Mathematics: Practice set: 001

1. Change to rectangular coordinates for $\left(3, \frac{\pi}{4}\right)$.
2. Define skew-symmetric matrix with an example.
3. Find the eccentricity of the ellipse: $4x^2 + 9y^2 = 36$.
4. Obtain the parametric equation of the line joining the points $P(1, 1, 0)$ and $Q(0, 2, 3)$.
5. Find the unit vector perpendicular to two vectors $\vec{a} = \vec{\imath} - \vec{\jmath} + \vec{k}$ and $\vec{b} = \overrightarrow{2\imath} - 3\vec{\jmath} - \vec{k}$.
6. Evaluate: $\lim_{x \to 0} \frac{a^x - 1}{x}$
7. Find the radius of curvature at the point (x, y) of the curve $y^2 = 4ax$.
8. Find the area bounded by $y = \sin x$, the $x-$ axis and the ordinates at $x = 0$ and $x = 2\pi$.
9. Verify Euler's theorem for the function $f(x, y) = x^3 + 3x^2y + 5xy^2 - y^3$.
10. Change to spherical polar and cylindrical polar coordinates for $x^2 + y^2 + z = 0$
11. Find the equation of the plane through the point $(2, 3, 4)$ and parallel to the plane $2x - 6y - 7z = 6$.
12. Find the image of the point $(1, 2, 5)$ in the plane $2x - y - z + 3 = 0$.
13. Solve the following system of linear equations, by using matrix inversion method:

$$4x + 9y - z = 14$$
$$5x + 8y - 2z = 28$$
$$x + 2y - z = -1$$

14. Find the maximum and minimum value of the function $f(x) = 2x^3 - 15x^2 - 36x + 10$.
15. Prove that $\frac{x}{a} - \frac{y}{b} = 1$ touches the curve $y = be^{-\frac{x}{a}}$ at the point where the curve crosses the axis of y.
16. Find the entire length of the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$.
17. Find the asymptotes of the curve $y^3 - x^2y + 2y^2 + 4y + x = 0$.
18. Expand $e^{\sin x}$ by Maclaurin's theorem up to the term containing x^4.
19. The ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is revolved about $x-$ axis. Find the volume of the solid thus generated.
20. Find the equation of line of intersection of two planes $3x + y + 2z = 2$ and $x + 2y + 4z = 8$.