Vectors and Solid Geometry

Jnaneshwar Chalise
M.A. (Mathematics)

Virtual Class,Mathematics - I

Bachelor of Information Technology

Kathmandu, Nepal

Vector and Vector Algebra

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Scalar and Vector quantities

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Quantity that can be characterized by magnitude only is known as *a scalar quantity* or simply a *scalar*.
Quantity which can be characterized by magnitude as well as direction is known as *a vector quantity* or simply *a vector*.

The concept of a vector

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Notation

Generally, vectors are denoted by bold faced type of letters. But due to inconvenience of this method to indicate the vector in writing, generally a vector will be represented by a letter or a combination of two letters with an arrow over it. But a scalar is denoted by the same letter or letters without any arrow over it.

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Representation

A vector is represented by a directed line segment. The initial point of the line segment representing a vector is known as the *origin* and end point is known as the *terminal point* or *terminus*.

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- **Zero Vector:** A zero vector is a vector with magnitude zero and its direction is indeterminate. In a zero vector, the origin and the terminal point coincide.
- **Negative of a vector:** The negative of a vector $\vec{a}$ denoted by $-\vec{a}$ is a vector whose magnitude is same as that of $\vec{a}$ and whose direction is opposite to $\vec{a}$.

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- **Localised vectors:** A vectors, passing through a given point and parallel to the given vector, is said to be a localised vector.

The concept of a vector

Vectors in terms of coordinates

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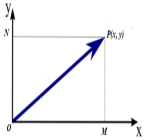
Vectors in terms of coordinates

Let OX and OY the two mutually perpendicular lines, represent $x-$axis and $y-$axis respectively.



Let $P(x, y)$ be a point on the plane such that
$OM =$ projection of OP on $x-$axis $= x$
$MP = ON$ projection of OP on $y-$axis $= y$

The concept of a vector

To displace from O to M and then from M to P is same as to displace from O to P. That is a horizontal displacement $\vec{OM}$ together with a vertical displacement $\vec{MP}$ gives the displacement $\vec{OP}$. So, $\vec{OP}$ is defined by an order pair (x, y). Thus $\vec{OP}= (x, y)$. Here $\vec{OP}$ is said to be the position vector of P and also known as plane vector.

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In the same way $\vec{OP} = (x, y, z)$ is the vector represented by the directed line segment OP where x, y and z are the projections of OP on $x-$axis, $y-$axis and $z-$axis respectively. In this case the vector known as space vector.

Addition and Subtraction of vectors

Addition and Subtraction of vectors

Addition of two vectors

Let $\vec{a}= (a_1, a_2, a_3)$ and $\vec{b}= (b_1, b_2, b_3)$ be two vectors, then the sum of the vectors $\vec{a}$ and $\vec{b}$ is defined by

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$$\vec{a} + \vec{b} = (a_1, a_2, a_3) + (b_1, b_2, b_3) = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$$

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$\vec{a} - \vec{b} = (a_1, a_2, a_3) - (b_1, b_2, b_3) = (a_1 - b_1, a_2 - b_2, a_3 - b_3)$

Some Important Result

Multiplication of vector by a scalar

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Multiplication of a vector $\overrightarrow{a}$ by a scalar k denoted by $k\overrightarrow{a}$ is a vector whose magnitude is k times that of $\overrightarrow{a}$ and whose direction is same as $\overrightarrow{a}$ if $k > 0$ and is in the opposite direction if $k < 0$.

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Collinear Vectors

Any number of vectors are said to be collinear when all of them are parallel to the same line whatever their magnitudes may be. Any vector $\vec{r}$ collinear with a given vector $\vec{a}$ can be expressed as $k\vec{a}$ where k is a scalar.

Some Important Result

Coplanar and non-coplanar vectors

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A system of vectors is said to be coplanar if a plane can be drawn parallel to all of them. Otherwise, they are said to be non-coplanar.

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- If $\vec{a}$ and $\vec{b}$ be two non-zero and non-collinear vectors and x, y the scalars such that $x\vec{a} + y\vec{b} = 0$ then $x = 0, y = 0$

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- If $\vec{a}, \vec{b}, \vec{c}$ be three non-zero, non-coplanar vectors and if x, y, z be three scalars such that $x\vec{a} + y\vec{b} + z\vec{c} = 0$ then, $x = y = z = 0$

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- Any vector $\vec{r}$ in the space can uniquely be expressed as the sum of the three non-coplanar vectors parallel to $\vec{a}, \vec{b}, \vec{c}$ as $\vec{r} = x\vec{a} + y\vec{b} + z\vec{c}$ where x, y, z are scalars

Resolution of vectors

Resolution of vectors

Rectangular resolution of a vector

$\vec{i}$, $\vec{j}$, $\vec{k}$, the unit vectors along with three rectangular axes are defined by

$\vec{i} = (1, 0, 0)$, $\vec{j} = (0, 1, 0)$ and $\vec{k} = (0, 0, 1)$ respectively.

Then the vector $\vec{a} = (a_1, a_2, a_3)$ can be written as

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Magnitude of a vector

If $\vec{r} = x \vec{i} + y \vec{j} + z \vec{k}$ then the magnitude of $\vec{r}$ is denoted by $| \vec{r} |$ and defined as $| \vec{r} | = \sqrt{x^2 + y^2 + z^2}$

Resolution of vectors

Direction cosines of a line

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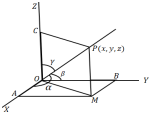

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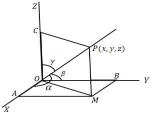

If α, β, γ are the angles made by the line OP with three mutually perpendicular lines OX, OY and OZ respectively, then $\cos\alpha, \cos\beta$ and $\cos\gamma$ are said to be the direction cosines of the line OP

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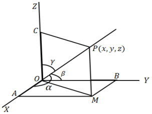

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m=cos$\beta = \frac{y}{\sqrt{x^2+y^2+z^2}}$

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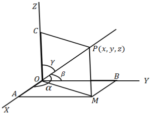

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Since $\vec{OP} = x\,\vec{i} + y\,\vec{j} + z\,\vec{k}$

The unit vector along $\vec{OP}$ is denoted by $\widehat{OP}$and defined by $\widehat{OP} = \frac{\vec{OP}}{|\vec{OP}|}$

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and direction cosines are
$\frac{x_2-x_1}{d}, \frac{y_2-y_1}{d}, \frac{z_2-z_1}{d}$ Where $d = |\overrightarrow{PQ}|$

Some Solved Problems

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Q. If the position vector of M and N are $3\vec{i} + \vec{j} - 3\vec{k}$ and $4\vec{i} - 2\vec{j} + \vec{k}$ respectively, find $\overrightarrow{MN}$ and determine its direction cosines.

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Let O be the origin.

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Now, $\vec{MN}=\vec{ON}-\vec{OM}=\vec{i}-3\vec{j}+4\vec{k}$

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and $|\vec{MN}|=\sqrt{(1)^2+(-3)^2+(4)^2}=\sqrt{26}$

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$\widehat{MN}=\frac{\vec{MN}}{|\vec{MN}|}$

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and $|\vec{MN}|=\sqrt{(1)^2+(-3)^2+(4)^2}=\sqrt{26}$

$\widehat{MN}=\frac{\vec{MN}}{|\vec{MN}|}=\frac{1}{\sqrt{26}}\vec{i}-\frac{3}{\sqrt{26}}\vec{j}+\frac{4}{\sqrt{26}}\vec{k}$

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$\therefore$ the direction cosines of the line MN are:

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Some solved Problems

Q. Three vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ are given by $\vec{a} = \vec{i} - 2\vec{j} + 3\vec{k}$ $\vec{b} = \vec{i} - 3\vec{j}$ $\vec{c} = 2\vec{i} - \vec{j} + 5\vec{k}$. Evaluate $\vec{a} + \vec{b} + \vec{c}$, $\vec{a} - 2\vec{b}$. Also find the unit vectors $\widehat{a}$, $\widehat{b}$ and $\widehat{c}$

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Given, $\vec{a} = \vec{i} - 2\vec{j} + 3\vec{k}$ $\vec{b} = \vec{i} - 3\vec{j}$ $\vec{c} = 2\vec{i} - \vec{j} + 5\vec{k}$

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Then, $\vec{a} + \vec{b} + \vec{c} = 4\vec{i} - 6\vec{j} + 8\vec{k}$

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$\vec{a} - 2\vec{b}$

Some solved Problems

Q. Three vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ are given by $\vec{a} = \vec{i} - 2\vec{j} + 3\vec{k}$ $\vec{b} = \vec{i} - 3\vec{j}$ $\vec{c} = 2\vec{i} - \vec{j} + 5\vec{k}$. Evaluate $\vec{a} + \vec{b} + \vec{c}$, $\vec{a} - 2\vec{b}$. Also find the unit vectors $\hat{a}, \hat{b}$ and $\hat{c}$

Given, $\vec{a} = \vec{i} - 2\vec{j} + 3\vec{k}$ $\vec{b} = \vec{i} - 3\vec{j}$ $\vec{c} = 2\vec{i} - \vec{j} + 5\vec{k}$

Then, $\vec{a} + \vec{b} + \vec{c} = 4\vec{i} - 6\vec{j} + 8\vec{k}$

$\vec{a} - 2\vec{b} = (\vec{i} - 2\vec{j} + 3\vec{k}) - 2(\vec{i} - 3\vec{j})$

Some solved Problems

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$\vec{a} + \vec{b} + \vec{c}$, $\vec{a} - 2\vec{b}$. Also find the unit vectors $\widehat{a}, \widehat{b}$ and $\widehat{c}$

Given, $\vec{a} = \vec{i} - 2\vec{j} + 3\vec{k}$ $\vec{b} = \vec{i} - 3\vec{j}$ $\vec{c} = 2\vec{i} - \vec{j} + 5\vec{k}$
Then, $\vec{a} + \vec{b} + \vec{c} = 4\vec{i} - 6\vec{j} + 8\vec{k}$
$\vec{a} - 2\vec{b} = (\vec{i} - 2\vec{j} + 3\vec{k}) - 2(\vec{i} - 3\vec{j}) = -\vec{i} + 4\vec{j} + 3\vec{k}$

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Further, $|\vec{a}| = \sqrt{14}$,

Some solved Problems

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Some solved Problems

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Some solved Problems

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Further, $|\vec{a}| = \sqrt{14}$, $|\vec{b}| = \sqrt{10}$, $|\vec{c}| = \sqrt{30}$

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Then, $\vec{a}+\vec{b}+\vec{c}=4\vec{i}-6\vec{j}+8\vec{k}$

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$$\widehat{a}=\frac{\vec{a}}{|\vec{a}|}=\frac{1}{\sqrt{14}}\vec{i}-\frac{2}{\sqrt{14}}\vec{j}+\frac{3}{\sqrt{14}}\vec{k}$$

$$\widehat{b}=\frac{\vec{b}}{|\vec{b}|}=\frac{1}{\sqrt{10}}\vec{i}-\frac{3}{\sqrt{10}}\vec{j}$$

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$$\widehat{b} = \frac{\vec{b}}{|\vec{b}|} = \frac{1}{\sqrt{10}}\vec{i} - \frac{3}{\sqrt{10}}\vec{j}$$

$$\widehat{c} = \frac{\vec{c}}{|\vec{c}|} = \frac{2}{\sqrt{30}}\vec{i} - \frac{1}{\sqrt{30}}\vec{j} + \frac{5}{\sqrt{30}}\vec{k}$$

Some Solved Problems

Q. Find all values of λ such that $(x, y, z) \neq (0, 0, 0)$and $(\vec{i} + \vec{j} + 3\vec{k})x + (3\vec{i} - 3\vec{j} + \vec{k})y + (-4\vec{i} + 5\vec{j})z = \lambda(x\vec{i} + y\vec{j} + z\vec{j})$ where $\vec{i}, \vec{j}, \vec{k}$ are unit vectors along the coordinate axes.

