Power spectral density approximations of longitudinal road profiles

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Abstract: The power spectral density (PSD) representation of road profiles can be used both to assess the road roughness, and as an input to vehicle dynamics. The PSD is often approximated with a simple function, using only a few parameters. The present paper presents a literature survey of such PSD approximations, and tests four approximations on longitudinal road profile data from the entire Swedish state road network.

Keywords: literature survey; longitudinal road profile; power spectrum density; spectral analysis; Swedish road network.

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1 Introduction

The more or less general availability of high-speed profilometers has given dramatically increased possibilities to analyse imperfections in road profiles and surfaces. The most common reason for the collection of longitudinal road profile data (called just road profiles in the following) is to form a basis for the analysis of road evenness that a rational pavement management requires. To make such an analysis possible the very large amount of data made available by modern assessment techniques must be reduced to manageable, and more informative, dimensions. This is normally done by the continuous computation of some kind of evenness index. In Sweden, the index normally used is the International Roughness Index (IRI). The IRI is adequately related to the comfort experience in a private car but does not allow much room for analysis of road surface dependent effects experienced by the road user. Mathematical representations of road profiles can also be used as input to mathematical vehicle models intended for studies of vehicle dynamics. These types of analyses require a more thorough knowledge of the distribution of frequencies with accompanying amplitudes. One such description, much used for road profiles, is the power spectral density (PSD). The PSD is often approximated with a simple function, using only a few parameters. This PSD approximation can be used both as a concise description of the road roughness level, and used either directly in vehicle dynamics or as a basis for road profile generation.

The present paper presents a literature survey of PSD approximations. Four approximations are applied on a large amount of road profile data from routine measurements carried out annually by the Swedish Road Administration (SRA). Routine measurements of the state road network have been carried out since 1987, and road profiles have been stored since 2001. In the present paper the right wheel path road profiles from 2001 to 2003 have been used. It should be noted that SRA has had the policy to measure its entire road network, i.e. not a subset considered to be statistically representative.

2 Use of power spectral density for road profiles

No information on how to compute or interpret the PSD will be given in the present paper. The mathematical theory is described in detail by e.g. Bendat and Piersol (1986), and the computational implementation by e.g. Press et al. (1992). Formulae for the approximation are presented with a uniform notation, taken from the ISO 8608 standard (ISO, 1995-09-01). In formulae below G_d is the displacement PSD, C is the general roughness parameter, n is the wavelength, and w a dimensionless parameter.

The earliest motivation for performing PSD calculations on road or runway profiles was their usefulness in vehicle dynamics. However, many researchers noted that the PSD would also be a convenient way to classify road roughness and deterioration, even though '[. . .] these points do not refer directly to our theory, they do suggest ways in which our results can be put to practical use' (Cote et al., 1966). One of the first uses of spectral analysis for roughness in civil engineering is presented in an introductory paper by Houbolt et al. (1955), and later extended by Houbolt (1962). Already in 1955 the straight-line approximation $G_d(n) = Cn^{-2}$ was proposed, and the authors conclude that '[t]he use of spectral techniques seems to be a rather concise way of presenting the characteristics of runway roughness'. In Cote et al. (1966) a more detailed description of the PSD concept is given 'because of the newness of this approach in this particular field'. The concept of roughness limits in form of straight lines in the PSD graph is introduced.

PSDs from elevation profiles for 'typical plowed ground' was considered by Bogdanoff and Kozin (1961) and Kozin and Bogdanoff (1961) for the study of off-road vehicles. A more thorough description of the off-road vehicle study is given in a series of papers from 1965–1966 (Bogdanoff et al., 1965; Cote et al., 1965, 1966; Kozin et al., 1966). In the conclusions it is stated that 'p.s.d. estimates from a variety of ground types have a very similar form' and 'we take hope in the possibility that ground p.s.d.'s may be described in terms of a few readily estimated parameters [. . .]'.

Kanesige (1960) proposed to derive the road roughness spectra from the vehicle vibration spectral density and noted that the spectra can be approximated as

$G_d(n) = Cn^{-w}$. The same idea was used by Quinn and Zable (1966) in one of the very first papers describing the use of PSD primarily for highways. The signal processing is somewhat dated, but the general concept is competently explained. Quinn and Hagen (1967) described some problems related to the measurement and computation of the PSD, and the need for a standardised process to compute the PSD is expressed.

