Introduction to Image Quality

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What Do We Mean by Image Quality

- The property of images that gives rise to the ability of expert observers to detect the presence of abnormalities or to estimate properties of abnormalities
- Low-contrast object in noise background
- Noise from limited number of x-ray quanta

Image Quality is Not About …

- pretty images
- making abnormalities obviously visible

- Limited by harmful effect of x-ray and costs associated with imaging

Image Quality is About

- Low-contrast object in noise background
- Noise from limited number of x-ray quanta
- Any additional detector noise

Outline

- Conceptual introduction
- Attributes of image quality
- Physical quantification of image quality
- Review of useful concepts
- Linear transfer of signal and noise
- SNR Calculations

Attributes of Image Quality

- Contrast
- Resolution
- Noise

Contrast

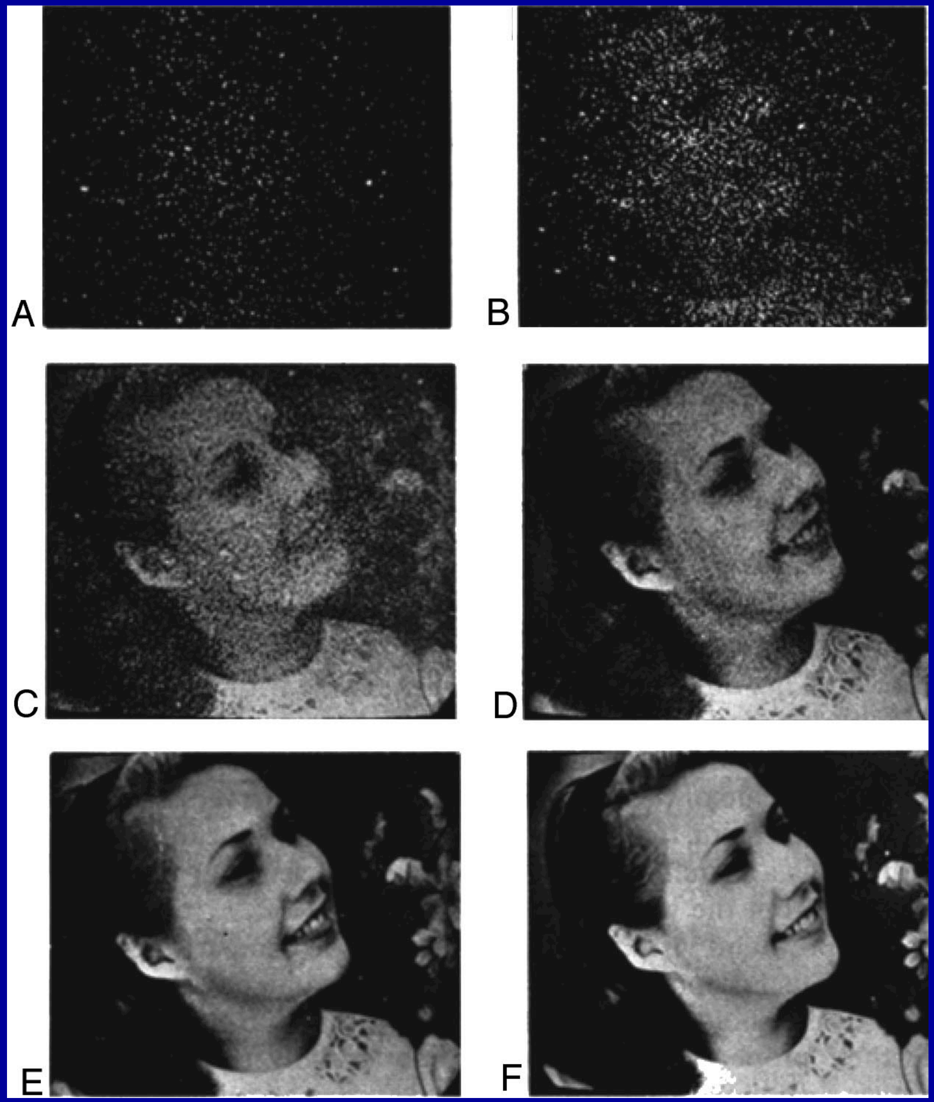
- Mammography lower kVp, high dose
- Contrast-enhanced
  - I, ultrasound micro-bubbles, Gd, dye