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$$[(1-\lambda)x + 3y - 4z]\vec{i} + [x - (3+\lambda)y + 5z]\vec{j} + [3x + y - \lambda z]\vec{k} = 0$$

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$(1-\lambda)x + 3y - 4z = 0$

x-(3+λ)$y + 5z = 0$

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3x+y-$\lambda z = 0$

For a nontrivial solution, we must have

Some Solved Problems

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$$\begin{vmatrix} 1-\lambda & 3 & -4 \\ 1 & -(3+\lambda) & 5 \\ 3 & 1 & -\lambda \end{vmatrix} = 0$$

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On simplification we gets: $\lambda = -1$ and $\lambda = 0$

Some Solved Problems

Q. Two points A and B have position vectors $\vec{a}$ and $\vec{b}$ relative to O as origin. If a point R divides the distance AB in the ratio $m : n$, find the position vector of R.

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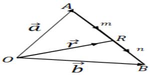

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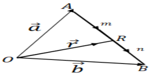

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$$\frac{\vec{AR}}{\vec{RB}} = \frac{m}{n}$$

From the triangle OAR we have,

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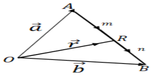

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From the triangle OAR we have, $\vec{AR}=\vec{OR} - \vec{OA}=\vec{r} - \vec{a}$

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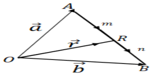

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Similarly, from the triangle triangle OBR we have,

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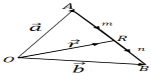

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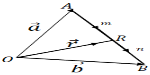

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Some Solved Problems

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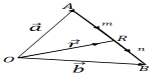

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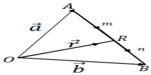

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Thus,

$\frac{\vec{AR}}{\vec{RB}} = \frac{m}{n} \Rightarrow \frac{\vec{r} - \vec{a}}{\vec{b} - \vec{r}} = \frac{m}{n}$

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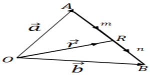

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Thus,

$\frac{\vec{AR}}{\vec{RB}} = \frac{m}{n} \Rightarrow \frac{\vec{r} - \vec{a}}{\vec{b} - \vec{r}} = \frac{m}{n} \Rightarrow \vec{r} = \frac{n\vec{a} + m\vec{b}}{m+n}$

Some Solved Problems

Q. If $\vec{a} = 2\vec{i} - \vec{j} + \vec{k}$, $\vec{b} = \vec{i} + 3\vec{j} - 2\vec{k}$, $\vec{c} = -2\vec{i} + \vec{j} - 3\vec{k}$ and $\vec{d} = 3\vec{i} + 2\vec{j} - 5\vec{k}$, find the scalars p, q, r such that $\vec{d} = p\vec{a} + q\vec{b} + r\vec{c}$, and also show that $p = q + r$.

Some Solved Problems

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We have, $\vec{d} = p\vec{a} + q\vec{b} + r\vec{c}$

Some Solved Problems

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We have, $\vec{d} = p\vec{a} + q\vec{b} + r\vec{c}$

$$\Rightarrow 3\vec{i} + 2\vec{j} - 5\vec{k} = p(2\vec{i} - \vec{j} + \vec{k}) + q(\vec{i} + 3\vec{j} - 2\vec{k})$$
$$+\text{r}(-2\vec{i} + \vec{j} - 3\vec{k})$$

Some Solved Problems

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$$\Rightarrow 3\vec{i} + 2\vec{j} - 5\vec{k} = (2p + q - 2r)\vec{i} + (-p + 3q + r)\vec{j} + (p - 2q - 3r)\vec{k}$$

Some Solved Problems

Q. If $\vec{a} = 2\vec{i} - \vec{j} + \vec{k}$, $\vec{b} = \vec{i} + 3\vec{j} - 2\vec{k}$, $\vec{c} = -2\vec{i} + \vec{j} - 3\vec{k}$ and $\vec{d} = 3\vec{i} + 2\vec{j} - 5\vec{k}$, find the scalars p, q, r such that $\vec{d} = p\vec{a} + q\vec{b} + r\vec{c}$, and also show that $p = q + r$.

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Equating the coefficient of like vectors, we obtain

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$\Rightarrow 3\vec{i} + 2\vec{j} - 5\vec{k} = (2p + q - 2r)\vec{i} + (-p + 3q + r)\vec{j} + (p - 2q - 3r)\vec{k}$

Equating the coefficient of like vectors, we obtain

$2p + q - 2r = 3,$

Some Solved Problems

Q. If $\vec{a} = 2\vec{i} - \vec{j} + \vec{k}$, $\vec{b} = \vec{i} + 3\vec{j} - 2\vec{k}$, $\vec{c} = -2\vec{i} + \vec{j} - 3\vec{k}$ and $\vec{d} = 3\vec{i} + 2\vec{j} - 5\vec{k}$, find the scalars p, q, r such that $\vec{d} = p\vec{a} + q\vec{b} + r\vec{c}$, and also show that $p = q + r$.

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Equating the coefficient of like vectors, we obtain

$2p + q - 2r = 3, \quad -p + 3q + r = 2,$

Some Solved Problems

Q. If $\vec{a} = 2\vec{i} - \vec{j} + \vec{k}$, $\vec{b} = \vec{i} + 3\vec{j} - 2\vec{k}$, $\vec{c} = -2\vec{i} + \vec{j} - 3\vec{k}$ and $\vec{d} = 3\vec{i} + 2\vec{j} - 5\vec{k}$, find the scalars p, q, r such that $\vec{d} = p\vec{a} + q\vec{b} + r\vec{c}$, and also show that $p = q + r$.

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Equating the coefficient of like vectors, we obtain

$2p + q - 2r = 3, \quad -p + 3q + r = 2, \quad p - 2q - 3r = -5$

Some Solved Problems

Q. If $\vec{a} = 2\vec{i} - \vec{j} + \vec{k}$, $\vec{b} = \vec{i} + 3\vec{j} - 2\vec{k}$, $\vec{c} = -2\vec{i} + \vec{j} - 3\vec{k}$ and $\vec{d} = 3\vec{i} + 2\vec{j} - 5\vec{k}$, find the scalars p, q, r such that $\vec{d} = p\vec{a} + q\vec{b} + r\vec{c}$, and also show that $p = q + r$.

We have, $\vec{d} = p\vec{a} + q\vec{b} + r\vec{c}$

$$\Rightarrow 3\vec{i} + 2\vec{j} - 5\vec{k} = p(2\vec{i} - \vec{j} + \vec{k}) + q(\vec{i} + 3\vec{j} - 2\vec{k}) + r(-2\vec{i} + \vec{j} - 3\vec{k})$$

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Equating the coefficient of like vectors, we obtain

$2p + q - 2r = 3, \quad -p + 3q + r = 2, \quad p - 2q - 3r = -5$

Solving we get, $p = -3, q = 1$ and $r = -4$.

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Q. If $\vec{a} = 2\vec{i} - \vec{j} + \vec{k}$, $\vec{b} = \vec{i} + 3\vec{j} - 2\vec{k}$, $\vec{c} = -2\vec{i} + \vec{j} - 3\vec{k}$ and $\vec{d} = 3\vec{i} + 2\vec{j} - 5\vec{k}$, find the scalars p, q, r such that $\vec{d} = p\vec{a} + q\vec{b} + r\vec{c}$, and also show that $p = q + r$.

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Equating the coefficient of like vectors, we obtain

$2p + q - 2r = 3, \quad -p + 3q + r = 2, \quad p - 2q - 3r = -5$

Solving we get, $p = -3, q = 1$ and $r = -4$. Further it is also true that $p = q + r$.

Some Solved Problems

Q. Show that the vectors $\vec{a} - 2\vec{b} + 3\vec{c}$, $-2\vec{a} + 3\vec{b} - 4\vec{c}$ and $-\vec{b} + 2\vec{c}$ are coplanar; $\vec{a}, \vec{b}, \vec{c}$ being any vectors.

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Let $\vec{r_1} = \vec{a} - 2\vec{b} + 3\vec{c}, \vec{r_2} = -2\vec{a} + 3\vec{b} - 4\vec{c}$ and $\vec{r_3} = -\vec{b} + 2\vec{c}$.

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$\vec{r_3} = x\vec{r_1} + y\vec{r_2}$ .....(i)

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$\Rightarrow -\vec{b} + 2\vec{c} = x(\vec{a} - 2\vec{b} + 3\vec{c}) + y(-2\vec{a} + 3\vec{b} - 4\vec{c})$

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$x - 2y = 0,$

Some Solved Problems

Q. Show that the vectors $\vec{a} - 2\vec{b} + 3\vec{c}$, $-2\vec{a} + 3\vec{b} - 4\vec{c}$ and $-\vec{b} + 2\vec{c}$ are coplanar; $\vec{a}$, $\vec{b}$, $\vec{c}$ being any vectors.