Macaulay (1963) also proposes the $G_d(n) = Cn^{-w}$ approximation, but adds that w should be 2.5 for wavelengths between about 100 and 3 m, and 1.5 for shorter wave-lengths.

Braun (1966) presents an early wide-ranging use of the PSD to evaluate road roughness. More than 30 test sections had their road profiles measured, and PSD computed. A straight line fit of the form $G_d(n) = Cn^{-w}$ was used. Braun also presents a comprehensive review of the use of PSD in road research to that date (which was limited to a few short test sections), and even reproduces much of the others' results.

Soon after, La Barre et al. (1969) presents a similarly sized test. Fifty four test sections in Britain, Belgium, France and Germany were measured, and the PSD were computed. No approximation was used but the PSDs were detrended with a Cn^{-2} line.

With experiences from his work in road surface modelling Dodds proposed a way, by authoring the BSI standardisation proposal (BSI, 1972), to classify road roughness. (Although the BSI standardisation proposal document does not have an explicit author, Dodds puts himself as the author in the reference list in his 1973 paper (Dodds and Robson, 1973), which indeed seems likely.) Agreement was finally reached in 1995 with the ISO 8608 standard (ISO, 1995-09-01). These will be covered in detail below.

3 Requirements for correct PSD

In order for a spectral analysis estimate to be valid certain requirements need to be fulfilled. Preferably, the road profile should be a member of a stationary random process. (A stationary random process can be broadly defined as a signal whose statistical properties do not vary with time, or rather length for road profiles. See Bendat and Piersol (1986) for a detailed explanation.) Whether road profiles, or more generally road surfaces, can be regarded as a random process has been discussed in a number of papers starting with the somewhat classical paper by Dodds and Robson (1973). In this paper it is stated that '[i]t is shown that typical road surfaces may be considered as realisations of homogeneous and isotropic two-dimensional Gaussian random processes'. Further on it is said that 'occasional large irregularities' like potholes must first be removed for this to be true.

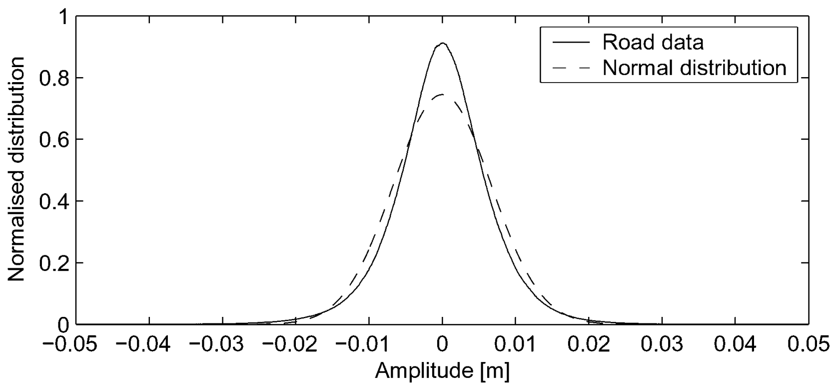
When the road profile is 'homogeneous' and 'isotropic' a complete road surface can be modelled with a single road profile PSD as input. The concept, applications and implications of homogeneity and isotropy are analysed and explained in great detail by Kamash and Robson (1977, 1978). An improved model has been presented by Heath (1989).

The statistical properties of road profiles have been investigated by Bruscella et al. (1999), Heath (1988) and Rouillard et al. (1996). The conclusion from these papers is that once large transients in the road profile have been removed the distribution is nearly Gaussian. However, it can be shown that PSD estimates are quite robust even for non-Gaussian distributions (see e.g. Blackman and Tukey, 1958).

Even though the profile spectra might not always be a statistically correct description of the road (due to a lack of randomness) they can nevertheless be used as an average classification of the roughness. As Walker and Hudson (1971) noted, the stationarity argument is valid not only for the PSD but for any description of road roughness.

