PURBANCHAL UNIVERSITY

2014 (New)

B.E.(Computer)/Fourth Semester/*Final*

Time: 03:00 hrs. Full Marks: 80 /Pass Marks: 32

BEG274CO: Discrete Structure

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Group A

Answer TWO questions. Umesh **2×12=24**

1(a) Define regular graph, complete graph and connected graph. Explain methods to represent graph in a computer.

(b) Explain transport network with an example.

2. What is finite state antomata? Design a deterministic finite automata (DFA) that accepts strings consisting of symbols 0 and 1 and ending with a substring 01.

3. Discuss recurrence relation, homogeneous recurrence relation and Fibonacci sequence.

Group B

Answer SEVEN questions. **7×8=56**

4. Suppose that a box contains 15 balls, of which 8 are red and 7 are black. In how many ways can 5 balls be chosen so that:
   (a) all five are red
   (b) all five are black

5. Prove the following statement by mathematical induction

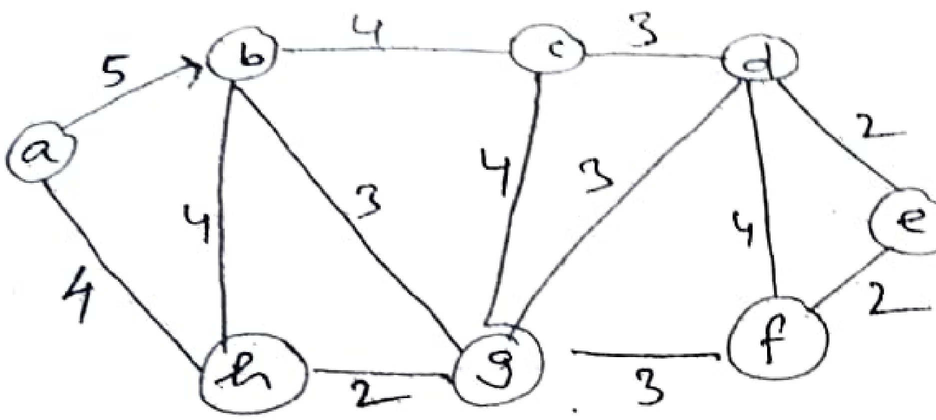
$$1+2+3+.......+n = \frac{n(n+1)}{2}$$

6. What do you mean by diagraph of a relation? Let A = {a, b, c, d} and R = {(a, b), (a, c), (b, a), (b, c), (c, d), (d, a)}.

   Find the transitive closure of R using warshall's algorithm.

7. What do you mean by minimal spanning tree? Using prim's algorithm find the minimal spanning tree for the following graph

Contd. ...



8. Define, union, intersection and difference of two sets with examples.

If A = {x/x is a positive integer < 8}

B = {x/x is an integer such that $2 \leq x \leq 4$}

C = {x/x is an integer such that $x^2<16$}

Then verify that:

$N(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$.

9. What do you mean by Proposition and truth tables? What is tautology? Prove that $\neg p \vee (p \vee q)$ is a tautology.

10. What are equivalence relation. Let A] {1, 2, 3, 4, 5}.

R = {(1, 1), (2, 2), (1, 2), (2, 1), (3, 3), (3, 4), (4, 3), (4, 4), (5, 5), (5, 4), (4, 5)}.

Determine whether R is a equivalence relation or not.

11. Define pigeonhole principle. What is extended pigeonhole principle? Show that if 9 colours are used to paint 100 horses, atleast 12 horses will be of same colour.

12. Write short notes on any TWO:
(a) Generating functions
(b) Strings and languages
(c) Composite function

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PURBANCHAL UNIVERSITY

2014 (New)

B.E.(Computer)/Fourth Semester/*Chance*

Time: 03:00 hrs. Full Marks: 80 /Pass Marks: 3[illegible]

