Spatio-angular Transfer Functions

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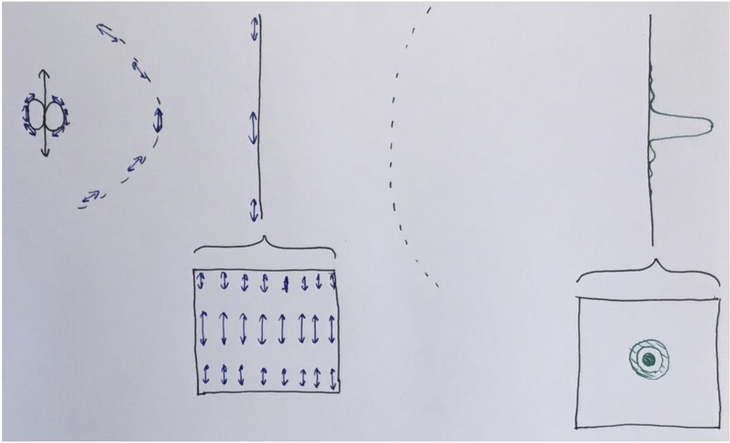
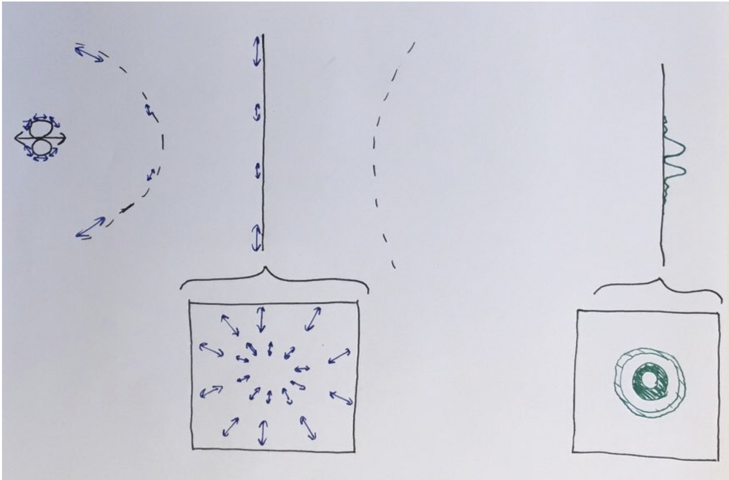
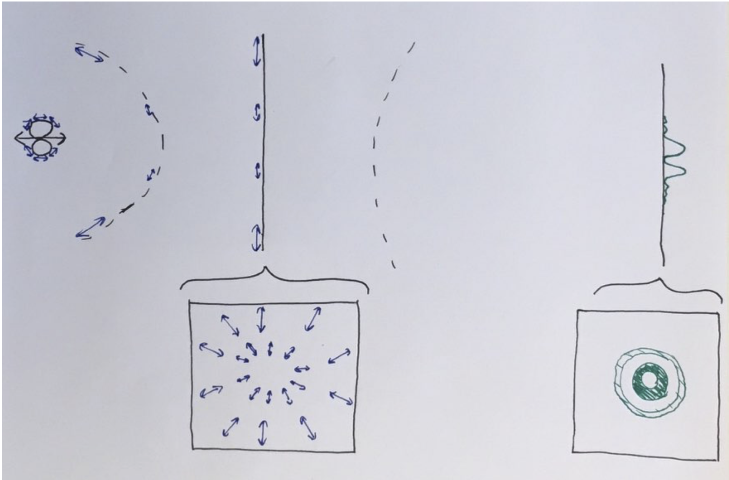
x-oriented dipole point spread function



z-oriented dipole point spread function



Point spread functions—Novotny



Point spread functions—Novotny

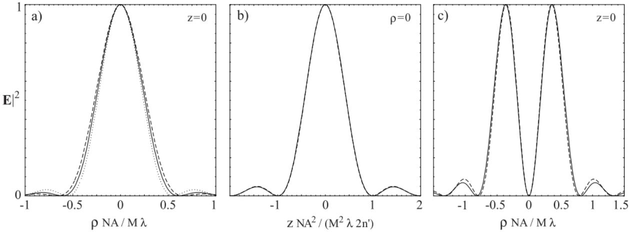
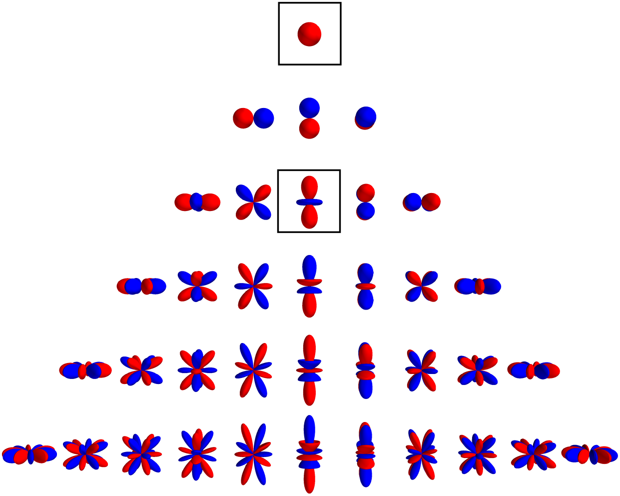


Figure 4.2: a) Point spread function depicted in the image plane ($z=0$) of a dipole with moment $\boldsymbol{\mu}=p_x\mathbf{n}_x$. The solid curve is the paraxial approximation whereas the dashed and dotted curves are the results of exact calculations for a $NA=1.4$ ($n=1.518$) objective lens. The dashed curve has been evaluated along the x-axis and the dotted curve along the y-axis. b) Point spread function evaluated along the optical axis z ($\boldsymbol{\mu}=\mu_x\mathbf{n}_x$). The solid curve is the paraxial approximation and the dashed curve is the exact result for $NA=1.4$. c) Point spread function depicted in the image plane of a dipole with moment $\boldsymbol{\mu}=\mu_z\mathbf{n}_z$. The solid curve is the paraxial approximation and the dashed curve is the exact result for $NA=1.4$. The figures demonstrate that the paraxial point spread function is a good approximation even for high NA objective lenses!