It is beyond the scope of the present paper to present a detailed statistical analysis of the road profiles used in the analysis below. The amplitude distribution of high-pass filtered profiles given in Figure 1 follows the Gaussian distribution with the same mean and variance fairly well. The somewhat 'tighter' curve has also been presented by Bruscella et al. (1999) and by Heath (1988).

Figure 1 Normalised amplitude distribution for Swedish road network elevation profiles



4 PSD approximations

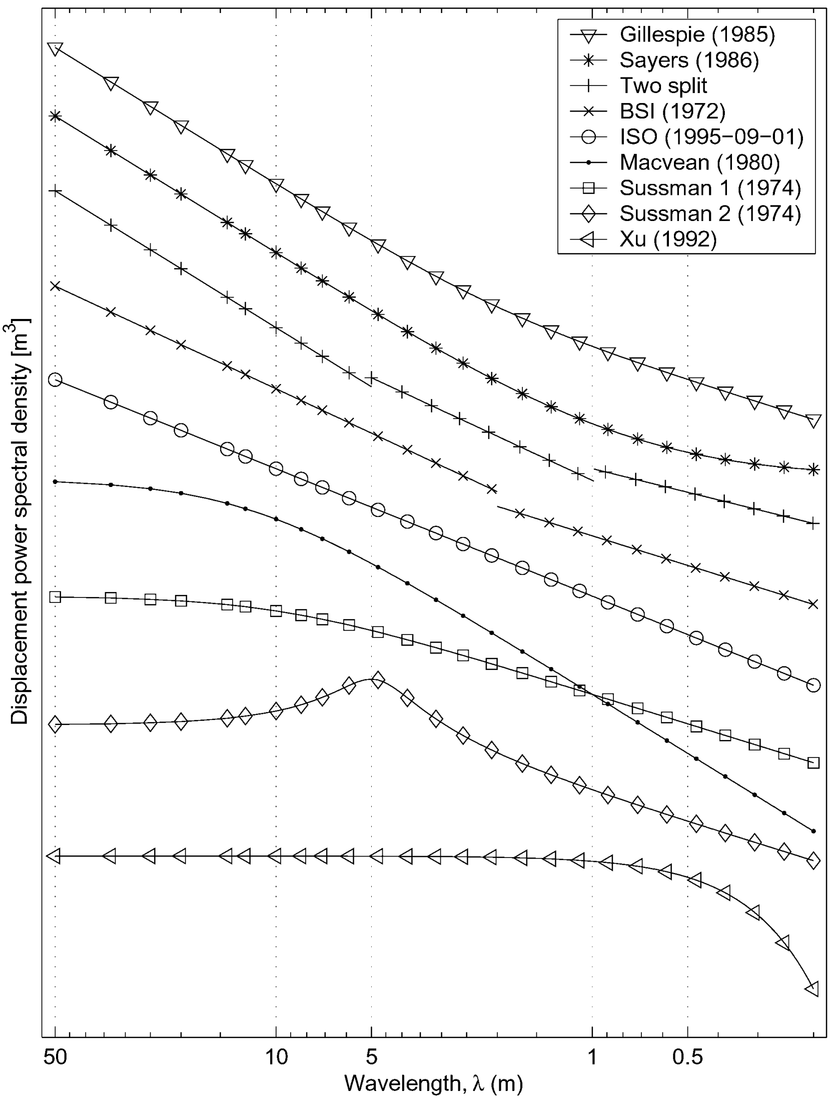
All approximations to the PSD found in the literature will be presented. The fit of the approximations on the entire Swedish road network is evaluated with the residual from a least square minimisation. The data fit, and hence the residual presented below, is performed on the logarithm of the PSD.

The general behaviour of all approximations is illustrated in Figure 2, with the main reference given in the legend. All approximations are also given in Table 1.

4.1 ISO 8608

The straight line approximation $G_d(n) = Cn^{-w}$ has been proposed a number of times in the literature. To begin with the exponent w was fixed to 2 (Houbolt et al., 1955), but the more general formulation with a varying exponent was soon proposed (Kanesige, 1960; Macaulay, 1963). With the straight line fit the complete road can be described by only two numbers: the general roughness C and the wavelength distribution w.