Resolution

- Mammography
- Bone fracture
- Pneumothorax
- Histology

Noise

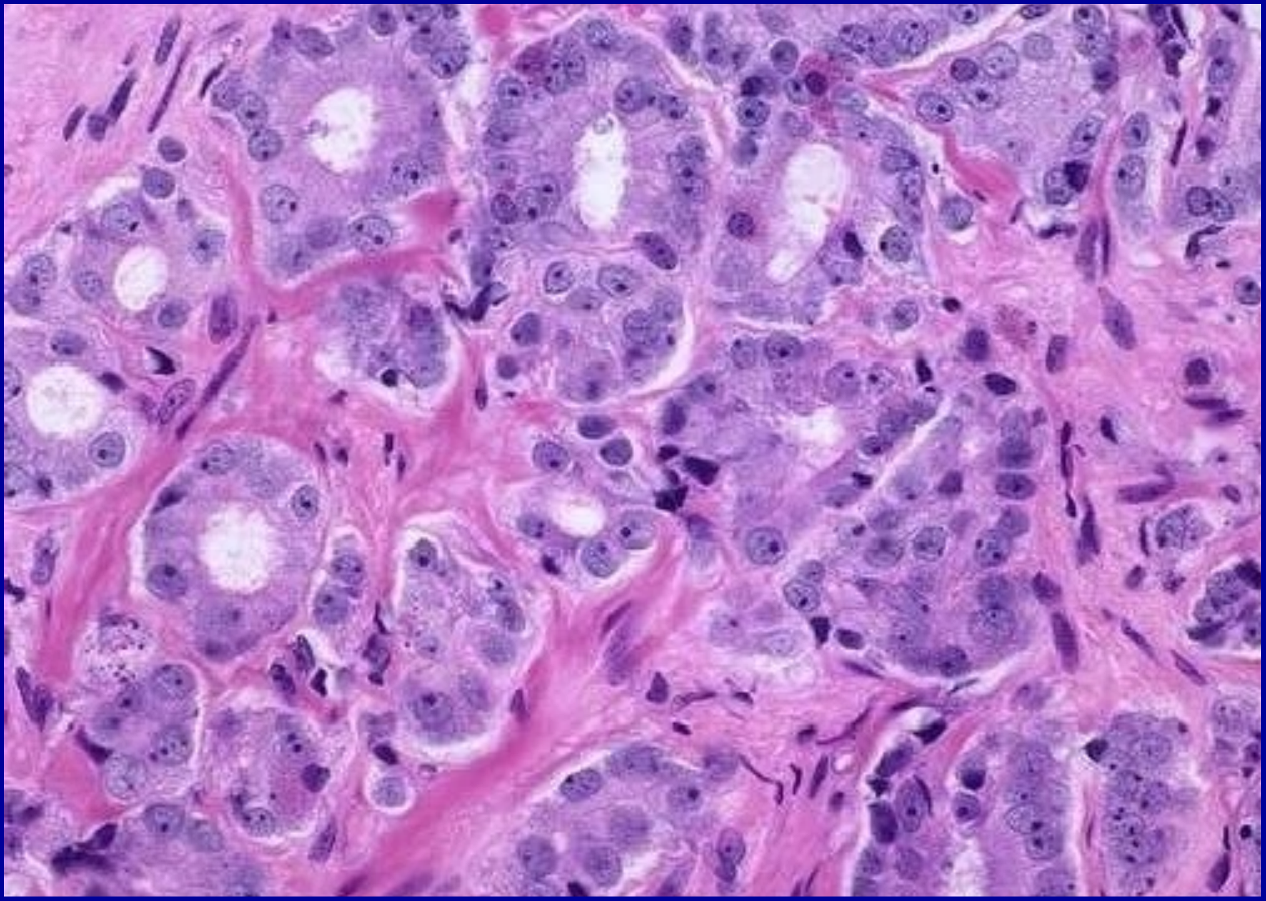
Noise



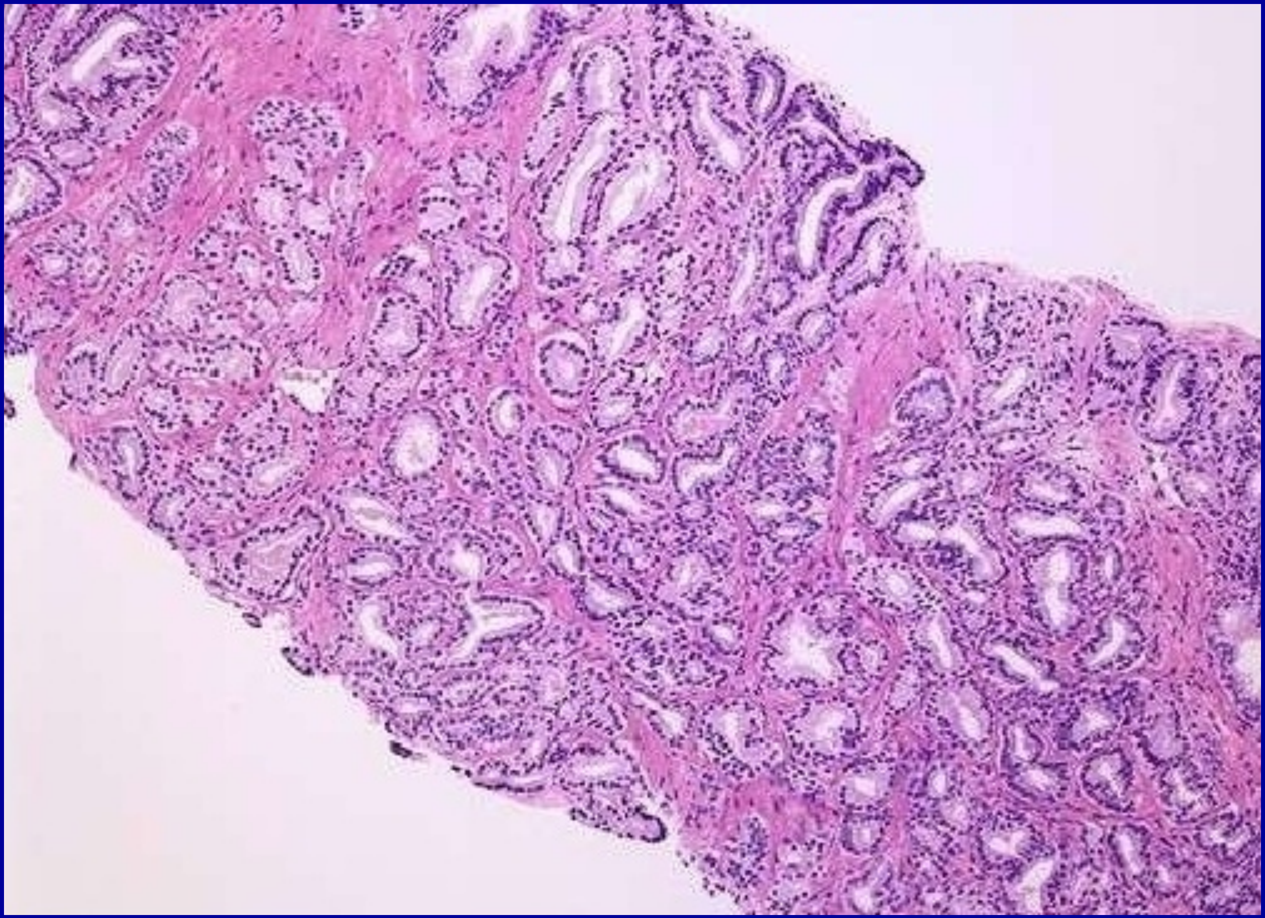
Remarks

- The optimal system is not necessarily one with the highest resolution
- System optimization depends on task

High-power Image of Prostate Cancer



Low-power Image of Prostate Cancer



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Signal-to-Noise Ratio

$$SNR = \frac{\mu}{\sigma}$$

- **Signal transfer**
- **Noise transfer**

Linear-System Analysis

- **Linear shift-invariant system**
- **Modeled as linear filter**
- **Spatial frequency domain: Fourier transform of the output is Fourier transform of the input times Fourier transform of the system PSF**
- **Spatial domain: output is the input convolved with the system PSF**

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Review of Several Important Concepts

- Central-limit theorem
- Gaussian random variable
- Poisson statistics
- Stationarity
- Ergodicity
- Ensemble average

Central Limit Theorem

- The probability density of a random variable that is the sum of a large number N other random variables approaches a Gaussian distribution as N tends to infinity, regardless of the probability density functions of the constituent random variables.
- Useful to understand the behavior of Gaussian random processes.

Ensemble Average

- Expectation (average) from a hypothetical ensemble of identical systems of random processes.
- Fundamental concept with respect to statistical expectation (average).

Stationarity

- A stationary random process has joint probability distribution that does not change when shifted in time or space (thus only depends on the shift).
- A wide-sense stationary random process has (weaker requirement of) only the mean and covariance that do not change when shifted.

Ergodicity

- A random process is said to be ergodic if statistical averages can be replaced by time averages.
- Make spatial-domain analysis the same as time-domain analysis
- Ergodic processes are stationary.

Poisson Random Variable

