Mathematics-II

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Differential equation of the first order

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Degree of differential equation

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1. $(y')^2 + 5y = \sin x$ is the differential equation of degree 2.
2. $y''' + 2(y'')^2 - y' = 0$ is the differential equation of degree 1, but order of the equation is 3.

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- **Particular Solution:** A solution obtained by giving particular values of arbitrary constants in the general solution is called the particular solution.
  If we consider a differential equation $y' = \cos x$ Then $y = \sin x + c$ is the general solution, however the equation $y = \sin x + 5$ is particular solution.

Differential equation of the first order

Solving a differential equation is not an easy matter. There is no systematic technique that enables us to solve all differential equations that have arisen during the investigation of real-life problems, although differential equations are the most important of all the applications of integral calculus.
A differential equation is said to be integrable by quadratures if its general solution can be obtained as a result of a finite sequence of elementary actions on the known functions and integrations of those functions. In this chapter we will discuss on the following differential equations which are integrable by quadratures.

1. Variable separable
2. Exact differential equations
3. Homogeneous equations
4. Linear differential equation

Variables Separable Method

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1. $x\,dx + y\,dy = 0$
2. $\frac{dy}{dx} = \frac{x^2+1}{y^2+1}$

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3. $\frac{dy}{dx} + 4x = 2e^{2x}$

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4. $(1+x)\left(1+y^2\right)\,dx+(1+y)\left(1=x^2\right)\,dy=0$

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Put $y = vx$, then we have $\frac{dy}{dx} = v + x\frac{dv}{dx}$

Now, given differential equatin becomes

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$\Rightarrow v + x\frac{dv}{dx} = v + \frac{v^2}{3}$

$\Rightarrow x\frac{dv}{dx} = \frac{v^2}{3}$

$\Rightarrow \frac{3\,dv}{v^2} = \frac{dx}{x}$

$\Rightarrow \frac{-3}{v} = \log x + \log c$

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$y - 2x = cx^2 y$ which is the required solution.

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8. $\frac{dy}{dx} = \frac{y}{x} + \tan\left(\frac{y}{x}\right)$
9. $x \sin\left(\frac{y}{x}\right) dy = \left(y \sin\left(\frac{y}{x}\right) - x\right)\, dx$

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$\therefore \ \frac{dY}{dX} = \frac{a(X+h)+b(Y+k)+c}{A(X+h)+B(Y+k)+C} = \frac{aX+bY+(ah+bk+c)}{AX+BY+(Ah+Bk+C)}$

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In this case we put $ax + by = v$. Then the variable can be easily separated.

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Now Substituting $X = x - 1$ and $Y = y - 1$ and solving we get

$x + y - 2 = C(x - y)^3$ Which is the required solution.

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A differential equation of the form $M\,dx = N\,dy = 0$, where M and N are function of x and y, is said to be exact when there is a function $f(x, y)$ such that $M\,dx + N\,dy = df(x, y)$. i.e. when $M\,dx + N\,dy$ is a perfect differential.

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4. If N has no term which is free from x, then $\int N\,dy$ is taken as zero

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In some cases there are certain rules to find integrating factor. But we can often find out the solution easily by suitable grouping of terms to form a perfect differential. In many cases we can also obtain the integrating factors by inspection. we give a few exact differentials which will be useful in making suitable groups or in finding an integrating factor by inspection.

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$\frac{\partial M}{\partial y} = -2$ and $\frac{\partial N}{\partial x} = -2$

$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

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$= \int (3x - 2y + 1)\,dx = \frac{3x^2}{2} - 2xy + x$

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$(3x-2y+1)\,dx+(3y-2x-1)\,dy=0$
Here, $M=3x-2y+1,\;\; N=3y-2x-1$
$\frac{\partial M}{\partial y}=-2$ and $\frac{\partial N}{\partial x}=-2$
$\therefore\ \frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$
Hence the equation is exact.
Now,
$\int M\,dx$ taking y as constant
$=\int(3x-2y+1)\,dx=\frac{3x^2}{2}-2xy+x$
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Solve the following differential equation

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   Solution: The given equation can be written as

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   $\Rightarrow \frac{-1}{xy} = \frac{-1}{x}\left(\log x + 1\right) + c$
   $\Rightarrow 1 + cxy = y(\log x + 1)$

Exact differential equation

Solve: $(x+y)(dx-dy)=dx+dy$

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Given equation can be written as