BEG274CO: Discrete Structure

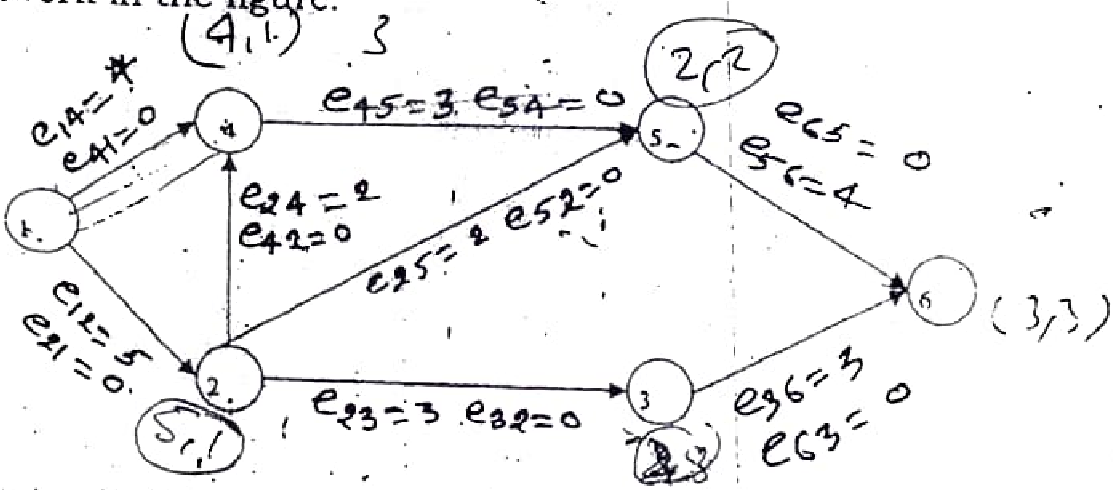
Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Group A:

[A]nswer TWO questions. Umesh $2\times12=24$

1. Use the labeling algorithm to find a maximum flow for the network in the figure:



),(5,5)}

2. Let A = {1, 2, 3, 4},

R = (1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (3, 4), (4, 3), (4, 4), (5, 5) and

S = {1, 1), (2, 2), (3, 3), (4, 4), (4, 5), (5, 4), (5, 5)}

Prove that R is an equivalence relation. Use Warshall's algorithm to compute the transitive closure containing R and S. 4+8

3. (a) Show that the following argument is valid: If today is Tuesday, I have a test in Mathematics or Economics. If my Economics Professor is sick, I will not have a test in Economics. Today is Tuesday and my Economics Professor is sick. Therefore I have a test in Mathematics. 8

(b) Obtain the principal disjunctive and principal conjunctive normal form of ~pvq. 4

Group B:

Answer SEVEN questions. $7 \times 8 = 56$.

4(a) Prove that, if a and b be two positive integers, then GCD(a, b).LCM(a, b) = ab. 4

(b) Compute GCD(273, 98) using the Euclidean algorithm. 4

5. State Pigeonhole principle. Show that if any eight positive integers are chosen, two of them will have the same remainder when divided by 7. 2+6

6. State principle of mathematical induction. Using the principle of mathematical induction prove that $1^2 + 3^2 + 5^2 + \ldots + (2n-1)^2$

$$= \frac{n(2n+1)(2n-1)}{3}$$

7. Let A = {1, 2, 3, 4} and B = {a, b, c}.

Let R = {(1, a), (1, b), (2, b), (2, c), (3, b), (4, a)} and

S = {(1, a), (2, c), (3, b), (4, b)}

Compute: (a) $\overline{R}$ (b) $R \cap C$ (c) $R \cup C$ and (d) R^{-1}.

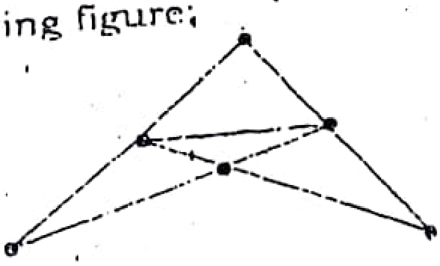
8. Let M be the finite state machine with state table appearing in the table:

S \ A	f: a	f: b	f: c	g: a	g: b	g: c
s0	s0	s1	s2	0	1	0
s1	s1	s1	s0	1	1	1
s2	s2	s1	s0	1	0	0

Find the input set A, the state set S, the output set O and initial state of M. Draw the state diagram of M. 2+3+3

Find the output string for the input string aabbc.