- **pdf** $$p_{\mathrm{x}}\left(\mathbf{X}=k\right)=\frac{\lambda^{k}e^{-\lambda}}{k!}$$
- **Mean** $$E\left\{p_{\mathrm{x}}\left(\mathbf{X}=k\right)\right\}=\lambda$$
- **variance** $$Var\left\{p_{\mathrm{x}}\left(\mathbf{X}=k\right)\right\}=\lambda$$

Gaussian Random Variable

- **pdf** $$f_x(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$
- **Mean** $$E\{f_x(x)\} = \mu$$
- **variance** $$Var\{f_x(x)\} = \sigma^2$$

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Signal Transfer

- Characterize signal (input) by its Fourier components
- Object of interest can be described complete by its Fourier components

Linear-System Transfer of Signal

- Spatial frequency domain:
  - Fourier transform of the input times the Fourier transform of the system PSF
- Spatial domain:
  - Convolution of input with the system PSF

Signal Transfer

$$
\begin{aligned}
w_{out}(t) &= psf(t) \otimes w_{in}(t) \\
&= \int_{-\infty}^{\infty} dt' \; psf(t) \;\; w_{in}(t - t')
\end{aligned}
$$

$$
\langle \mathbf{w}_{out}(t) \rangle = psf(t) \otimes w_{in}^{ave}(t)
$$

Noise Transfer

- Characterize noise by autocorrelation function
- Imagine a uniform (flat) object
- Any spatial variation in the image is caused by statistical (stochastic) noise
- Image variance characterizes noise magnitude
- Autocorrelation characterizes noise texture

Autocorrelation Function

$$R_{in}(\tau) = \left\langle \mathbf{w}_{in}(t)\,\mathbf{w}_{in}(t+\tau) \right\rangle$$

$$R_{in}(\tau) =$$

$$\int_{-\infty}^{\infty} dw_{in}(t) \int_{-\infty}^{\infty} dw_{in}(t+\tau)\; w_{in}(t)\, w_{in}(t+\tau)\, p\left[w_{in}(t), w_{in}(t+\tau)\right]$$

Transfer of Autocorrelation Function

$$w_{out}\left(t\right) = psf\left(t\right) \otimes w_{in}\left(t\right)$$

$$= \int_{-\infty}^{\infty} dt' \; psf\left(t\right) \; w_{in}\left(t - t'\right)$$

$$R_{out}\left(\tau\right) = \left\langle \mathbf{w}_{out}\left(t\right) \mathbf{w}_{out}\left(t + \tau\right) \right\rangle$$

$$= \left\langle \int_{-\infty}^{\infty} dt' \int_{-\infty}^{\infty} dt'' PSF\left(t'\right) PSF\left(t''\right) \mathbf{w}_{in}\left(t - t'\right) \mathbf{w}_{in}\left(t + \tau - t''\right) \right\rangle$$

