Measurement of Planck's Constant via the Photoelectric Effect

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When a metal is hit with a photon of high enough energy, it emits an electron. This phenomena is known as the photoelectric effect. By measuring the maximum kinetic energy of electrons ejected from an alkali metal exposed to different frequencies of light, we were able to arrive at values for Planck's constant, h, and the work function of the metal, ϕ, through two different analysis methods. We found from the first method that $h = (2.11 \pm 0.14_{\mathrm{stat}} \pm 0.07_{\mathrm{sys}}) \times 10^{-15}$ eV $\cdot$ s and $\phi = (1.03 \pm 0.10_{\mathrm{stat}} \pm 0.05_{\mathrm{sys}})$ eV; from the second method we found that $h = (2.29 \pm 0.06_{\mathrm{stat}} \pm 0.15_{\mathrm{sys}}) \times 10^{-15}$ eV $\cdot$ s and $\phi = (1.15 \pm 0.04_{\mathrm{stat}} \pm 0.22_{\mathrm{sys}})$ eV.

I. INTRODUCTION

The photoelectric effect is a well-known phenomena in which a metal surface illuminated by a high enough frequency of light emits an electron. In the early twentieth century, Albert Einstein made a crucial insight into the underlying physics of this process. Expanding on the work of Max Planck, he proposed that light carries discrete amounts of energy equal to $h\nu$, where h is Planck's constant and ν is the light's frequency [1]. Today, we know the energy quanta of light as photons.

The kinetic energy of electrons ejected from the metal surface would therefore be $K = h\nu - \phi$, where ϕ is a constant, the 'work function', which varies according to properties of the metal. From this, we can deduce that increasing the intensity of the light source in a photoelectric experiment would result in more electrons getting ejected from the metal as more photons hit it; however, each of these electrons would have the same kinetic energy as before the intensity was increased because the energy of each photon does not change. Similarly we can see that increasing the frequency of light would result in the same number of electrons being ejected as before, each with a higher energy.

These effects, as well as Einstien's model for the kinetic energy of ejected electrons, $K = h\nu - \phi$, are observed experimentally. These results won Einstein the Nobel Prize in 1921, and helped introduce the existence of photons. This played a major role in the acceptance of quantized energy and the development of quantum mechanics [2].

II. EXPERIMENTAL SET-UP

We set out to perform an experiment of the photoelectric effect, measuring the kinetic energy of electrons ejected from an alkali metal surface for several wavelengths of light. From this data, we can calculate the value of Planck's constant as well as the work function of the metal.

A block diagram of our experimental set-up can be seen in Figure 1. The mercury lamp outputs only frequencies in mercury's emission spectrum, so filtering for a certain range of light allows only one wavelength to hit to photosurface. The photosurface itself is a smooth piece of potassium, enclosed in a glass chamber under vacuum with the anode ring. Our power supply was an Agilent variable DC Power Supply, and we used a Keithley Electrometer.

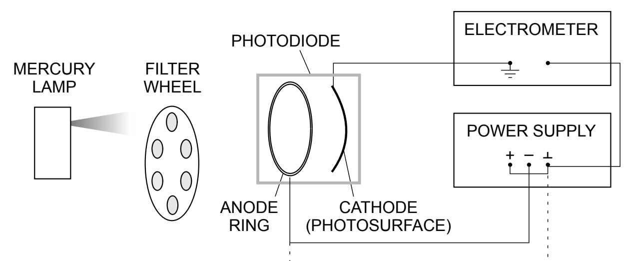

FIG. 1. The experimental set-up. A single wavelength from the lamp reaches the cathode through the filter, ejecting an electron onto the anode ring and inducing a current. The voltage from the power supply applies an electric field that decreases the current. At the cut-off voltage, the measured current reads zero. The dotted line is an extra grounding wire added during the measurements for 404.7 nm light to keep the measured current from fluctuating. (Figure based off of diagram from [1].)

When our single-frequency light hits the cathode, the electrons ejected land on the anode ring, creating a current measured by the electrometer. When the power supply is turned on, it applies an electric field to slow down that current, decreasing the measured value. The voltage from the power supply is therefore referred to as the "retarding voltage". As the retarding voltage is increased, the current should decrease linearly until it reaches zero, at which point it will stay constant. The voltage at which the current first hits zero is the cut-off voltage, V_{co}.

The cut-off voltage is proportional to the kinetic energy of the electrons: $Ve = K$, where e is the charge of an electron [1]. We can substitute this into the equation for the kinetic energy of electrons ejected from the metal to

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find that

$$V_{co} = \frac{h}{e}\nu - \frac{\phi}{e} \tag{1}$$

Thus, our goal is to find V_{co} at different frequencies; a plot of frequency vs. cut-off voltage will have a slope proportional to Planck's constant and an intercept proportional to the work function of our photosurface.

