Measurement of the Mean Speed and Lifetime of Cosmic-Ray Muons

Tal Scully*

MIT Department of Physics

(Dated: October 3, 2017)

Cosmic-ray muons travel near the speed of light and allow us to test the predictions of special relativity. We can detect muons in the laboratory and measure their mean speed and lifetime. We calculated the muons' mean speed to be $(4.955 \pm 0.066_{\mathrm{stat}} \pm 0.020_{\mathrm{sys}}) \times 10^8$ m/s and found the lifetime to be $2.240 \pm 0.003_{\mathrm{stat}} + 0.089_{\mathrm{sys}}$ μs.

1. INTRODUCTION

In 1905, Einstein proposed his theory of special relativity to describe the motion of particles and objects moving near the speed of light [1]. Unlike Newtonian dynamics, special relativity allows for all the laws of physics – including the universal speed of light, c, implicit in Maxwell's equations – to apply in any inertial frame.

By measuring the speed and lifetime of cosmic-ray muons, we can provide experimental evidence to support Einstein's theory of special relativity. Muons are formed 15 km above sea level when cosmic rays interact with gas molecules in Earth's atmosphere [1]. They then travel near the speed of light until they decay into an electron or positron and two neutrinos [2].

We will find that, using Newtonian physics, the speed and lifetime of the muon are such that we would not expect it to reach the ground at all. Special relativity tells us that the lifetime of a muon in motion will be related to the lifetime when it is at rest by $\tau_{\mathrm{motion}} = \gamma\tau_{\mathrm{rest}}$ where $\gamma = (1 - v^2/c^2)^{-1/2}$ and v is the muon speed [3]. Since $v < c$, we see that *gamma* > 1 and $\tau_{\mathrm{motion}} > \tau_{\mathrm{rest}}$. The fact that we can observe muons at sea level is explained by this lengthening of the muon's lifetime.

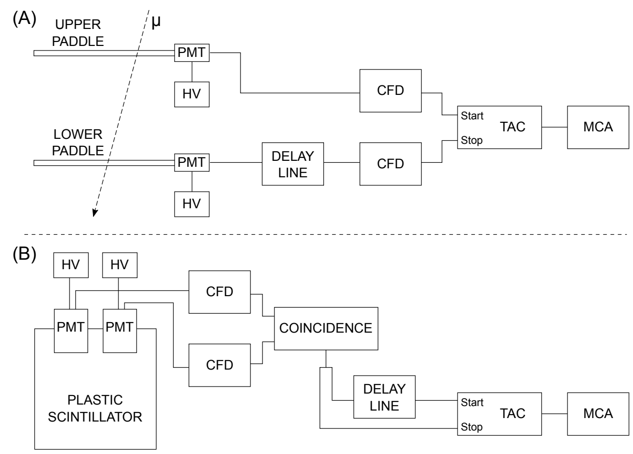

FIG. 1: (A) This is the set-up to measure the muon speed. The PMTs detected muons as they passed through the paddles. The signals were processed and sent to the TAC, which reported the time interval between the signals, allowing us to find the time between the muon hitting each paddle. (B) This shows the set-up for measuring the muon lifetime. Both PMTs detected when a muon entered the plastic scintillator and when it decayed. The PMT signal was split and fed to the TAC such that it measured the time between the muon coming to rest and decaying.

2. EXPERIMENTAL SET-UP

There were two separate experiments to measure the average speed and lifetime of the muons. The set-up for measuring the muon speed is shown in Figure 1(A). Several muons per second that reach the lab pass through the upper and lower rectangular scintillation paddles, each (0.60 ± 0.01) m by (0.40 ± 0.01) m. The distance between the two paddles was adjustable. The photomultiplier tubes (PMT) attached to each paddle detected when a muon passed through the paddle. By measuring the time between signals from the PMTs, we can calculate the time it takes for the muon to travel between the paddles.