Figure 2 General behaviour of the PSD approximations



According to the standard, the PSD should first be smoothed in octave bands from the lowest calculated frequency up to the centre frequency of 0.0312 cycles/m, then third octave bands to a centre frequency of 0.25 cycles/m; and twelfth octave bands up to the highest calculated frequency. This smoothing procedure is not needed for the straight line approximation, and for consistency with the other approximations only results from the unsmoothed fit are reported below.

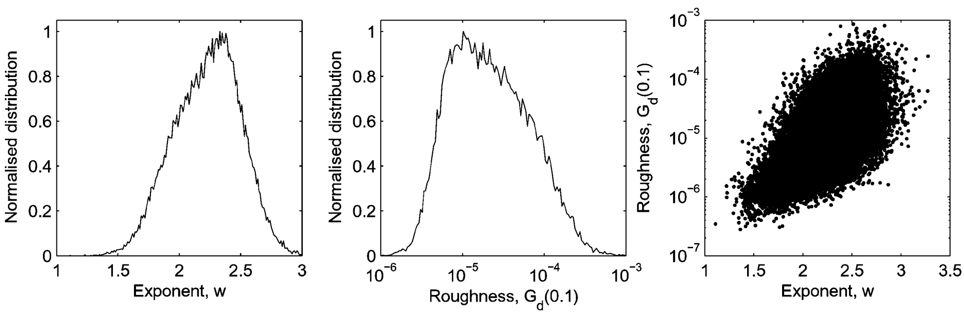
The normalised distributions of the PSD slope w, the roughness level G_d and their correlation on the entire Swedish road network are given in Figure 3. The correlation coefficient between w and $\log(G_d)$ is 0.67.

Table 1 Approximations of road profile roughness spectra

Name	*PSD approximation*	*Wavenumber*
ISO (1995-09-01)	$G_d(n) = Cn^{-w}$	$0 \le n \le \infty$
BSI (1972)	$G_d(n) = \begin{cases} Cn^{-w_1} \\ Cn^{-w_2} \end{cases}$	$0 \le n \le n_0$ $n_0 \le n \le \infty$
Two Split	$G_d(n) = \begin{cases} Cn^{-w_1} \\ Cn^{-w_2} \\ Cn^{-w_3} \end{cases}$	$0 \le n \le n_1$ $n_1 \le n \le n_2$ $n_2 \le n \le \infty$
Sayers (1986)	$G_d(n) = C_1/n^4 + C_2/n^2 + C_3$	$0 \le n \le \infty$
Gillespie (1985)	$G_d(n) = C\left(1 + (0.066/n)^2\right)/n^2$	$0 \le n \le \infty$
Marcondes et al. (1991)	$G_d(n) = \begin{cases} C_1 \exp(-kn^p) \\ C_2(n - n_0)^q \end{cases}$	$0 \le n \le n_0$ $n_0 \le n \le \infty$
Sussman (1974)	$G_d(n) = \dfrac{C}{\alpha^2 + n^2}$	$0 \le n \le \infty$
Macvean (1980)	$G_d(n) = \dfrac{C}{(\alpha^2 + n^2)^2}$	$0 \le n \le \infty$
Sussman (1974)	$G_d(n) = \dfrac{C(n^2 + \alpha^2 + \beta^2)}{(n^2 + \alpha^2 + \beta^2)^2 + 4n^2\alpha^2}$	$0 \le n \le \infty$
Xu et al. (1992)	$G_d(n) = A/2a \exp\left(-n^2/\left((2a)^2\right)\right)$	$0 \le n \le \infty$
Kozin and Bogdanoff (1961)	$G_d(n) = A/a \exp(-n^2/a^2)$	$0 \le n \le \infty$

Note: C, C_1, C_2, C_3, p, k, q, α and β are real positive constants

Figure 3 Normalised distributions of the exponent w, the general roughness term $G_d\,(0.1)$ and their correlation