Transfer of Autocorrelation Function

$$R_{out}(\tau) = PSF(\tau) * PSF(\tau) \otimes R_{in}(\tau)$$

$$S_{out}(\upsilon) = \left|P(\upsilon)\right|^2 S_{in}(\upsilon)$$

Noise Power Spectrum (NPS)

- **Wiener power spectral density**

$$S(\upsilon) = Fourier\left\{R_{in}(\tau)\right\}$$

$$S_{out}(\upsilon) = \left|P(\upsilon)\right|^2 S_{in}(\upsilon)$$

$$H_{out}(\upsilon) = P(\upsilon)\ H_{in}(\upsilon)$$

Variance in the Image

$$\sigma^2(\vec{r}) = \left[PSF(\vec{r})\right]^2 \otimes w_{in}^{ave}(\vec{r})$$

$$\sigma^2(\upsilon) = \left|PSF(\upsilon)\right|^2 H(\upsilon)$$

Word of Caution on Autocorrelation Function and NPS

- Notation not always consistent
- Often with regard to zero-mean process
- Not normalized by the variance
- Units

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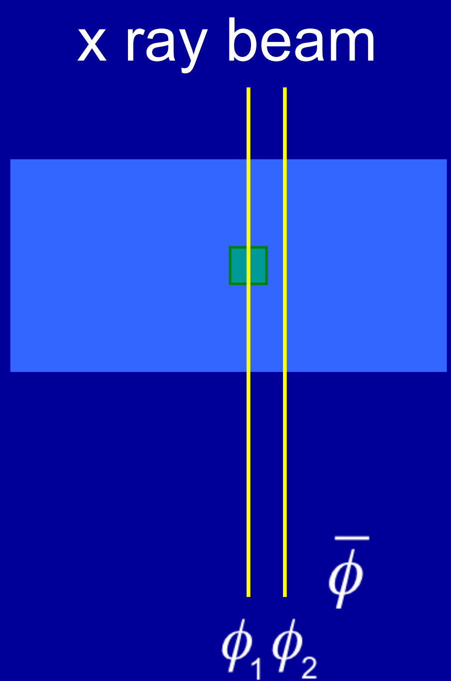
Rose Model SNR



$$\phi_1 A - \phi_2 A = \Delta\phi A$$

$$\sigma = \sqrt{\bar{\phi} A}$$

$$\mathrm{SNR} = \frac{\mu}{\sigma} = \frac{\Delta\phi A}{\sqrt{\bar{\phi} A}} = \frac{\Delta\phi}{\bar{\phi}}\sqrt{\bar{\phi} A} = C\sqrt{\bar{\phi} A}$$

SNR $\neq$ contrast-to-noise ratio

SNR $>$ 5

Comments on SNR

- SNR increases with increase in exposure
  - Signal increases
  - Contrast does not increase
  - Noise increases
  - Noise increases slower than signal

$$\text{SNR} = \frac{\Delta\phi A}{\sqrt{\bar{\phi} A}} = C\sqrt{\bar{\phi} A}$$

Rose Model

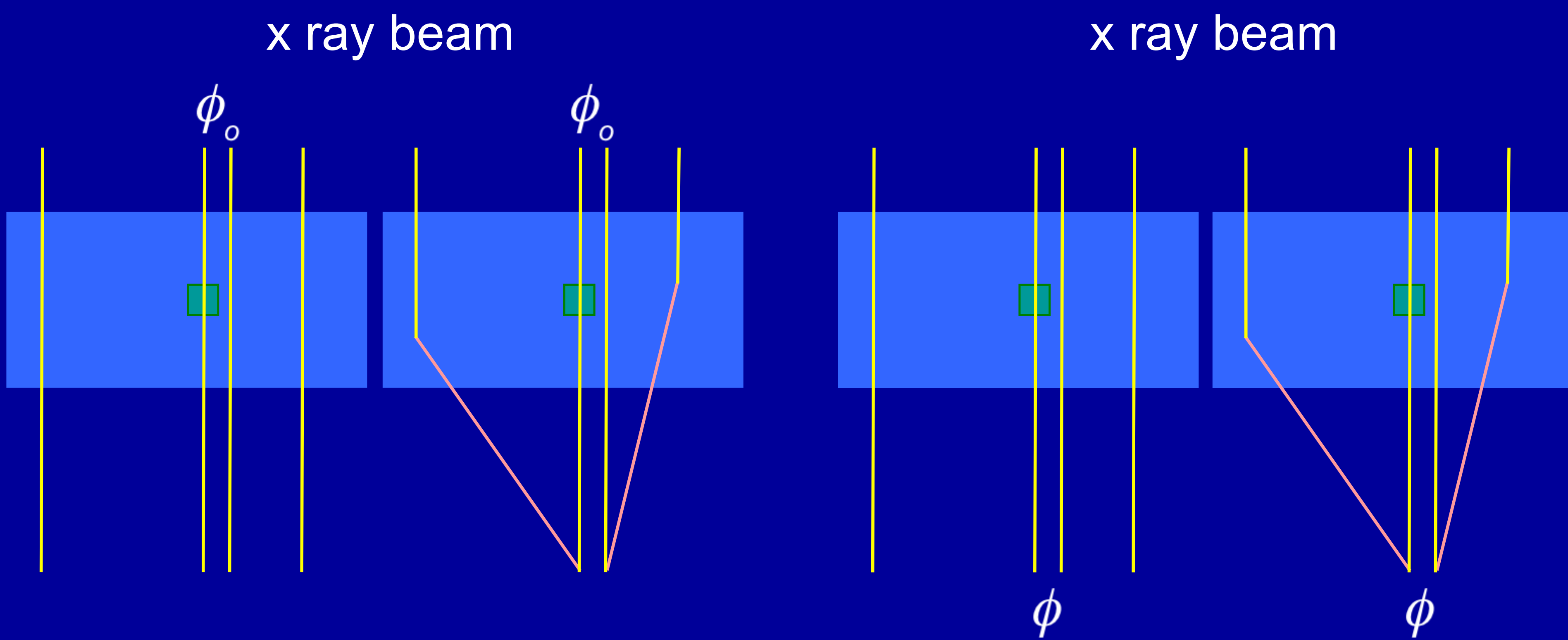
- **Flat signal on flat background**
- **“SKE” and “BKE”**

