Landauer Experiment
Recent results and discussions 2

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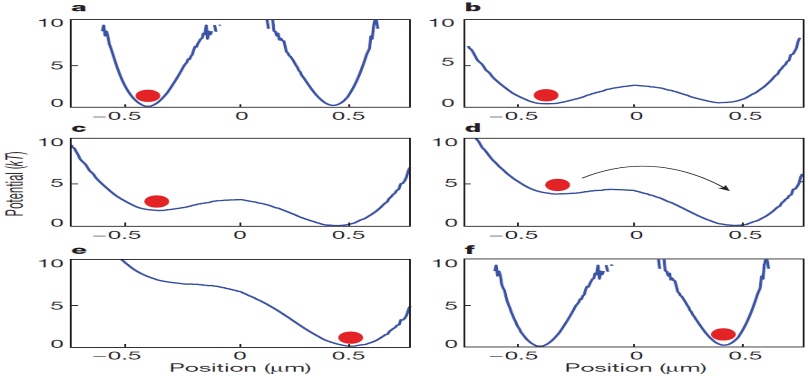
Outline

What has been done in Nature paper-1



What has been done in Nature paper-2

For fig. a,b and f the plots are found using experimental data as:

- Well potential calculated using pdf is used via.

$$U_0(x, I_L) = -K_B T \; ln[(P(x, I_L)]$$

- According to paper, the measured $U_0(x, I_L)$ is plotted in fig.a,b,f and can be fitted by 8^{th} order polynomial as :

$$U_0(x, I_L) = \sum_{n=0}^{8} u_n(I_L, d_f)x^n$$

where d_f is the distance between the two points over which laser is switched (1450 nm here)

What has been done in Nature paper-3

For fig. c,d and e the plots are found using following calculations :

- Total time of erasure = Time of stage motion with increasing velocity = τ
- Amplitude of viscous force is increased linearly during time τ : $F(t) = \pm F_{max} t/\tau$
- Intermediate plots during transfer is calculated at three different values of time t :

$$U(x,t) = U_0(x,l_L) - F_{max}(t/\tau).x$$

- *I suspect, the plot in fig.b is also an* 8^{th} *order fit, since the contours in the plots from b to e are the same*

Our approach-1

Points of discussion last time :

- How did they obtain good curve for saddle points ?
- Their is not only a polynomial fit, but involves calculation using the 8^{th} order fitted curve $U_0(x, I_L)$ as :

$$U(x,t) = U_0(x, I_L) - F(t)$$

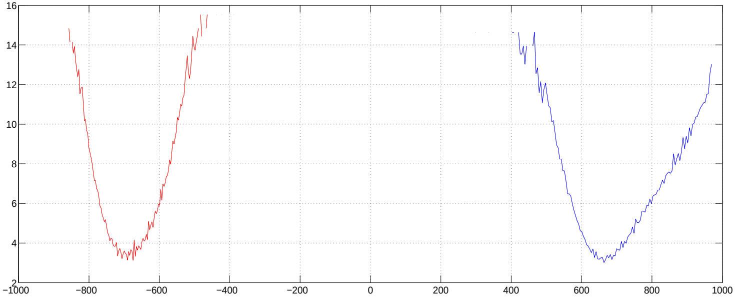
- This is why they obtain such good curve at the saddle point

Our approach to match their plots :

- Obtain fig. a and f using the photodiode data as they have done
- Obtain the fig. d **which is the exact point of transfer** using actual data and fitted curve

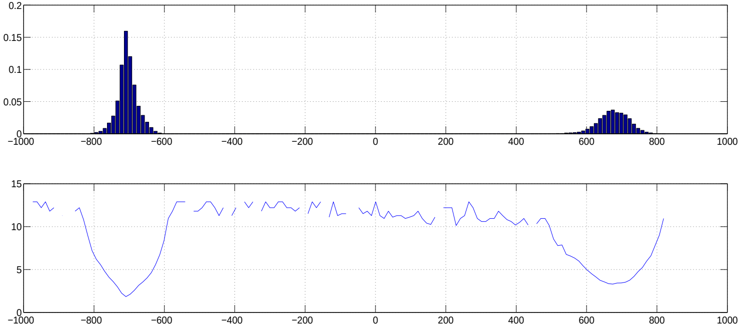
Our approach-2

Replicated fig.a and f with R=3.2 $K_B T$, SD = 48.3 nm, L = 3.4 $K_B T$, SD = 37.1 nm