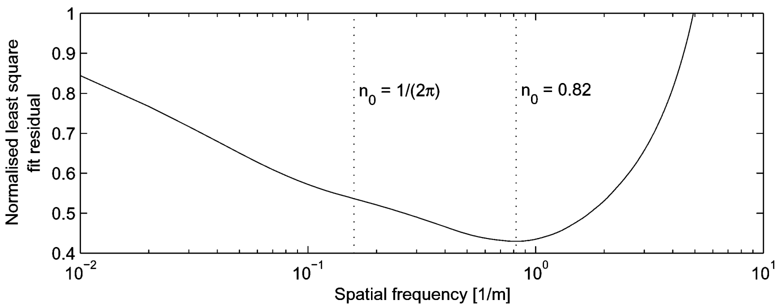
4.2 BSI 1972

As mentioned above, the split straight lines approximation was proposed by Macaulay (1963), albeit with fixed exponents. Using the results from La Barre et al. (1969) it was found that the single straight line approximation was not suitable for many roads, and the more general form was proposed by Dodds (BSI, 1972).

$$G_d(n) = \begin{cases} Cn^{-w_1} & \text{for} \quad n \leq n_0 \\ Cn^{-w_2} & \text{for} \quad n \geq n_0 \end{cases} \tag{1}$$

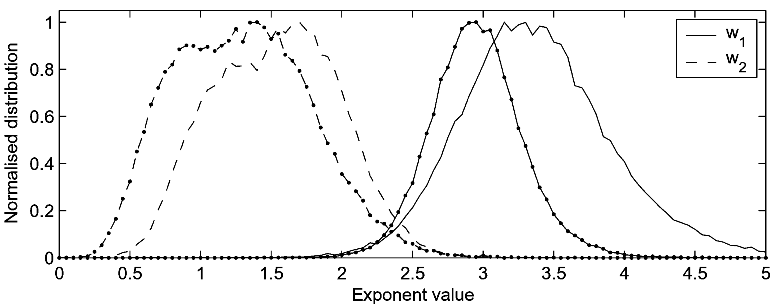
The discontinuity between the two lines is set to the $1/2\pi$ frequency, or a wavelength of about 6.3 m, by Dodds et al. (BSI, 1972; Dodds and Robson, 1973). In order to achieve an overall minimisation of the least square residual error for the Swedish road network a significantly higher frequency of 0.82 cycle/m was optimal, as illustrated in Figure 4. It is not clearly expressed in any document describing the split line approximation whether the lines segments should be connected or not. In the present analysis the line segments are not connected.

Figure 4 Least square residual error as a function of the break frequency



The distribution of the exponents w_1 and w_2 is illustrated in Figure 5. The average value of about 3 for w1 agrees well with the values reported by Dodds (BSI, 1972). The value for w_2 is reported to be about 2 by Dodds. For the Swedish road network 1.4 seems to be better. In fact, the values from the Swedish road network agree fairly well with the values suggested by Macaulay (1963). (N.B. the values in (BSI, 1972) should be multiplied by 1.5, as suggested by Kamash and Robson (1978).)

Figure 5 Normalised distribution of the w_1 and w_2 exponents. The dotted lines are for the error minimising frequency 0.82 cycles/m. The higher frequency break point is placed at the flatter part of the PSD, making both exponents lower



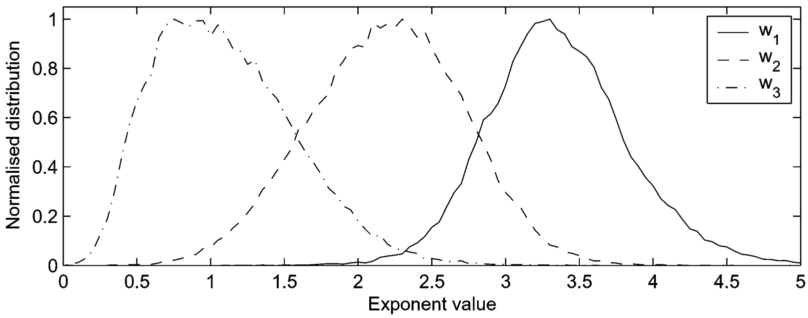
4.3 Two split

A natural next step from the straight line and the split straight lines is to break the line in two places. It is noted in the 8608 standard that: 'In the literature a two or more straight line fitting is often used, but the standardisation of a method which guarantees a unique solution is practically impossible'. However, no references are given and no publication has been found describing an actual use of the two split approximation.