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$x - 2y = 0, \quad -2x + 3y = -1,$

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$\vec{r_3} = x\vec{r_1} + y\vec{r_2}$ .....(i)

$\Rightarrow -\vec{b} + 2\vec{c} = x(\vec{a} - 2\vec{b} + 3\vec{c}) + y(-2\vec{a} + 3\vec{b} - 4\vec{c})$ Equating the coefficient of like vectors, we obtain the following system of equation

$x - 2y = 0, \quad -2x + 3y = -1, \quad 3x - 4y = 2$

Some Solved Problems

Q. Show that the vectors $\vec{a} - 2\vec{b} + 3\vec{c}$, $-2\vec{a} + 3\vec{b} - 4\vec{c}$ and $-\vec{b} + 2\vec{c}$ are coplanar; $\vec{a}$, $\vec{b}$, $\vec{c}$ being any vectors.

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$x - 2y = 0, \quad -2x + 3y = -1, \quad 3x - 4y = 2$

Solving we obtain $x = 2, y = 1$.

Some Solved Problems

Q. Show that the vectors $\vec{a} - 2\vec{b} + 3\vec{c}$, $-2\vec{a} + 3\vec{b} - 4\vec{c}$ and $-\vec{b} + 2\vec{c}$ are coplanar; $\vec{a}, \vec{b}, \vec{c}$ being any vectors.

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$\vec{r_3} = x\vec{r_1} + y\vec{r_2}$ .....(i)

$\Rightarrow -\vec{b} + 2\vec{c} = x(\vec{a} - 2\vec{b} + 3\vec{c}) + y(-2\vec{a} + 3\vec{b} - 4\vec{c})$ Equating the coefficient of like vectors, we obtain the following system of equation

$x - 2y = 0, \quad -2x + 3y = -1, \quad 3x - 4y = 2$

Solving we obtain $x = 2, y = 1$. Hence from (i) $\vec{r_3} = 2\vec{r_1} + \vec{r_2}$

Hence the three vectors are coplanar.

Self Exercise

Self Exercise

Exercise:1

Self Exercise

Exercise:1

1. The vertices A, B, C of a triangle are $2\vec{i} - \vec{j} - 3\vec{k}$, $4\vec{i} + 2\vec{j} + 3\vec{k}$ and $6\vec{i} + 3\vec{j} + 4\vec{k}$ respectively, Compute $\vec{AB}$ and $\vec{AC}$. Also, show that $AB = 7$ and $AC = 9$

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2. If $\vec{OP} = \vec{i} + 3\vec{j} - 7\vec{k}$ and $\vec{OQ} = 5\vec{i} - 2\vec{j} + 4\vec{k}$, find $\vec{PQ}$ and determine its direction cosines.
3. If $\vec{a} = (3, -1, -4)$, $\vec{b} = (-2, 4, -3)$ and $\vec{c} = (-5, 7, -1)$ find the direction cosines of the vector $\vec{a} - 2\vec{b} + \vec{c}$.

Self Exercise

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2. If $\overrightarrow{OP} = \vec{i} + 3\vec{j} - 7\vec{k}$ and $\overrightarrow{OQ} = 5\vec{i} - 2\vec{j} + 4\vec{k}$, find $\overrightarrow{PQ}$ and determine its direction cosines.
3. If $\vec{a} = (3, -1, -4)$, $\vec{b} = (-2, 4, -3)$ and $\vec{c} = (-5, 7, -1)$ find the direction cosines of the vector $\vec{a} - 2\vec{b} + \vec{c}$.
4. Show that the following vectors are collinear: $\vec{i} + 2\vec{j} + 4\vec{k}$, $2\vec{i} + 5\vec{j} - \vec{k}$, $\overset{3\rightarrow}{i} + 8\vec{j} - 6\vec{k}$

Self Exercise

Self Exercise

Exercise:2

Self Exercise

Exercise:2

1. If the position vector of M and N are $3\vec{i}+\vec{j}-3\vec{k}$ and $4\vec{i}-2\vec{j}+\vec{k}$ respectively, find $\vec{MN}$ and determine its direction cosines.

Self Exercise

Exercise:2

1. If the position vector of M and N are $3\vec{i}+\vec{j}-3\vec{k}$ and $4\vec{i}-2\vec{j}+\vec{k}$ respectively, find $\vec{MN}$ and determine its direction cosines.
2. Find all values of λ such that $(x, y, z) \neq (0, 0, 0)$and $(\vec{i}+\vec{j}+3\vec{k})x+(3\vec{i}-3\vec{j}+\vec{k})y+(-4\vec{i}+5\vec{j})z=\lambda(x\vec{i}+y\vec{j}+z\vec{j})$ where $\vec{i}, \vec{j}, \vec{k}$ are unit vectors along the coordinate axes.

Self Exercise

Exercise:2

1. If the position vector of M and N are $3\vec{i}+\vec{j}-3\vec{k}$ and $4\vec{i}-2\vec{j}+\vec{k}$ respectively, find $\vec{MN}$ and determine its direction cosines.
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3. Three vectors $\vec{a},\vec{b},\vec{c}$ are given by $\vec{a}=\vec{i}-2\vec{j}+3\vec{k}$ $\vec{b}=\vec{i}-3\vec{j}$ $\vec{c}=2\vec{i}-\vec{j}+5\vec{k}$. Evaluate $\vec{a}+\vec{b}+\vec{c}$, $\vec{a}-2\vec{b}$. Also find the unit vectors $\widehat{a},\widehat{b}$ and $\widehat{c}$

Self Exercise

Self Exercise

Exercise:3

Self Exercise

Exercise:3

1. If $\vec{a} = 2\vec{i} - \vec{j} + \vec{k}$, $\vec{b} = \vec{i} + 3\vec{j} - 2\vec{k}$, $\vec{c} = -2\vec{i} + \vec{j} - 3\vec{k}$ and $\vec{d} = 3\vec{i} + 2\vec{j} - 5\vec{k}$, find the scalars p, q, r such that $\vec{d} = p\vec{a} + q\vec{b} + r\vec{c}$

Self Exercise

Exercise:3

1. If $\vec{a}= 2\vec{i} - \vec{j} + \vec{k}$, $\vec{b}= \vec{i} + 3\vec{j} - 2\vec{k}$, $\vec{c}= -2\vec{i} + \vec{j} - 3\vec{k}$ and $\vec{d}= 3\vec{i} + 2\vec{j} - 5\vec{k}$, find the scalars p, q, r such that $\vec{d}= p\vec{a} + q\vec{b} + r\vec{c}$

2. Two points A and B have position vectors $\vec{a}$ and $\vec{b}$ relative to O as origin. If a point R divides the distance AB in the ratio $m : n$, find the position vector of R

Self Exercise

Exercise:3

1. If $\vec{a} = 2\vec{i} - \vec{j} + \vec{k}$, $\vec{b} = \vec{i} + 3\vec{j} - 2\vec{k}$, $\vec{c} = -2\vec{i} + \vec{j} - 3\vec{k}$ and $\vec{d} = 3\vec{i} + 2\vec{j} - 5\vec{k}$, find the scalars p, q, r such that $\vec{d} = p\vec{a} + q\vec{b} + r\vec{c}$
2. Two points A and B have position vectors $\vec{a}$ and $\vec{b}$ relative to O as origin. If a point R divides the distance AB in the ratio $m : n$, find the position vector of R
3. Show that the vectors $\vec{a} - 2\vec{b} + 3\vec{c}$, $-2\vec{a} + 3\vec{b} - 4\vec{c}$ and $-\vec{b} + 2\vec{c}$ are coplanar; $\vec{a}, \vec{b}, \vec{c}$ being any vectors.

Self Exercise

Self Exercise

Exercise:4

Self Exercise

Exercise:4

1. Prove that the following vectors are coplanar:
$\vec{a} - 3\vec{b} + 5\vec{c}$, $\vec{a} - 2\vec{b} + 3\vec{c}$, $-2\vec{a} + 3\vec{b} - 4\vec{c}$

Self Exercise

Exercise:4

1. Prove that the following vectors are coplanar:
   $\vec{a} - 3\vec{b} + 5\vec{c}$, $\vec{a} - 2\vec{b} + 3\vec{c}$, $-2\vec{a} + 3\vec{b} - 4\vec{c}$
2. The points $A(2, 4, -1)$, $B(4, 5, 1)$ and $C(3, 6, -3)$ are the vertices of a triangle ABC. Find AB, BC, CA and show that the triangle ABC is a right-angled triangle. Find the direction cosines of $\vec{AB}$.

Self Exercise

Exercise:4

1. Prove that the following vectors are coplanar:
   $\vec{a} - 3\vec{b} + 5\vec{c}$, $\vec{a} - 2\vec{b} + 3\vec{c}$, $-2\vec{a} + 3\vec{b} - 4\vec{c}$
2. The points $A(2, 4, -1)$, $B(4, 5, 1)$ and $C(3, 6, -3)$ are the vertices of a triangle ABC. Find AB, BC, CA and show that the triangle ABC is a right-angled triangle. Find the direction cosines of $\vec{AB}$.
3. Three vectors of lengths $1, 2, 3$ units meeting at the corner of a cube are directed along the diagonals of its three faces meeting at the corner. Find their resultant.

Product of two vectors

Product of two vectors

Scalar or Dot Product of two vectors

Product of two vectors

Scalar or Dot Product of two vectors

Vector or Cross Product of two vector

Scalar or Dot Product of two Vectors

Scalar or Dot Product of two Vectors

Definition

Scalar or Dot Product of two Vectors

Definition

If $\vec{a} = (a_1, a_2, a_3)$ and $\vec{b} = (b_1, b_2, b_3)$ be two vectors then the scalar product of these two vectors denoted by

Scalar or Dot Product of two Vectors

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If $\vec{a} = (a_1, a_2, a_3)$ and $\vec{b} = (b_1, b_2, b_3)$ be two vectors then the scalar product of these two vectors denoted by $\vec{a} \,.\, \vec{b}$

Scalar or Dot Product of two Vectors

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$\vec{a} \,.\, \vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3.$

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Scalar or Dot Product of two Vectors

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$\vec{a} \, . \, \vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$.
In other words, If $\vec{a}$ and $\vec{b}$ be two vectors, then their scalars product denoted by $\vec{a} \, . \, \vec{b}$ defined by

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In other words, If $\vec{a}$ and $\vec{b}$ be two vectors, then their scalars product denoted by $\vec{a} \,.\, \vec{b}$ defined by

$\vec{a} \,.\, \vec{b} = |\vec{a}|\,|\vec{b}|\cos\theta = ab\cos\theta$

Scalar or Dot Product of two Vectors

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If $\vec{a} = (a_1, a_2, a_3)$ and $\vec{b} = (b_1, b_2, b_3)$ be two vectors then the scalar product of these two vectors denoted by $\vec{a} \, . \, \vec{b}$ is a scalar defined by

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In other words, If $\vec{a}$ and $\vec{b}$ be two vectors, then their scalars product denoted by $\vec{a} \, . \, \vec{b}$ defined by

$\vec{a} \, . \, \vec{b} = |\,\vec{a}\,|\,\vec{b}\,| \cos\theta = ab\cos\theta$

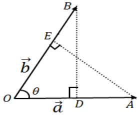
Where θ is the angle between two vectors $\vec{a}$ and $\vec{b}$.