Our approach-3

Replicated fig.d *at exact point of transfer* with blinks L=7 ,R=1 ; L = 2 K_BT,R = 3.3 K_BT, Height = 12.5 K_BT



Our approach-4

To obtain good curve at saddle points, following is done :

- Several datasets depicting the exact points of transfer are obtained
- Each individual potential well calculated using pdf is used via.

$$U_0(x, l_L) = -K_B T \; ln[(P(x, l_L)]$$

- Several datasets averaged to depict actual wells
- Polynomial fitted for comparison

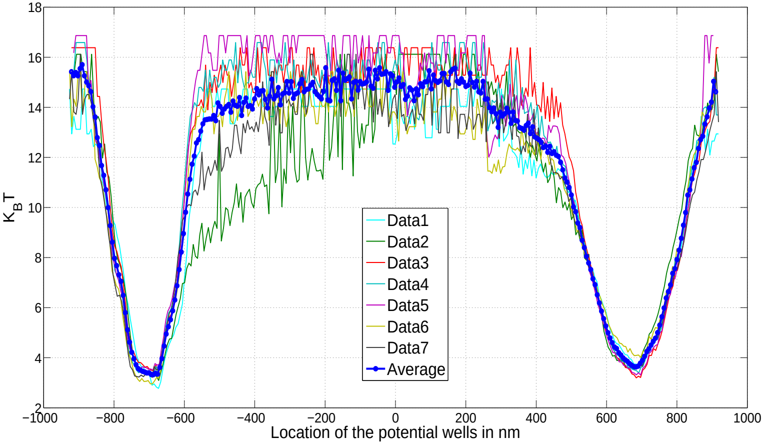
Outline

Multiple Transfers - Several examples

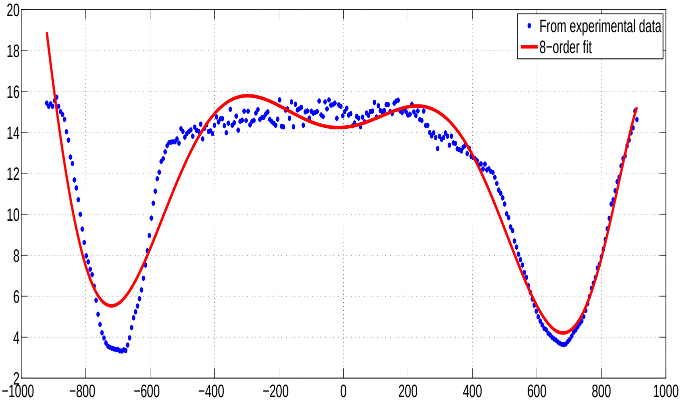
- Trap bead at +700; potential well formed at +700
- Total 12 blinks fixed : for equal potential, 6 blinks for each well, each blink for 20 μs
- R$\rightarrow$L transition seen from 8,4 onwards i.e. 8,4...9,3...10,2 and 11,1
- To see the minimum tilt needed for this transition, 8,4 is fixed
- Modulate the on times as :

Total	Left	Right
12	6	6
12	7	5
12	8	4
12	6	6

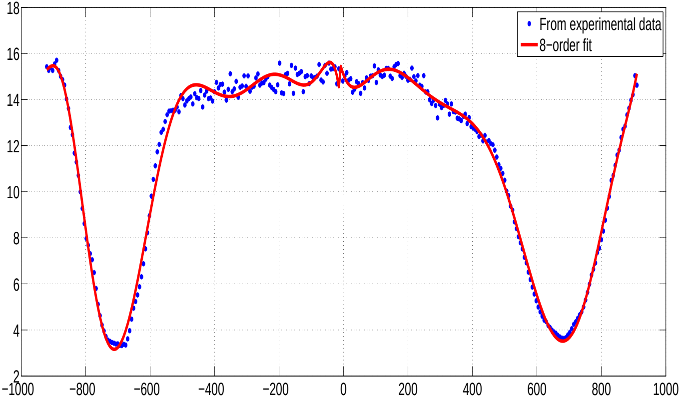
Averaging 7 instances of R→L transfer at 8,4 blinks