$$G_d(n) = \begin{cases} C_1 n^{-w_1} & \text{for} \quad n \leq n_1 \\ C_2 n^{-w_2} & \text{for} \quad n_1 \leq n \leq n_2 \\ C_3 n^{-w_3} & \text{for} \quad n_2 \leq n \end{cases} \tag{2}$$

A lower break frequency of 0.21 cycles/m, and the higher break frequency at 1.22 cycles/m (approximately 4.76 and 0.82 m wavelengths) produced the lowest total least square error for the Swedish road network. The distribution of the exponents is given in Figure 6. It is obvious from these distributions that the PSD is steep at longer wavelengths, and gets flatter at the shorter wavelengths.

Figure 6 Normalised distribution of the w_1, w_2 and w_3 exponents

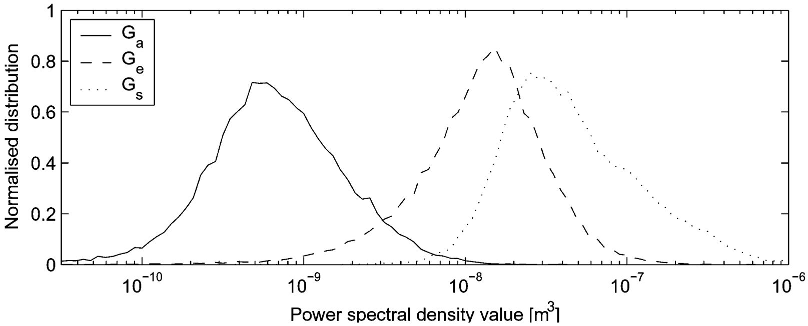


4.4 Sayers

Sayers (1986) proposed the elevation PSD $G_d(n) = C_1/n^4 + C_2/n^2 + C_3$. A step-wise method is used to fit the approximation to measured data. The overall roughness amplitude C_2 coefficient is calculated as the mean value for the slope PSD covering wavelengths from 0.08 to 0.5 cycles/m (2–12.5 m). The C_2 value is then subtracted from the PSD for all wavelengths in order to retain only the residual roughness. C_1 is then determined from the acceleration PSD over the 0.003 to 0.05 cycles/m range (20–333 m), and C_3 is calculated from the elevation PSD over the 0.7 to 3 cycles/m (0.33–1.43 m).

Apart from this not much is documented on the implementation of the algorithm. Some improvements in fit are possible by using the C_1, C_2 and C_3 coefficients from Sayers method as seeds to a minimum-least-square algorithm. The normalised distributions of C_1, C_2 and C_3 from the 'improved' implementation used in the present study are given in Figure 7.

Figure 7 Normalised distribution of the C_1, C_3 and C_2 coefficients



4.5 Other approximations

With the aim that '[a] model should have just one parameter that establishes the roughness [...]' the following PSD approximation was proposed by Gillespie (1985) and Gillespie et al. (1980) for flexible constructions

$$G_d(n) = C\left(1 + (0.66/n)^2\right)/n^2.$$

For rigid constructions the cutoff wavenumber 0.066 cycles/m should be replaced with 0.16. The one-parameter approach would of course be ideal, but results indicate that two or three parameters are needed to describe the road roughness in any detail. Preliminary results suggest a correlation between the roughness level and roughness distribution for the straight line fit (see Figure 3), which could result in a better one-parameter approximation. More work is needed to fully understand the nature of this correlation.

A new type of PSD approximation is proposed by Marcondes et al. (1991). This model was developed as no existing approximation would fit their measured data. However, the reason for the lack of success with the existing approximations was that the 'sections profiles exhibited some peculiarities in the form of concentrated roughness', where 'concentrated roughness' is explained as 'railroad tracks, potholes and other inhomogeneities' (Marcondes et al., 1991). This might of course be the very reason no fit was possible. The validity of a PSD analysis depends on its randomness, which clearly has been violated here. (This approximation is not illustrated in Figure 2 due to uncertainties in the implementation.)

$$G_d(n) = \begin{cases} C_1 \exp(-kn^p) & \text{for} \quad n \leq n_1 \\ C_2(n - n_0)^q & \text{for} \quad n > n_1 \end{cases}$$

An approximation originally developed for elevated guideways carrying high-speed trains was proposed by Sussman (1974).

$$G_d(n) = \frac{A}{n_0^2 + n^2}.$$

This approximation is probably not quite applicable to roads. For a guideway it is fair to assume that the PSD could peak at the wavelength for the guideway span length.

