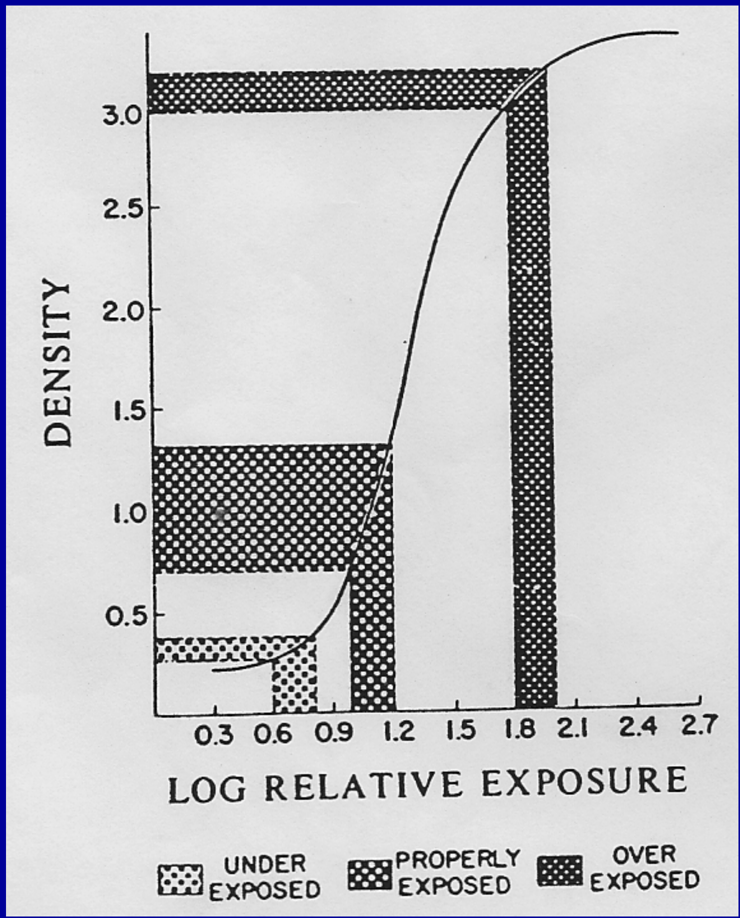
H&D Curve



DENSITY
LOG RELATIVE EXPOSURE
D2 = 2.0
D1 = 1.5
Log E1 = 1.2
Log E2 = 1.35
Gamma = (D2 - D1) / (log E2 - log E1)
= (2.0 - 1.5) / (1.35 - 1.2)
= 0.5 / 0.15
Gamma = 3.33
2.2
0.45
Log E1 = 0.85
Log E2 = 1.4
Average Gradient =
(2.2 - 0.45) / (1.4 - 0.85) = 1.75 / 0.55
Average Gradient = 3.2

Important Properties of Film

- Speed
- Gamma
- Latitude
- Reciprocity failure



Reciprocity Failure

- Over a range of normal exposure, optical density proportional to exposure (the product of exposure rate and time)
- Analogous to the quantity of x-ray output expressed in mAs
- Reciprocity fails at low (and high) exposure rates
- Failure occurs in film exposed to optical light but not in film exposed to x-ray