9. Use Fleury's algorithm to construct an Euler circuit for the graph in the following figure:



vertex can be repeated

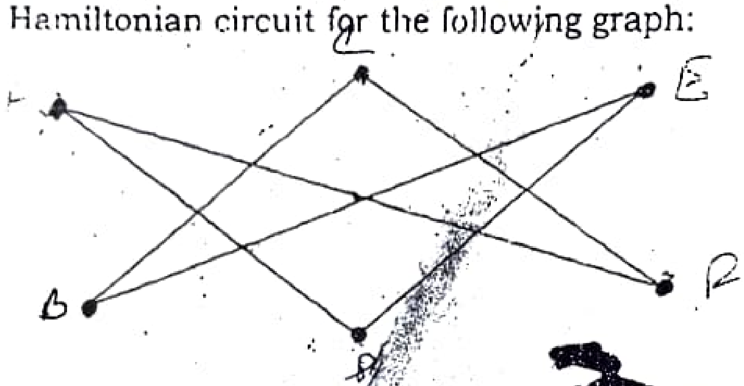
Contd. ...

10. Solve the recurrence relation:
$a_n - 9a_{n-1} + 20a_{n-2} = 0$, $a_0 = -3$, $a_1 = -10$.

11. Use generating function to solve the following: $a_n = 3a_{n-1} + 2$, $a_0 = 1$.

12(a) Compute the truth table of the statement $(p \Rightarrow q) \Leftrightarrow (\sim q \Rightarrow \sim p)$

(b) Find a Hamiltonian circuit for the following graph: 4+4



edge can be repeated

For basic step
$n = 1$

$$\frac{1(2\times1+1)(2\times1-1)}{3}$$

$$= \frac{1(3)(1)}{3}$$

$$= \frac{3}{3} = 1 \text{ which is true}$$

For inductive step
Let it is true for K

$$\frac{K(2K+1)(2K-1)}{3} \text{ is true}$$

we have to show that it is true for $K+1$

$$\frac{(K+1)(2(K+1)+1)(2(K+1)-1)}{3}$$

$$= 1^2 + 3^2 + 5^2 + \cdots + K^2 + (K+1)^2$$

$$= \frac{K(2K+1)(2K-1)}{3} + (K+1)^2$$

So So

$$= \frac{K(2K+1)(2K-1) + 3(K+1)^2}{3}$$

PURBANCHAL UNIVERSITY

2015

B.E.(Computer)/Fourth Semester/*Final*

Time: 03:00 hrs. Full Marks: 80 /Pass Marks: 32

BEG274CO: Discrete Structure *(New Course)*

Candidates are required to give their answers in their own words as far as practicable.

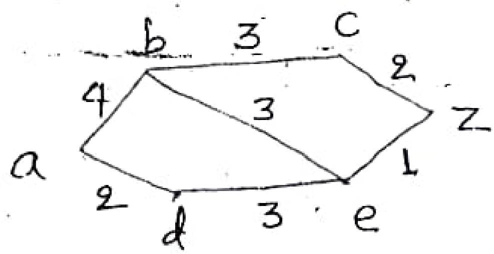
The figures in the margin indicate full marks.

Group A

Answer TWO questions. Umesh **2×12=24**

1. What is Logic? What are propositions? Differentiate between Tautology and contradiction. Draw a truth table for $(p\rightarrow q)\wedge(\wedge q\rightarrow\wedge p)$.

2. Define bipartite graph with example. Find the shortest path between a and z in the weighted graph shown below.



3. (a) State the definition of regular expression and list the set represented by te expression $(0+1)^*1$.

   (b) Define deterministic finite automata. Design a DFA that accepts the set of binary stings which have 101 as substring.

Group B

Answer SEVEN questions. **7×8=56**

4. State the product rule in counting with an example. Find the number of ways to draw 2 red and 4 white balls from a bag containing 10 balls, of which 5 are red and 5 are white, when 6 balls are drawn.

5. Prove the following by the method of induction.

$$\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+\cdots\cdots\frac{1}{(n-1)n}=\frac{3}{2}-\frac{1}{n}$$

Contd. ...

6. Use warshall's algorithm to find the transitive closure of the relation R= {(1,2), (1,3), (1,4), (2,3), (2,4), (3,4)}.

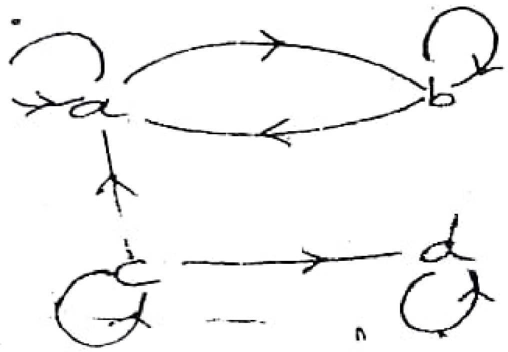
7. Find the explicit formulas for the following recurrence relation.