Scalar or Dot Product of two vectors

Geometrical Interpretation of Scalar product of two vectors

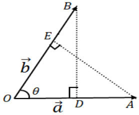
Scalar or Dot Product of two vectors

Geometrical Interpretation of Scalar product of two vectors



Scalar or Dot Product of two vectors

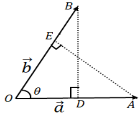
Geometrical Interpretation of Scalar product of two vectors



Let $\overrightarrow{OA} = \overrightarrow{a}$, $\overrightarrow{OB} = \overrightarrow{b}$ and θ be the angle between the two vectors.

Scalar or Dot Product of two vectors

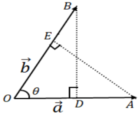
Geometrical Interpretation of Scalar product of two vectors



Let $\overrightarrow{OA} = \overrightarrow{a}$, $\overrightarrow{OB} = \overrightarrow{b}$ and θ be the angle between the two vectors. From A and B draw AD and BE perpendiculars to OB and OA respectively.

Scalar or Dot Product of two vectors

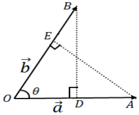
Geometrical Interpretation of Scalar product of two vectors



Let $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$ and θ be the angle between the two vectors. From A and B draw AD and BE perpendiculars to OB and OA respectively.
Now, $\vec{a} \,.\, \vec{b} = |\vec{a}||\vec{b}|\cos\theta = (OA)(OB\cos\theta) = (OA)(OE)$

Scalar or Dot Product of two vectors

Geometrical Interpretation of Scalar product of two vectors



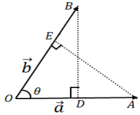
Let $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$ and θ be the angle between the two vectors. From A and B draw AD and BE perpendiculars to OB and OA respectively.

Now, $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\theta = (OA)(OB\cos\theta) = (OA)(OE)$

$\Rightarrow \vec{a} \cdot \vec{b} = (\textit{magnitude of } \vec{a})(\textit{projection of } \vec{b} \textit{ on } \vec{a})$

Scalar or Dot Product of two vectors

Geometrical Interpretation of Scalar product of two vectors



Let $\overrightarrow{OA} = \vec{a}, \overrightarrow{OB} = \vec{b}$ and θ be the angle between the two vectors. From A and B draw AD and BE perpendiculars to OB and OA respectively.

Now, $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\theta = (OA)(OB\cos\theta) = (OA)(OE)$

$\Rightarrow \vec{a} \cdot \vec{b} = (magnitude\ of\ \vec{a})(projection\ of\ \vec{b}\ on\ \vec{a})$

Similarly, $\vec{a} \cdot \vec{b} = (magnitude\ of\ \vec{b})(projection\ of\ \vec{a}\ on\ \vec{b})$

Conclusion: Scalar product of two vector is the product of magnitude of one vector and the projection of second vector on the first.

Scalar or Dot Product of two vectors

Relation between three mutually perpendicular unit vectors

Scalar or Dot Product of two vectors

Relation between three mutually perpendicular unit vectors

If $\vec{i}, \vec{j}, \vec{k}$ be three perpendicular unit vectors, then we have the following relation between them:

Scalar or Dot Product of two vectors

Relation between three mutually perpendicular unit vectors

If $\vec{i}, \vec{j}, \vec{k}$ be three perpendicular unit vectors, then we have the following relation between them:

- $\vec{i} \,.\, \vec{i} = \vec{j} \,.\, \vec{j} = \vec{k} \,.\, \vec{k} = 1$

Scalar or Dot Product of two vectors

Relation between three mutually perpendicular unit vectors

If $\vec{i}, \vec{j}, \vec{k}$ be three perpendicular unit vectors, then we have the following relation between them:

- $\vec{i} \cdot \vec{i} = \vec{j} \cdot \vec{j} = \vec{k} \cdot \vec{k} = 1$
- $\vec{i} \cdot \vec{j} = \vec{j} \cdot \vec{k} = \vec{k} \cdot \vec{i} = 0$

Scalar or Dot Product of two vectors

Relation between three mutually perpendicular unit vectors

If $\vec{i}, \vec{j}, \vec{k}$ be three perpendicular unit vectors, then we have the following relation between them:

- $\vec{i} \,.\, \vec{i} = \vec{j} \,.\, \vec{j} = \vec{k} \,.\, \vec{k} = 1$
- $\vec{i} \,.\, \vec{j} = \vec{j} \,.\, \vec{k} = \vec{k} \,.\, \vec{i} = 0$

Note: $\vec{a} \,.\, \vec{a} = |\vec{a}|^2 = a^2$

Scalar or Dot Product of two vectors

Properties of scalar product of two vectors

Scalar or Dot Product of two vectors

Properties of scalar product of two vectors

If $\vec{a}$, $\vec{b}$, $\vec{c}$ be any three vectors then

Scalar or Dot Product of two vectors

Properties of scalar product of two vectors

If $\vec{a}, \vec{b}, \vec{c}$ be any three vectors then

- $\vec{a} \,.\, \vec{b} = \vec{b} \,.\, \vec{a}$ (Commutative)

Scalar or Dot Product of two vectors

Properties of scalar product of two vectors

If $\vec{a}, \vec{b}, \vec{c}$ be any three vectors then

- $\vec{a} \,.\, \vec{b} = \vec{b} \,.\, \vec{a}$ (Commutative)
- $\vec{a} \,.(\vec{b} + \vec{c}) = \vec{a} \,.\, \vec{b} + \vec{a} \,.\, \vec{c}$ (Distributive)

Scalar or Dot Product of two vectors

Properties of scalar product of two vectors

If $\vec{a}, \vec{b}, \vec{c}$ be any three vectors then

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- $\vec{a} \,.(\vec{b} + \vec{c}) = \vec{a} \,.\, \vec{b} + \vec{a} \,.\, \vec{c}$ (Distributive)
- $m\vec{a} \,.n\vec{b} = mn(\vec{a} \,.\, \vec{b}) = mn\vec{a} \,.\, \vec{b} = \vec{a} \,.mn\vec{b}$ where m and n are scalars. (Associative)

Scalar or Dot Product of two vectors

Properties of scalar product of two vectors

If $\vec{a}$, $\vec{b}$, $\vec{c}$ be any three vectors then

- $\vec{a}\,.\,\vec{b} = \vec{b}\,.\,\vec{a}$ (Commutative)
- $\vec{a}\,.(\vec{b} + \vec{c}) = \vec{a}\,.\,\vec{b} + \vec{a}\,.\,\vec{c}$ (Distributive)
- $m\,\vec{a}\,.n\,\vec{b} = mn(\vec{a}\,.\,\vec{b}) = mn\,\vec{a}\,.\,\vec{b} = \vec{a}\,.mn\,\vec{b}$ where m and n are scalars. (Associative)

Some simple identities:

Scalar or Dot Product of two vectors

Properties of scalar product of two vectors

If $\vec{a}, \vec{b}, \vec{c}$ be any three vectors then

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Some simple identities:

- $(\vec{a} + \vec{b})^2 = a^2 + 2\vec{a} \,.\, \vec{b} + b^2$

Scalar or Dot Product of two vectors

Properties of scalar product of two vectors

If $\vec{a}$, $\vec{b}$, $\vec{c}$ be any three vectors then

- $\vec{a} \,.\, \vec{b} = \vec{b} \,.\, \vec{a}$ (Commutative)
- $\vec{a} \,.(\vec{b} + \vec{c}) = \vec{a} \,.\, \vec{b} + \vec{a} \,.\, \vec{c}$ (Distributive)
- $m\vec{a} \,.n\vec{b} = mn(\vec{a} \,.\, \vec{b}) = mn\vec{a} \,.\, \vec{b} = \vec{a} \,.mn\vec{b}$ where m and n are scalars. (Associative)

Some simple identities:

- $(\vec{a} + \vec{b})^2 = a^2 + 2\,\vec{a} \,.\, \vec{b} + b^2$
- $(\vec{a} - \vec{b})^2 = a^2 - 2\,\vec{a} \,.\, \vec{b} + b^2$

Scalar or Dot Product of two vectors

Properties of scalar product of two vectors

If $\vec{a}$, $\vec{b}$, $\vec{c}$ be any three vectors then

- $\vec{a}\,.\,\vec{b} = \vec{b}\,.\,\vec{a}$ (Commutative)
- $\vec{a}\,.(\vec{b} + \vec{c}) = \vec{a}\,.\,\vec{b} + \vec{a}\,.\,\vec{c}$ (Distributive)
- $m\vec{a}\,.n\vec{b} = mn(\vec{a}\,.\,\vec{b}) = mn\vec{a}\,.\,\vec{b} = \vec{a}\,.mn\vec{b}$ where m and n are scalars. (Associative)

Some simple identities:

- $(\vec{a} + \vec{b})^2 = a^2 + 2\vec{a}\,.\,\vec{b} + b^2$
- $(\vec{a} - \vec{b})^2 = a^2 - 2\vec{a}\,.\,\vec{b} + b^2$
- $(\vec{a} + \vec{b}).(\vec{a} - \vec{b}) = a^2 - b^2$

Some Solved Problems

Some Solved Problems

Q. If $\vec{a} = \vec{i} + \vec{j} - 2\vec{k}$ and $\vec{b} = 2\vec{i} - \vec{j} - \vec{k}$ are any two vectors, find $a, b, \vec{a} \cdot \vec{b}$ and the angle between the two vectors.

Some Solved Problems

Q. If $\vec{a} = \vec{i} + \vec{j} - 2\vec{k}$ and $\vec{b} = 2\vec{i} - \vec{j} - \vec{k}$ are any two vectors, find a, b, $\vec{a} \cdot \vec{b}$ and the angle between the two vectors.

