Chapter7 : Reasoning in Artificial intelligence

Reasoning:

The reasoning is the mental process of deriving logical conclusions and making predictions from available knowledge, facts, and beliefs. Or we can say, "**Reasoning is a way to infer facts from existing data**." It is a general process of thinking rationally, to find valid conclusions.

In artificial intelligence, reasoning is essential so that the machine can also think rationally as a human brain, and can perform like a human.

Types of Reasoning

In artificial intelligence, reasoning can be divided into the following categories:

- Deductive reasoning
- Inductive reasoning
- Abductive reasoning
- Common Sense Reasoning
- Monotonic Reasoning
- Non-monotonic Reasoning

> Note: Inductive and deductive reasoning are the forms of propositional logic.

1. Deductive reasoning:

- Deductive reasoning is deducing new information from logically related known information. It is the form of valid reasoning, which means the argument's conclusion must be true when the premises are true.
- Deductive reasoning is a type of propositional logic in AI, and it requires various rules and facts. It is sometimes referred to as top-down reasoning, and contradictory to inductive reasoning.
- In deductive reasoning, the truth of the premises guarantees the truth of the conclusion.

- Deductive reasoning mostly starts from the general premises to the specific conclusion, which can be explained as below example.

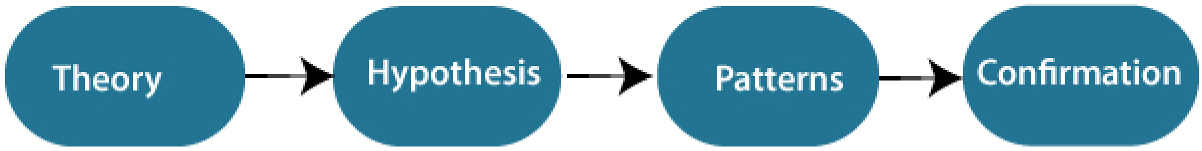
Example:

Premise-1: All the human eats veggies

Premise-2: Suresh is human.

Conclusion: Suresh eats veggies.

The general process of deductive reasoning is given below:



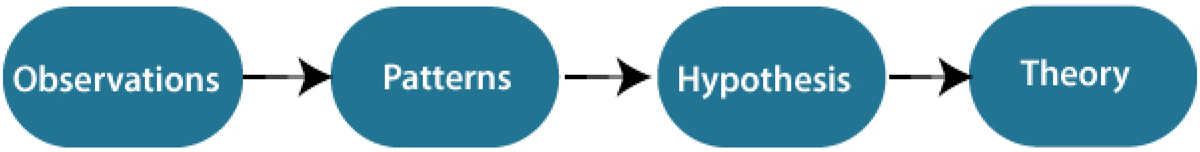
2. Inductive Reasoning:

- Inductive reasoning is a form of reasoning to arrive at a conclusion using limited sets of facts by the process of generalization. It starts with a series of specific facts or data and reaches a general statement or conclusion.
- Inductive reasoning is a type of propositional logic, which is also known as cause-effect reasoning or bottom-up reasoning.
- In inductive reasoning, we use historical data or various premises to generate a generic rule, for which premises support the conclusion.
- In inductive reasoning, premises provide probable support to the conclusion, so the truth of premises does not guarantee the truth of the conclusion.

Example:

Premise: All of the pigeons we have seen in the zoo are white.

Conclusion: Therefore, we can expect all the pigeons to be white.



3. Abductive reasoning:

- Abductive reasoning is a form of logical reasoning which starts with single or multiple observations then seeks to find the most likely explanation or conclusion for the observation.
- Abductive reasoning is an extension of deductive reasoning, but in abductive reasoning, the premises do not guarantee the conclusion.

Example:

Implication: Cricket ground is wet if it is raining

Axiom: Cricket ground is wet.

Conclusion It is raining.

4. Common Sense Reasoning

- Common sense reasoning is an informal form of reasoning, which can be gained through experiences.
- Common Sense reasoning simulates the human ability to make presumptions about events which occur on every day.
- It relies on good judgment rather than exact logic and operates on **heuristic knowledge** and **heuristic rules**.

Example:

1. **One person can be at one place at a time.**
2. **If I put my hand in a fire, then it will burn.**

The above two statements are examples of common sense reasoning which a human mind can easily understand and assume.

