

Update on multiframe polarized light microscope singular spectra

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CC-CD Model: $\mathbb{L}_2(\mathbb{R}^2 \times \mathbb{S}^2) \rightarrow \mathbb{L}_2(\mathbb{R}^{2N})$

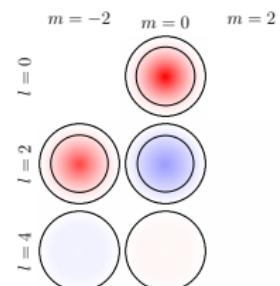
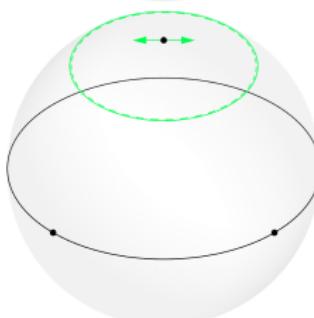
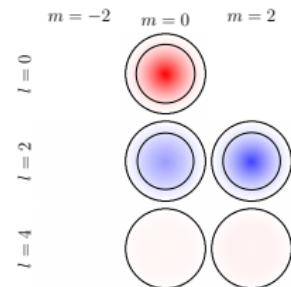
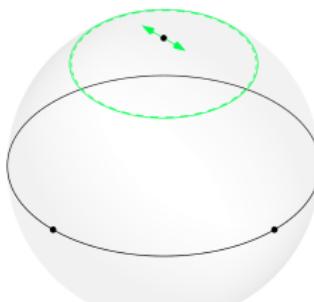
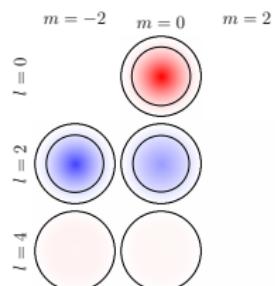
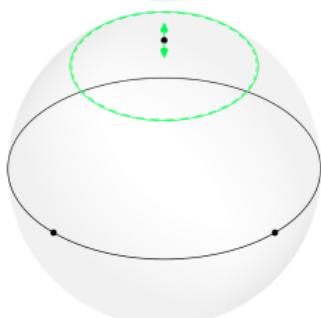
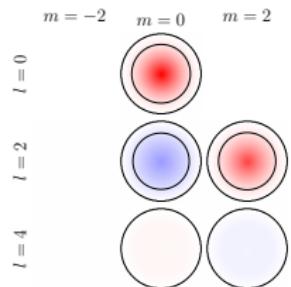
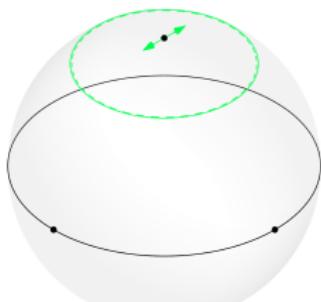
Forward model:

$$[\mathcal{H}f]_n(\mathbf{r}_d) = \int_{\mathbb{S}^2} d\hat{\mathbf{s}}_o \int_{\mathbb{R}^2} d\mathbf{r}_o h_n(\mathbf{r}_d - \mathbf{r}_o, \hat{\mathbf{s}}_o) f(\mathbf{r}_o, \hat{\mathbf{s}}_o), \quad n = 1, 2, \dots, N,$$