Given, $\vec{a} = \vec{i} + \vec{j} - 2\vec{k}$ and $\vec{b} = 2\vec{i} - \vec{j} - \vec{k}$

Some Solved Problems

Q. If $\vec{a} = \vec{i} + \vec{j} - 2\vec{k}$ and $\vec{b} = 2\vec{i} - \vec{j} - \vec{k}$ are any two vectors, find $a, b, \vec{a}\,.\,\vec{b}$ and the angle between the two vectors.

Given, $\vec{a} = \vec{i} + \vec{j} - 2\vec{k}$ and $\vec{b} = 2\vec{i} - \vec{j} - \vec{k}$

$a = |\vec{a}|$

Some Solved Problems

Q. If $\vec{a} = \vec{i} + \vec{j} - 2\vec{k}$ and $\vec{b} = 2\vec{i} - \vec{j} - \vec{k}$ are any two vectors, find $a, b, \vec{a} \cdot \vec{b}$ and the angle between the two vectors.

Given, $\vec{a} = \vec{i} + \vec{j} - 2\vec{k}$ and $\vec{b} = 2\vec{i} - \vec{j} - \vec{k}$

$a = |\vec{a}| = \sqrt{1+1+4}$

Some Solved Problems

Q. If $\vec{a} = \vec{i} + \vec{j} - 2\vec{k}$ and $\vec{b} = 2\vec{i} - \vec{j} - \vec{k}$ are any two vectors, find a, b, $\vec{a} \cdot \vec{b}$ and the angle between the two vectors.

Given, $\vec{a} = \vec{i} + \vec{j} - 2\vec{k}$ and $\vec{b} = 2\vec{i} - \vec{j} - \vec{k}$

$a = |\vec{a}| = \sqrt{1+1+4} = \sqrt{6}$

Some Solved Problems

Q. If $\vec{a} = \vec{i} + \vec{j} - 2\vec{k}$ and $\vec{b} = 2\vec{i} - \vec{j} - \vec{k}$ are any two vectors, find a, b, $\vec{a} \cdot \vec{b}$ and the angle between the two vectors.

Given, $\vec{a} = \vec{i} + \vec{j} - 2\vec{k}$ and $\vec{b} = 2\vec{i} - \vec{j} - \vec{k}$

$a = |\vec{a}| = \sqrt{1+1+4} = \sqrt{6}$ and

$b = |\vec{b}|$

Some Solved Problems

Q. If $\vec{a} = \vec{i} + \vec{j} - 2\vec{k}$ and $\vec{b} = 2\vec{i} - \vec{j} - \vec{k}$ are any two vectors, find a, b, $\vec{a} \,.\, \vec{b}$ and the angle between the two vectors.

Given, $\vec{a} = \vec{i} + \vec{j} - 2\vec{k}$ and $\vec{b} = 2\vec{i} - \vec{j} - \vec{k}$

$a = |\vec{a}| = \sqrt{1+1+4} = \sqrt{6}$ and

$b = |\vec{b}| = \sqrt{4+1+1}$

Some Solved Problems

Q. If $\vec{a} = \vec{i} + \vec{j} - 2\vec{k}$ and $\vec{b} = 2\vec{i} - \vec{j} - \vec{k}$ are any two vectors, find $a, b, \vec{a}\,.\,\vec{b}$ and the angle between the two vectors.

Given, $\vec{a} = \vec{i} + \vec{j} - 2\vec{k}$ and $\vec{b} = 2\vec{i} - \vec{j} - \vec{k}$

$a = |\vec{a}| = \sqrt{1+1+4} = \sqrt{6}$ and

$b = |\vec{b}| = \sqrt{4+1+1} = \sqrt{6}$

Some Solved Problems

Q. If $\vec{a} = \vec{i} + \vec{j} - 2\vec{k}$ and $\vec{b} = 2\vec{i} - \vec{j} - \vec{k}$ are any two vectors, find a, b, $\vec{a} \cdot \vec{b}$ and the angle between the two vectors.

Given, $\vec{a} = \vec{i} + \vec{j} - 2\vec{k}$ and $\vec{b} = 2\vec{i} - \vec{j} - \vec{k}$

$a = |\vec{a}| = \sqrt{1+1+4} = \sqrt{6}$ and

$b = |\vec{b}| = \sqrt{4+1+1} = \sqrt{6}$

$\vec{a} \cdot \vec{b}$

Some Solved Problems

Q. If $\vec{a} = \vec{i} + \vec{j} - 2\vec{k}$ and $\vec{b} = 2\vec{i} - \vec{j} - \vec{k}$ are any two vectors, find $a, b, \vec{a}\,.\,\vec{b}$ and the angle between the two vectors.

Given, $\vec{a} = \vec{i} + \vec{j} - 2\vec{k}$ and $\vec{b} = 2\vec{i} - \vec{j} - \vec{k}$

$a = |\vec{a}| = \sqrt{1+1+4} = \sqrt{6}$ and

$b = |\vec{b}| = \sqrt{4+1+1} = \sqrt{6}$

$\vec{a}\,.\,\vec{b} = (\vec{i} + \vec{j} - 2\vec{k}).(2\vec{i} - \vec{j} - \vec{k})$

Some Solved Problems

Q. If $\vec{a} = \vec{i} + \vec{j} - 2\vec{k}$ and $\vec{b} = 2\vec{i} - \vec{j} - \vec{k}$ are any two vectors, find a, b, $\vec{a} \cdot \vec{b}$ and the angle between the two vectors.

Given, $\vec{a} = \vec{i} + \vec{j} - 2\vec{k}$ and $\vec{b} = 2\vec{i} - \vec{j} - \vec{k}$

$a = |\vec{a}| = \sqrt{1+1+4} = \sqrt{6}$ and

$b = |\vec{b}| = \sqrt{4+1+1} = \sqrt{6}$

$\vec{a} \cdot \vec{b} = (\vec{i} + \vec{j} - 2\vec{k}).(2\vec{i} - \vec{j} - \vec{k}) = 2 - 1 + 2$

Some Solved Problems

Q. If $\vec{a} = \vec{i} + \vec{j} - 2\vec{k}$ and $\vec{b} = 2\vec{i} - \vec{j} - \vec{k}$ are any two vectors, find $a, b, \vec{a}\,.\,\vec{b}$ and the angle between the two vectors.

Given, $\vec{a} = \vec{i} + \vec{j} - 2\vec{k}$ and $\vec{b} = 2\vec{i} - \vec{j} - \vec{k}$

$a = |\vec{a}| = \sqrt{1+1+4} = \sqrt{6}$ and

$b = |\vec{b}| = \sqrt{4+1+1} = \sqrt{6}$

$\vec{a}\,.\,\vec{b} = (\vec{i} + \vec{j} - 2\vec{k}).(2\vec{i} - \vec{j} - \vec{k}) = 2 - 1 + 2 = 3$

Some Solved Problems

Q. If $\vec{a} = \vec{i} + \vec{j} - 2\vec{k}$ and $\vec{b} = 2\vec{i} - \vec{j} - \vec{k}$ are any two vectors, find a, b, $\vec{a} \cdot \vec{b}$ and the angle between the two vectors.

Given, $\vec{a} = \vec{i} + \vec{j} - 2\vec{k}$ and $\vec{b} = 2\vec{i} - \vec{j} - \vec{k}$

$a = |\vec{a}| = \sqrt{1+1+4} = \sqrt{6}$ and

$b = |\vec{b}| = \sqrt{4+1+1} = \sqrt{6}$

$\vec{a} \cdot \vec{b} = (\vec{i} + \vec{j} - 2\vec{k}).(2\vec{i} - \vec{j} - \vec{k}) = 2 - 1 + 2 = 3$

If θ be the angle between the two vectors, then

Some Solved Problems

Q. If $\vec{a} = \vec{i} + \vec{j} - 2\vec{k}$ and $\vec{b} = 2\vec{i} - \vec{j} - \vec{k}$ are any two vectors, find a, b, $\vec{a} \cdot \vec{b}$ and the angle between the two vectors.

Given, $\vec{a} = \vec{i} + \vec{j} - 2\vec{k}$ and $\vec{b} = 2\vec{i} - \vec{j} - \vec{k}$

$a = |\vec{a}| = \sqrt{1+1+4} = \sqrt{6}$ and

$b = |\vec{b}| = \sqrt{4+1+1} = \sqrt{6}$

$\vec{a} \cdot \vec{b} = (\vec{i} + \vec{j} - 2\vec{k}).(2\vec{i} - \vec{j} - \vec{k}) = 2 - 1 + 2 = 3$

If θ be the angle between the two vectors, then

$\cos\theta$

Some Solved Problems

Q. If $\vec{a} = \vec{i} + \vec{j} - 2\vec{k}$ and $\vec{b} = 2\vec{i} - \vec{j} - \vec{k}$ are any two vectors, find a, b, $\vec{a}\,.\,\vec{b}$ and the angle between the two vectors.

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$a = |\vec{a}| = \sqrt{1+1+4} = \sqrt{6}$ and

$b = |\vec{b}| = \sqrt{4+1+1} = \sqrt{6}$

$\vec{a}\,.\,\vec{b} = (\vec{i} + \vec{j} - 2\vec{k}).(2\vec{i} - \vec{j} - \vec{k}) = 2 - 1 + 2 = 3$

If θ be the angle between the two vectors, then

$\cos\theta = \frac{\vec{a}\,.\,\vec{b}}{ab}$

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$\cos\theta = \frac{\vec{a}\,.\,\vec{b}}{ab} = \frac{3}{\sqrt{6}\sqrt{6}} = \frac{1}{2}$
$\therefore \quad \theta = \frac{\pi}{3}$

Some Solved Problems

Q. Using scalar product, Find the unit vector, which is perpendicular to the two vectors $\vec{a} = 2\vec{i} + \vec{j} + 2\vec{k}$ and $\vec{b} = 3\vec{i} - \vec{j} - \vec{k}$

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Some Solved Problems

If $\vec{a} \, . \, \vec{b} = \vec{a} \, . \, \vec{c}$, show that either $a = 0$ or $b = c$ or $\vec{a}$ is orthogonal to $(\vec{b} - \vec{c})$

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So, we have $|\vec{a}||\vec{b}-\vec{c}|\cos\theta=0.....(*)$ Where θ is the angle between $\vec{a}$ and $(\vec{b}-\vec{c})$

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So, either $a=0$ or $b=c$ or the vectors $\vec{a}$ is orthogonal to $(\vec{b}-\vec{c})$

Some Solved Problems

Using the dot product, prove the law of cosines
$c^2 = a^2 + b^2 + 2ab\cos\theta$.