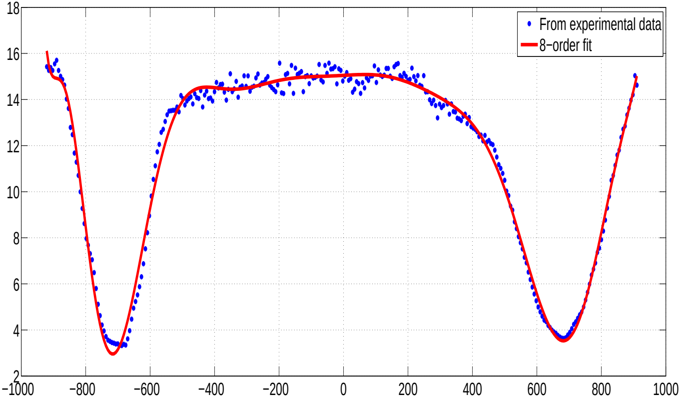
Polynomial fit- 8 th order, both wells together



Polynomial fit- 8 th order,R and L separately



Polynomial fit- 15 th order,both wells together



Inference

- Averaging multiple transfers gives good graphical representation of the 'saddle point'
- Fitting polynomial to both wells together needs higher order fit (at least 15^{th} order here)
- Separately fitting R and L wells can be accomplished with lower order polynomials

Outline

Idea

Directly using calculated work done :

- Calculate work done $W = \int_{t_1}^{t_2} F.dx$; Heat dissipated $Q = -\int_{t_1}^{t_2} F.dx$
- $< Q >$ over several iterations should approach Landauer's limit $K_B Tln(2)$

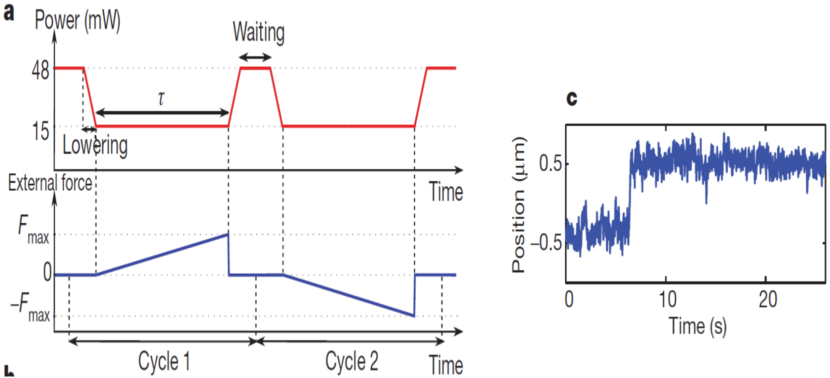
Using Jarzynski's equality :

- For a quasi-static process $< W > \approx \Delta F$
- Using $< e^{-\beta W} > = \ e^{-\beta \Delta F}$, find ΔF, the free energy difference
- The value $\Delta F_{eff} = -ln(< e^{-\beta W} >)$ is always close to the Landauer's limit $K_B Tln(2)$, *independent of the maximal force or procedure duration**

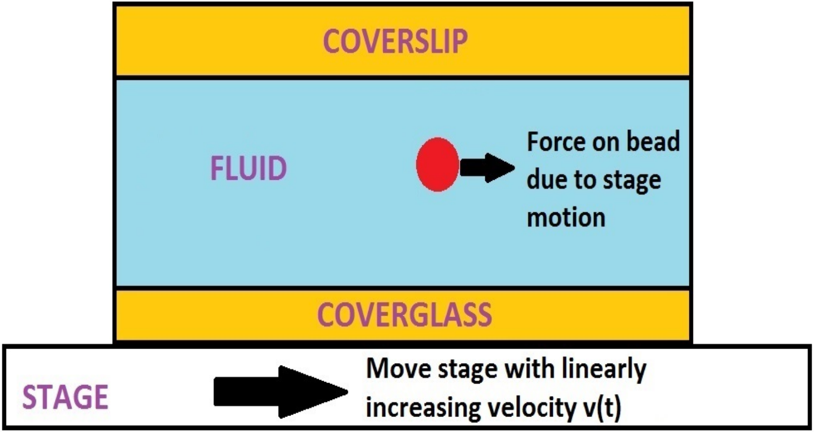
Detailed Jarzynski equality applied to logically irreversible procedure, A.Berut et al, EPL, 2013

