DQE and NEQ

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The Definition of DQE and NEQ

$$\mathrm{DQE} = \left[\frac{\mathrm{SNR}(\mathrm{out})}{\mathrm{SNR}(\mathrm{in})}\right]^2 \qquad \mathrm{DQE} = \frac{\mathrm{NEQ}}{\Phi}$$

- **DQE is a relative quantity**
- **NEQ is an absolute quantity**

Detective Quantum Efficiency (DQE)

- The fraction of incident photons that would have to be detected without additional noise to yield **the same signal-to-noise ratio** as is actually observed by the detector

$$DQE = \left[\frac{SNR(out)}{SNR(in)}\right]^2$$

Noise equivalent quanta (NEQ)

- The equivalent number of input quanta per unit area required by an ideal imaging system to give **the same SNR** achieved by an actual system
  – Barrett & Myers, P866

$$\mathrm{DQE} = \frac{\mathrm{NEQ}}{\Phi}$$

Signal-to-Noise Ratio

$$SNR = \frac{\mu}{\sigma}$$

$$SNR\left(\vec{r}\right) = \frac{PSF\left(\vec{r}\right) \otimes w_{in}^{ave}\left(\vec{r}\right)}{\sqrt{\left[PSF\left(\vec{r}\right)\right]^2 \otimes w_{in}^{ave}\left(\vec{r}\right)}}$$

Outline

- Review basic concepts
- Derive DQE
- Derive DQE for digital systems
- Derive DQE for screen-film systems
- Derive NEQ

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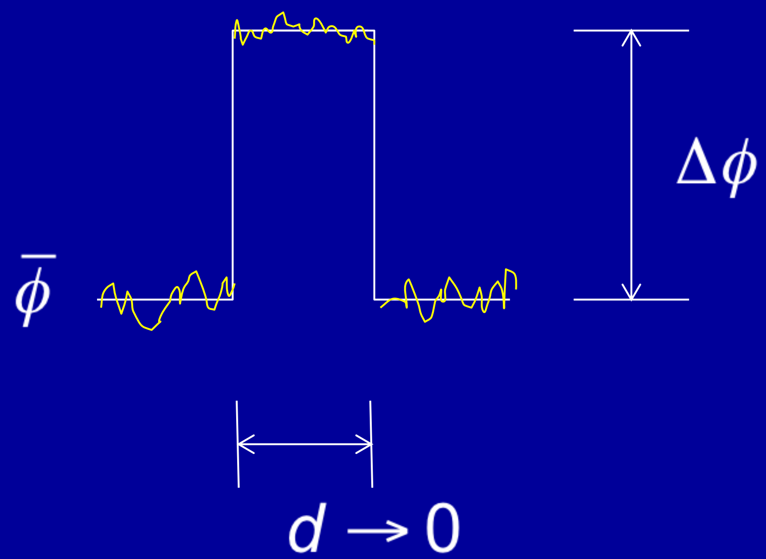
Consider a Poisson Impulse Object

- Signal: ensemble average of the impulse
- Noise: autocovariance of the impulse

$$\mu_{in}(\vec{r}) = \langle \mathbf{w}_{in}(\vec{r}) \rangle = \Delta\phi \ \delta(\vec{r} - \vec{r}_o) \qquad \mu_{in}(\vec{\rho}) = \Delta\phi$$

$$\sigma^2_{in}(\vec{r}) = \langle S_{in}(\vec{r}) \rangle = \bar{\phi} \ \delta(0) \qquad \sigma^2_{in}(\vec{\rho}) = W_{in}(\vec{\rho}) = \bar{\phi}$$

Schematic of the Impulse



SNR of the Impulse

$$SNR_{in}\left(\vec{r}\right) = \frac{\mu_{in}\left(\vec{r}\right)}{\sqrt{\sigma_{in}^{2}\left(\vec{r}\right)}} = \frac{\Delta\phi}{\bar{\phi}}\sqrt{\bar{\phi}}\ \delta\left(\vec{r}-\vec{r}_{o}\right) = C\sqrt{\bar{\phi}}\ \delta\left(\vec{r}-\vec{r}_{o}\right)$$

$$SNR_{in}\left(\vec{\rho}\right) = \frac{\mu_{in}\left(\vec{\rho}\right)}{\sqrt{\sigma_{in}^{2}\left(\vec{\rho}\right)}} = \frac{\Delta\phi}{\bar{\phi}}\sqrt{\bar{\phi}} = C\sqrt{\bar{\phi}}$$