We collected data from three wavelengths: 365.0 nm, 404.7 nm and 435.7 nm. Starting with the power supply at zero, we increased the retarding voltage stepwise by 0.05 V, measuring the current output on the electrometer at each step. We repeated these scans from 0 to 2 V five times for every wavelength.

It should be noted that the signal shown on the electrometer was extremely sensitive. We measured currents on the picoamp scale, so waving one's hand near the apparatus or jostling a wire slightly would cause the displayed current to fluctuate. To avoid disturbing the setup, we taped down the wires so they could not move. In addition, while taking measurements for the 404.7 nm light, we added a grounding wire (marked as a dotted line in Fig. 1) to keep the current signal steady. Lastly, we put a black cloth over the lamp, filters, and photodiode to keep out ambient light.

III. ANALYSIS

The main challenge in analyzing this data is calculating the cut-off voltage. There are two main differences between the observed data and what the theoretical model predicts. Firstly, the plot curves down to zero instead of being sharply cut off; this can be caused by a dislodged electron not landing on the anode. Secondly, the constant value the current reaches will not always be zero, but may be some other positive or negative value. We will refer to this as the "steady current", or I_{steady}. This may be caused by photons hitting the anode instead of the cathode and starting a photoelectric effect there [1].

We have used two methods for determining the cut-off voltage from our data. The first involves drawing linear fits to predict when the current would have hit zero according to the theoretical model. The second method entails determining when the current reaches a certain percentage of its initial value and defining that voltage as the cut-off voltage.

Method 1: Linear Interpolation

In Figure 2A, we have displayed a plot of retarding voltage vs. current for 365.0 nm light. While the data ultimately begins to curve, the first several data points do appear to be linear. We want to identify the linear portion of the data and fit a line to it. We will then draw a horizontal line at I_{steady}, defining the cut-off voltage to be where those two lines intersect.

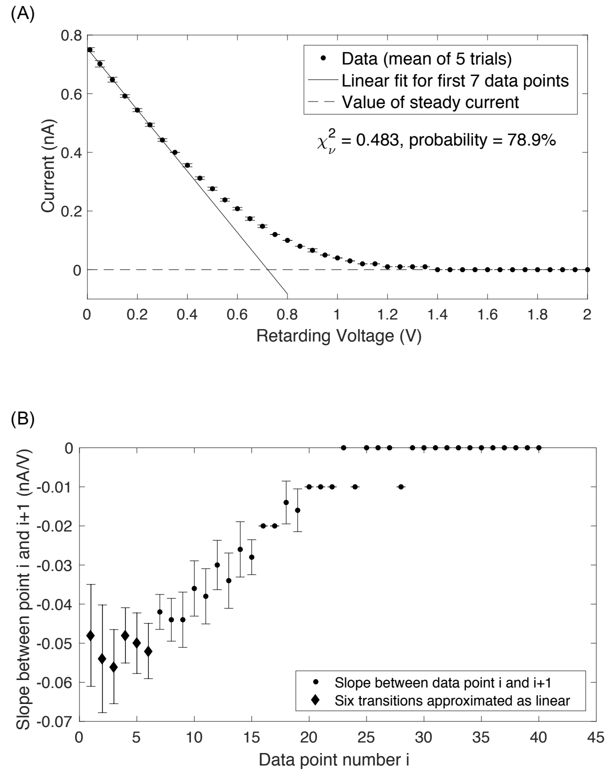

FIG. 2. In (A), we see a plot of the data gathered with 365.0 nm light where each point is the average of five data points from five trials. The error bars denote the standard deviation of each point. The plot begins linearly, then curves down to zero. (B) is a plot of the slope between each point in (A). We see that the first six calculated slopes are approximately the same, leading us to approximate the first seven data points as linear. The resulting fit is plotted in (A). The fit has a reduced χ^2_ν value of 0.483, indicating it matches the data well.

To determine what part of the data can be approximated as linear, we calculated the slope between each pair of data points. We define the slope between points i and $i+1$ to be

$$m_i = \frac{I_{i+1} - I_i}{V_{i+1} - V_i} \tag{2}$$

where I_i is the current and V_i is the retarding voltage for point i. Plotting m_i for the 365.0 nm data (Fig. 2B), we see that the first six transitions have a somewhat constant slope. We therefore define the current at the seventh data point to be I_{curve}, with uncertainties equal to the uncertainties of that data point. For data from 404.7 nm and 435.7 nm, we approximated the first four points as linear.

To find steady current value, I_{steady}, we use the lowest current we measured for this value in each wavelength. Finally, we need to find where the linear fit crosses I_{steady}; for a linear fit of the form $I = aV + b$, where I is current, V is retarding voltage, and a and b are parameters of the fit, we see that

$$V_{co} = I_{\text{steady}} - \frac{b}{a} \tag{3}$$

The error from I_{steady} and and fit parameters was propagated using formulae from [3].

