Assignment (Including all the Unit ) for final Evaluation of Internal Marks

**Date of Submission: August 31, 2021**

1. Transform the equation $x^2 + y^2 = x$ to cylindrical coordinates.
2. Transform the equation $x^2 + y^2 - z^2 = 4$ by using Spherical polar coordinates.
3. Transform the equation $x^2 + y^2 = 4x$ to polar coordinates.
4. Prove that the distance between two points in a plane with polar coordinates $P(r_1, \theta_1)$ and $Q(r_2, \theta_2)$ is given by $d^2 = {r_1}^2 + {r_2}^2 - 2r_1 r_2 \cos(\theta_1 - \theta_2)$, where $d$ is the distance between the points $P$ and $Q$.
5. Define conic. When does it become ellipse?
6. Define eccentricity of conic. What would you conclude if $e < 1$.
7. Define eccentricity of conic. What would you conclude if $e > 1$.
8. Find the centre, eccentricity, foci and directories of the hyperbola

$$9x^2 - 16y^2 + 72x - 32y - 16 = 0$$

9. If $\vec{a} = \vec{\imath} + 3\vec{\jmath} - \vec{k}$ and $\vec{b} = 2\vec{\imath} - \vec{\jmath} + 3\vec{k}$, find $\vec{a} \,.\, \vec{b}$ and $\vec{a} \times \vec{b}$.
10. If $\vec{a} = \overrightarrow{2\imath} - \vec{\jmath} + 3\vec{k}$ and $\vec{b} = \vec{\imath} + 2\vec{\jmath} - 3\vec{k}$, find $\vec{a} \,.\, \vec{b}$ and $\vec{a} \times \vec{b}$.
11. Find the parametric equations of the line joining the points $P_1(1, 1, 0)$ and $P_2(0, 2, 3)$.
12. If $\vec{a} = \vec{\imath} - 2\vec{\jmath} + 3\vec{k}$ and $\vec{b} = 2\vec{\imath} + \vec{\jmath} - \vec{k}$, find $\vec{a} \,.\, \vec{b}$ and $\vec{a} \times \vec{b}$.
13. Find the equation of a plane containing the point $P(1, 2, 3)$ and perpendicular to the line containing $A(5, 2, 1)$ and $B(2, 1, -1)$.
14. Find the equation of a plane containing the point $P(1, 2, 3)$ and perpendicular to the line containing $A(5, 2, 1)$ and $B(2,\ 2, -1)$.
15. Find the volume of the parallelepiped whose concurrent edges are represented by $3\vec{\imath} - 3\vec{\jmath} + 3\vec{k}$, $\vec{\imath} + 2\vec{\jmath} - \vec{k}$, and $3\vec{\imath} - \vec{\jmath} + 2\vec{k}$.
16. Find the value of $c$ so that the lines $\frac{x-1}{-3} = \frac{y-1}{2c} = \frac{z-3}{2}$ and $\frac{x-1}{3c} = \frac{y-5}{1} = \frac{z-6}{-5}$ are perpendicular to each other.
17. Given $A = (-1, 1, 2), B = (0, 1, 3), C = (2, 3, 4)$ and $D = (-1, 3, 3)$, find the volume of the parallelepiped with $\overrightarrow{AB}$, $\overrightarrow{AC}$ and $\overrightarrow{AD}$ as three of its edges.
18. State Mean Value Theorem.
19. State Rolle's Theorem.
20. State Generalized Mean Value Theorem.
21. Compute the area bounded by a loop of the lemniscate $r^2 = a^2 \cos 2\theta$.
22. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ when $z = x^3 + 3x^2 y - y^3$.
23. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ when $z = x^3 - 3x^2 y + y^3$.
24. Define Orthogonal matrix with an example.
25. Define Skew-Hermitian matrix with an example.
26. Solve by Gauss-Jordan method the following system of equations:

$$3x + 6y - 3z = 6$$
$$x + 2y - z = 2$$
$$x + 3y - 2z = 1$$

27. Solve by Gauss-Jordan method the following system of equations:

$$3x + 6y + z = 18$$
$$x + 2y - z = 2$$
$$x + 3y - 2z = 1$$

28. Solve by Cramer's rule, the following system of equations:

$$3x + 6y + z = 18$$
$$x + 2y - z = 2$$
$$x + 3y - 2z = 1$$

29. Compute the area bounded by a loop of the lemniscate $r^2 = a^2 \cos 2\theta$.
30. Evaluate: $\lim\limits_{x \to \frac{\pi}{2}} (\sin x)^{\tan x}$
31. Evaluate: $\lim\limits_{x \to 1} (1 - x) \tan \frac{\pi x}{2}$
32. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ when $z = x^3 + y^3 + 3axy$.
33. Find the coordinates of the vertices, the eccentricity and the coordinates of the foci of the hyperbola $9x^2 - 16y^2 + 36x + 32y - 124 = 0$.
34. Find the maximum and minimum values of the function defined by $f(x) = \frac{40}{3x^4 - 8x^3 - 18x^2 + 60}$.
35. Show that at any point of the parabola $y^2 = 4ax$, the subnormal is constant and the subtangent varies as the abscissa of the point of contact.
36. Find the radius of curvature at any point of the parabola $y^2 = 4ax$ and hence show that the radius of curvature at its vertex is equal to its semi-latus rectum.