$f_n = f_{n-1} + f_{n-1}$ if $n>2$

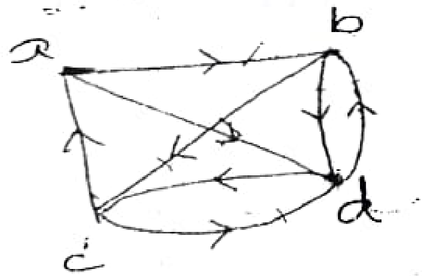
$= 0$ if $n=1$

$= 1$ if $n=2$

8. Determine whether relation represented by following di-graph are reflexive, symmetric, antisymmetric, and /or transitive.



9. Define Hamiltonian path and circuit. Determine whether the following directed graph has an euler path and/or euler circuit.



10. Define union, intersection and complement of set.

If A = {x | x is a positive integer <4}

B= {x^2 | x is an integer and $2 \leq x \leq 5$}, then verify DeMorgan's laws.

11. Write short notes on any TWO:

(a) Adjacency matrix & adjacency list.

(b) Pigeon hole principle

(c) Transport network

PURBANCHAL UNIVERSITY
2016

B.E.(Computer)/Fourth Semester/*Final*

Time: 03:00 hrs. Full Marks: 80 /Pass Marks: 32

BEG274CO: Discrete Structure *(New Course)*

Candidates are required to give their answers in their own words as far as practicable.

All questions carry equal marks. The marks allotted for each sub-question is specified along its side.

Answer EIGHT questions. Umesh 8×10=80

1(a) Prove that, if d is GCD)(a, b), then: 5
(i) **d=sa+tb** for some integers s and t.
(ii) If c is any other common divisor of a and b, then c divides d.

(b) Prove that if n pigeons are assigned to m pigeonholes, then one of the pigeonholes must contain at least $\left[\frac{(n-1)}{m}\right]+1$ pigeons. 5

2(a) Define converse, contra-positive and inverse. What are the contra-positive, the converse and the inverse of the conditional statement: "The home team wins whenever it is raining"? 5

(b) State De Morgan's laws for logical equivalences. Determine whether the logical equivalence $\cancel{(p \to q) \land} \to q$ is a tautology, contradiction or contingency. 5

$(p \to q) \equiv (p \land \sim q)$

3(a) Define principal disjunctive and conjunctive normal forms. Find principal conjunctive normal form for $[\cancel{(p \lor q) \land \sim p \to \sim q}]$. 5

$(p \lor q) \land \sim p$

(b) State principle of mathematical induction. Show that $1+2+3+\dots+x=\frac{x(x+1)}{2}$ by mathematical induction. 5

4(a) Define symmetric and transitive properties on a set A. Show that if R1 and R2 are equivalence relations on A then R1 R2 is an equivalence relation. 5

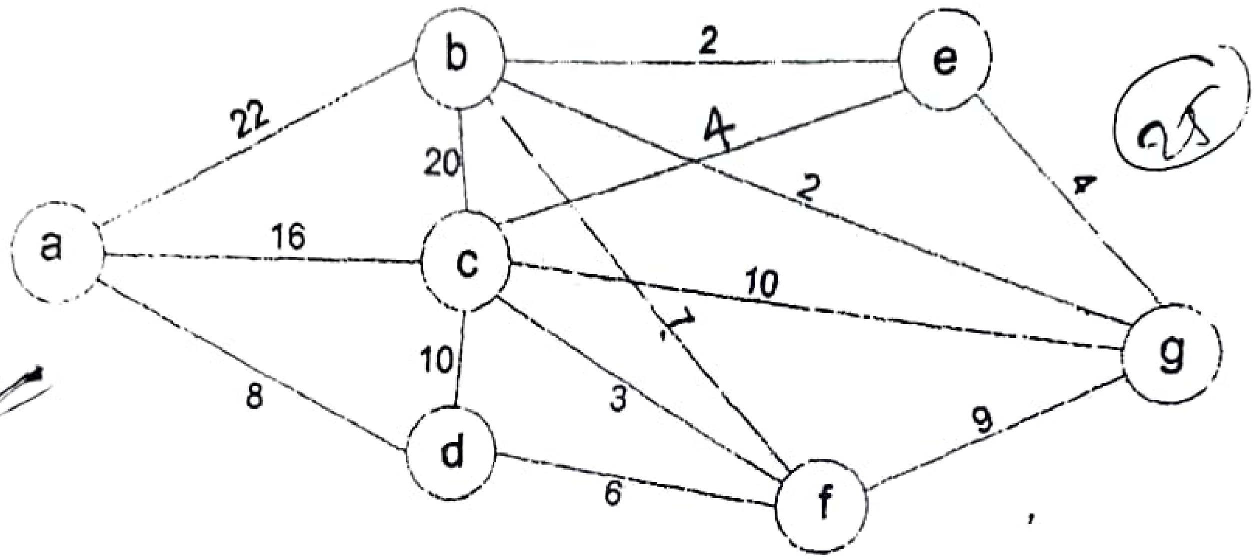
(b) Let A={1,2,3,4} and let R={(1, 4),(3,2),(4,3)} and S={(1, 1), (1, 2), (2,2), (3,3), (4,2), (4, 4)} be the relations on A. Use Warshall's algorithm to compute the transitive closure of R∪S. 5

