**CMSC-411 Final Report: CORDIC Algorithm in ARM**

Stephanie Tam, Evan Andre, Ben Hazlett

**Total Computer Cycles**

branch - 3 cycles compare - 1 cycle

add, sub, mov, shift (normal) - 1 cycle shift (from register) - 2 cycles

load - 2 cycles load (offset) - 3 cycles

store – 2 cycles store (scaled) – 3 cycles

These are my cycles per instruction that we pulled from <http://infocenter.arm.com/help/index.jsp?topic=/com.arm.doc.ddi0214b/ch09s01s01.html>

This is for an ARMv7 machine and our simulator is for an ARMv5 machine, but this should still produce a reasonable estimate.

From these we went through our code and estimated the expected number of cycles for **sin and cos**. Our results were **346 total cycles**.

For tan our results were **238 total cycles**.

**A note on sine and cosine**: These values were computed simultaneously as the values of sine and cosine were required during the algorithm. This is why they have the same number of instructions and the same number of total cycles.

**Estimated CPI**

186 instructions to calculate sin(x) and cos(x)   
CPI = 346 / 186 = 1.8602 cycles / instruction

134 additional instructions to calculate tan(x)

CPI = (346 + 238) / (186 + 134) = 1.825 cycles / instruction

**Estimated Total Processing Time**

Assuming a system clock of the following:

* 32kHz:
  + sin and cos: 186 \* 1.8602 / 32kHz = 0.0108124125 seconds
  + tan: (346 + 238) \* 1.825 / 32kHz = 0.03330625 seconds
* 1MHz:
  + sin and cos: 186 \* 1.8602 / 1MHz = 0.0003459972 seconds
  + tan: (346 + 238) \* 1.825 / 1MHz = 0.0010658 seconds
* 1GHz
  + sin and cos: 186 \* 1.8602 / 1GHz = 0.000000345972 seconds
  + tan: (346 + 238) \* 1.825 / 1GHz = 0.0000010658 seconds

**Description of Implemented Algorithms**   
To calculate *sin(x)* and *cos(x)* we used the CORDIC algorithm.

**The CORDIC Algorithm for Sine and Cosine**

Sine and Cosine are estimated through this algorithm using only shifts, adds, and subtracts. The key for this is the convergence on a specific angle by either adding to our target angle when our target angle is negative and subtracting from our target angle when our target angle is positive. The values for sine and cosine are initialized to 0 and 0.6072529350 respectively. To set up our table of angles we took the arctan(2i) where i started at 0 and counted up to 11. Then from there the new cosine and new sine values are calculated as such:

If current angle < 0,

current angle += angle table[i]

new cosine = current cosine + (current sine >> i)

new Sine = current sine  - (current cosine >> i)

If current angle >= 0,

current angle -= angle table[i]

new cosine = current cosine - (current sine >> i)

new sine   = current sine  + (current cosine >> i)

current cosine = new cosine

current sine = new sine

This was then iterated through the whole angle table which contained twelve angle values.

After the twelve iterations the current cosine and current sine values will result in our estimate for our cosine and sine values for that angle. One important method we used when implementing this algorithm in ARM is shifting all of the numbers left by 16 to the left. This is done to avoid having floating point values (and registers) to instead use the regular registers. Then in the interpretation of the final value, we take the hexadecimal value stored in the register and interpret it as a decimal value.

In implementing division in ARM LEGv8,

**The Shift Subtract algorithm for Division**

We used division in ARM to calculate the tangent values from our estimated sine and cosine values. To divide we used a simple shift subtract algorithm. First we took our calculate sine and cosine values and prepared them for the shift add algorithm by shifting our sine value by 4 to the right as to avoid potential overflow. Then we had a comparison if cosine is greater than sine we shift cosine to the left until cosine is larger. Then our algorithm is set up to start the shifting and subtracting as follows:

If sine > cosine

sine = sine – cosine

tan += number of carries

cosine = cosine >> 1

continue with the above until sine is less than cosine.

Then we store our values into memory, sine, cosine, and tangent.

**Sample Input & Output**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **Angle, z** | **Cos(z)**  **Hex** | **Sin(z)**  **Hex** | **Cos(z)**  **Approx.** | **Sin(z)**  **Approx.** | **Actual**  **Cos(z)** | **Actual**  **Sin(z)** |
| **5.2954** | **0000.fee3** | **0000.17be** | **0.9956512** | **0.0927429** | **0.9957321** | **0.0922906** |
| **80.1** | **0000.2c0a** | **0000.fc2e** | **0.1720275** | **0.9850769** | **0.1719291** | **0.9851093** |
| **24.23** | **0000.e989** | **0000.68d4** | **0.9122467** | **0.4094848** | **0.9119053** | **0.4104005** |
| **48.52** | **0000.a96f** | **0000.bfe9** | **0.6618499** | **0.7496490** | **0.6623585** | **0.7491869** |
| **10.001** | **0000.fc17** | **0000.2c88** | **0.9847259** | **0.1739501** | **0.9848047** | **0.1736653** |
| **70.86159** | **0000.53f1** | **0000.f1d5** | **0.3278961** | **0.9446563** | **0.3278513** | **0.9447293** |
| **30.0** | **0000.ddbd** | **0000.7ff0** | **0.8661651** | **0.4997558** | **0.8660254** | **0.5** |
| **86.5** | **.0b043615** | **.efe84526** | **0.0430330** | **0.9371379** | **0.0610485** | **0.9981347** |
| **90** | **00000000** | **00000001** | **0** | **1** | **0** | **1** |
| **0** | **00000001** | **00000000** | **1** | **0** | **1** | **0** |
| **4.3667** | **.efb4bda** | **.0ec37009** | **0.9363511** | **0.0576696** | **0.9970970** | **0.0761407** |
| **88.667** | **.021bd47a** | **.f0269e12** | **0.0082371** | **0.9380832** | **0.0232631** | **0.9997293** |
| **45** | **0000.b4f6** | **0000.b513** | **0.7068786** | **0.7073211** | **0.7071067** | **0.7071067** |
| **60** | **0000.7ff0** | **0000.ddbd** | **0.4997558** | **0.8661651** | **0.5** | **0.8660254** |