H&D Curve of Film Exposed to x-ray

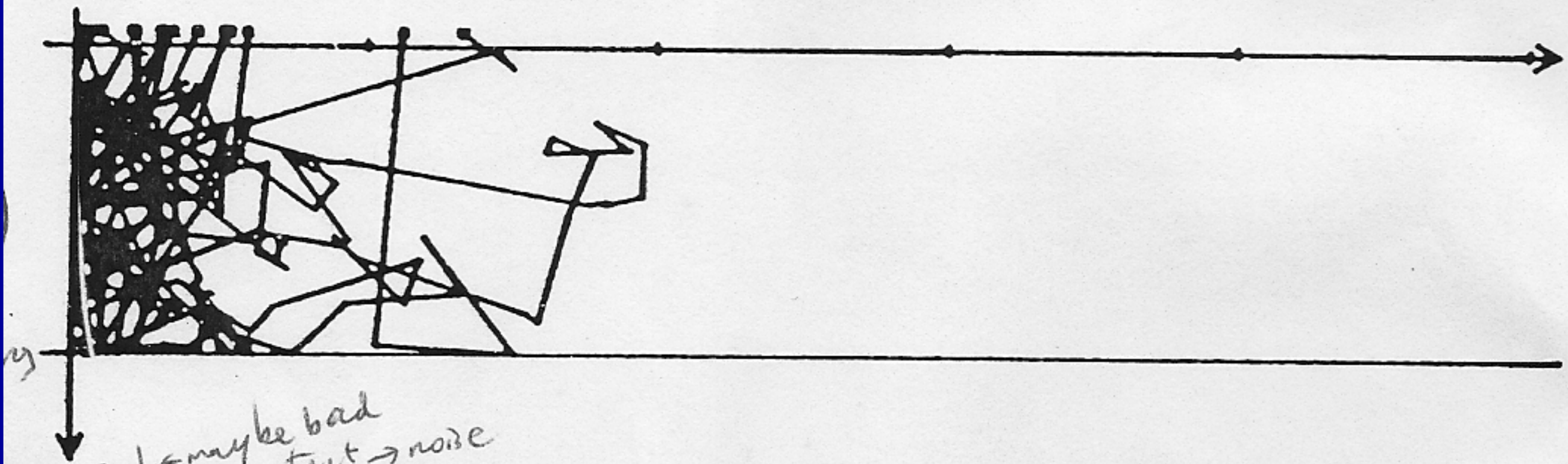
Intensifying Screen

- Absorbs x-ray
  - Much more efficiently than film
- Converts energy to light photons
  - More gain than film

Intensifying Screen

- Benefits
  - Reduce patient exposure
  - Reduce tube loading
  - Reduce effect of patient motion
- Cost
  - Degradation to MTF from light scattering

Light Scatter in Screen



t intensity ← may be bad
riation in light output → noise

Simulated light photon paths. The arrowhead indicates the direction of the x-ray beam.

DQE of Film or Screen-Film

$$\mathrm{DQE} = \left[\frac{\mathrm{SNR}_{\mathrm{out}}}{\mathrm{SNR}_{\mathrm{in}}}\right]^2$$

$$\mathrm{DQE} = \frac{(0.434\gamma)^2}{\Phi A \sigma_D^2}$$

$$\mathrm{S}_{\mathrm{in}} = \Delta\Phi\ \mathrm{A} = (\Phi_2 - \Phi_1)\ \mathrm{A}$$

$$\mathrm{S}_{\mathrm{out}} = \Delta D = (D_2 - D_1)$$

$$\mathrm{N}_{\mathrm{in}} = \sqrt{\Phi\ \mathrm{A}}$$

$$\mathrm{N}_{\mathrm{out}} = \sigma_D$$

MTF for Film or Screen-Film

- Consider low contrast for linear-system theory to work

Derivation of MTF for Film or Screen-Film

$$E(x) = \bar{E} + \Delta E \cos(2\pi\xi x) \qquad \Delta E > 0$$

$$T(x) = \bar{T} - \Delta T \cos(2\pi\xi x) \qquad \Delta T > 0$$

$$D(x) = \bar{D} + \Delta D \cos(2\pi\xi x) \qquad \Delta D > 0$$

Modulation of Low-Contrast Signal

$$M_{in}^{E} = \frac{\left[E\left(x\right)\right]_{max} - \left[E\left(x\right)\right]_{min}}{\left[E\left(x\right)\right]_{max} + \left[E\left(x\right)\right]_{min}} = \frac{\Delta E}{\overline{\overline{E}}}$$

$$M_{out}^{T} = \frac{\Delta T}{\overline{\overline{T}}}$$

$$M_{out}^{D} = \left[D\left(x\right)\right]_{max} - \left[D\left(x\right)\right]_{min} = \Delta D$$