5(a) Solve the following recurrence relation using generating function: 5

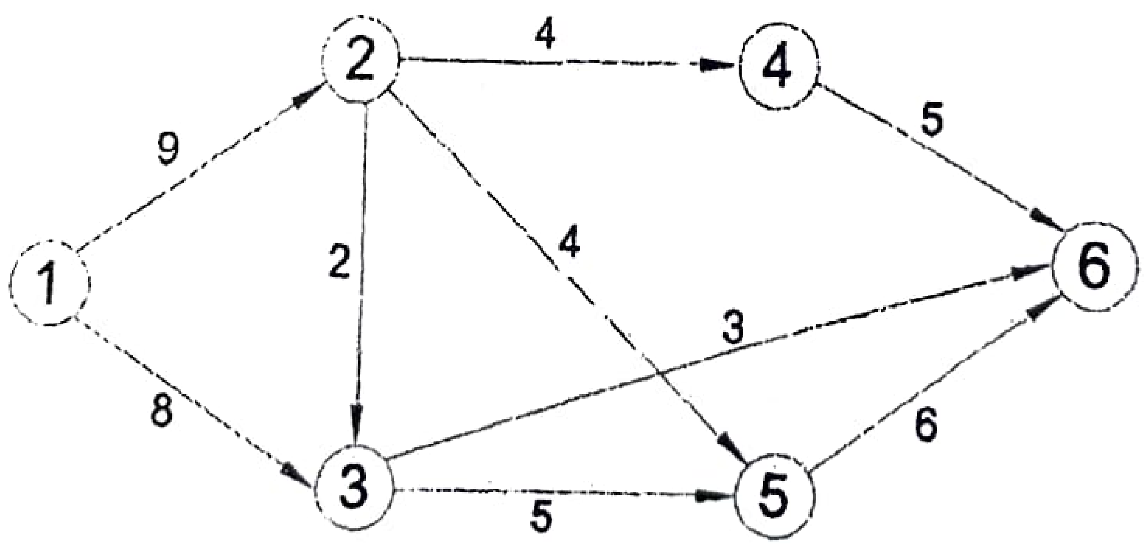
$a_n - 9a_{n-1} + 20a_{n-2} = 0$ $a_0 = 2$, $a_1 = 1$

(b) Solve: $a_{n+2} - 4a_{n+1} + 4a_n = 2^n$ 5

6. Determine a shortest path between, the vertices a to g as shown below: 10



7. Find a maximum flow in the given network by using the labeling algorithm. 10



8(a) What are the strings in the regular sets specified by the regular expressions:

10^*, $(10)^*$, $0 \cup 01$, $0(0 \cup 1)^*$ and $(01)^*$? 5

Contd. ...

(b) Define finite Machines with output. Draw the state diag the finite-state machine with the state table:

State	f Input		g Input	
	0	1	0	1
s_0	s_1	s_0	0	0
s_1	s_2	s_0	1	1
s_2	s_0	s_3	0	1
s_3	s_1	s_3	1	0

9. What do you understand by permutation and combination? bag contains 4 red, 5 black and 6 white balls. In how ma ways:

(i) 2 red and 1 black balls can be drawn?

(ii) 2 black and 1 white balls can be drawn?

(iii) 1 ball of each colour can be drawn?