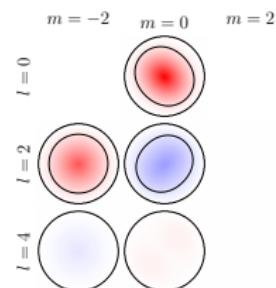
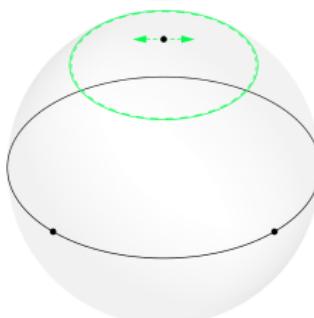
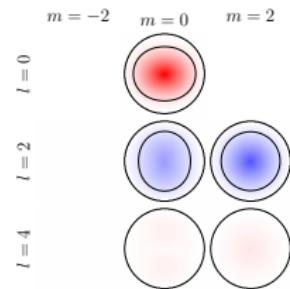
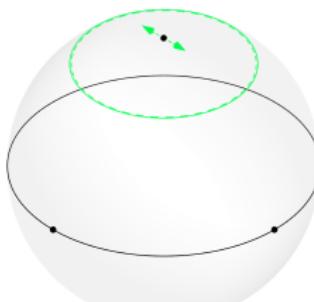
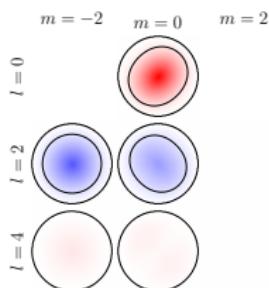
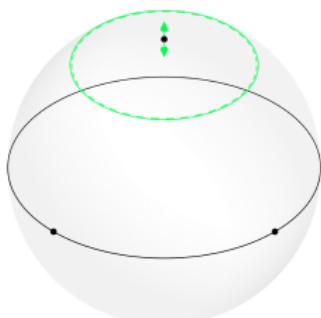
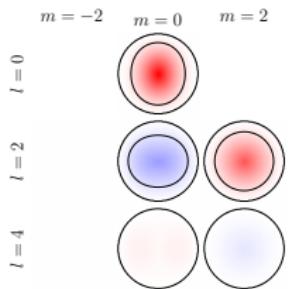
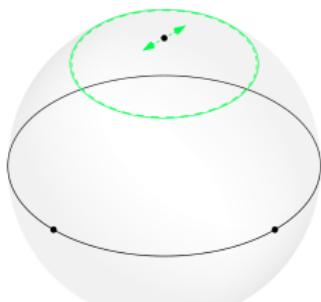
Spatio-angular OTF:

$$H_{l,n}^m(\boldsymbol{\nu}) = \int_{\mathbb{S}^2} d\hat{\mathbf{s}}_o \int_{\mathbb{R}^2} d\mathbf{r}_o h_n(\mathbf{r}_d - \mathbf{r}_o, \hat{\mathbf{s}}_o) y_l^m(\hat{\mathbf{s}}_o) e^{-i2\pi \mathbf{r}_o \cdot \boldsymbol{\nu}}.$$

