Update on multiframe polarized light microscope singular spectra

Talon Chandler

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Product of functions in harmonic coefficient space

$$f(\hat{\mathbf{p}}) = \sum_{n=0}^{\infty} c_n z_n(\hat{\mathbf{p}}), \qquad f'(\hat{\mathbf{p}}) = \sum_{n'=0}^{\infty} c'_{n'} z_{n'}(\hat{\mathbf{p}}). \tag{1}$$

The product of these two functions is

$$f''(\hat{\mathbf{p}}) = f(\hat{\mathbf{p}}) f'(\hat{\mathbf{p}}) = \sum_{n=0}^{\infty} \sum_{n'=0}^{\infty} c_n c'_{n'} z_n(\hat{\mathbf{p}}) z_{n'}(\hat{\mathbf{p}}) = \sum_{n''=0}^{\infty} c''_{n''} z_{n''}(\hat{\mathbf{p}}), \tag{2}$$

Product of harmonics in coefficient space

$$c''_{n''} = \sum_{n=0}^{\infty} \sum_{n'=0}^{\infty} P_{n,n',n''} c_n c'_{n'}, \quad \text{or} \tag{3}$$

$$c''^{n''} = P^{n''}_{n,n'} c^n c'^{n'} \quad \text{in Einstein notation.} \tag{4}$$

$$P_{n,n',n''} = \int_{\mathbb{S}^1} d\hat{\mathbf{p}}\, z_n(\hat{\mathbf{p}}) z_{n'}(\hat{\mathbf{p}}) z_{n''}(\hat{\mathbf{p}}). \tag{5}$$

Product of circular and spherical harmonics

$$c''^{n'',j''} = P_{n,n'}^{n''} G_{j,j'}^{j''} c^{n,j} c'^{n',j'}. \tag{6}$$

where

$$P_{n,n',n''} = \int_{\mathbb{S}^1} d\hat{\mathbf{p}}\, z_n(\hat{\mathbf{p}}) z_{n'}(\hat{\mathbf{p}}) z_{n''}(\hat{\mathbf{p}}). \tag{7}$$

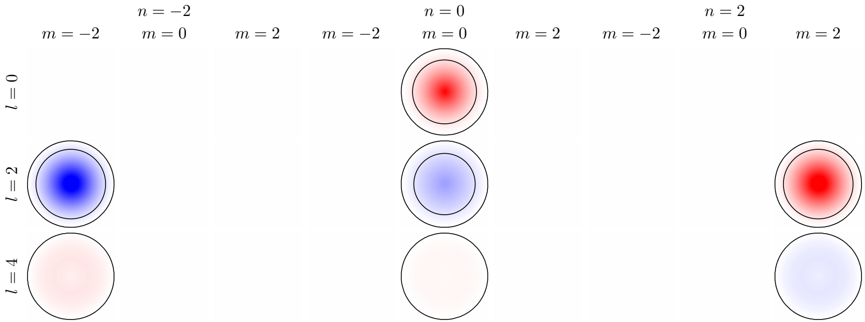
$$G_{j,j',j''} = \int_{\mathbb{S}^2} d\hat{\mathbf{s}}\, y_j(\hat{\mathbf{s}}) y_{j'}(\hat{\mathbf{s}}) y_{j''}(\hat{\mathbf{s}}). \tag{8}$$

.

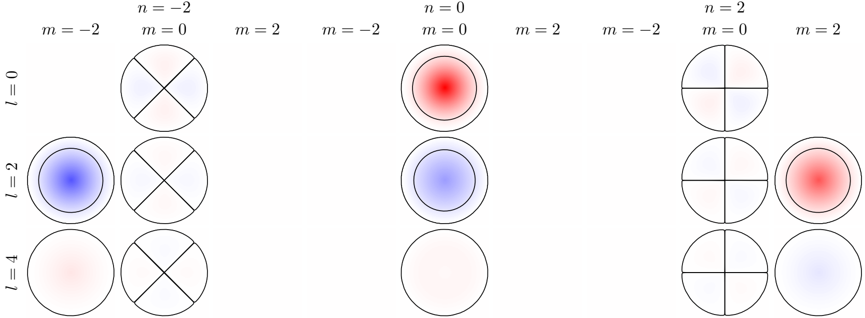
In `numpy`:

```
np.einsum('abc,def,ad,be->cf', P, G, c1, c2)    (9)
```

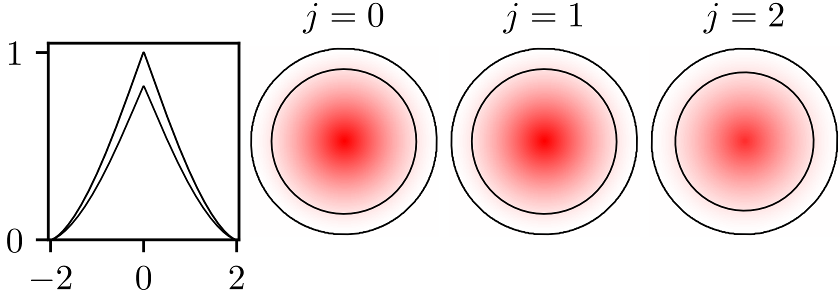
Single-view polarized illumination transfer function



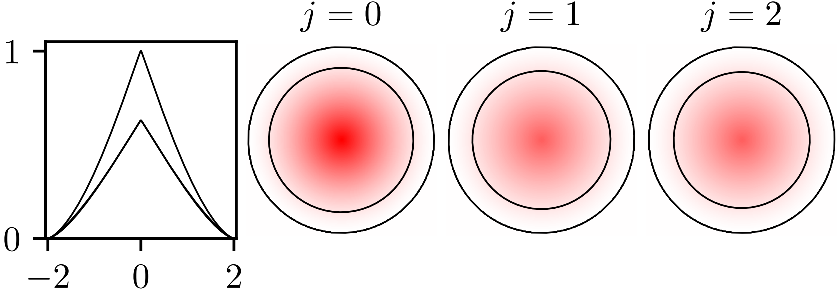
Single-view polarized detection transfer function



Single-view polarized illumination singular spectrum



Single-view polarized detection singular spectrum



Symmetry group for both polarized illumination and detection