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Self Exercise

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Exercise:1

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1. If $\vec{a} = \vec{i} - 2\vec{j} + 3\vec{k}$ and $\vec{b} = \vec{i} + \vec{j} + 2\vec{k}$ are any two vectors, find a, b, $\vec{a} \cdot \vec{b}$ and the angle between the two vectors.

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2. If $\vec{a} = (3, 1, 2)$ and $\vec{b} = (2, -2, 4)$ are any two vectors, find a, b, $\vec{a} \cdot \vec{b}$ and the angle between the two vectors.

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Self Exercise

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Exercise:2

Self Exercise

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1. Using the dot product, prove the law of cosines $a^2 = b^2 + c^2 - 2bc\cos A$, in any triangle ABC.

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3. Using the dot product, prove the law of cosines $c^2 = a^2 + b^2 - 2ab\cos C$, in any triangle ABC.

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3. Using the dot product, prove the law of cosines $c^2 = a^2 + b^2 - 2ab\cos C$, in any triangle ABC.
4. Define scalar product of two vectors and give its geometrical meaning.

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4. Define scalar product of two vectors and give its geometrical meaning.
5. Define Scalar product of two vectors. Also derive the condition of orthogonality of them.

Self Exercise

Self Exercise

Exercise:3

Self Exercise

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1. Using scalar product, Find the unit vector, which is perpendicular to the two vectors $\vec{a} = 2\vec{i} + \vec{j} + 2\vec{k}$ and $\vec{b} = 3\vec{i} - \vec{j} - \vec{k}$

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2. If $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c}$, show that either $a = 0$ or $b = c$ or $\vec{a}$ is orthogonal to $(\vec{b} - \vec{c})$
3. Using Scalar product, Find the vector which is perpendicular to the two vectors $\vec{a} = \vec{i} - 3\vec{j} + 2\vec{k}$ and $\vec{b} = 2\vec{i} - \vec{j} + \vec{k}$ and is of length 5.

Vector or Cross product of two vectors

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The vector product of two vectors $\vec{a}$ and $\vec{b}$ denoted by $\vec{a} \times \vec{b}$ is a vector normal to the plane of $\vec{a}$ and $\vec{b}$ and the magnitude of $\vec{a} \times \vec{b}$ is $ab\sin\theta$ where a and b are the magnitude of $\vec{a}$ and $\vec{b}$, and θ the angle between them,

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Vector or Cross product of two vectors

Definition:

The vector product of two vectors $\vec{a}$ and $\vec{b}$ denoted by $\vec{a} \times \vec{b}$ is a vector normal to the plane of $\vec{a}$ and $\vec{b}$ and the magnitude of $\vec{a} \times \vec{b}$ is $ab\sin\theta$ where a and b are the magnitude of $\vec{a}$ and $\vec{b}$, and θ the angle between them, Symbolically, $\vec{a} \times \vec{b} = ab\sin\theta\ \vec{\eta}$, where $\vec{\eta}$ is the unit vector normal to the plane of $\vec{a}$ and $\vec{b}$

Note: $|\ \vec{a} \times \vec{b}\ | = ab\sin\theta$

Vector or Cross Product of two vectors

Geometrical interpretation of vector product of two vectors

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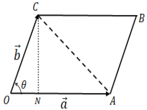

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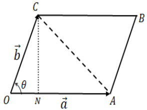

Let $\overrightarrow{OA} = \overrightarrow{a}, \overrightarrow{OC} = \overrightarrow{b}$ and $\angle AOC = \theta$.

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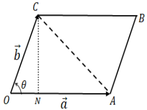

Let $\overrightarrow{OA}=\overrightarrow{a}$, $\overrightarrow{OC}=\overrightarrow{b}$ and $\angle AOC=\theta$. Draw a parallelogram $OABC$ with $OA=a$ and $OC=b$ as its adjacent sides.

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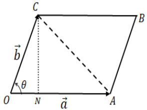

Let $\overrightarrow{OA} = \overrightarrow{a}, \overrightarrow{OC} = \overrightarrow{b}$ and $\angle AOC = \theta$. Draw a parallelogram $OABC$ with $OA = a$ and $OC = b$ as its adjacent sides. From C, draw CN perpendicular to OA.

$\overrightarrow{a} \times \overrightarrow{b} = ab \sin\theta \, \overrightarrow{\eta}$

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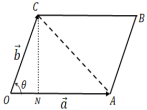

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$\overrightarrow{a} \times \overrightarrow{b} = ab \sin\theta \, \overrightarrow{\eta}$

$|\overrightarrow{a} \times \overrightarrow{b}| = ab \sin\theta = (OA)(OC)\sin\theta = (OA)(CN)$

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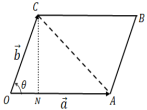

Let $\overrightarrow{OA}=\vec{a}$, $\overrightarrow{OC}=\vec{b}$ and $\angle AOC=\theta$. Draw a parallelogram $OABC$ with $OA=a$ and $OC=b$ as its adjacent sides. From C, draw CN perpendicular to OA.

$\vec{a}\times\vec{b}=ab\sin\theta\,\vec{\eta}$

$|\vec{a}\times\vec{b}|=ab\sin\theta=(OA)(OC)\sin\theta=(OA)(CN)$

$\therefore\ |\vec{a}\times\vec{b}|=$ area of the parallelogram $OABC$

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- $\vec{i} \times \vec{j} = \vec{k}, \vec{j} \times \vec{k} = \vec{i}, \vec{k} \times \vec{i} = \vec{j}$

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Vector product of mutually perpendicular unit vectors

If $\vec{i}, \vec{j}, \vec{k}$ be three mutually perpendicular unit vectors. Then

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- $\vec{i} \times \vec{j} = \vec{k}, \vec{j} \times \vec{k} = \vec{i}, \vec{k} \times \vec{i} = \vec{j}$
- $\vec{j} \times \vec{i} = -\vec{k}, \vec{k} \times \vec{j} = -\vec{i}, \vec{i} \times \vec{k} = -\vec{j}$

Vector or Cross Product of two vectors

Vector product of two vectors in the determinant form

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Vector product of two vectors in the determinant form

Let $\vec{a} = (a_1, a_2, a_3)$ and $\vec{b} = (b_1, b_2, b_3)$ be two given vectors. Then the vector product of $\vec{a}$ and $\vec{b}$ will be expressed in the determinant form as follows:

$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

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Note: $\vec{a} \times \vec{b} = (a_2 b_3 - a_3 b_2)\,\vec{i} - (a_1 b_3 - a_3 b_1)\,\vec{j} + (a_1 b_2 - a_2 b_1)\,\vec{k}$.

Some Solved Problems

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Q. Using the cross product of two vectors, find a unit vector perpendicular to $\vec{a}$ and $\vec{b}$, where $\vec{a} = 2\vec{i} + \vec{j} + 2\vec{k}$ and $\vec{b} = 3\vec{i} - \vec{j} - \vec{k}$.

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$\vec{a} \times \vec{b}$

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Now,

$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 1 & 2 \\ 3 & -1 & -1 \end{vmatrix}$$

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Further, $|\vec{a} \times \vec{b} = \sqrt{90}|$

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So, $\frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|} = \frac{\vec{i} + 8\vec{j} - 5\vec{k}}{\sqrt{90}}$ Which is the required unit vector

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Q. Find the area of the parallelogram determine by the vectors $\vec{i}+2\vec{j}+3\vec{k}$ and $-3\vec{i}-2\vec{j}+\vec{k}$

Given vectors are: $\vec{a}=\vec{i}+2\vec{j}+3\vec{k}$ and
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Now, $\vec{a}\times\vec{b}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 1 & 2 & 3\\ -3 & -2 & 1\end{vmatrix}=8\vec{i}-10\vec{j}+4\vec{k}$

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Now, $|\vec{a}\times\vec{b}|=6\sqrt{5}$

Thus area of parallelogram is $6\sqrt{5}$ square unit.

Some Solved Problems

If $\vec{a} + \vec{b} + \vec{c} = 0$, prove that
$\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$

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$\vec{a} \times (\vec{a} + \vec{b} + \vec{c}) = 0 \Rightarrow \vec{a} \times \vec{a} + \vec{a} \times \vec{b} + \vec{a} \times \vec{c} = 0$

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$\Rightarrow \vec{a}\times\vec{b}=-(\vec{c}\times\vec{a})$

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If $\vec{a} + \vec{b} + \vec{c} = 0$, prove that $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$

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Multiplying $(*)$ by $\vec{a}$ we get,

$\vec{a} \times (\vec{a} + \vec{b} + \vec{c}) = 0 \Rightarrow \vec{a} \times \vec{a} + \vec{a} \times \vec{b} + \vec{a} \times \vec{c} = 0$

$\Rightarrow \vec{a} \times \vec{b} = -(\vec{c} \times \vec{a}) \Rightarrow \vec{a} \times \vec{b} = \vec{c} \times \vec{a} ....(**)$

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Thus from $(**)$ and $(***)$ we obtain,

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Self Exercise

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1. Find the area of a parallelogram whose adjacent sides are determined by the vectors $\vec{i} - 2\vec{j} + 3\vec{k}$ and $2\vec{1} + 3\vec{k} - \vec{j}$

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3. Find a unit vector perpendicular to $\vec{a}$ and $\vec{b}$, where $\vec{a} = 2\vec{i} + \vec{j} + 2\vec{k}$ and $\vec{b} = 3\vec{i} - \vec{j} - \vec{k}$.

Equation of Line and Plane

Equation of Line and Plane

Equation of line

Equation of Line and Plane

Equation of line

Equation of Plane

Equation of Line

Standard Equation of Line

Equation of Line

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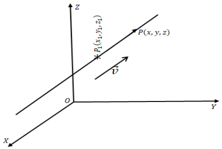
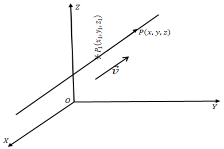


Equation of Line

Standard Equation of Line



Let L be a line in space passing through a given point $P_1(x_1, y_1, z_1)$ and is parallel to a given nonzero vector $\vec{v} = a\ \vec{i}\ + b\ \vec{j}\ + c\ \vec{k}$;

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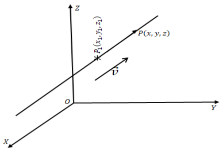

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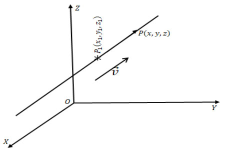

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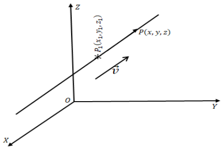

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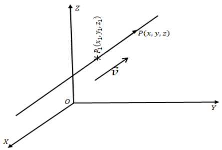

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$(x - x_1) = at, \quad (y - y_1) = bt, \quad (z - z_1) = ct \quad \ldots.(i)$ where t is a parameter.