This procedure was repeated for all four wavelengths of light. Details on the linear fits, as well as the reduced χ^2 value for each wavelength, can be found in the following table:

Wavelength	Number of data points included in fit	Linear fit χ^2_ν
365.0 nm	7	0.483
404.7 nm	4	0.415
435.7 nm	4	1.57

Method 2: Fractional Decrease in Current

For this method, we want to pick a reference point during the decrease in current that can be identified for all of our wavelengths. Since all the data starts approximately linear, that is a good characteristic off of which to base our reference point. We will define I_{curve} to be the point before which the data is linear and after which the data is non-linear, defining the same section of data to be linear as in method 1.

Again, we used the lowest current we measured for each wavelength to find the steady current, I_{steady}. To find the cut-off voltage, we will look for the retarding voltage at the point when the current has decreased from I_{curve} to some fraction f of its initial value. We call this the cut-off current, I_{co}, where

$$I_{co} = f(I_{\text{curve}} - I_{\text{steady}}) + I_{\text{steady}} \tag{4}$$

Unfortunately, this method of finding V_{co} does not have as strong a physical motivation as method 1, which was based off of the theoretically expected plot. There is little to suggest what value should be chosen for f. We chose $f = 37\%$, as it matched the data from method 1 most closely; this implicitly incorporates a bit of the physical theory from method 1 into this calculation.

We now need to find the retarding voltage at I_{co}. We identified the two data points closest to that current value, a and b, and approximated the current to be linear between them, as seen in Figure 3. We can now calculate V_{co}:

$$I_{co} = \frac{I_b - I_a}{V_b - V_a} V_{co} + \left(I_a - \frac{I_b - I_a}{V_a - V_b} V_a \right)$$

$$V_{co} = \frac{V_b - V_a}{I_b - I_a} \left(I_{co} - I_a + \frac{I_b - I_a}{V_a - V_b} V_a \right) \tag{5}$$

where V_a and V_b are the voltages and I_a and I_b are the currents at points a and b. This analysis was repeated for data from all three wavelengths. For the 365.0 nm data, the seventh data point was used to determine I_{curve}, and for the 404.7 nm and 435.7 nm data, the fourth data points were used.

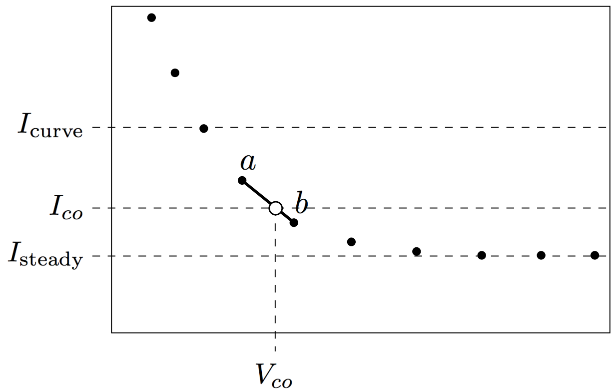

FIG. 3. This is a close-up diagram of how method 2 calculates the cut-off voltage. We define I_{curve} to be the current at which the data stops being linear; I_{co} is 37% of I_{curve} for $I_{\text{steady}} = 0$. Points a and b are closest to I_{co}, so we draw a line and define the cut-off voltage, V_{co} to be the retarding voltage at the point where that line is equal to I_{co}.

Systematic Error

As was discussed briefly in the experimental set-up, the measurements of current by the electrometer were very sensitive. Precautions were taken to minimize some sources of data variation, but not all systematic error was accounted for. The electrometer signal would fluctuate wildly if someone walked past the experiment or if a laptop was turned on nearby. Tapping a wire with a finger or adjusting the cloth placed over part of the set-up also would shift the displayed currents by a small amount. For all of these effects, we assign a systematic error of ± 0.01 nA for the 365.0 nm data and ± 1 pA for the 404.7 nm and 435.7 nm data.

In addition, the electrometer signal appeared to be steadily lowering while we took data for the 404.7 nm light. It was not until the last two trials that we saw variations as small as we had seen when measuring the 365.0 nm and 435.7 nm currents, suggesting that the electrometer had only then reached a stable state. To account for the variability of the electrometer's output, we assign an additional systematic error of ± 2 pA to the 404.7 nm data.

We propagated these errors through our calculations in different ways depending on the calculation. For method 2, we had systematic uncertainty for I_{co}, I_a, and I_b, so we used the general error propagation formula found in [3] based off of equation (5).