Outline

Work done in the Nature paper



Work done in the Nature paper



Work done in the Nature paper

- Stage velocity v, viscous force on the bead $F = -\gamma v$
- During erasure, magnitude of force increased linearly during time τ as $F(t) = \pm F_{max}.(t/\tau)$
- Heat dissipated during tilt :

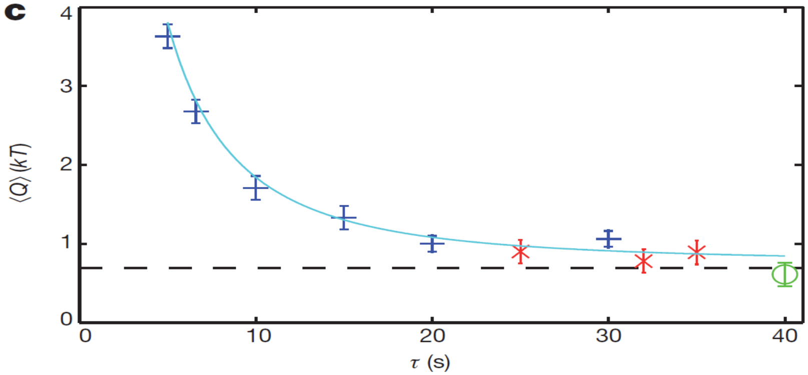
$$Q = -\int_0^{\tau_{cycle}} F(t)\dot{x}(t)\, dt = \pm \int_0^{\tau} F_{max}(t/\tau)\dot{x}(t)\, dt$$

- Velocity is calculated using the discretization

$$\dot{x}(t + \Delta t/2) \approx [x(t + \Delta t) - x(t)]/\Delta t$$

- For ideal quasi static erasure process ($\tau \to \infty$) the dissipated heat equals Landauer's value

Work done in the Nature paper



plus sign r≥ 90% ; cross sign r≥ 85% ; circle sign r≥ 75%
where r is the success rate of the erasure protocol

Outline

Free energy using Jarzynski

When $r \to 1$ we have $< e^{-\beta W} >= 1/2$, then $\Delta F_{eff} \approx K_B T ln(2)$

All we need is the **work done**

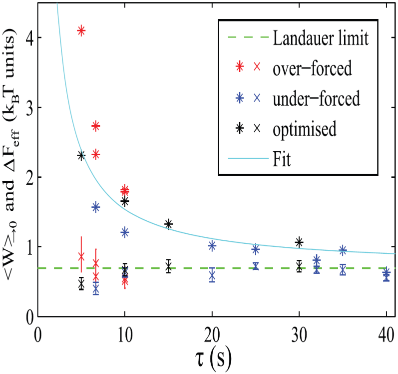


Fig. 3: (Colour on-line) Mean of the work (∗) and effective free energy difference (×) for different procedures. The over-forced procedures (red) have a proportion of success $P_S \sim 95\%$, the optimized procedures (black) have $P_S > 91\%$, the under-forced procedures have $P_S > 83\%$ (except the last point, that has $P_S \approx 75\%$). The fit (blue line) is $\langle W(\tau)\rangle_{\to 0} = k_B T \ln(2) + B/\tau$ with B a constant. For reader's convenience the error-bars on the mean work (∗) are not shown but were estimated to be $\pm 0.15 k_B T$.