5. Monotonic Reasoning:

- In monotonic reasoning, once the conclusion is taken, then it will remain the same even if we add some other information to existing information in our knowledge base. In monotonic reasoning, adding knowledge does not decrease the set of prepositions that can be derived.
- To solve monotonic problems, we can derive the valid conclusion from the available facts only, and it will not be affected by new facts.
- Monotonic reasoning is not useful for the real-time systems, as in real time, facts get changed, so we cannot use monotonic reasoning.

- Monotonic reasoning is used in conventional reasoning systems, and a logic-based system is monotonic.
- Any theorem proving is an example of monotonic reasoning.

Example:

  - **Earth revolves around the Sun.**

It is a true fact, and it cannot be changed even if we add another sentence in the knowledge base like, "The moon revolves around the earth" Or "Earth is not round," etc.

Advantages of Monotonic Reasoning:

  - In monotonic reasoning, each old proof will always remain valid.
  - If we deduce some facts from available facts, then it will always remain valid.

Disadvantages of Monotonic Reasoning:

  - We cannot represent real world scenarios using Monotonic reasoning.
  - Hypothesis knowledge cannot be expressed with monotonic reasoning, which means facts should be true.
  - Since we can only derive conclusions from the old proofs, so new knowledge from the real world cannot be added.

6. Non-monotonic Reasoning

- In Non-monotonic reasoning, some conclusions may be invalidated if we add some more information to our knowledge base.
- Logic will be said as non-monotonic if some conclusions can be invalidated by adding more knowledge into our knowledge base.
- Non-monotonic reasoning deals with incomplete and uncertain models.
- "Human perceptions for various things in daily life, "is a general example of non-monotonic reasoning.

Example: Let suppose the knowledge base contains the following knowledge:

  - **Birds can fly**
  - **Penguins cannot fly**

- **Pitty is a bird**

So from the above sentences, we can conclude that **Pitty can fly**.

However, if we add one another sentence into the knowledge base "**Pitty is a penguin**", which concludes "**Pitty cannot fly**", it invalidates the above conclusion.

Advantages of Non-monotonic reasoning:

- For real-world systems such as Robot navigation, we can use non-monotonic reasoning.
- In Non-monotonic reasoning, we can choose probabilistic facts or can make assumptions.

Disadvantages of Non-monotonic Reasoning:

- In non-monotonic reasoning, the old facts may be invalidated by adding new sentences.
- It cannot be used for theorem **proving**.

Probabilistic reasoning in Artificial intelligence

Uncertainty in Reasoning:

Till now, we have learned knowledge representation using first-order logic and propositional logic with certainty, which means we were sure about the predicates. With this knowledge representation, we might write A→B, which means if A is true then B is true, but consider a situation where we are not sure about whether A is true or not then we cannot express this statement, this situation is called uncertainty.

So to represent uncertain knowledge, where we are not sure about the predicates, we need uncertain reasoning or probabilistic reasoning.

Causes of uncertainty:

Following are some leading causes of uncertainty to occur in the real world.

1. Information occurred from unreliable sources.
2. Experimental Errors
3. Equipment fault
4. Temperature variation
5. Climate change.

Probabilistic reasoning:

- Probabilistic reasoning is a way of knowledge representation where we apply the concept of probability to indicate the uncertainty in knowledge. In probabilistic reasoning, we combine probability theory with logic to handle the uncertainty.
- We use probability in probabilistic reasoning because it provides a way to handle the uncertainty that is the result of someone's laziness and ignorance.
- In the real world, there are lots of scenarios, where the certainty of something is not confirmed, such as "It will rain today," "behavior of someone for some situations," "A match between two teams or two players." These are probable sentences for which we can assume that it will happen but are not sure about it, so here we use probabilistic reasoning.

Need of probabilistic reasoning in AI:

- When there are unpredictable outcomes.
- When specifications or possibilities of predicates become too large to handle.
- When an unknown error occurs during an experiment.

In probabilistic reasoning, there are two ways to solve problems with uncertain knowledge:

- **Bayes' rule**
- **Bayesian Statistics**

> Note: We will learn the above two rules in later chapters.

As probabilistic reasoning uses probability and related terms, so before understanding probabilistic reasoning, let's understand some common terms:

Probability: Probability can be defined as a chance that an uncertain event will occur. It is the numerical measure of the likelihood that an event will occur. The value of probability always remains between 0 and 1 that represent ideal uncertainties.

