Assignment: One

BIT 1st Semester

Subject: Mathematics-I

Answer all the questions.

1. Transform to polar coordinates: $(\sqrt{3}, -1)$.
2. Find the distance between the polar points $\left(4, \frac{\pi}{2}\right)$ and $\left(5, \frac{7\pi}{6}\right)$.
3. Transform the equation $x^2 + y^2 + z^2 = 2z$ by spherical polar coordinates.
4. Define conic? When does it become Parabola?
5. If the equation of a hyperbola is $4x^2 - 9y^2 = 36$, find its eccentricity and length of latus rectum.
6. Transform the equation $y^2 - x^2 = 4$ by rotating the coordinate axes x and y through an angle of $45°$.
7. Define Scalar (or dot) product of two non-zero vectors. If $\vec{a} = \vec{i} + \vec{j} + 3\vec{k}$ & $\vec{b} = 3\vec{i} - 3\vec{j} + \vec{k}$ then show that $\vec{a} \,.\, \vec{b} = \vec{b} \,.\, \vec{a}$.
8. Find the parametric equations of the line joining the points $P_1(1, 1, 0)$ and $P_2(0, 2, 3)$.
9. Define symmetric and skew-symmetric matrices with example.
10. If $A = \begin{pmatrix} 1 & 3 \\ -5 & 2 \end{pmatrix}$ & $B = \begin{pmatrix} 3 & 2 \\ 1 & -1 \end{pmatrix}$ compute $(AB)^T$.
11. Transform the equation $x^2 + y^2 = x$ to Cylindrical coordinates.
12. Prove that the distance between two points in a plane with polar coordinates (r_1, θ_1) and (r_2, θ_2) is given by $d^2 = {r_1}^2 + {r_2}^2 - 2r_1 r_2 \cos(\theta_2 - \theta_1)$.
13. Find the equation of the plane passing through the point $(1, 2, -1)$ and perpendicular to the planes $x + y - 2z = 5$ and $3x - y + 4z = 12$.
14. Find the shortest distance between the two skew lines $\frac{x-8}{3} = \frac{y+9}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$
15. Find the volume of the parallelepiped whose concurrent edges are represented by $3\vec{i} - 3\vec{j} + 3\vec{k}, \vec{i} + 2\vec{j} - \vec{k},$ and $3\vec{i} - \vec{j} + 2\vec{k}$.

16. Find the centre, eccentricity, foci and directories of the hyperbola $9x^2 - 16y^2 + 72x - 32y - 16 = 0$.

17. Find the value of c so that the lines $\frac{x-1}{-3} = \frac{y-1}{2c} = \frac{z-3}{2}$ and $\frac{x-1}{3c} = \frac{y-5}{1} = \frac{z-6}{-5}$ are perpendicular to each other.

18. Find the equation of the plane through the points $A(2,2,1)$, $B(3,4,2)$ and $C(7,0,6)$.

19. If $\vec{a} + \vec{b} + \vec{c} = 0$, prove that $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$.

20. Find the centre and eccentricity of the ellipse $9x^2 + 25y^2 - 18x - 100y - 116 = 0$.

21. Transform the equation $x^2 - z^2 = 4$ by using Spherical polar coordinates.

22. Given $\vec{a} = \vec{\imath} - 2\vec{\jmath} + 3\vec{k}$ and $\vec{b} = 2\vec{\imath} + \vec{\jmath} - \vec{k}$, find $\vec{a} . \vec{b}$, $\vec{a} \times \vec{b}$ and $\vec{b} \times \vec{a}$.

23. Given $A(-1,1,2)$, $B(0,1,3)$, $C(2,3,4)$ & $D(-1,3,3)$, find the volume of the parallelepiped with $\overrightarrow{AB}, \overrightarrow{AC}$ and $\overrightarrow{AD}$ as three of its edge.

24. State Generalized Mean Value Theorem.

25. Show that at any point of the parabola $y^2 = 4ax$, the subnormal is constant and the subtangent varies as the abscissa of the point of contact.