Detector Output of the Impulse

$$\mu_{out}\left(\vec{r}\right) = \left\langle PSF\left(\vec{r}\right) \otimes \mathbf{w}_{in}\left(\vec{r}\right)\right\rangle$$

$$\mu_{out}\left(\vec{\rho}\right) = MTF\left(\vec{\rho}\right)\Delta\phi G$$

$$\sigma^2_{out}\left(\vec{\rho}\right) = W_{out}\left(\vec{\rho}\right)$$

SNR at the Detector Output

$$SNR_{out}(\vec{\rho}) = \frac{\mu_{out}(\vec{\rho})}{\sqrt{\sigma^2_{out}(\vec{\rho})}} = \frac{MTF(\vec{\rho})\Delta\phi G}{\sqrt{W_{out}(\vec{\rho})}} = \frac{MTF(\vec{\rho})C\bar{\phi}G}{\sqrt{W_{out}(\vec{\rho})}}$$

Derivation of DQE

$$DQE(\vec{\rho}) = \frac{SNR_{out}^2(\vec{\rho})}{SNR_{in}^2(\vec{\rho})} = \frac{MTF^2(\vec{\rho})(C\bar{\phi}G)^2}{W_{out}(\vec{\rho})} \frac{1}{\left(C\sqrt{\bar{\phi}}\right)^2}$$

$$= \frac{MTF^2(\vec{\rho})\bar{\phi}G^2}{W_{out}(\vec{\rho})}$$

Consider Poisson Noise Only —If No Added Detector Noise

$$W_{out_Q}(0) = \frac{k'^2\left(\bar{\phi}A_Q\right)}{A_S}$$

$$W_{out_Q}(\vec{\rho}) \propto NTF_Q^2(\vec{\rho})$$

$$W_{out_Q}(\vec{\rho}) = \frac{k'^2\bar{\phi}A_Q NTF_Q^2(\vec{\rho})}{A_S}$$

DQE Not Dependent on Fluence

$$DQE(\vec{\rho}) = \frac{MTF^2(\vec{\rho})\,\overline{\phi}\,G^2}{W_{out}(\vec{\rho})}$$

$$W_{out_Q}(\vec{\rho}) = \frac{k'^2\,\overline{\phi}\,A_Q\,NTF_Q^2(\vec{\rho})}{A_S}$$

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Digital Systems

$$G = k$$

$$DQE\left(\vec{\rho}\right) = \frac{MTF^2\left(\vec{\rho}\right)\bar{\phi}k^2}{W_{out}\left(\vec{\rho}\right)}$$

Screen-Film Systems

$$G = \frac{\gamma \log_{10} e}{\bar{\phi}}$$

$$\gamma = \frac{\Delta D}{\Delta \log \phi} = \frac{\Delta D}{\log \frac{\bar{\phi} + \Delta\phi}{\bar{\phi}}} = \frac{\Delta D}{\log_{10} e \;\; \ln\left(1 + \frac{\Delta\phi}{\bar{\phi}}\right)} = \frac{\Delta D}{\log_{10} e \;\; \frac{\Delta\phi}{\bar{\phi}}}$$

$$\frac{\Delta D}{\Delta\phi} = \frac{\gamma \log_{10} e}{\bar{\phi}}$$

Screen-Film Systems

$$G = \frac{\gamma \log_{10} e}{\bar{\phi}}$$

$$DQE(\vec{\rho}) = \frac{MTF^2(\vec{\rho})\gamma^2 \log_{10}^2 e}{W_{out}(\vec{\rho})\bar{\phi}}$$

Comparison between Digital and Screen-Film Systems

- **Digital:**

$$DQE(\vec{\rho}) = \frac{MTF^2(\vec{\rho})\bar{\phi}k^2}{W_{out}(\vec{\rho})}$$

- **Screen-film:**

$$DQE(\vec{\rho}) = \frac{MTF^2(\vec{\rho})\gamma^2 \mathbf{log}_{10}^2 e}{W_{out}(\vec{\rho})\bar{\phi}}$$