Error propagation for method 1 had a two parts to it. First, we added our systematic error to the intercept of the linear fit and recorded the change in V_{co}. Second, we considered how changes to the number of points approximated as linear affected the final values. Specifically, we found the mean slope between the points we initially identified as linear and added extra data points if their slope fell within one standard deviation of that mean slope. The final systematic error on V_{co} was the sum of these two values.

While calculating Planck's constant and the cathode's work function, we used a similar technique for error propagation to that of method 1. At this point there was different systematic error for each data point, so we added and subtracted the uncertainty from each data point in different combinations; one half of the range of how much the final result for h and ϕ changed from these perturbations was defined as the propagated systematic error.

IV. RESULTS

Now that we have a value for our cut-off voltages, we can create a plot of light frequency vs. cut-off voltage. Equation (1) tells us this should be a linear relationship with the slope equal to h/e and intercept equal to ϕ/e.

We plotted the results from methods 1 and 2 separately, as seen in Figure 4. The calculated values for Planck's constant and the work function of the metal are shown below with subscripts indicating which method the value came from:

$$
\begin{aligned}
h_1 &= (2.11 \pm 0.14_{\text{stat}} \pm 0.07_{\text{sys}}) \times 10^{-15}\ \text{eV} \cdot \text{s} \\
h_2 &= (2.29 \pm 0.06_{\text{stat}} \pm 0.15_{\text{sys}}) \times 10^{-15}\ \text{eV} \cdot \text{s} \\
\phi_1 &= (1.03 \pm 0.10_{\text{stat}} \pm 0.05_{\text{sys}})\ \text{eV} \\
\phi_2 &= (1.15 \pm 0.04_{\text{stat}} \pm 0.22_{\text{sys}})\ \text{eV}
\end{aligned}
$$

The accepted value for Planck's constant is 4.135×10^{-15} eV $\cdot$ s, so we have calculated it to the correct order of magnitude, off by 49.0% and 44.7% in methods 1 and 2, respectively. According to [1], the work function of the metal should be close to 2.3 eV, which matches our data.

The reduced χ^2 value in Fig. 2(A) is 20.40, suggesting the data from method 1 does not fit the function well. It is plausible that this caused our calculated h to be off. The most likely culprit for the bad fit is the 404.7 nm data point, as it is significantly below the fit line on the plot for method 1 but not for method 2. The data for that wavelength was taken on a different day under slightly different conditions; future repetitions of this experiment should attempt to take all the data in one day to avoid this.

We have successfully calculated Plank's constant and the work function of our cathode metal to the correct order of magnitude. In addition, we have confirmed that the kinetic energy of electrons ejected from a metal has a linear relationship with the frequency of light, supporting Einstein's predictions. This result, as discussed earlier, implies that light carries energy in discrete quanta, laying the foundation for quantum mechanics.

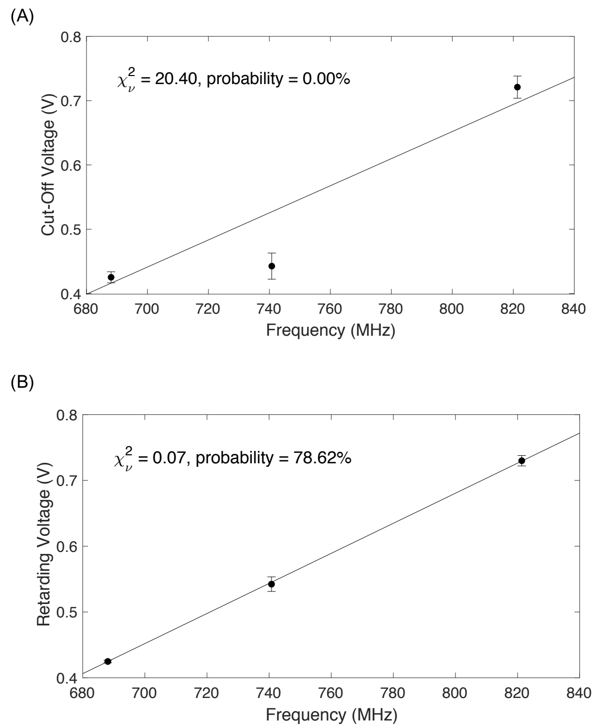

FIG. 4. (A) shows a plot of frequency vs. V_{co} from method 1, and (B) is the equivalent plot from method 2. It is clear from the reduced χ^2 value that the fit on method 1 is not very good; this may be due to the data for the 404.7 nm light being taken on a different day under slightly different conditions.

V. ACKNOWLEDGEMENTS

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[2] R. Field, "Quantum mechanics - historical background," (2013), lecture notes from Physical Chemistry class at MIT.

[3] P. Bevington and D. Robinson, *Data Reduction and Error Analysis for the Physical Sciences* (McGraw-Hill, 2003).