Update on multiframe polarized light microscope singular spectra

Talon Chandler

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Continuous model $\mathbb{L}_2(\mathbb{R}^2 \times \mathbb{S}^2) \to \mathbb{L}_2(\mathbb{R}^2 \times \mathbb{S}^1)$

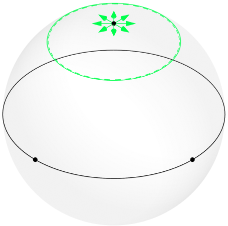
Forward model:

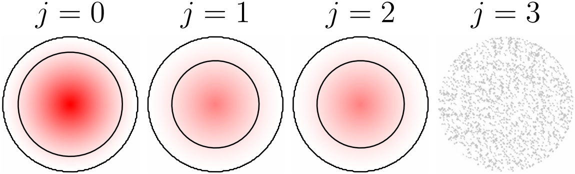
$$[\mathcal{H}f](\mathbf{r}_d, \hat{\mathbf{p}}) = \int_{\mathbb{S}^2} d\hat{\mathbf{s}}_o \int_{\mathbb{R}^2} d\mathbf{r}_o \, h(\mathbf{r}_d - \mathbf{r}_o, \hat{\mathbf{s}}_o; \hat{\mathbf{p}}) f(\mathbf{r}_o, \hat{\mathbf{s}}_o),$$

Adjoint:

$$[\mathcal{H}^\dagger g](\mathbf{r}_o, \hat{\mathbf{s}}_o) = \int_{\mathbb{S}^1} d\hat{\mathbf{p}} \int_{\mathbb{R}^2} d\mathbf{r}_d \, h(\mathbf{r}_d - \mathbf{r}_o, \hat{\mathbf{s}}_o; \hat{\mathbf{p}}) g(\mathbf{r}_d, \hat{\mathbf{p}}).$$

4-frame polarized illumination singular spectrum





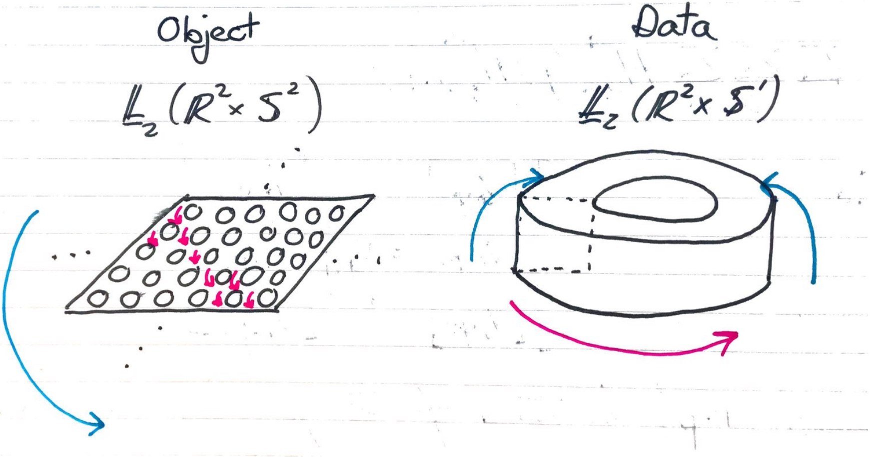
Polarized illumination SVD

$$\mu_{\boldsymbol{\rho},0} = \{H_{0,0}^{0}(\rho)\}^2 + \{H_{2,0}^{0}(\rho)\}^2 + \{H_{4,0}^{0}(\rho)\}^2,$$
$$\mu_{\boldsymbol{\rho},1} = \mu_{\boldsymbol{\rho},2} = \{H_{2,2}^{2}(\rho)\}^2 + \{H_{4,2}^{2}(\rho)\}^2,$$

$$u_{\boldsymbol{\rho},0}(\mathbf{r}_o, \hat{\mathbf{s}}_o) = \mathrm{e}^{i2\pi\boldsymbol{\rho}\cdot\mathbf{r}_o}[H_{0,0}^{0}(\rho)y_0^0(\hat{\mathbf{s}}_o) + H_{2,0}^{0}(\rho)y_2^0(\hat{\mathbf{s}}_o) + H_{4,0}^{0}(\rho)y_4^0(\hat{\mathbf{s}}_o)],$$
$$u_{\boldsymbol{\rho},1}(\mathbf{r}_o, \hat{\mathbf{s}}_o) = \mathrm{e}^{i2\pi\boldsymbol{\rho}\cdot\mathbf{r}_o}[H_{2,-2}^{-2}(\rho)y_2^{-2}(\hat{\mathbf{s}}_o) + H_{4,-2}^{-2}(\rho)y_4^{-2}(\hat{\mathbf{s}}_o)],$$
$$u_{\boldsymbol{\rho},2}(\mathbf{r}_o, \hat{\mathbf{s}}_o) = \mathrm{e}^{i2\pi\boldsymbol{\rho}\cdot\mathbf{r}_o}[H_{2,2}^{2}(\rho)y_2^2(\hat{\mathbf{s}}_o) + H_{4,2}^{2}(\rho)y_4^2(\hat{\mathbf{s}}_o)].$$

$$v_{\boldsymbol{\rho},0}(\mathbf{r}_d, \hat{\mathbf{p}}) = \mathrm{e}^{i2\pi\boldsymbol{\rho}\cdot\mathbf{r}_d}z_0(\hat{\mathbf{p}}),$$
$$v_{\boldsymbol{\rho},1}(\mathbf{r}_d, \hat{\mathbf{p}}) = \mathrm{e}^{i2\pi\boldsymbol{\rho}\cdot\mathbf{r}_d}z_{-2}(\hat{\mathbf{p}}),$$
$$v_{\boldsymbol{\rho},2}(\mathbf{r}_d, \hat{\mathbf{p}}) = \mathrm{e}^{i2\pi\boldsymbol{\rho}\cdot\mathbf{r}_d}z_2(\hat{\mathbf{p}}).$$

Symmetries



Object
$\mathbb{L}_2(\mathbb{R}^2 \times S^2)$
Data
$\mathbb{L}_2(\mathbb{R}^2 \times S^1)$