Inference

1. Work done calculated using $\int F.dx$ and averaged over several cycles should approach Landauer's limit
2. This work done is the **work done by external agency on the bead during the tilting of the wells**
3. The free energy difference calculated using Jarzynski equality converges to the limit independent of the value of the maximal force or procedure duration, so **from experimental viewpoint, Jarzynski equality enables us to perform experiments over a shorter duration**
4. *Bottom-line* : We need to calculate work done in our case where tilting of wells is done by modulating the dwell times of the laser

Outline

Using drag force on the bead

- Force on bead due to viscous drag

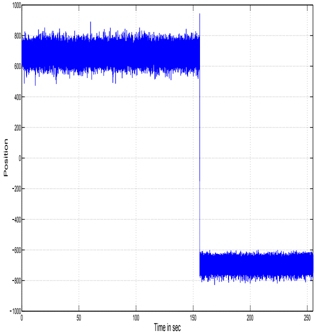
$$F_{drag} = -\gamma v = -6\pi\eta R v$$

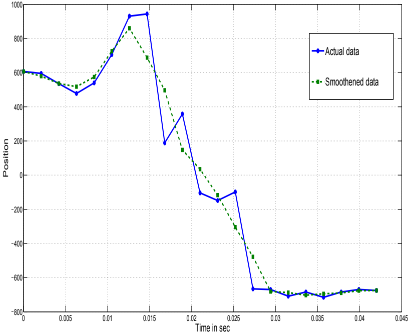
  where η is viscosity coefficient for water, R is bead radius and v is relative velocity between bead and medium
- Work done = $\int_0^{\tau} F_{drag} dx = \int_0^{\tau} -(\gamma v) dx$
- Two calculations are done :(1) Using unfiltered data and (2) Using filtered data with a 3-point MA filter*
- Question : How to decide τ? We first considered total time, then time just around the transfer

* In the nature paper, they've processed the images to increase precision before using them

Using drag force on the bead

Illustration on sample dataset





Calculations-1

$\eta = 8.9 \times 10^{-4}$Pa.s for distilled water at 25°C
R = 500 nm and v = velocity of bead = dx/dt
$dt = 0.002$ sec $(500H_z)$

Dataset	Total data (K_BT)		Data around transfer (K_BT)	
	Unfiltered	Filtered	Unfiltered	Filtered
1	-111850	-17956	-2963.2	-814.87
2	-204420	-33144	-1502	-568.5
3	-169840	-32520	-12470	-382.8
4	-265370	-38830	-689.3	-251.93
5	-352990	-70021	-2351.3	-600.51
6	-112830	-19527	-34864	-590.9
7	-161950	-29409	-168937	-579.40

Clearly these are very high values !

Analysis

Issue I:

- The work done by drag force is dissipative
- Motion of bead in either + or - direction will add to the dissipative work
- Also, the more data points are considered, higher is be the work done calculated by this method
- Bead position detected has some noisy component as well, which gets captured. It is clearly evident that filtering helps, but still not significantly enough

Analysis

Issue II:

- Coefficient of friction used by Nature Paper is $\gamma = 1.89 \times 10^{-10}$Ns/m for 2$\mu m$ diameter bead, which translates to $\eta = 1.003 \times 10^{-5}$Pa.s for our 1$\mu m$ bead
- It is about **an order lower** than the η we use, which is the standard coefficient for distilled water
- To see its effect, I recalculated the work done using this lower η

Calculations-2

$\eta = 1.003 \times 10^{-5}$Pa.s for distilled water at 25°C
R = 500 *nm* and v = velocity of bead = dx/dt

Dataset	Total data ($K_B T$)		Data around transfer ($K_B T$)	
	Unfiltered	Filtered	Unfiltered	Filtered
1	-1253.5	-201.22	-33.2	-9.131
2	-2290.8	-371.42	-16.83	-6.37
3	-1903.3	-364.42	-13.97	-4.289
4	-2301.5	-435.13	-7.724	-2.823
5	-3955.7	-784.68	-26.416	-6.729
6	-1264.4	-218.82	-39.06	-6.622
7	-1814.8	-329.56	-18.93	-6.4931

Analysis

- Clearly, the lower value of η has significant effect
- When combined with filtering and using data around transfer, work done values seem practical

Several questions arise, which have been listed next

Questions-1

1. Is this approach using drag forces (dissipative work) correct ? In that case, it makes sense to **use only the data around transfer.**
2. Perhaps truncating the data around transfer might help even more ?*
3. For calculating the drag force F_{drag} we use $F_{drag} = -\gamma v$ where v is the velocity of bead.
   According to Stokes law, v is the relative velocity between fluid and bead; so our assumption here is that the fluid is stationary(as we're not moving the stage). Is that correct ?