The signal from the lower PMT was sent through a delay line of 6 ft of cable in order to make the time interval larger and easier to measure. Both signals were then processed by a constant fraction discriminator (CFD). The signals from the upper and lower paddles were sent into the "Start" and "Stop" inputs of the time to amplitude converter (TAC), respectively. The TAC sends a voltage signal with amplitude proportional to the time between receiving the "Start" and "Stop" pulses. This signal was then sent through a multichannel analyzer (MCA) to a computer [1].

The computer displayed a histogram of the voltage signals received from the TAC; by studying the distribution in the MCA channels, which is related to the distribution of time it took for muons to travel between the upper and lower paddles, we can calculate the mean speed of the muons. To find a conversion between MCA channels and time measured by the TAC, we sent signals from a time calibrator (TC), which sends pulses separated by known time intervals, to the TAC [1]. For the speed measurements the TAC had a range of 100 ns, meaning it waited a maximum of 100 ns to receive a "Stop" pulse after each "Start" signal, and our delay line was 6 ft of cable.

The set-up for measuring the lifetime of a muon is shown

*Electronic address: tals@mit.edu

in Figure 1(B). Muons entering the plastic scintillator ($[\mathrm{CH}_2]_n$, density ≈ 1.2 g/cm^3) lost approximately 50 MeV of energy, which was enough to bring their speed to zero [1]. The event of a muon stopping in the scintillator was measured by both PMTs. The signal was discriminated by the CFDs and sent to the coincidence circuit, which outputs a signal only when it receives two signals at the same time; this eliminated noise that was detected by only one PMT.

The signal from the coincidence circuit was then split. One signal arrived at the "Stop" input on the TAC and was ignored. The other signal was delayed and put into the "Start" input on the TAC, starting a timing sequence.

The PMTs also detected the decay of the muon that had been stopped in the scintillator. The signal traveled through the same path, but this time the signal into the "Stop" input of the TAC ended the timing sequence. At this point the TAC output a voltage proportional to the time interval between the "Start" and "Stop" signals, equivalent to the time interval between the muon coming to rest in the scintillator and the muon decaying.

As before, these voltages were sent to the MCA and displayed as a histogram on a computer. For the lifetime measurement, the TAC had a range of 20 μs and our delay line was 64 ft of cable. We used the TC to re-calibrate the set-up again with these settings.

3. ANALYSIS

The first step in our analysis is to calibrate the MCA channels so that we know what bins correspond to what time intervals measured by the TAC. For the speed measurement TAC settings, we calibrated by outputting time intervals that were multiples of 0.01 μs from the TC; for the lifetime measurement settings we outputted time intervals that were multiples of 1.28 μs.

The MCA histogram displayed sharp peaks ranging over anywhere from 1 to 11 channels in width. We defined the channel corresponding to each time interval to be the weighted average for all the counts in the relevant peak. The standard deviation of those counts will be incorporated into systematic error in our final calculations for speed and lifetime.

Now we can plot the MCA channels against the time intervals we know they represent. This plot for the calibration of the lifetime set-up is shown in Figure 2. The relationship between MCA channel and the time interval measured by the TAC is quite clearly linear: the speed set-up's calibration had a χ^2_ν value of 0.111, and the lifetime set-up's calibration had a χ^2_ν value of 0.631. We can fit linear functions to our data to convert any MCA channel to its corresponding time interval. Now that we know how to make this conversion, we can use our MCA histograms to calculate the muons' mean speed.

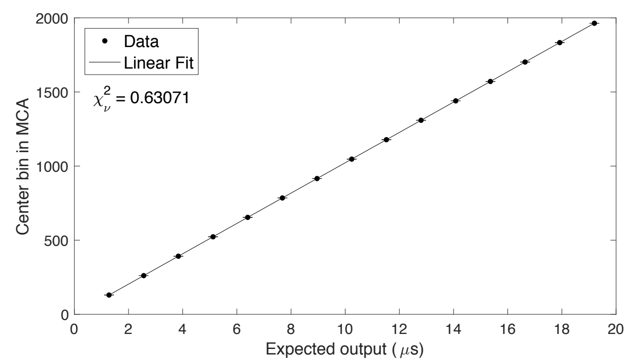

FIG. 2: This is the time interval sent to the TAC plotted against the MCA channel at which the corresponding signal appeared. This data is from the time calibration for the lifetime measurement TAC settings. We see there is a linear relationship between time interval and MCA channel ($\chi^2_\nu = 0.111$).