$dx-dy=\frac{dx+dy}{x+y}$

$\Rightarrow dx-dy=\frac{d(x+y)}{x+y}$

Integrating we get

$x-y=\log(x+y)+c$

Exact differential equation

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Solve: $\left(x^2+y^2+2x\right)dx+2y\,dy=0$

Given differential equation can be written as
$\left(x^2+y^2\right)dx+2x\,dx+2y\,dy=0$
$\Rightarrow\left(x^2+y^2\right)dx+d\left(x^2+y^2\right)=0$
Dividing by x^2+y^2 we get
$dx+\frac{d\left(x^2+y^2\right)}{x^2+y^2}=0$

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$\Rightarrow 3x^4 8x^3+6x^2y^2=c$

Linear Differential Equation

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- Integrating both sides we get

$$ye^{\int Pdx} = \int Qe^{\int Pdx} dx + c$$

which is the solution of the given differential equation

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Now, $I.F. = e^{\int P dx} = e^{\int 2\, dx} = e^{2x}$
Multiplying equation (i) by $I.F.$ we get
$e^{2x}\frac{dy}{dx} + 2e^{2x}y = 4xe^{2x}$

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Integrating both sides, we get
$ye^{2x} = 4\int xe^{2x}dx$
$ye^{2x} = 4\left[\frac{xe^{2x}}{2} - \frac{e^{2x}}{4}\right] + C$
$\Rightarrow y = (2x - 1) + Ce^{-2x}$ which is the required solution.

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3. $\frac{dy}{dx} + \frac{2xy}{1+x^2} = \frac{1}{\left(1+x^2\right)^2}$
4. $x\left(x - 1\right) \frac{dy}{dx} - y = x^2 \left(x - 1\right)^2$

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3. $\frac{dy}{dx} + \frac{2xy}{1+x^2} = \frac{1}{\left(1+x^2\right)^2}$
4. $x\left(x - 1\right) \frac{dy}{dx} - y = x^2 \left(x - 1\right)^2$
5. $\left(1 + x\right) \frac{dy}{dx} - xy = 1 - x$
6. $\frac{dy}{dx} + y \cot x = 2 \cos x$
7. $\frac{dy}{dx} + y \tan x = \sec x$
8. $\cos x \frac{dy}{dx} + y \sin x = \sec^2 x$

Linear differential equation

Solve the following differential equation:

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Note:

If a linear differential equation is of the form $\frac{dx}{dy} + Px = Q$, where P and Q are functions of y alone or constants, then the $I.F. = e^{\int P dy}$.

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Now, multiplying equation (i) by $I.F.$ we get
$\frac{d}{dy}(xe^{-y}) = e^{-y}(y + 1)$
Integrating we get
$xe^{-y} = \int e^{-y}(y + 1)dy$
On solving we get,
$(x + y + 2) = Ce^{y}$ which is the required solution.

Linear differential equation

Linear differential equation

Equation Reducible to linear form

An equation of the form $\frac{dy}{dx} + Py = Qy^n$, where P and Q are function of x alone is called Bernoulli's equation.

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- Dividing by y^n, the equation becomes

$$\frac{1}{y^n}\frac{dy}{dx} + Py^{1-n} = Q$$

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- Dividing by y^n, the equation becomes

$$\frac{1}{y^n}\frac{dy}{dx} + Py^{1-n} = Q$$

- Put $y^{1-n} = v$, then $(1-n)y^{-n}\frac{dy}{dx} = \frac{dv}{dx}$
- The equation is reduced to $\frac{dv}{dx} + (1-n)Pv = (1-n)Q$, which is a linear equation.

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Dividing both sides of given equation by y^2, it becomes
$\frac{1}{y^2} \cdot \frac{dy}{dx} + \frac{1}{y}\frac{1}{x} = 1 \quad ....(i)$

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Put, $\frac{1}{y} = v$ then we have, $-\frac{1}{y^2} \cdot \frac{dy}{dx} = \frac{dv}{dx}$

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$-\frac{dv}{dx} + \frac{1}{x}.v = 1 \; \Rightarrow \frac{dv}{dx} - \frac{1}{x}.v = -1 \; ...(ii)$,
Which is linear in v
Now, $I.F. = e^{-\int \frac{1}{x}\,dx} = \frac{1}{x}$

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Now multiplying equation (i) by $I.F.$ and integrating we get,
$v.\frac{1}{x} = -\int \frac{1}{x} dx \ \Rightarrow \frac{v}{x} = -\log x + c$

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Substituting $v = \frac{1}{y}$ and solving we get

$xy\,(C - \log x) = 1$ which is the required solution.

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THANK YOU

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