Refer to rightmost column of table in the calculations-2 slide

Questions-2

1. The coefficient of viscosity η used in Nature paper (bidistilled water) is an order lower than ours (de-ionized water).*Deionization produces a high purity water that is generally similar to distilled water*. Which to use ?
2. Filtering (3-point MA filter, the smallest one possible) has significant effect. How to decide which one to use ?
3. In our case where tilting is done due to different dwell times of blinking, what is the external parameter ?
4. How do we calculate **work done on the bead** due to the variation of this external parameter ?

Outline

Using total force on the bead

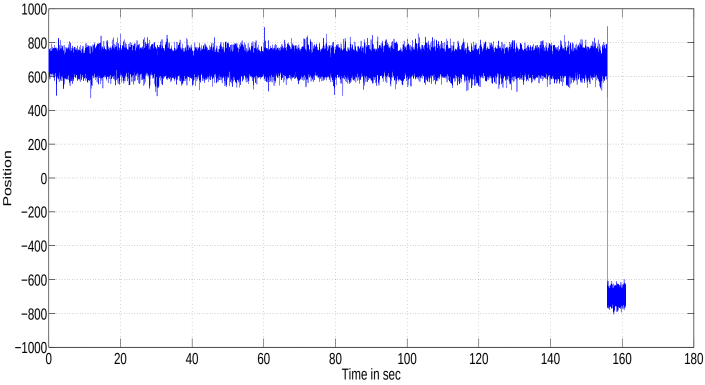
- Total force on bead at given instant:

$$F_{net} = m\ddot{x} = \frac{-\partial U}{\partial x} - \gamma v + noise$$

- For given polystyrene bead with diameter $1\mu m$ bead density =$1.8189 \times 10^{12} beads/gm$; so individual bead mass = $5.4978 \times 10^{-16} kg$
- Work done = $\int_0^\tau F_{net} dx$ where τ is *time until just after the bead transfer* and $F_{net} = m\ddot{x}$
- Work is calculated using unfiltered and MA filtered(window=3) bead position data

Using total force on the bead

Illustration on sample dataset. Transfer at 156 s, data till $\tau =$161 s



Using total force $\rightarrow \tau$ just after transfer

τ is till just after transfer

Dataset	Work Done ($K_B T$)		Well Potential ($K_B T$)		Δ U
	Unfiltered	Filtered	U_2(Left well)	U_1(Right well)	$U_2 - U_1$ ($K_B T$)
1	-0.767	-0.075	2.8	3.68	-0.83
2	-3.21	-0.289	3.2	3.55	-0.35
3	-4.402	-0.4246	3.5	3.25	0.25
4	-4.4706	-0.4336	3.35	3.45	-0.10
5	-7.1814	-0.7003	3.55	3.35	0.20
6	-1.0911	-0.1037	3.0	4.05	-1.05
7	-2.855	-0.269	3.25	3.65	-0.40

Using total force $\rightarrow \tau$ is entire dataset

τ is entire dataset

Dataset	Work Done (K_BT)		Well Potential (K_BT)		Δ U
	Unfiltered	Filtered	U_2(Left well)	U_1(Right well)	$U_2 - U_1$ (K_BT)
1	-1.3901	-0.1303	2.8	3.68	-0.83
2	-7.8841	-0.6667	3.2	3.55	-0.35
3	-7.015	-0.6507	3.5	3.25	0.25
4	-8.5449	-0.7862	3.35	3.45	-0.10
5	-11.6882	-1.091	3.55	3.35	0.20
6	-2.8291	-0.2251	3.0	4.05	-1.05
7	-6.034	-0.5412	3.25	3.65	-0.40

Mean heat dissipated

Using work done : Heat dissipated $Q = -\int_0^{\tau} Fdx$

	Unfiltered	Filtered
Data around transfer	3.4253	0.3279
Total data	6.4836	0.5845

Using Jarzynski: $\Delta F_{eff} = -ln < e^{-\beta W} >$

	Unfiltered	Filtered
Data around transfer	2.0265	0.3087
Total data	3.1107	0.5386