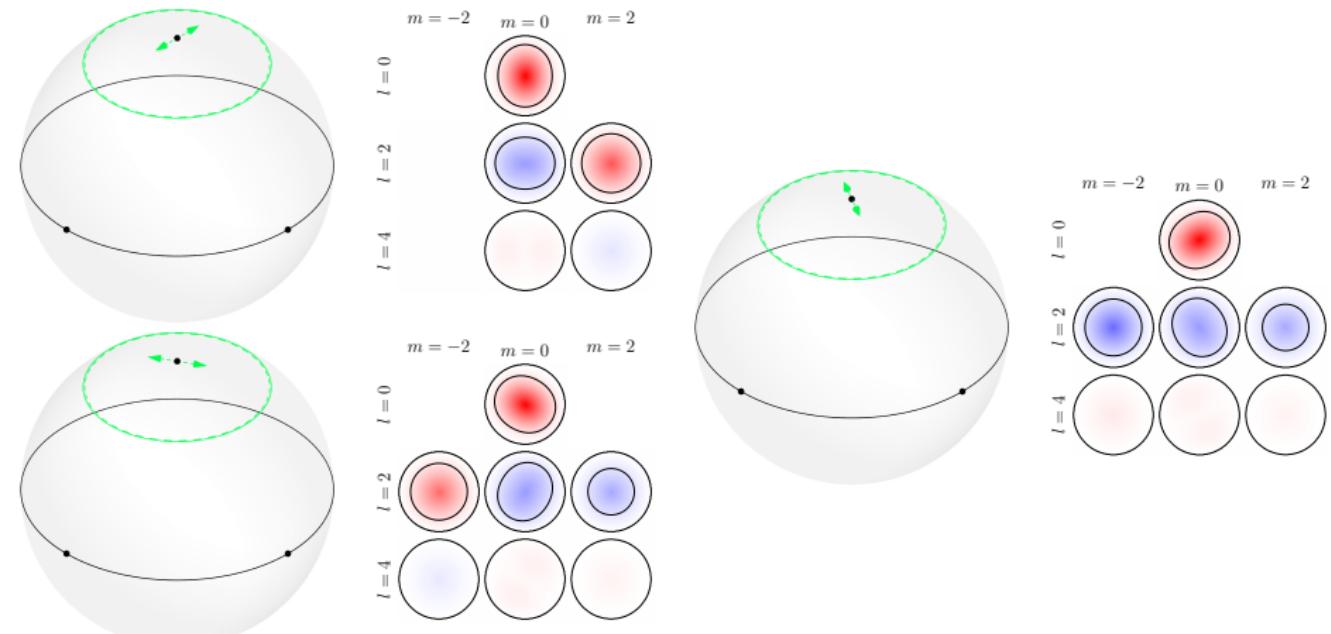
4-frame polarized illumination OTFs



4-frame polarized detection OTFs



3-frame polarized detection OTFs



Singular system

Forward model:

$$[\mathcal{H}f]_n(\mathbf{r}_d) = \int_{\mathbb{S}^2} d\hat{\mathbf{s}}_o \int_{\mathbb{R}^2} d\mathbf{r}_o h_n(\mathbf{r}_d - \mathbf{r}_o, \hat{\mathbf{s}}_o) f(\mathbf{r}_o, \hat{\mathbf{s}}_o), \quad n = 1, 2, \dots, N,$$

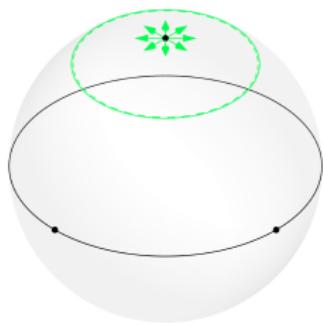
Adjoint:

$$[\mathcal{H}^\dagger \mathbf{g}](\mathbf{r}_o, \hat{\mathbf{s}}_o) = \sum_{j=1}^N \int_{\mathbb{R}^2} d\mathbf{r}_d h_n(\mathbf{r}_d - \mathbf{r}_o, \hat{\mathbf{s}}_o) g_n(\mathbf{r}_d).$$

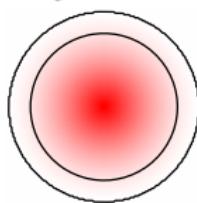
Eigenvalue problems:

$$\begin{aligned}\mathcal{H}^\dagger \mathcal{H} u_{\rho,j}(\mathbf{r}_o, \hat{\mathbf{s}}_o) &= \mu_{\rho,j} u_{\rho,j}(\mathbf{r}_o, \hat{\mathbf{s}}_o) \\ \mathcal{H} \mathcal{H}^\dagger \mathbf{v}_{\rho,j}(\mathbf{r}_d) &= \mu_{\rho,j} \mathbf{v}_{\rho,j}(\mathbf{r}_d)\end{aligned}$$

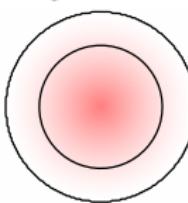
4-frame polarized illumination singular spectrum