Now eliminating t from (i) we get

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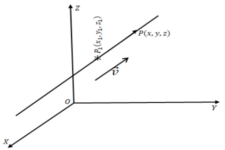

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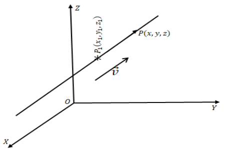

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Which is the standard form of the equation of the line.

Some Solved Problems

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Find the parametric equation of the line joining the points$P_1(1,1,0)$ and $P_2(0,2,3)$.

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We have the parametric equation as follow:

$x - 1 = t, \;\; y - 1 = t, \;\; z = 3t,$ Where t is a parameter and $P(x, y, z)$ is any point on the line.

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Find the value of c so that the lines $\frac{x-1}{-3} = \frac{y-1}{2c} = \frac{z-3}{2}$ and $\frac{x-1}{3c} = \frac{y-5}{1} = \frac{z-6}{-5}$ are perpendicular to each other.

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$\Rightarrow -3(3c) + 2c(1) + 2(-5) = 0$

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$\Rightarrow c = -\frac{10}{7}$

Self Exercise

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4. Derive the standard equation of line in a space(by using vector method).

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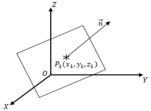

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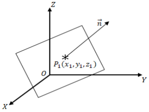

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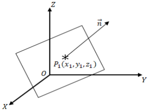

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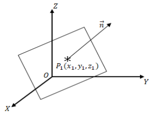

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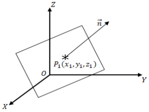

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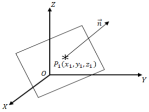

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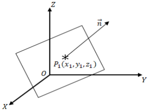

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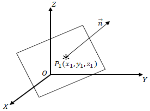

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In general the equation of plane can be written as

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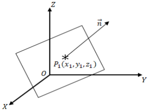

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$ax + by + cz + d = 0 \quad \ldots.eqn(1)$ where the vector $a\vec{i} + b\vec{j} + c\vec{k}$ is normal to the plane.

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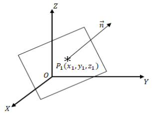

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Further $eqn(1)$ can be written as
$\frac{x}{A} + \frac{y}{B} + \frac{z}{C} = 1$ Where A, B, C are the intercepts made by plane with three coordinate planes.

Some Solved Problems

Find the equation of a plane containing the point$P(1, 2, 3)$ and perpendicular to the line containing $A(5, 2, 1)$ and $(B(2, 1, -1))$.

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Thus the required equation of plane is: $3x + y + 2z - 11 = 0$

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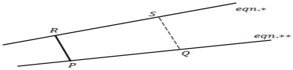

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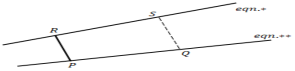

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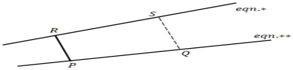

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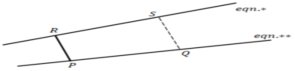

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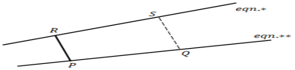

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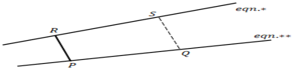

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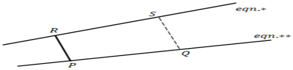

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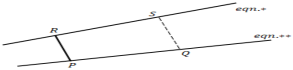

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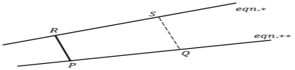

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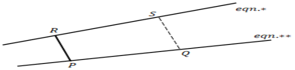

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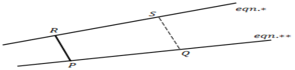

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Finally, Projection of $\vec{SQ}$ on $\vec{RP}=\left|\frac{(\vec{a}\times\vec{b}).\vec{SQ}}{|\vec{a}\times\vec{b}|}\right|$

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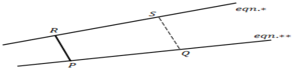

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Here, $\overrightarrow{a}\times\overrightarrow{b}=24\overrightarrow{i}+36\overrightarrow{j}+72\overrightarrow{k}$ and $|\overrightarrow{a}\times\overrightarrow{b}|=84$

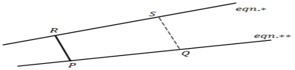
And, $(\overrightarrow{a}\times\overrightarrow{b}).\overrightarrow{SQ}=1176$

Finally, Projection of $\overrightarrow{SQ}$ on $\overrightarrow{RP}=\left|\frac{(\overrightarrow{a}\times\overrightarrow{b}).\overrightarrow{SQ}}{|\overrightarrow{a}\times\overrightarrow{b}|}\right|=14$

Some Solved Problems

Find the shortest distance between the two skew lines $\frac{x-8}{3}=\frac{y+9}{-16}=\frac{z-10}{7}$ and $\frac{x-15}{3}=\frac{y-29}{-16}=\frac{z-5}{-5}$

Given two skew lines are:
$\frac{x-8}{3}=\frac{y+9}{-16}=\frac{z-10}{7}......*$ and $\frac{x-15}{3}=\frac{y-29}{-16}=\frac{z-5}{-5}......**$



RP be the shortest distance between the given lines. Then RP is perpendicular to both the lines $*$ and $**$ and is the projection of SQ, Where the coordinates of Q and S are respectively $(15, 29, 5)$ and $(8, -9, 10)$

So, $\overrightarrow{SQ}= 7\overrightarrow{i}+38\overrightarrow{j}-5\overrightarrow{k}$ Now, the vector $\overrightarrow{a}=3\overrightarrow{i}-16\overrightarrow{j}+7\overrightarrow{k}$ is parallel to line $*$ and the vector $\overrightarrow{b}=3\overrightarrow{i}-+8\overrightarrow{j}-5\overrightarrow{k}$ is parallel to line $**$

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Finally, Projection of $\overrightarrow{SQ}$ on $\overrightarrow{RP}=\left|\frac{(\overrightarrow{a}\times\overrightarrow{b}).\overrightarrow{SQ}}{|\overrightarrow{a}\times\overrightarrow{b}|}\right|=14$

Thus the shortest distance between two given skew lines is14 units

Self Exercise

Exercise:1

Self Exercise

Exercise:1

1. Find the equation of plane passing through the three points $(-1, 1, 2), (2, 0, -3)$ and $(5, 1, 2)$.

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2. Find the equation of the plane passing through the point $(3, 3, 5)$ and having normal vector $2\vec{i} + \vec{j} + 2\vec{k}$.
3. Find the equation of the plane passing through the point $(1, 2, -1)$ and perpendicular to the planes $x + y - 2z = 5$ and $3x - y + 4z = 12$.

Self Exercise

Exercise: 2

Self Exercise

Exercise: 2

1. Find the equation of the plane containing the point $P(1, 2, 3)$ and perpendicular to the line containing points $(5, 2, 1)$ and $(2, 1, -1)$.

Self Exercise

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1. Find the equation of the plane containing the point $P(1, 2, 3)$ and perpendicular to the line containing points $(5, 2, 1)$ and $(2, 1, -1)$.
2. Find the distance of the point $(2, 3, 4)$ from the plane $3x - 6y + 2z + 11 = 0$

Self Exercise

Exercise: 2

1. Find the equation of the plane containing the point $P(1, 2, 3)$ and perpendicular to the line containing points $(5, 2, 1)$ and $(2, 1, -1)$.
2. Find the distance of the point $(2, 3, 4)$ from the plane $3x - 6y + 2z + 11 = 0$
3. Find the line of intersection of the two planes $x - 4y + 2z + 7 = 0$ and $3x + 2y - z - 2 = 0$.

Self Exercise

Exercise: 2

1. Find the equation of the plane containing the point $P(1, 2, 3)$ and perpendicular to the line containing points $(5, 2, 1)$ and $(2, 1, -1)$.
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3. Find the line of intersection of the two planes $x - 4y + 2z + 7 = 0$ and $3x + 2y - z - 2 = 0$.
4. Find the shortest distance between the two skew lines $\frac{x-8}{3} = \frac{y+9}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{-16} = \frac{z-5}{-5}$

Product of three or more vectors

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When we consider products of three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$. Then the following cases should arises:

- $(\vec{a} \,.\, \vec{b})\, \vec{c}$
- $\vec{a} \,.(\vec{b} \times \vec{c})$ Scalar triple product.
- $\vec{a} \times(\vec{b} \times \vec{c})$ Vector triple product.

Scalar Triple Product

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Definition

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The scalar product of two vectors, one of which being again a vector product of two vectors is a scalar and is known as the scalar triple product.

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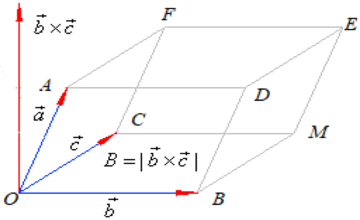
If $\vec{a}, \vec{b}$ and $\vec{a}$ are three vectors, then $\vec{b} \times \vec{c}$ is a vector perpendicular to the plane of $\vec{b}$ and $\vec{c}$. Again $\vec{a}$ and $\vec{b} \times \vec{c}$ both being vectors, their dot product denoted by $\vec{a} .(\vec{b} \times \vec{c})$ is a scalar and hence is known as the scalar triple product.

Scalar Triple Product

Scalar Triple Product

Geometrical Interpretation of Scalar Triple Product

Consider a parallelepiped with three concurrent edges OA, OB and OC which represent in magnitude and direction the three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ respectively.