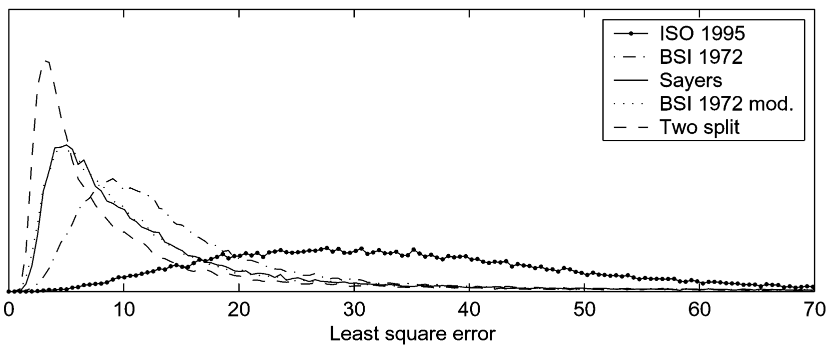
For roads, however, there is no tendency of this behaviour, except perhaps for very long wavelengths. If the cutoff frequency is set high enough the approximation is practically reduced to the straight line fit. Sussman (1974) also proposed the approximation illustrated as 'Sussman 2' in Figure 2. This approximation is explicitly intended for filtered data, making it even less relevant.

Macvean (1980) proposes a PSD approximation $G_d(n) = A/(\alpha^2 + n^2)^2$ for calculating the response of an accelerating vehicle. In order to make the PSD function even and have a zero slope at the zero frequency Xu et al. (1992) proposed an approximation $G_d(n) = A/2a\exp(-n^2/((2a)^2))$. A similar equation was proposed by Kozin and Bogdanoff (1961) (the $2a$ term is substituted by a, making it decrease even faster at short wavelengths).

5 Brief comparison of four approximations

The present paper is intended mainly as an overview of existing methods to approximate the calculated PSD from a road profile with a simple function. However, the approximations judged to be of interest for future work were tested on the Swedish state road network. These were the ISO 8608, the BSI 1972, the two split, and the approximation proposed by Sayers. An overall comparison between the approximations is presented in Figure 8. The least square fit residual on the logarithm of the PSD is used as a measure on the match of the approximations. A small error indicates a good approximation.

Figure 8 Normalised distribution of least square residuals for different approximations



The most simple ISO 8608 straight line fit, quite naturally, came out worst in the comparison. The modified split straight line (with error minimising break frequency) and Sayers' approximation have very similar distributions. The two split approximation had the smallest error overall.

The Swedish state road network is divided into three classes. Motorways are numbered given numbers less than 100, principal roads road numbers from 100 to 499, and minor roads from 500 and upwards. The least square residual of the approximation for the road types is given in Table 2. The errors are reported in relation to the ISO 8608 approximation. As can be seen, the behaviour is similar for all road types.

Table 2 Relative least square fit residuals (values in percent)

	ISO 8608	*BSI 1972*	*Sayers*	*BSI 1972 mod*	*Two split*
Motorways	100.00	47.33	40.82	37.96	34.52
Principal roads	100.00	49.42	42.47	40.43	35.42
Minor roads	100.00	55.05	45.65	46.40	35.35

6 Conclusions

A literature survey of power spectral density (PSD) approximations has been presented. Three of then ten approximations found have been applied to road profile data from the entire Swedish state road network. A fourth approximation found in the literature, which was only mentioned but not actually used, has also been applied to the data material.

The data material used seems to be a fair realisation of a random process, making the spectral analysis valid. Judging from the least square residual the fit of the approximations to measured data increases with increasing complexity of the approximation function. The needed level of complexity of the approximation depends on the application. For road roughness identification a two or three parameter approximation is probably enough, but a definite answer requires some more research.

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