Difference in Fluence Dependence Comes from Characteristic Curve

$$\Delta P = k\Delta\phi$$

$$\sigma^2_{\Delta P} = k^2\sigma^2_{\Delta\phi}$$

$\Rightarrow$ $W_{out_{\Delta P}} \propto \bar{\phi}$

$$\Delta D = \frac{\gamma \log_{10} e}{\bar{\phi}}\Delta\phi$$

$$\sigma^2_{\Delta D} = \left(\frac{\gamma \log_{10} e}{\bar{\phi}}\right)^2 \sigma^2_{\Delta\phi}$$

$\Rightarrow$ $W_{out_{\Delta D}} \propto \frac{1}{\bar{\phi}}$

Poisson Noise Only—Digital Systems

$$W_{out_Q}\left(\vec{\rho}\right) = \frac{k'^2\overline{\phi}A_Q NTF_Q^2\left(\vec{\rho}\right)}{A_S}$$

$$G = k'A_Q$$

$$W_{out_Q}\left(\vec{\rho}\right) = \frac{k^2\overline{\phi}NTF_Q^2\left(\vec{\rho}\right)}{A_Q A_S}$$

$$DQE_{\Delta P}\left(\vec{\rho}\right) = A_Q A_S \frac{MTF^2\left(\vec{\rho}\right)}{NTF_Q^2\left(\vec{\rho}\right)}$$

Poisson Noise Only—Screen-Film Systems

$$W_{out_Q}(\vec{\rho}) = \frac{k'^2 \bar{\phi} A_Q NTF_Q^2(\vec{\rho})}{A_S}$$

$$G = \frac{\gamma \log_{10} e}{\bar{\phi}}$$

$$W_{out_Q}(\vec{\rho}) = \frac{\gamma^2 \log_{10}^2 e NTF_Q^2(\vec{\rho})}{\bar{\phi} A_Q A_S}$$

$$DQE_{\Delta D}(\vec{\rho}) = A_Q A_S \frac{MTF^2(\vec{\rho})}{NTF_Q^2(\vec{\rho})}$$

Comparison between Digital and Screen-Film Systems

- **Digital:** $$DQE_{\Delta P}\left(\vec{\rho}\right) = A_Q A_S \frac{MTF^2\left(\vec{\rho}\right)}{NTF_Q^2\left(\vec{\rho}\right)}$$

- **Screen-film:** $$DQE_{\Delta D}\left(\vec{\rho}\right) = A_Q A_S \frac{MTF^2\left(\vec{\rho}\right)}{NTF_Q^2\left(\vec{\rho}\right)}$$

Consider Additional Detector Noise

$$W_{out}\left(\vec{\rho}\right) = W_{out_Q}\left(\vec{\rho}\right) + W_{out_D}\left(\vec{\rho}\right) + W_{out_{SQ}}\left(\vec{\rho}\right)$$

$$DQE\left(\vec{\rho}\right) = \frac{MTF^2\left(\vec{\rho}\right)G^2}{W_{out}\left(\vec{\rho}\right)} = \frac{MTF^2\left(\vec{\rho}\right)G^2}{W_{out_Q}\left(\vec{\rho}\right)} \frac{W_{out_Q}\left(\vec{\rho}\right)}{W_{out}\left(\vec{\rho}\right)}$$

$$= A_Q A_S \frac{MTF^2\left(\vec{\rho}\right)}{NTF_Q^2\left(\vec{\rho}\right)} \frac{W_{out_Q}\left(\vec{\rho}\right)}{W_{out}\left(\vec{\rho}\right)}$$

The Ideal Detector is Quantum Noise Limited

- Implies no additional detector noise
- SNR can be increased simply by increasing exposure
- For non-ideal detectors, not all increase in exposure translate into increase in SNR

Important Components of DQE

- Modular transfer function (of signal)
- Quantum noise transfer function
- Spatial-frequency dependent fraction of total noise power as quantum noise
- Spatial frequency dependence of DQE

Important Components of DQE

- Quantum detection efficiency
- Swank factor

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Derivation of NEQ

$$\mathrm{DQE}(\vec{\rho}) = \frac{\mathrm{NEQ}(\vec{\rho})}{\bar{\phi}}$$

$$DQE(\vec{\rho}) = \frac{MTF^2(\vec{\rho})\bar{\phi}G^2}{W_{out}(\vec{\rho})}$$

$$\mathrm{NEQ}(\vec{\rho}) = \frac{MTF^2(\vec{\rho})\bar{\phi}^2 G^2}{W_{out}(\vec{\rho})}$$