Only two spherical harmonics in the paraxial spatio-angular point spread function



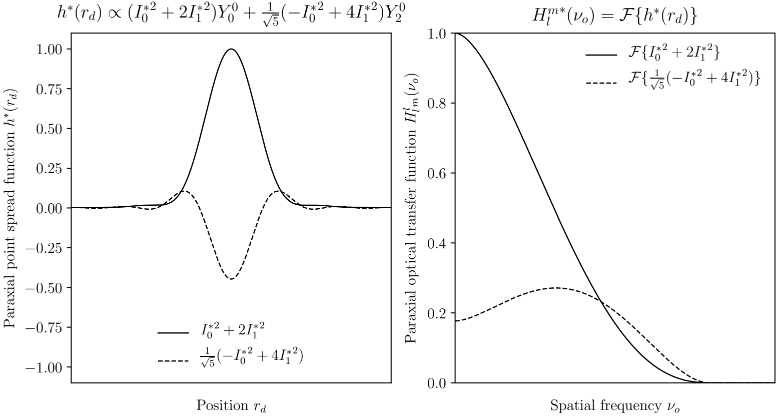
Paraxial spatio-angular point spread function

$$h^*(\mathbf{r}_d; \mathbf{r}_o, \hat{\mathbf{s}}_o) \propto ({I_0^*}^2 + 2{I_1^*}^2) Y_0^0(\hat{\mathbf{s}}_o) + \frac{1}{\sqrt{5}} \left(-{I_0^*}^2 + 4{I_1^*}^2 \right) Y_2^0(\hat{\mathbf{s}}_o)$$

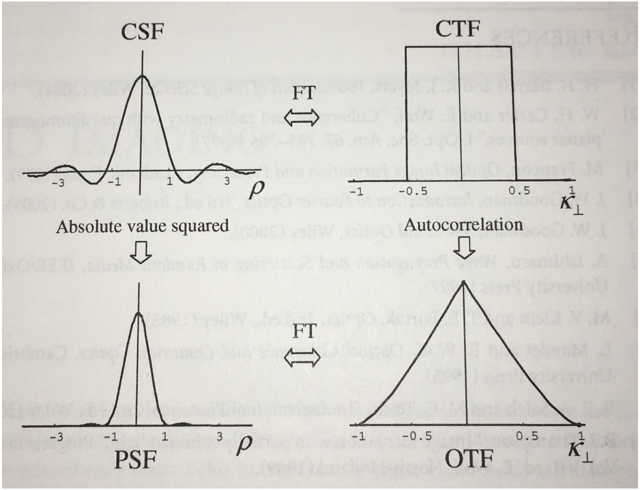
$$I_0^* = \left(\frac{\mathrm{NA}}{n_o} \right)^2 \left[\frac{2J_1(\tilde{r}_d)}{\tilde{r}_d} \right], \qquad I_1^* = \left(\frac{\mathrm{NA}}{n_o} \right)^3 \left[\frac{2J_2(\tilde{r}_d)}{\tilde{r}_d} \right],$$

$$\tilde{r}_d = \frac{2\pi \mathrm{NA}\, r_d}{M\lambda}, \qquad \mathrm{NA} = n_o \sin\theta_{\max}, \qquad M = \frac{n_o}{n_b} \frac{f_t}{f_o}.$$

Paraxial spatio-angular PSF and OTF



Scalar optics transfer functions—Mertz, Goodman



Electromagnetic optics transfer functions

$$
\begin{tikzcd}[column sep=huge, row sep=huge]
\begin{array}{c}\text{CSF}\\ \mathbf{e}_d(\mathbf{r}_d;\mathfrak{r}_o,\hat{\mathbf{s}}_o)\end{array} \arrow[r, leftrightarrow, "\begin{array}{c}\text{Tube Lens}\\ \mathcal{F}_{\mathbb{R}^2}\end{array}"] \arrow[d, "\begin{array}{c}\text{Detector}\\ |\mathbf{e}_d(\mathbf{r}_d;\mathfrak{r}_o,\hat{\mathbf{s}}_o)|^2_{\mathbb{R}^2}\end{array}"'] & \begin{array}{c}\text{CTF}\\ \mathbf{e}_b(\mathbf{r}_b;\mathfrak{r}_o,\hat{\mathbf{s}}_o)\end{array} \arrow[d, "\mathcal{F}_{\mathbb{S}^2}[\mathbf{e}_b \star_{\mathbb{R}^2} \mathbf{e}_b]"] \\
\begin{array}{c}\text{PSF}\\ h(\mathfrak{r}_o,\hat{\mathbf{s}}_o)\end{array} \arrow[r, leftrightarrow, "\mathcal{F}_{\mathbb{R}^3\times\mathbb{S}^2}"'] & \begin{array}{c}\text{OTF}\\ H_l^m(\boldsymbol{\nu})\end{array}
\end{tikzcd}
$$