Scalar Triple Product

Scalar Triple Product

Then $\vec{b} \times \vec{c}$ is the vector perpendicular to the plane of $\vec{OB}$ and $\vec{OC}$ whose magnitude is the area of the parallelogram $OBMC$. Then

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$\vec{a}\,.(\vec{b} \times \vec{c}) =$ (magnitude of $\vec{b} \times \vec{c}$)(projection of $\vec{a}$) on $\vec{b} \times \vec{c}$

Scalar Triple Product

Then $\vec{b} \times \vec{c}$ is the vector perpendicular to the plane of $\vec{OB}$ and $\vec{OC}$ whose magnitude is the area of the parallelogram $OBMC$. Then
$\vec{a}.(\vec{b} \times \vec{c}) =$ (magnitude of $\vec{b} \times \vec{c}$)(projection of $\vec{a}$) on $\vec{b} \times \vec{c}$
$=$ (Area of the parallelogram $OBMC$)(AT)

Scalar Triple Product

Then $\vec{b} \times \vec{c}$ is the vector perpendicular to the plane of $\vec{OB}$ and $\vec{OC}$ whose magnitude is the area of the parallelogram $OBMC$. Then
$\vec{a}.(\vec{b} \times \vec{c})$ = (magnitude of $\vec{b} \times \vec{c}$)(projection of $\vec{a}$) on $\vec{b} \times \vec{c}$
= (Area of the parallelogram $OBMC$)(AT)
= Area of the parallelogram $OBMC \times$ Height

Scalar Triple Product

Then $\vec{b} \times \vec{c}$ is the vector perpendicular to the plane of $\vec{OB}$ and $\vec{OC}$ whose magnitude is the area of the parallelogram $OBMC$. Then

$\vec{a}.(\vec{b} \times \vec{c}) =$ (magnitude of $\vec{b} \times \vec{c}$)(projection of $\vec{a}$) on $\vec{b} \times \vec{c}$

$=$ (Area of the parallelogram $OBMC$)(AT)

$=$ Area of the parallelogram $OBMC\times$ Height

$=$ Volume of the parallelepiped

Scalar Triple Product

Then $\vec{b} \times \vec{c}$ is the vector perpendicular to the plane of $\vec{OB}$ and $\vec{OC}$ whose magnitude is the area of the parallelogram $OBMC$. Then

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Scalar Triple Product

Then $\vec{b} \times \vec{c}$ is the vector perpendicular to the plane of $\vec{OB}$ and $\vec{OC}$ whose magnitude is the area of the parallelogram $OBMC$. Then

$\vec{a}\,.(\vec{b} \times \vec{c}) =$ (magnitude of $\vec{b} \times \vec{c}$)(projection of $\vec{a}$) on $\vec{b} \times \vec{c}$

$=$ (Area of the parallelogram $OBMC$)(AT)

$=$ Area of the parallelogram $OBMC\times$ Height

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$\therefore \vec{a}\,.(\vec{b} \times \vec{c}) = V$

Scalar Triple Product

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Notation

A special Symbol of abbreviation is conventionally used to denote the scalar triple product $\vec{a}.(\vec{b} \times \vec{c})$ by $(\vec{a}\ \vec{b}\ \vec{c})$ or $[\vec{a}\ \vec{b}\ \vec{c}]$ which indicates the three vectors and their cyclic order.

Scalar Triple Product

Scalar Triple Product in Determinant Form:

Scalar Triple Product

Scalar Triple Product in Determinant Form:

Let $A(a_1, a_2, a_3), B(b_1, b_2, b_3)$ and $C(c_1, c_2, c_3)$ be three points in space. Let O be the origin. Then

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$OA = \vec{a} = a_1 \vec{i} + a_2 \vec{j} + a_3 \vec{k}$

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Some Solved Problems

Some Solved Problems

Q. If $\vec{a} = 2\vec{i} - 3\vec{j} + \vec{k}$, $\vec{b} = \vec{i} - \vec{j} + 2\vec{k}$ and $\vec{c} = 2\vec{i} + \vec{j} - \vec{k}$. Compute $\vec{a}.(\vec{b} \times \vec{c})$.

Some Solved Problems

Q. If $\vec{a} = 2\vec{i} - 3\vec{j} + \vec{k}$, $\vec{b} = \vec{i} - \vec{j} + 2\vec{k}$ and $\vec{c} = 2\vec{i} + \vec{j} - \vec{k}$. Compute $\vec{a}.(\vec{b} \times \vec{c})$.

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$$\vec{a}.(\vec{b} \times \vec{c}) = \begin{vmatrix} 2 & -3 & 1 \\ 1 & -1 & 2 \\ 2 & 1 & -1 \end{vmatrix}$$

Some Solved Problems

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Some Solved Problems

Some Solved Problems

Q.Given $A = (-1, 1, 2)$, $B = (0, 1, 3)$, $C = (2, 3, 4)$ and $D = (-1, 3, 3)$, find the volume of the parallelepiped with $\vec{AB}$, $\vec{AC}$ and $\vec{AD}$ are three of its edges.

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Let $\overrightarrow{OA}$, $\overrightarrow{OB}$, $\overrightarrow{OC}$and $\overrightarrow{OD}$ are position vectors of given points. Then

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Let $\vec{OA}$, $\vec{OB}$, $\vec{OC}$and $\vec{OD}$ are position vectors of given points. Then

$\vec{AB}=\vec{OB} - \vec{OA}= (1, 0, 1)$

$\vec{AC}=\vec{OC} - \vec{OA}= (3, 2, 2)$

Some Solved Problems

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Let $\vec{OA}$, $\vec{OB}$, $\vec{OC}$and $\vec{OD}$ are position vectors of given points. Then

$\vec{AB}$=$\vec{OB}$ − $\vec{OA}$= $(1, 0, 1)$
$\vec{AC}$=$\vec{OC}$ − $\vec{OA}$= $(3, 2, 2)$
$\vec{AD}$=$\vec{OD}$ − $\vec{OA}$= $(1, 0, 1)$
are the three vectors of edges.

Some Solved Problems

We know, Volume $V = \overrightarrow{AB} \cdot (\overrightarrow{AC} \times \overrightarrow{AD})$

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We know, Volume $V = \overrightarrow{AB}.(\overrightarrow{AC} \times \overrightarrow{AD})$ So we have

$$V = \begin{vmatrix} 1 & 0 & 1 \\ 3 & 2 & 2 \\ 0 & 2 & 1 \end{vmatrix} = 4 \text{ cubic unit.}$$

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We know, Volume $V = \vec{AB} . (\vec{AC} \times \vec{AD})$ So we have

$$V = \begin{vmatrix} 1 & 0 & 1 \\ 3 & 2 & 2 \\ 0 & 2 & 1 \end{vmatrix} = 4 \text{ cubic unit.}$$

Q. Find the volume of the parallelepiped whose concurrent edges are represented by $3\vec{i} - 3\vec{j} + 3\vec{k}$, $\vec{i} + 2\vec{j} - \vec{k}$ and $3\vec{i} - \vec{j} + 2\vec{k}$

Some Solved Problems

Prove that the following four points are coplanar: $2\vec{i}+3\vec{j}-\vec{k}$, $\vec{i}-2\vec{j}+3\vec{k}$, $3\vec{i}+4\vec{j}-2\vec{i}$ and $\vec{i}-6\vec{j}+6\vec{k}$.

Some Solved Problems

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$$\vec{OA} = 2\vec{i}+3\vec{j}-\vec{k}$$
$$\vec{OB} = \vec{i}-2\vec{j}+3\vec{k}$$
$$\vec{OC} = 3\vec{i}+4\vec{j}-2\vec{k}$$
$$\vec{OD} = \vec{i}-6\vec{j}+6\vec{k}$$

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So we have,

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Now, $\vec{AB}.(\vec{BC} \times \vec{CD}) = \begin{vmatrix} -1 & -5 & 4 \\ 2 & 6 & -5 \\ -2 & -10 & 8 \end{vmatrix} = 0$

Which Shows that the given four points are coplanar.

Vector Triple Product

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For all vectors $\vec{a}, \vec{b}, \vec{c}$, Prove that
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Sphere, Cylinder, Cone & Quadratic Surface

Sphere

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When sphere passes through origin from equation (i) we get $k = 0$

Some Solved Problems on Sphere

When Sphere passes through $(a, 0, 0)$ from equation (i) we get $a(a + 2g) = 0$

Some Solved Problems on Sphere

When Sphere passes through $(a, 0, 0)$ from equation (i) we get $a(a + 2g) = 0$ but $a \neq 0$

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Thus from (i) substituting the values of g, f, h and k we obtain
$x^2 + y^2 + z^2 - ax - by - cz = 0.$ which is the required equation of sphere.

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- If the guiding curve is an ellipse the cylinder is called an *elliptic cylinder*.
- If the guiding curve is a hyperbola the cylinder is called a *hyperbolic cylinder*.

Cylinder

Theorem:

Cylinder

Theorem: An equation in Cartesian coordinates, from which a variable is missing, represents a cylinder parallel to the axis of the missing variable. The curve corresponding to this equation is the guiding curve of the cylinder.

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Proof: Left as an exercise.

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This equation represent an ellipse with centre at the origin and with semi-axes 4 and 2. The missing variable is z and therefore the axis of the cylinder is the $z-$ axis. Hence, the surface represent an elliptic cylinder.

Example: 2

Cylinder

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Example: 2

The surface $y^2 + 4z^2 = 16$ is an elliptic cylinder

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Example: 2

The surface $y^2 + 4z^2 = 16$ is an elliptic cylinder whose axis parallel to the $x-$ axis. The cross - section of the cylinder are ellipses with centre on the $x-$ axis

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This equation represent an ellipse with centre at the origin and with semi-axes 4 and 2. The missing variable is z and therefore the axis of the cylinder is the $z-$ axis. Hence, the surface represent an elliptic cylinder.

Example: 2

The surface $y^2 + 4z^2 = 16$ is an elliptic cylinder whose axis parallel to the $x-$ axis. The cross - section of the cylinder are ellipses with centre on the $x-$ axis and hence the $x-$ axis is the axis of the cylinder.

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If we take the circle $x^2 + y^2 = a^2$ in the $xy-$ plane as the guiding curve and any point on the $z-$ axis, say $(0, 0, h)$ as the vertex, the cone obtained is called the right - circular cone.

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The equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$
represent a surface known as *ellipsoid*.

Self Exercise:1

Parabola

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Parabola

1. Define conic. When does it becomes parabola?

Contact Information

Contact Information

E-mail : math.gyanu@gmail.com

Contact Information

E-mail : math.gyanu@gmail.com

Call: 00977 9857064657

Contact Information

E-mail : math.gyanu@gmail.com

Call: 00977 9857064657

https://www.facebook.com/jnaneshwar.chalise

Contact Information

E-mail : math.gyanu@gmail.com

Call: 00977 9857064657

https://www.facebook.com/jnaneshwar.chalise

If you have any queries regarding to this material, content or you need any help on this content; please feel free to contact me at any time.

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THANK YOU

JN Chalise