$$SNR > 5$$

Calculation of Effect of Scatter on SNR

For screen-film system, exposure to image detector must be approximately constant due to the H&D curve

Calculation of Effect of Scatter on SNR



Calculation of Effect of Scatter on SNR

- Derive an expression for $\Delta\phi$
- Calculate SNR

Calculation of Effect of Scatter on SNR

$$\phi_1 = \phi_o \, e^{-\mu L}$$

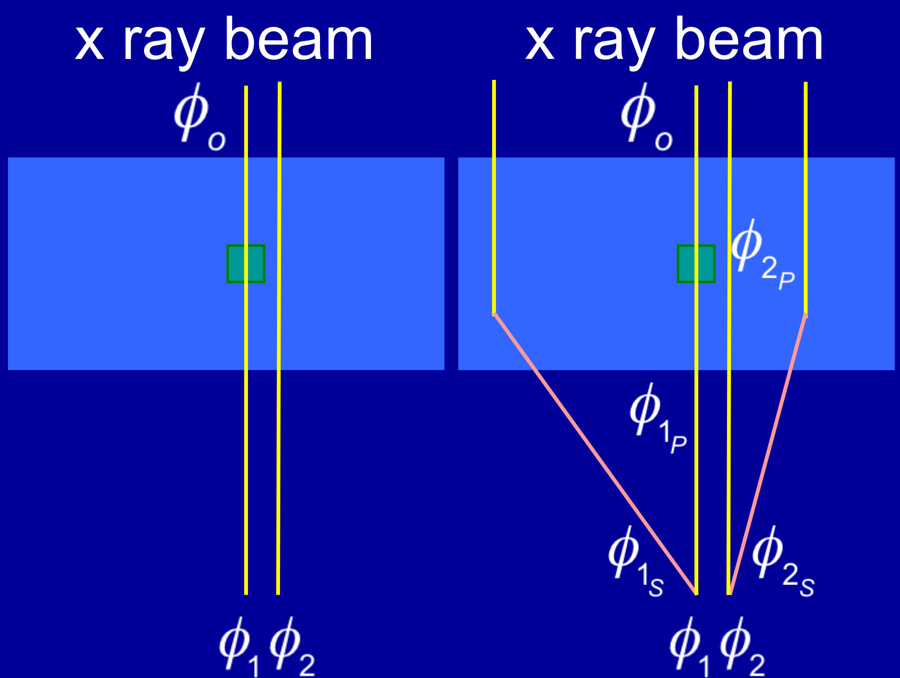
$$\phi_2 = \phi_o \, e^{-\mu(L-d)-\mu_d d}$$

$$\Delta\phi = \phi_2 - \phi_1 = \phi_o\left[e^{-\mu(L-d)-\mu_d d} - e^{-\mu L}\right] = \phi_o e^{-\mu L}\left[e^{-(\mu_d-\mu)d} - 1\right]$$

$$\Delta\phi = \phi_o e^{-\mu L}\left[e^{-(\mu_d-\mu)d} - 1\right] = \phi_o e^{-\mu L}\left[1-(\mu_d - \mu)d - 1\right]$$

$$\Delta\phi = -(\mu_d - \mu)d\phi_o e^{-\mu L} = -e^{-\mu L}\,\Delta\mu\, d\, \phi_o$$