Interpretation of NEQ as Detector Output SNR

$$DQE(\vec{\rho}) = \frac{SNR_{out}^{2}(\vec{\rho})}{SNR_{in}^{2}(\vec{\rho})} = \frac{SNR_{out}^{2}(\vec{\rho})}{C^{2}\overline{\phi}}$$

$$\mathrm{NEQ}(\vec{\rho}) = \frac{SNR_{out}^{2}(\vec{\rho})}{C^{2}}$$

Interpretation of NEQ as Detector Output SNR

- NEQ can be interpreted as the SNR squared at the detector out for unit contrast object

Secondary Quantum Noise (Gain Noise)

Jiang – Lecture 4 $\boxed{\text{pg. 496-497}}$

$$\sigma^2 = \langle (\tilde{w}_{out}(t))^2 \rangle - \langle \tilde{w}_{out}(t) \rangle^2$$

or square pulse..?

$$= R_{out}(\gamma=0)$$

$$R_{out}(\gamma) = psf(\gamma) * psf(\gamma) \otimes R_{in}(\gamma)$$

All for delta input..?

$$\sigma^2 = psf^2 \otimes \langle w_{in} \rangle$$

$$\underset{FT^{-1}}{\overset{FT}{\rightleftarrows}}$$

$$\mu_{in}(\vec{r}) = \langle \tilde{w}_{in}(\vec{r}) \rangle = \Delta\phi\,\delta(r-r_0) \qquad \mu_{in}(\vec{p}) = \Delta\phi$$

$$\sigma^2_{in}(\vec{r}) = \langle S_{in}(0) \rangle = \bar{\phi}\,\delta(0) \qquad (\text{Variance} \propto \phi) \qquad \sigma^2(\vec{p}) = \bar{\Phi}$$

$$SNR_{in}(\vec{r}) = \frac{\mu_{in}(\vec{r})}{\sqrt{\sigma^2_{in}(\vec{r})}} = \frac{\Delta\phi\,\delta(\vec{r}-\vec{r}_0)}{\sqrt{\bar{\Phi}}} = \frac{\Delta\phi}{\bar{\Phi}}\sqrt{\bar{\Phi}}\,\delta(\vec{r}-\vec{r}_0) = C\sqrt{\bar{\Phi}}\,\delta(\vec{r}-\vec{r}_0)$$

$$SNR_{in}(\vec{p}) = \frac{\mu_{in}(\vec{p})}{\sqrt{\sigma^2_{in}(\vec{p})}} = \frac{\Delta\phi}{\sqrt{\bar{\Phi}}} = \frac{\Delta\phi}{\bar{\Phi}}\sqrt{\bar{\Phi}} = C\sqrt{\bar{\Phi}}$$

$$\mu_{out}(\vec{r}) = \langle psf(\vec{r}) \otimes \tilde{w}_{in}(\vec{r}) \rangle$$

$$\mu_{out}(\vec{p}) = MTF(\vec{p}) \cdot \Delta\phi \cdot k \qquad , \quad k = \text{system gain}$$

$k = k' \cdot A_Q$ $\qquad$ A_Q = what fraction ... pass through?

k' = only x-rays that interact w/ detector

$$SNR_{out}(\vec{p}) = \frac{\mu_{out}(\vec{p})}{\sqrt{\sigma^2_{out}(\vec{p})}} = \frac{MTF(\vec{p})\,\Delta\phi\,k}{\sqrt{W_{out}(\vec{p})}} = \frac{MTF(\vec{p})\,C\bar{\Phi}\,k}{\sqrt{W_{out}(\vec{p})}}$$

$$DQE(\vec{p}) = \frac{SNR^2_{out}(\vec{p})}{SNR^2_{in}(\vec{p})} = \frac{MTF^2(\vec{p})\,(C\bar{\Phi}k)^2}{W_{out}(\vec{p})} \cdot \frac{1}{C^2\bar{\Phi}}$$