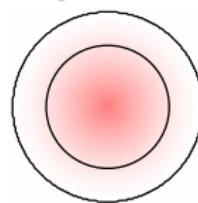
$j = 0$



$j = 1$



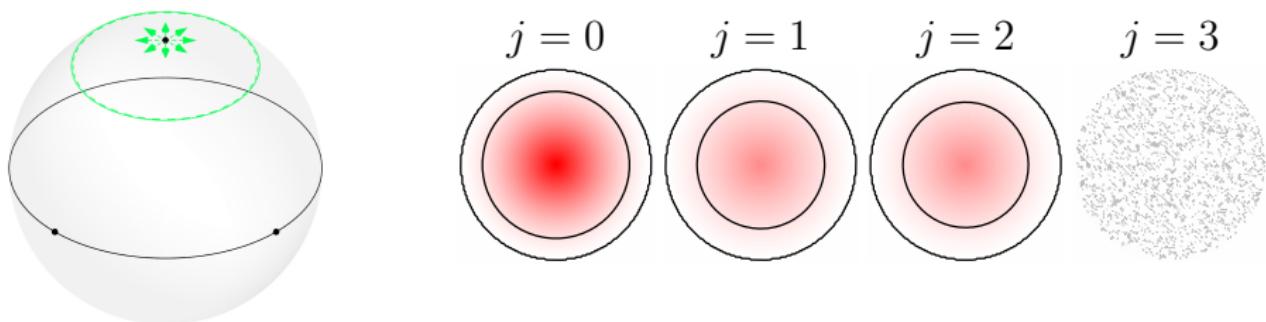
$j = 2$



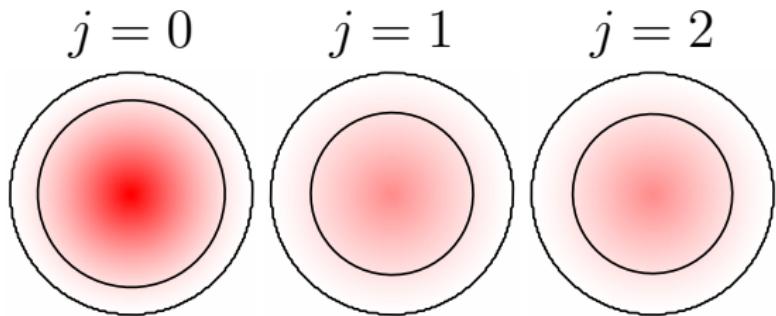
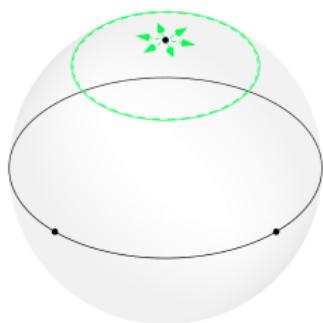
$j = 3$



4-frame polarized detection singular spectrum



3-frame polarized detection singular spectrum



More insight from CC-CC model?

$$\mathbb{L}_2(\mathbb{R}^2 \times \mathbb{S}^2) \rightarrow \mathbb{L}_2(\mathbb{R}^2 \times \mathbb{S}^1)$$

Forward model:

$$[\mathcal{H}f](\mathbf{r}_d, \hat{\mathbf{p}}) = \int_{\mathbb{S}^2} d\hat{\mathbf{s}}_o \int_{\mathbb{R}^2} d\mathbf{r}_o h(\mathbf{r}_d - \mathbf{r}_o, \hat{\mathbf{s}}_o; \hat{\mathbf{p}}) f(\mathbf{r}_o, \hat{\mathbf{s}}_o),$$

Adjoint:

$$[\mathcal{H}^\dagger g](\mathbf{r}_o, \hat{\mathbf{s}}_o) = \int_{\mathbb{S}^1} d\hat{\mathbf{p}} \int_{\mathbb{R}^2} d\mathbf{r}_d h(\mathbf{r}_d - \mathbf{r}_o, \hat{\mathbf{s}}_o; \hat{\mathbf{p}}) g(\mathbf{r}_d, \hat{\mathbf{p}}).$$