3.1. Measuring Muon Mean Speed

We took data on the time it takes for muons to travel from the top to the bottom scintillation paddle with the paddles in four positions: 0.52 ± 0.01 m, 1.00 ± 0.01 m, 1.50 ± 0.01 m, and 2.51 ± 0.01 m apart. We collected at least 3000 counts at each distance, with integration times ranging from approximately 17 minutes for the smallest paddle separation to 24 hours for the largest separation.

The data from the paddles being separated by 0.52 ± 0.01 m is shown in Figure 3. To find the mean time interval measured by the TAC, we fit a gaussian to the distribution and use the center as the mean time. Note, however, that the time interval measured by the TAC is not the time interval for the muon to travel between the paddles; we have not accounted for the effects of the delay line, or for any other systematic errors that could arise from, for example, the PMTs having different response times to signals.

We can express the time interval measured by the TAC for the i^{th} muon, t_i, to be

$$t_i = t_0 + \frac{d_i}{v_i} \tag{1}$$

where t_0 is a constant accounting for the delay line and instrumental affects, d_i is the distance traveled, and v_i is the muon's velocity [1]. We rearrange equation (1) and take the average of these values for each paddle separation and find that

$$D = vT - vt_0 \tag{2}$$

where T is the average of the t_is, D is the average distance traveled, and v is the mean speed of the muons. We see from equation (2) that if we plot the mean distance traveled against the mean TAC time interval for each paddle separation, the slope will be equal to v, the mean

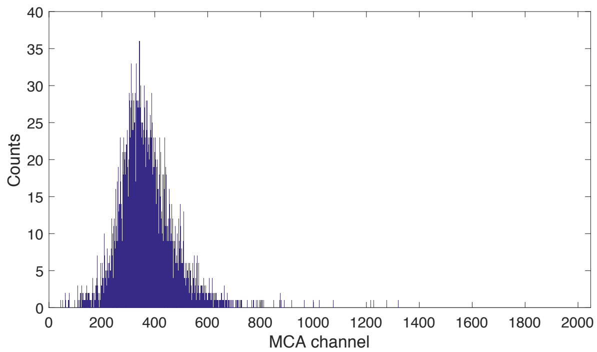

FIG. 3: This is the distribution of counts in each MCA channel for speed data collection with the paddles separated by 0.52 ± 1 m. We fit a gaussian to the data and use the center of the gaussian for the mean time it takes for the muons the travel between the paddles.

muon speed. The intercept, vt_0 will be a constant of the apparatus about which we need not worry.

We next must find the mean distance traveled by the muons for each paddle separation. This calculation is complicated by the fact that muons can come in at an angle, thus traveling a distance slightly larger than the paddle separation. The intensity of muons that reach sea level at a particular zenith angle ϕ is

$$I(\phi) = I_V \cos^2(\phi) \tag{3}$$

where $I_V = 0.83 \times 10^{-2}$ cm^{-2}s^{-1-2}str^{-1} is a constant [1].

To find the mean distance traveled, we wrote a Monte Carlo simulation that generates a muon at a random location on the upper paddle. The muon has an incoming zenith angle generated by a probability distribution proportional to the muon intensity in equation (3). Using basic geometry, we determine whether the simulated muon lands on the lower paddle; if it does, the distance traveled is recorded. For every paddle separation, a simulation of 10,000 muons was run. We took the average of the distances traveled by the simulated muons to be the mean distance traveled by muons in the lab. The variance in distance traveled was included as statistical error.