(i) $^4C_2 \times {^5C_1}$

(ii) $^5C_2 \times {^6C_1}$

(iii) $^4C_1 \times {^5C_1} \times {^6C_1}$

3 a $(p \vee q) \wedge \sim p \to \sim q$

2 (b) $\sim(p \to q) \equiv p \wedge \sim q$

PURBANCHAL UNIVERSITY
2017

B.E.(Computer)/Fourth Semester/*Final*

Time: 03:00 hrs. Full Marks: 80 /Pass Marks: 32

BEG274CO: Discrete Structure *(New Course)*

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All questions carry equal marks. The marks allotted for each sub-question is specified along its side.

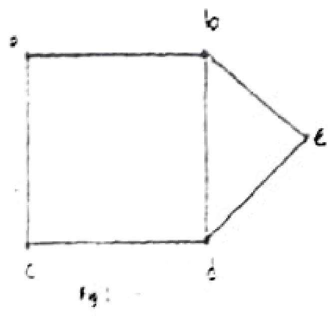
Answer EIGHT questions. **8×10=80**

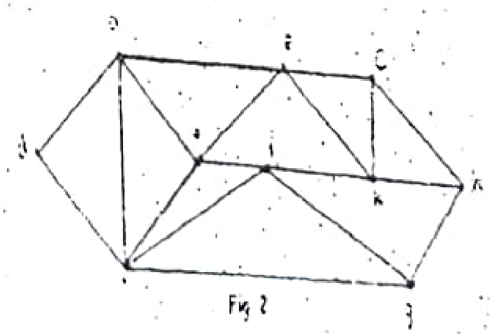
1(a) Use Warshall's Algorithm to find the transitive closure of the relation R on a set A = [1,2,3,4] where R = (1, 1), (1, 3), (2,3), (2,4), (3,2), (3,3), (4,1)). 5

(b) Suppose that the relation R on a set is represented by the matrix $M_R = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$. Is R reflexive, symmetric, and/or antisymmetric? 5

2. State the contra-positive and converse statement of the following statement: "If the triangle is equilateral then it is equiangular". Show that the statement $((p \rightarrow q) \wedge (q \leftrightarrow r)) \rightarrow (p \rightarrow r)$ is tautology and obtain the principle disjunctive normal form of $(\sim p \rightarrow r) \wedge (q \leftrightarrow p)$. 2+4+4

3(a) Determine whether the given graph has Hamilton circuit. If it does, find such a circuit. 2+4





(b) Define the following graph with example (any TWO): 2+2

(i) Regular graph (ii) Compete graph (iii) Bipartite graph

Contd. ...

4(a) Define generating functions. List out their areas of application. 2+2

(b) Solve the recurrence relation: 6

$F_n=5F_{n-1}-6F_{n-2}$ where

$F_0=1$ and $F_1=4$.

5(a) State multiplication principle of counting. In how many ways can 7 women be seated in a row if 2+5

(i) any person may sit next to any other?

(ii) men and women must occupy alternate seat?.

(b) Define composition of relation with an example. 3

6(a) If R be a relation on a set of integers Z defined by R = $\{(x,y): x, y \varepsilon Z, (x,y)$ is divisible by 6$\}$. Then prove that R is an equivalence relation. 5

(b) Let R= $\{(1,1),(2,1),(3,2),(4,3)\}$ be a relation on a set A = $\{1,2,3,4\}$. Find the powers R^n, $n \geq 1$. 5

7(a) Define mathematical induction. Prove that the statement given below is true using mathematical induction: 1+4

$$1^3+2^3+3^3+ \text{-----} +n^3 = \frac{n^2(n+1)^2}{4}$$

(b) Define walk, path and cycle with examples. 5

8(a) Give the formal definition of regular expression and grammar. 4

(b) Describe finite state automata with an example. 6

9(a) Describe the Euler circuit and path with example. 5

(b) Write short notes on: 2.5+2.5

(i) Fibonacci member (ii) Logical equivalent

10. Prove the validity of the following argument If I get the job and work hard, then I will get promoted. If I get promoted then I will be happy. I will not be happy. Therefore, either I will not get the job or I will not work hard'. 10

PURBANCHAL UNIVERSITY
2018

B.E.(Computer)/Fourth Semester/*Final*

Time: 03:00 hrs. Full Marks: 80 /Pass Marks: 32

BEG274CO: Discrete Structure *(New Course)*

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All questions carry equal marks. The marks allotted for each sub-question is specified along its side.