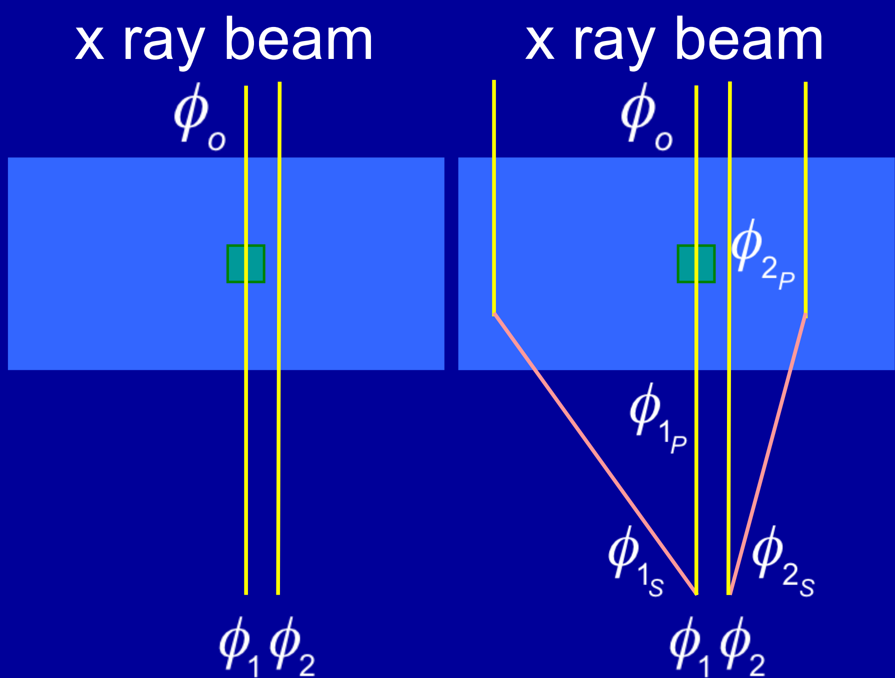
Constant Patient Exposure



$$C_{NS} = \frac{\Delta\phi}{\phi_P}$$

$$C_S = \frac{\Delta\phi}{\phi_P + \phi_S}$$

Constant Patient Exposure



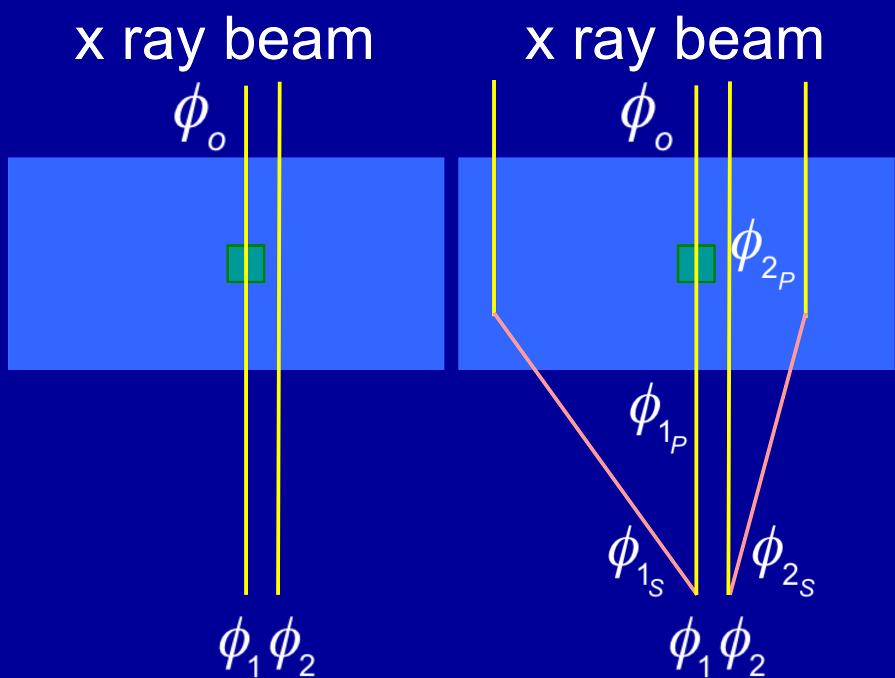
$$C_{NS} = \frac{\Delta\phi}{\phi_P}$$

$$C_S = \frac{\Delta\phi}{\phi_P + \phi_S}$$

$$SNR_{NS} = C_{NS}\sqrt{A\phi_P} = \frac{\Delta\phi}{\phi_P}\sqrt{A\phi_P}$$

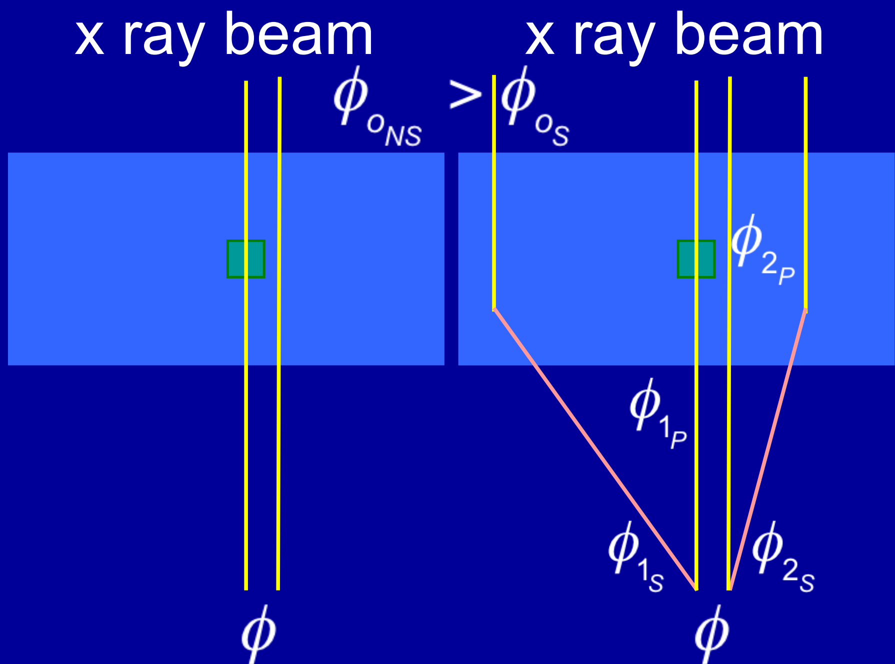
$$SNR_S = C_S\sqrt{A\left(\phi_P + \phi_S\right)} = \frac{\Delta\phi}{\phi_P + \phi_S}\sqrt{A\left(\phi_P + \phi_S\right)}$$

Constant Patient Exposure



$$\frac{SNR_S}{SNR_{NS}} = \sqrt{\frac{\phi_P}{\phi_P + \phi_S}} = \sqrt{1 - SF}$$