Normalization of MTF

$$M_{out}^{T}\left(\xi \to 0\right) = \gamma \frac{\Delta E}{\overline{E}} = \gamma M_{in}^{E}$$

$$M_{out}^{D}\left(\xi \to 0\right) = 0.434\gamma \frac{\Delta E}{\overline{E}} = 0.434\gamma M_{in}^{E}$$

$$\mathrm{MTF}^{T}\left(\xi\right) = \frac{1}{\gamma}\left|\frac{M_{out}^{T}}{M_{in}^{E}}\right|$$

$$\mathrm{MTF}^{D}\left(\xi\right) = \frac{1}{0.434\gamma}\left|\frac{M_{out}^{D}}{M_{in}^{E}}\right|$$

MTF of Film or Screen-Film

$$\mathrm{MTF}^{T}(\xi) = \frac{M_{out}^{T}(\xi)}{M_{in}^{E}(\xi)}$$

$$\mathrm{MTF}^{D}(\xi) = \frac{M_{out}^{D}(\xi)}{M_{in}^{E}(\xi)}$$

$$\mathrm{MTF}^{T}(\xi) = \mathrm{MTF}^{D}(\xi)$$

$$\mathrm{MTF}(\xi) = \frac{1}{\gamma}\frac{\Delta T}{\bar{T}} \Big/ \frac{\Delta E}{\bar{E}} = \frac{\Delta D}{0.434\gamma} \Big/ \frac{\Delta E}{\bar{E}}$$

MTF of Film or Screen-Film

- One can measure the MTF in term of film density
- The MTF can be applied to light transmission without modification
- Remarkable given the logarithmic transformation of film

Physics Model of Film Viewing

$$\frac{dI}{I} = -n\sigma_{view}\,dx$$

$$\frac{I_d}{I_o} = e^{-n\sigma_{view}d}$$

$$D = -\log\left(\frac{I_d}{I_o}\right) = 0.434\ n\sigma_{view}d$$

$$D_{max} = -\log\left(\frac{I_d}{I_o}\right) = 0.434\ N\sigma_{view}d$$

Physics Model of Ag Grain Density

$$\frac{dn_j(t)}{dt} = \sigma_{exposure}\frac{\Phi}{\tau}\left[n_{j-1}(t) - n_j(t)\right]$$

$$D(\sigma_{exposure}\Phi) = D\left(\frac{\Phi}{\Phi_o}\right) = D_{max}\left[1 - e^{-\frac{\Phi}{\Phi_o}}\sum_{j=0}^{m-1}\frac{\left(\frac{\Phi}{\Phi_o}\right)^j}{j!}\right]$$

Physics Model of Film

- Explains the H&D curve behavior
- Shows that important properties of film or screen-film can be explained from physics first principles

Conversion Efficiency of Intensifying Screen

$$\propto \ \eta_1 \left(\frac{m\varepsilon_o}{\varepsilon_x}\right) \alpha$$

- Efficiency of x-ray photon absorption
- Efficiency of x-ray photon to light photon conversion
- Efficiency of light photon absorbtion

Geometry of Screen for PSF Calculation

Model of Intensifying Screen PSF

$$dN = \Phi_x\left(x', y'\right) e^{-\mu z} \; m\mu \; dx'dy'dz$$

$$d\Phi_o\left(x, y\right) = \frac{dN \; \cos\theta}{4\pi \; l^2}$$

$$\Phi_o\left(x, y\right) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\Phi_x\left(x', y'\right)dx'dy'\int_0^d \frac{m\mu \; e^{-\mu z} \; \cos\theta}{4\pi \; l^2}dz$$

Model of Intensifying Screen PSF

$$\Phi_x(x', y') = \delta(x')\delta(y')$$

$$psf(x, y) = \frac{m\mu}{4\pi}\int_0^d \frac{e^{-\mu z}\ \cos\theta}{l^2} dz$$

$$\Phi_x(y') = \delta(y') \quad \rightarrow \quad lsf(y)$$

MTF of Screen-Film

- Can be modeled from first principle (physics and geometry)
- Measured experimentally in practice
- Rotational symmetry assumed

$$\mathrm{MTF}\left(\eta\right)$$

Image Detectors

- Ideal detectors
- Analog detectors
- Digital detectors

Digital Image Detectors

- Indirect-conversion digital image detectors
  – CsI coupled with CCD or TFT
- Direct-conversion digital image detectors
  – a-Se
- Photon-counting digital image detectors
- Computed radiography (CR)