Answer EIGHT questions. **8×10=80**

1(a) State pigeonhole principle. Find the minimum number of students in a class to be sure that four out of them are horn in the same month. 5

(b) A collection of 10 electric bulbs contain 3 defective ones. (i) In how many ways can a sample of four bulbs be selected? (ii) In how many ways can a sample of four bulbs be selected which contain 2 good bulbs and 2 defective one? 5

2(a) What do you mean by valid argument? Prove the validity of the argument. "If the market is free then there is no inflation. If there is no inflation then there are price controls. Since there are price controls therefore the market is free." 1+6

(b) Describe the law of modes ponen with example. 3

3(a) Consider the DFA M defined by the following next state table. 6

(i) Find the input smbo1s of M. the state set of M. thc initial state of M and accepting state of M.

(ii) Draw the state diagram of M.

	δ		
S \ Σ	a	b	c
S_0	S_1	S_0	S_2
S_1	S_0	S_3	S_0
S_2	S_3	S_2	S_0
S_3	S_1	S_0	S_1

(b) Give a formal definition of a regular expression and grammar. 4

4(a) Let A={1, 2, 3,.4} for e relation R whose matrix is given. Find the mat ix of transitiv closure by using warshall's algorithm. 5

$$M_R = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(b) Define partition set with an example. 2

(c) Let R be a relation from A to B and let S and T be relations from B to C. Prove or disprove. 3

(SUT)oR=(SoR)U(ToR)

5(a) What do you mean by generating function? Give example. 3

(b) Solve the recurrence relation: $a_{r+2} - 2a_{r+1} + a_r = 2^r$ given that $a_0 = 2$ and $a_1 = 1$. 7

6(a) Let R= {(1,2),(2,3),(3,1)} and A= {1,2,3}. Find reflexive. symmetric and transitive closure of R. using composition relation of R. 5

(b) Prove that if a and b are two positive integers, then gcd(a,b)×lcm(a,b)=ab. 5

7(a) Prove by mathematical induction that $1^2+2^2+3^2+$ ---- n^2 $= n(n+1)(2n+1)/6$. 5

(b) Define Euler circuit and path. Give an example of graph which is Euler circuit. 5

8(a) Define tautology, contradiction and contingency. Show that the statement ~(p↔q)≡((P∧~q)V(q∧~p)) is tautology. 3+4

(b) Define universal quantifier and existential quantifier. 3

9. Write short notes on any TWO: 5+5

(i) DNF and CNF (ii) Hamiltonian circuit

(iii) Transport network

PURBANCHAL UNIVERSITY
2019

B.E.(Computer)/Fourth Semester/*Final*
Time: 03:00 hrs. Full Marks: 80 /Pass Marks: 32

BEG274CO: Discrete Structure *(New Course)*

Candidates are required to give their answers in their own words as far as practicable.

All questions carry equal marks. The marks allotted for each sub-question is specified along its side.

Answer EIGHT questions. **8×10=80**

1(a) (a) Compute GCD(a,b) when a = 2385 and b=3233. If d= GCD (a,b), then express d =sa+tb for some integers s and t. 5

(b) State pigeonhole principle. How many friends must you have to guarantee at least 5 of them will have birthday in the same month? 2+3

2(a) Prove the statement using mathematical induction: $1^{2}+2^{2}+3^{2}+\ldots+x^{2}=\frac{x(x+1)(2x+1)}{6}$. 5

(b) In how many ways can 6 men and 6 women he seated in a row if (i) any person may sit next to any other? (ii) men and women occupy alternate seats?

3(a) Solve the recurrence relation $a_n = 2a_{n-1} + 31_{n-2} + 5^n, n \geq 2, given\ a_0 = -2, a_1 = 1$. 5

(b) Solve the recurrence relation by using generating functions $a_n = 4a_{n-1} - 3a_{n-2}, n \geq 2$, given $a_0=2, a_1=5$. 5

4(a) Define walk, path, trail and circuit. 4

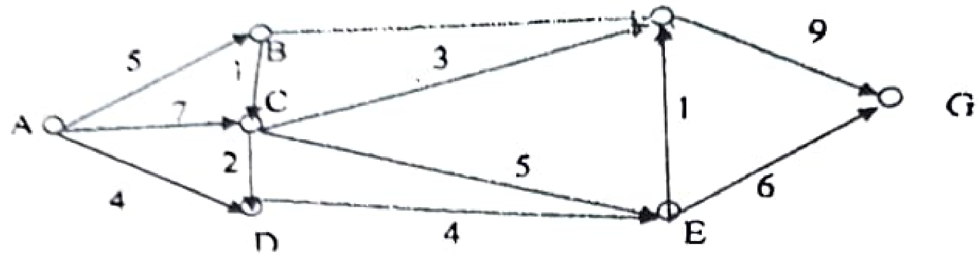
(b) Describe different types of a graph. 6