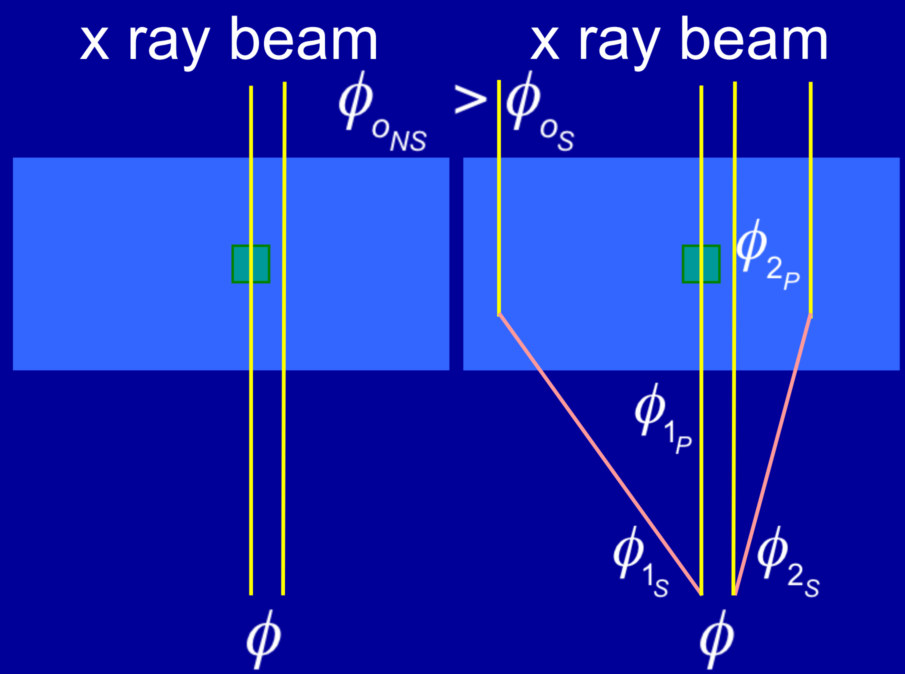
Constant Detector Entrance Exposure



$$C_{NS} = \frac{\alpha\phi_{o_{NS}}}{\phi}$$

$$C_S = \frac{\alpha\phi_{o_S}}{\phi}$$

Constant Detector Entrance Exposure



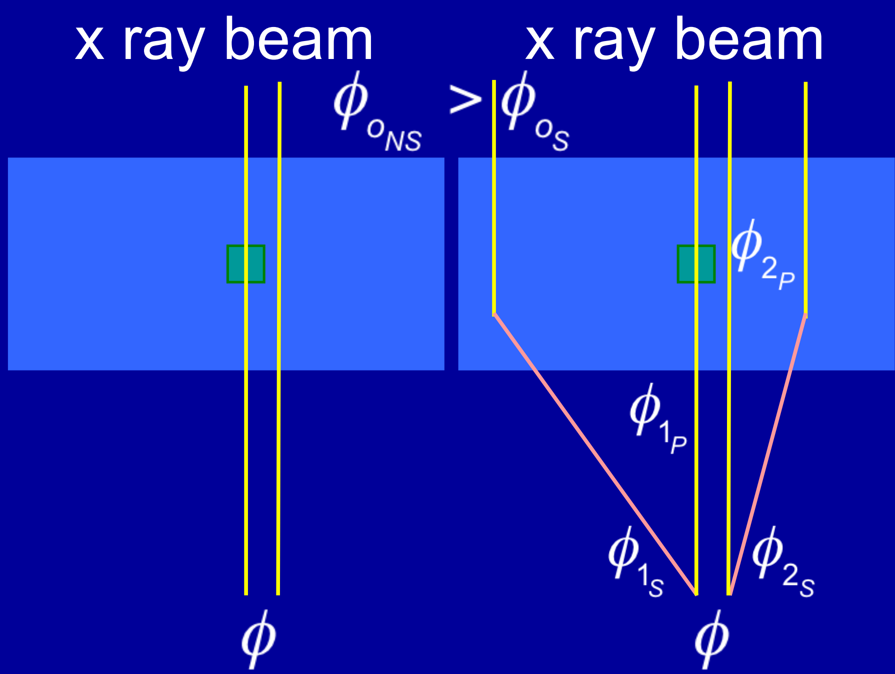
$$C_{NS} = \frac{\alpha\phi_{o_{NS}}}{\phi}$$

$$C_S = \frac{\alpha\phi_{o_S}}{\phi}$$

$$SNR_{NS} = C_{NS}\sqrt{A\phi} = \frac{\alpha\phi_{o_{NS}}}{\phi}\sqrt{A\phi}$$

$$SNR_S = C_S\sqrt{A\phi} = \frac{\alpha\phi_{o_S}}{\phi}\sqrt{A\phi}$$

Constant Detector Entrance Exposure



$$\frac{SNR_S}{SNR_{NS}} = \frac{\phi_{o_S}}{\phi_{o_{NS}}} = \frac{\phi_{P_S}}{\phi_{P_{NS}}} = \frac{\phi_{P_S}}{\phi_{P_S} + \phi_{S_S}} = \frac{1}{1+SF}$$

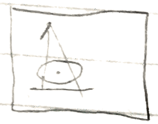
Summary

- Important image quality attributes include contrast, resolution, and noise properties of an imaging system
- SNR is a physical quantification of image quality
- Rose-model of SNR