CsI

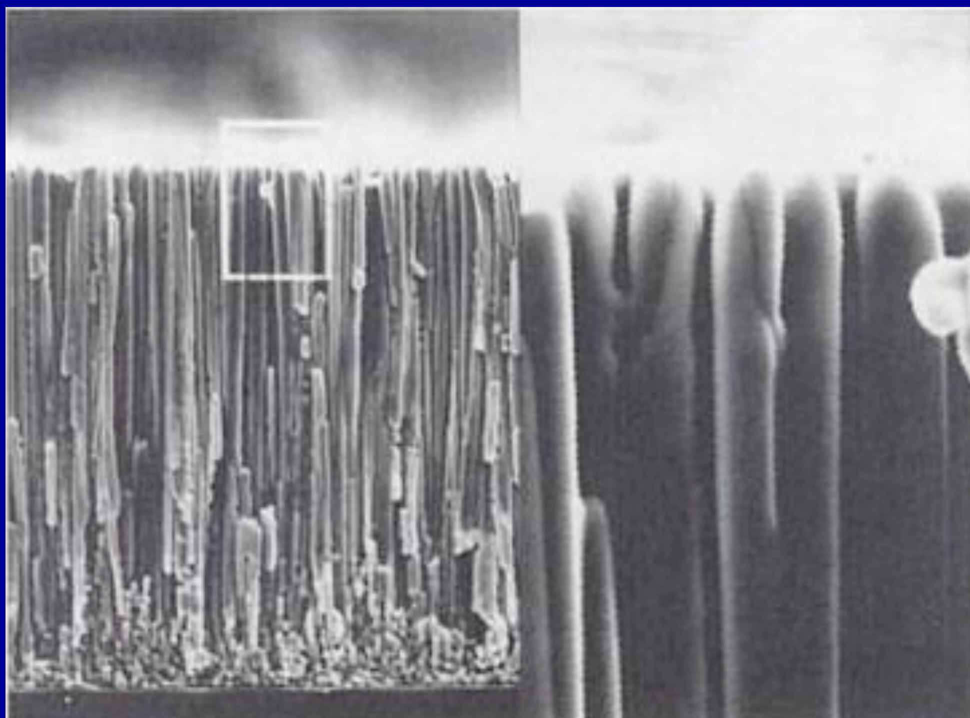
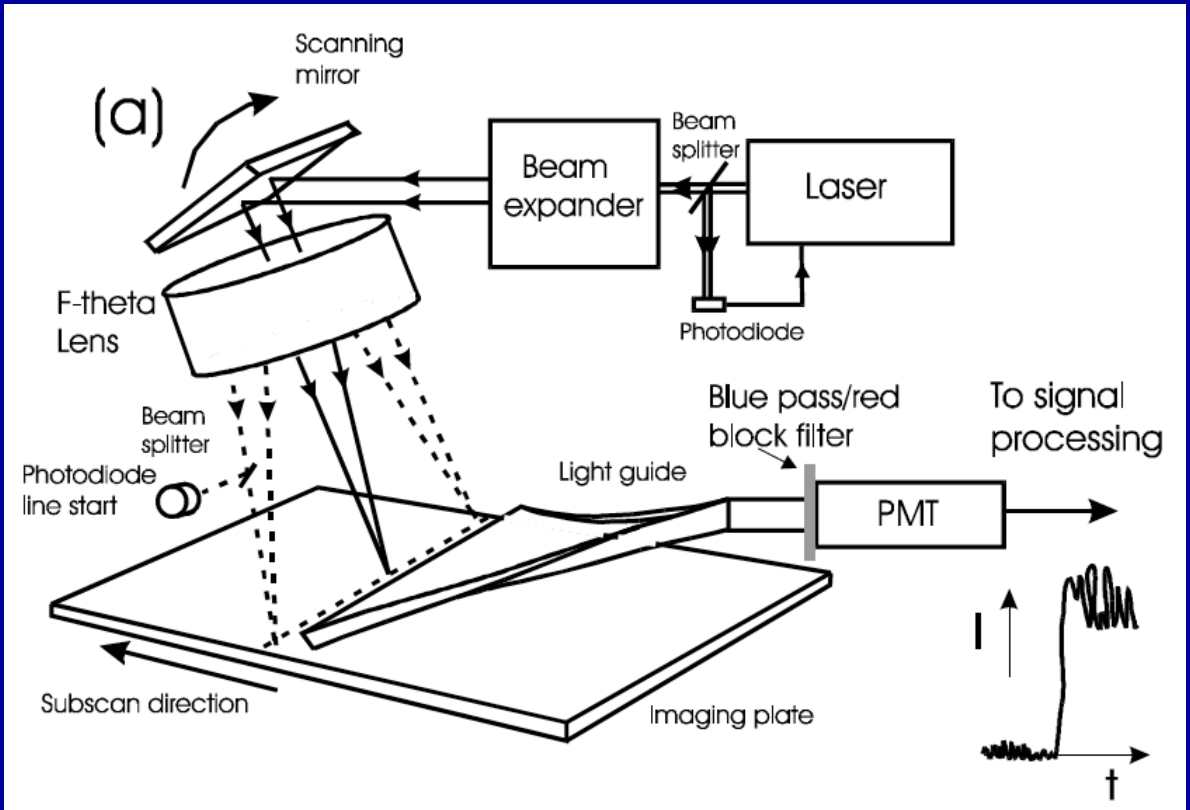