According to equation (2), a plot of mean distance vs. mean TAC time interval will have a slope equal to the muon velocity. This plot is shown in Figure 4. Unfortunately, we find the slope to be $(4.955 \pm 0.066) \times 10^8$ m/s $= (1.65 \pm 0.02)c$. It is, of course, not physical for muons to be traveling faster than the speed of light. We also see that our data does not fit the linear function very well, having a χ^2_ν value of 107. This suggests that there may be have been systematic effects altering our results for some of the data points.

The largest difference in the procedure used to take the data was that the data for paddle separation of 2.51 m was taken on a different day, potentially with different BNC cables. If we exclude this data point from our fit, as shown by the solid line in Figure 4, we find the muon speed to be $(3.36 \pm 0.13) \times 10^8$ m/s $= (1.12 \pm 0.04)c$. This result is still unphysical, but it is closer to a reasonable value for muon speed. Furthermore, the linear fit has a much improved χ^2_ν value of 21.6. It is questionable, however, whether we have sufficient motivation to exclude the 2.51 m data point as an outlier; future experiments should explore whether effects like weather and switching out BNC cables can affect the data as it appears to here.

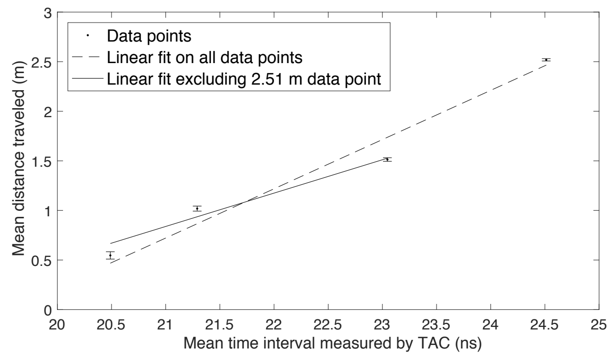

FIG. 4: This plots the mean distance traveled by muons against the mean time interval measured by the TAC. The slope of this line should be the mean speed of the muons. When fitting a line to all the data points, we find the muon speed to be $(4.955 \pm 0.066) \times 10^8$ m/s ($\chi^2_\nu = 107$). If we exclude the 2.51 m data point, which was taken on a different day under potentially different conditions, the muon speed is $(3.35 \pm 0.14) \times 10^8$ m/s ($\chi^2_\nu = 21.6$).

3.2. Measuring Muon Lifetime

Decay of muons, like decay of radioactive nuclei, is random and follows a Poisson distribution. The probability of observing n events in time t is therefore

$$P(n,t) = \frac{(rt)^n e^{-rt}}{n!} \tag{4}$$

where r is the average event rate. From this, we see that the probability of no events occurring in time t and one event occurring at exactly time t is

$$P(0,t) \cdot P(1,0) = r \cdot e^{-rt}. \tag{5}$$

This experiment measures the time between a muon coming to rest and decaying. The time between these events will follow the distribution in equation (5), which describes the distribution for a decay event occurring precisely at time t [2]. We therefore expect the MCA histogram for our lifetime measurements to fit an exponential decay. The lifetime, $\tau = 1/r$, can be calculated from the exponential fit of the histogram.

Our MCA data is shown in Figure 5, and it appears to be an exponential distribution as expected. There is some noise due to so-called "accidental stops" where, for example, a muon enters the scintillator while another muon still present in the scintillator has not yet decayed; in this case the TAC would measure the interval between muons arriving rather than the time before they decay. The distribution of these random events will also be an exponential, but for the TAC's range it can be approximated as a small constant. We account for this in our analysis by fitting the MCA data in Figure 5 to a function of the form $f = ae^{-t/\tau} + b$, where τ is the lifetime. When we perform this fit, we find that $\tau = 2.240 \pm 0.003$ μs.