5(a) Define the transitive closure of a relation R on a set A. Let A={1,2,3,4} and R=1(1,2), (2,3),(3,4),(2,1)}. Find the transitive closure of R by using Warshall's Algorithm. 2+5

(b) Define subgraph with an example. 3

Contd. ...

6. What is a transport network? Find the maximum flow in the given network by using the labeling algorithm. 3+7



7(a) Define grammar. Draw the state diagram for the finite state machine with the state table. 1+4

State	input	
	0	1
S_0	S_1	S_0
S_1	S_2	S_0
S_2	S_0	S_3
S_3	S_1	S_3

(b) Define regular expression. Write down the English sentence for regular expression given below: 2+3

(i) $aa^* \, bb^* \, cc^*$ (ii) $(a+b)^* \, ab \, (a+b)^*$ (iii) $ab \, (a+b)^* \, ab$

8(a) Determine the validity of the following argument by constructing a truth table.

If I study, then I will pass.

If I do not go to movie, then I will study

I failed.

Therefore, I went to movie

(b) What do you mean by logical equivalence of two propositions? Show that $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$. 5

9(a) Express p => $(q \wedge r)$ in disjunctive normal form. 5

(b) Let A={x/x is a positive integer <8}

B{x/x is an integer $2 \leq x \leq 4$}

C{x/x is an integer such that $x^2 < 16$}, Then find (i) AU $(B \cap C)$ (ii) A-B. 5

PURBANCHAL UNIVERSITY

2021

B.E.(Computer)/Fourth Semester/ *Final*

Time: 03:00 hrs. Full Marks: 80 / Pass Marks: 32

BEG274CO: Discrete Structure *(New Course)*

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All questions carry equal marks. The marks allotted for each sub-question is specified along its side.

Answer EIGHT questions. **8×10=80**

1(a) State pigeonhole principle. There are 15 staffs in the office. Find the minimum number of staffs that can have their joining in the same months. 2+3

(b) In a group of 7 boys and 5 girls, four children are to be selected. In how many different ways can they be selected such that at least one boy should be there? 5

2(a) What do you mean by valid argument? Prove the validity of the argument "If the market is free then there is no inflation. If there is no inflation then there are price controls. Since there are price controls therefore, the market is free" 1+6

(b) Write the inverse. converse and, contrapositive of the statement "If it is raining then the game is cancelled? 3

3(a) Define tautology and contradiction. Show that the statement. 2+4

[(p->(q->r)] ->[(p->q) -> (p->r] is tautology or not.

(b) Prove by mathematical induction: 4

$$1^1 + 2^2 + 3^3 + \ldots\ldots + n^2 = \frac{n(n+1)(2n+1)}{6}.$$

4(a) What is normal form? Obtain he conjunctive normal form of 1+4

$\neg((\neg p \rightarrow \neg q) \wedge r)$.

(b) Find GCD of 190 and 34 using Euclidean algorithm. 5

5(a) Let A= {1 , 2, 3, 4}, for the relation R whose matrix is given. Find the matrix of transitive closure by using warshall's algorithm. 2+5

Contd. ...

$$M_R = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

(b) Let A= {a,b} , R= {(a, a)(b, a)(b, b) and S= (a, b)(b, a)(b,b) then verify $(SoR)^{-1}$ = $(SoR)^{-1} = R^{-1}oS^{-1}$. 3

6(a) Let X {1,2 ....7) and R {x,y}:x-y is divisible by 3}, show that R is an equivalence relation. 3

(b) Solve the recurrence relation $a_{n+2}-5a_{n+1}+6a_n=2$, given that $a_0=1$ and $a_1=-1$. 7

7(a) Define walk, path and circuit with example. 6

(b) Define Hamiltonian circuit and path with example. 4

8(a) Describe types of graph with examples. 6

(b) Define regular expression and grammar. 4

9. Write short notes on any TWO: 5+5

(a) Generating function

(b) Euler path

(c) Finite state machine.

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