37. The area bounded by the parabola $y^2 = 4x$ and the straight line $4x - 3y + 2 = 0$ is rotated about the $Y$ −axis. Show that the volume of the solid formed is given by $\frac{\pi}{20}$.
38. State Euler's Theorem. Verify Euler's theorem for the function $f(x, y) = x^3 + y^3 + 3x^2y$.
39. Find the asymptotes of the curve: $y^3 - x^2y + 2y^2 + 4y + x = 0$.
40. Find the shortest distance between the two skew lines $\frac{x-8}{3} = \frac{y+9}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$.
41. If $\vec{a} = \vec{\imath} - 2\vec{\jmath} + \vec{k}, \vec{b} = 2\vec{\imath} + \vec{\jmath} + \vec{k}$ and $\vec{c} = \vec{\imath} + 2\vec{\jmath} - \vec{k}$ then verify that
$$\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \,.\, \vec{c})\vec{b} - (\vec{a} \,.\, \vec{b})\vec{c}.$$
42. If $\vec{a} = \vec{\imath} - 2\vec{\jmath} + \vec{k}, \vec{b} = \vec{\imath} + 2\vec{\jmath} - \vec{k}$ and $\vec{c} = 2\vec{\imath} + \vec{\jmath} + \vec{k}$ then verify that
$$\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \,.\, \vec{c})\vec{b} - (\vec{a} \,.\, \vec{b})\vec{c}.$$
43. Expand $e^{\sin x}$ by Maclaurin's theorem as far as the term containing $x^4$.
44. Find the coordinates of the vertices, the eccentricity and the coordinates of the foci of the hyperbola $9x^2 - 16y^2 + 36x + 32y - 124 = 0$.

45. Find the radius of curvature at any point of the parabola $y^2 = 4ax$ and hence show that the radius of curvature at its vertex is equal to its semi-latus rectum.
46. Find the area bounded by the curve $x^2 = 4y$ and the straight line $x = 4y - 2$.
47. State Euler's Theorem. Verify Euler's theorem for the function $f(x,y) = x^3 - 3x^2y + 5xy^2 - y^3$.
48. Find the asymptotes of the curve: $y^3 - 6xy^2 + 11x^2y - 6x^3 + x + y = 0$
49. Solve by Cramer's rule or Gauss-Jordan method, the following system of equations:

$$x + 2y - z = 2$$
$$3x + 6y + z = 1$$
$$3x + 3y + 2z = 3$$

50. For all vectors $\vec{a}, \vec{b}, \vec{c}$, prove that: $\vec{a} \times (\vec{b} \times \vec{c}) = \vec{b} \times (\vec{c} \times \vec{a}) = \vec{c} \times (\vec{a} \times \vec{b}) = 0$.
51. State Maclaurin's Theorem. Show that $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ \dots \ + \frac{x^n}{n!} \sin\frac{n\pi}{2} - \ \dots$ .
52. Transform the equation $x^2 - z^2 = 4$ by using Spherical polar coordinates.
53. Find the volume of the sphere $x^2 + y^2 + z^2 = a^2$.
54. Find the total length of the curve given by $x = a\cos^3\theta\,, y = a\sin^3\theta$
55. Transform the equation $x^2 - z^2 = 4$ by using Spherical polar coordinates.
56. Show that at any point of the hyperbola $xy = c^2$,the subtangent varies as the abscissa and the subnormal varies as the cube of the ordinate of the point contact.
57. Find the radius of curvature at any point of the parabola $y^2 = 4ax$ and hence show that the radius of curvature at its vertex is equal to its semi-latus rectum.
58. Find the asymptotes of the curve: $y^3 - x^2y + 2y^2 + 4y + x = 0$

59. The area bounded by the parabola $y^2 = 4x$ and the straight line $4x - 3y + 2 = 0$ is rotated about the $Y$ −axis. Show that the volume of the solid formed is given by $\frac{\pi}{20}$

60. Find the total length of the curve given by $x = a\cos^3\theta\,, y = a\sin^3\theta$
61. State Euler's Theorem. Verify Euler's theorem for the function $f(x,y) = x^3 + y^3 + 3x^2y$.

62. Find the shortest distance between the two skew lines $\frac{x-8}{3} = \frac{y+9}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$.
63. State Maclaurin's theorem. Expand $e^{\sin x}$ by Maclaurin's theorem as far as the term containing $x^4$.