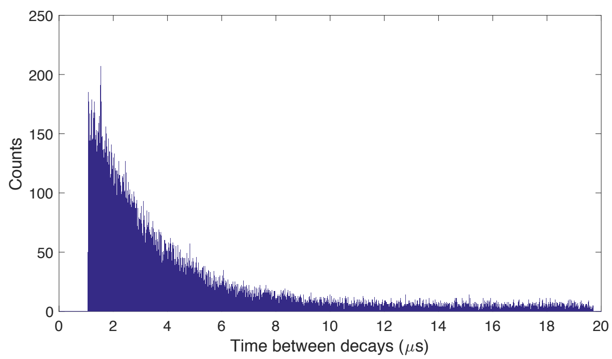

FIG. 5: This is the data collected for the muon lifetime measurement. We fit an exponential to this plot and found the lifetime of the muon to be $\tau = 2.240 \pm 0.003$ μs.

3.3. Sources of Systematic Error

The sources of systematic error are listed in Table I, along with their their sizes as a percent of the final values for the muon speed and lifetime. Uncertainty from the width of peaks in the time calibration, as discussed near the beginning of Section III, contributed a very small amount to uncertainty in both speed and lifetime. The largest contributor to error in the muon speed was uncertainty in measuring the separation between the scintillation paddles.

In the lifetime measurement, we also considered the effects of removing some outliers in the data: in Figure 5 there is a peak several channels wide that is higher than the rest of the noise in the exponential. This might come from an apparatus effect, so we ignored it and repeated the exponential fit to find τ. The lifetime changed by an extremely small amount, so this contributes nearly nothing to the systematic error. The last source considered is descibed in detail in Melissinos' book *Experiments in Modern Physics* [2]. Sometimes negative muons will be captured by carbon atoms in the plastic scintillator, causing the measured lifetime to be shorter than the actual lifetime. Melissinos suggests adding an error allowing the calculated value to increase by 4%.

TABLE I: Here are all the sources of systematic error accounted for in the calculation of the muon mean speed and lifetime. Sources of error are listed along with the size of the error they contributed, expressed as a percent of the final value.

Source of error	**Percent**
Mean speed:	
Uncertainty from time calibration	$\pm$ 0.08 %
Measurement uncertainty in paddle separation	$\pm$ 0.23 %
Measurement uncertainty in paddle dimensions	$\pm$ 0.10 %
Lifetime:	
Uncertainty from time calibration	$\pm \ll 1$ %
Ignoring or including the large peak in histogram	$\pm \ll 1$ %
Side process of negative muons being captured by scintillator atoms	+ 4 %

4. RESULTS

We found the mean speed of a muon to be $(4.955 \pm 0.066_{\text{stat}} \pm 0.020_{\text{sys}}) \times 10^8$ m/s and the lifetime to be $2.240 \pm 0.003_{\text{stat}} + 0.089_{\text{sys}}$ μs. Unfortunately, our result for the speed of the muon is unphysical, so we cannot perform the calculations suggested by special relativity to find the lifetime of a muon in motion. We have certainly shown that muons travel near the speed of light, and our value for the lifetime is very close to the accepted value of $(2.19703 \pm 0.00004) \times 10^{-6}$ s with a 2% error [2].

Future experiments should further explore the systematics of the muon speed experiment and strive to arrive at a more accurate measure for the speed. From this value, we will be able to examine whether using the equations of special relativity allow the muons to reach sea level from 15 km in the atmosphere before decaying.

Acknowledgments

A huge thank you goes to Carissa Skye, who has been a terrific partner to work with. I would also like to thank Prof. Joseph Formaggio, Dr. Sean Robinson, Kay Lowden, Spencer Tomarken, and Amelia Smith of the Junior Laboratory staff for all their patience and help.

[1] J. L. Staff, *The Speed and Decay of Cosmic-Ray Muons*, MIT Department of Physics (2014), lab manual.

[2] A. C. Melissinos and J. Napolitano, *Experiments in Modern Physics*, 2nd Ed. (Academic Press, 2003).

[3] T. Slatyer, *Time Dilation* (Lecture for MIT course 8.033: Relativity, 2015).