FIGURE 9-4. A scanning electron micrograph illustrates the needle-like structure of a CsI input phosphor.

CR Reader

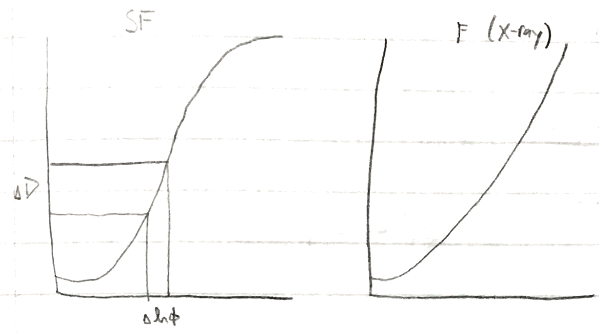


(a)
Scanning mirror
Beam expander
Beam splitter
Laser
F-theta Lens
Photodiode
Blue pass/red block filter
To signal processing
Beam splitter
Photodiode line start
Light guide
PMT
I
t
Subscan direction
Imaging plate

Summary

- Fundamentals of image detector
- Basic properties of film and screen-film
- Applicability of linear-system analysis
- First-principle modeling of film and screen properties
- Introduction to various types of digital detectors

Physics - Jiang 3/30



?

$$DQE = \left(\frac{SNR_{out}}{SNR_{in}}\right)^2$$

contrast? area? Poisson noise

$$SNR_{in} = \frac{S_{in}}{N_{in}} = \frac{\Delta\phi A}{\sqrt{\bar{\phi} A}}$$

$$SNR_{out} = \frac{S_{out}}{N_{out}} = \frac{\Delta D}{\sigma_D}$$

$$\Delta D = \gamma(\log_{10}e)\,\Delta(\ln\phi) = \gamma(\log_{10}e)\frac{\Delta\phi}{\bar{\phi}}$$ Taylor expan

$$DQE = \left[\frac{\gamma(\log_{10}e)\frac{\Delta\phi}{\bar{\phi}}}{\sigma_D}\right]^2 \frac{\bar{\Phi}A}{(\Delta\phi A)^2} = \frac{\gamma^2(\log_{10}e)^2}{\bar{\phi}A\sigma_D^2}$$