$$= \boxed{\frac{MTF^2(\vec{p})\,\bar{\Phi}\,k^2}{W_{out}(\vec{p})}}$$

$\tilde{W}_{in}(\vec{p}) \propto \bar{\Phi}$

$\tilde{W}_{out}(\vec{p}) \propto W_{in}(\vec{p})$

So not actually function of $\bar{\Phi}$

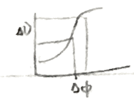
for only quantum N

$\boxed{\text{Digital (lin)}}$

$$k = \frac{\Delta P}{\Delta \phi}$$

$\boxed{\text{SF (H\&D)}}$

$$k = \frac{\Delta D}{\Delta \phi} = \frac{\gamma \log_{10}(e)}{\bar{\phi}}$$



$$\gamma = \frac{\Delta D}{\Delta \log_{10} \phi}$$

$$DQE(\vec{\rho}) = \frac{MTF^2(\vec{\rho})\,\gamma^2 \log_{10}^2 e}{W_{out}(\vec{\rho})\,\bar{\Phi}}$$

$$\Delta D = \frac{\gamma \log_{10}(e)}{\bar{\Phi}} \Delta \Phi$$

$$DQE(\vec{\rho}) = \frac{MTF^2(\vec{\rho})\,\bar{\Phi} k^2}{W_{out}(\vec{\rho})}$$

$$\sigma_D^2 = \left(\frac{\gamma \log_{10}(e)}{\bar{\Phi}}\right)^2 \sigma_\Phi$$

$$W_{out}(\vec{\rho}) \propto \bar{\Phi}$$

$$W_{out\,D} \propto \frac{1}{\bar{\Phi}}$$

how noise is transfered

$$W_{out\,Q}(\vec{\rho}) = \frac{k^2 \bar{\Phi} \quad NTF_Q(\vec{\rho})}{\underbrace{A_Q}_{QE} \underbrace{A_S}_{\text{secondary Q noise "swank"}}}$$

$$0 < A_S \leq 1$$
$$0 < A_Q \leq 1$$

SF (H&D)

$$DQE(\vec{\rho}) = \frac{MTF^2(\vec{\rho})\,\gamma^2 \log_{10}^2(e)}{W_{out\,D}(\vec{\rho})\,\bar{\Phi}}$$

$$k = \frac{\gamma \log_{10} e}{\bar{\Phi}}$$

$$W_{out}(\vec{\rho}) = \frac{(\gamma \log_{10} e)^2\, NTF^2(\vec{\rho})}{\bar{\Phi} \quad A_Q A_S}$$

$$DQE(\vec{\rho}) = \frac{MTF^2(\vec{\rho})}{NTF^2(\vec{\rho})} A_Q A_S$$

Digital (linear)

$$DQE(\vec{\rho}) = \frac{MTF^2\, k^2 \bar{\Phi}}{W_{out}(\vec{\rho})}$$

$$= \frac{MTF^2(\vec{\rho})}{NTF^2(\vec{\rho})} A_Q A_S$$

$$W_{out}(\vec{\rho}) = W_{out\,Q} + W_{out\,D} + W_{out\,SR}$$

$$DQE(\vec{\rho}) = \frac{MTF^2(\vec{\rho})\,\bar{\Phi} k}{W_{out}(\vec{\rho})} \cdot \frac{W_{out\,Q}(\vec{\rho})}{W_{out\,Q}(\vec{\rho})}$$

$$= \frac{MTF(\vec{\rho})}{NTF(\vec{\rho})} A_Q A_S \cdot \frac{W_{out\,Q}(\vec{\rho})}{W_{out}(\vec{\rho})}$$

$$DQE(\vec{\rho}) = \frac{NEQ(\vec{\rho})}{\bar{\Phi}}$$

$$DQE(\vec{\rho}) = \frac{MTF^2(\vec{\rho})\,\bar{\Phi}\,k}{W_{out}(\vec{\rho})} \qquad NEQ(\vec{\rho}) = \frac{MTF^2(\vec{\rho})\,\bar{\Phi}^2\,k}{W_{out}(\vec{\rho})}$$

$$DQE(\vec{\rho}) = \frac{SNR^2_{out}(\vec{\rho})}{SNR^2_{in}(\vec{\rho})} = \frac{SNR^2_{out}(\vec{\rho})}{c^2\,\Phi}$$

$$NEQ(\vec{\rho}) = \frac{SNR^2_{out}(\vec{\rho})}{c^2}$$