Jiang Lecture 3, 4-4-17

$psf(t)$

$\tilde{w}_{in}(t)$

$\tilde{w}_{in}(t)$ $\longrightarrow$  $\tilde{w}_{out}(t)$

autocorrelation

ensemble average

$$R_{in}(\tau) = \langle \tilde{w}_{in}(t)\, \tilde{w}_{in}(t+\tau) \rangle$$

$$= \int_{-\infty}^{\infty} dt\, \tilde{w}_{in}(t)\, \tilde{w}_{in}(t+\tau)\, p[\tilde{w}_{in}(t)\, \tilde{w}_{in}(t+\tau)]$$

Power spectrum

$$S_{in}(v) = F\{R_{in}(\tau)\}$$

$$w_{out}(t) = w_{in}(t) \otimes psf(t)$$

$$R_{out}(\tau) = \langle \tilde{w}_{out}(t)\, \tilde{w}_{out}(t+\tau) \rangle$$

$$= \langle \int_{-\infty}^{\infty} dt' \int_{-\infty} dt''\, psf(t')\, psf(t'')\, \tilde{w}_{in}(t-t')\, \tilde{w}_{in}(t+\tau-t'') \rangle$$

$\vdots$

$$psf(\tau) * psf(\tau) \otimes R_{in}(\tau)$$

mathematically: convolutions

different signals b/c $R_{in}(\tau)$ is s.p.

$$S_{out}(v) = |p(v)|^2\, S_{in}(v)$$

$$\langle \tilde{w}_{out}(t) \rangle = \langle psf(t) \otimes \tilde{w}_{in}(t) \rangle$$

$\vdots$

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$$= \int_{-\infty}^{\infty} dt'\, psf(t') \langle \tilde{w}_{in}(t) \rangle = P(0) \langle \tilde{w}_{in}(t) \rangle$$

$$\sigma_{out}^2 = \langle [\tilde{w}_{out}(t)]^2 \rangle - \langle \tilde{w}_{out}(t) \rangle^2$$

$$\cdots \quad = \int_{-\infty}^{\infty} |P(v)|^2 S_{in}(v)\, dv - [P(0) \langle \tilde{w}_{in}(t) \rangle]^2$$

$$SNR = \frac{\mu}{\sigma} = \frac{\Delta \Phi A}{\sqrt{\Phi A}} = \frac{\Delta \Phi}{\Phi}\sqrt{\Phi A} = C\sqrt{\Phi A}$$

Rose Model

$SNR \geq 5$ for people to detect

Constant patient exposure

$$\phi_1 = \phi_0 e^{-\mu L}$$

$$\phi_2 = \phi_0 e^{-\mu_1(L-d)-\mu_d d}$$

$$\Delta\phi = \phi_2 - \phi_1 = \phi_0 e^{-\mu_1 L}\left[e^{-(\mu_d-\mu_1)d} - 1\right]$$ "Low Contrast"

$$= \phi_0 e^{-\mu L}\left[1-(\mu_d-\mu)d - 1\right]$$ ← Taylor ($\Delta\mu$ small)

$$= -\Delta\mu d \phi_0 e^{-\mu L}$$

$$C_{NS} = \frac{\Delta\Phi}{\Phi_P} \qquad C_S = \frac{\Delta\Phi}{\Phi_P + \Phi_S}$$

$$SNR_{NS} = C_{NS}\sqrt{\Phi_P A} = \frac{\Delta\Phi}{\Phi_P}\sqrt{\Phi_P A}$$

$$SNR_S = C_S\sqrt{(\Phi_P+\Phi_S)A} = \frac{\Delta\phi}{\phi_P+\phi_S}\sqrt{(\phi_P+\phi_S)A}$$

$$\frac{SNR_S}{SNR_P} = \sqrt{\frac{\phi_P}{\phi_P+\phi_S}} = \sqrt{1-SF}, \quad SF = \frac{\phi_S}{\phi_P+\phi_S}$$

Constant Entrance Exposure (same exposure after pat before det → diff ϕ_0's)

$$\Delta\phi_{NS} = \Delta\mu d \phi_{0,NS} e^{-\mu L} = \alpha\phi_{0,NS} \qquad C_{NS} = \frac{\alpha\phi_{0,NS}}{\phi}$$

$$\Delta\phi_S = -\Delta\mu d \phi_{0S} e^{-\mu L} = \alpha\phi_{0,S} \qquad C_S = \frac{\alpha\phi_{0S}}{\phi}$$

$$SNR_{NS} = C_{NS}\sqrt{\phi A} = \frac{\alpha\phi_{0,NS}}{\phi}\sqrt{\phi A} \qquad \frac{SNR_S}{SNR_{NS}} = \frac{\phi_{0,S}}{\phi_{0,NS}} = \frac{\phi_{S,P}}{\phi_{NS,P}}$$

$$SNR_S = C_S\sqrt{\phi A} = \frac{\alpha\phi_{0,S}}{\phi}\sqrt{\phi A}$$

Entrance exposure =

$$\phi_{NS,p} = \phi_{s,p} + \phi_{s,s}$$

$$SF = \frac{\phi_{s,s}}{\phi_{s,p} + \phi_{s,s}}$$

$$\phi_{s,s} = \phi_{s,p}' \frac{SF}{1-SF}$$

$$= \phi_{s,p} \frac{1}{1-SF}$$

$$\frac{SNR_s}{SNR_{ns}} = 1 - SF$$