1. $0 \leq P(A) \leq 1$, where P(A) is the probability of an event A.
1. P(A) = 0, indicates total uncertainty in an event A.
1. P(A) =1, indicates total certainty in an event A.

We can find the probability of an uncertain event by using the below formula.

$$\textbf{Probability of occurrence} = \frac{\text{Number of desired outcomes}}{\text{Total number of outcomes}}$$

- $P(\neg A)$ = probability of a not happening event.
- $P(\neg A) + P(A) = 1$.

Event: Each possible outcome of a variable is called an event.

Sample space: The collection of all possible events is called sample space.

Random variables: Random variables are used to represent the events and objects in the real world.

Prior probability: The prior probability of an event is probability computed before observing new information.

Posterior Probability: The probability that is calculated after all evidence or information has been taken into account. It is a combination of prior probability and new information.

Conditional probability:

Conditional probability is a probability of occurring an event when another event has already happened.

Let's suppose, we want to calculate the event A when event B has already occurred, "the probability of A under the conditions of B", it can be written as:

$$P(A|B) = \frac{P(A \wedge B)}{P(B)}$$

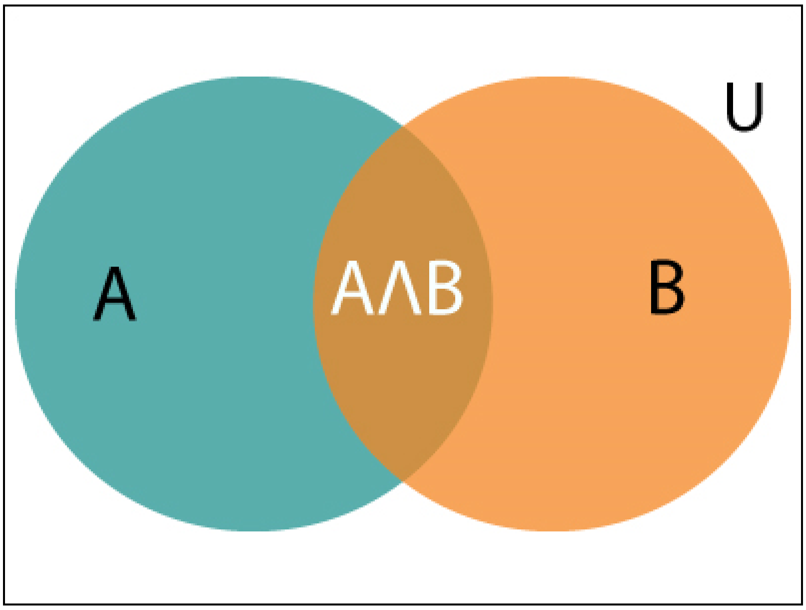
Where P(*A*∧*B*)= Joint probability of a and B

P(B)= Marginal probability of B.

If the probability of A is given and we need to find the probability of B, then it will be given as:

$$P(B|A) = \frac{P(A \wedge B)}{P(A)}$$

It can be explained by using the below Venn diagram, where B is occurred event, so sample space will be reduced to set B, and now we can only calculate event A when event B is already occurred by dividing the probability of **P(A∧*B*) by P(B)**.



Example:

In a class, there are 70% of the students who like English and 40% of the students who like English and mathematics, and then what is the percentage of students who like English and also like mathematics?

Solution:

Let, A is an event that a student likes Mathematics

B is an event where a student likes English.

$$P(A|B) = \frac{P(A \wedge B)}{P(B)} = \frac{0.4}{0.7} = 57\%$$

Hence, 57% of the students who like English also like Mathematics.

- ## Case Based Reasoning :

  Case-Based Reasoning(CBR) resolves new problems by adjusting previously fortunate solutions to similar problems. Roger Schank is widely held to be the beginning of CBR. He proposed an unalike sight on model-based reasoning stimulated by human logical and memory organization.

Basis of CBR :

Here, we will discuss the basic key parameters of CBR.

1. **Regularity-**

   The identical steps executed under the same circumstances will tend to have the same or alike outcomes.

2. **Typicality-**

   Experiences tend to repeat themselves.

3. **Consistency-**

   Minor switches in the circumstances require merely small changes in the explanation and in the effect.

4. **Adaptability-**

   When things replicate, the dissimilarities tend to be minute, and the small differences are uncomplicated to repay for.

Working Cycle of CBR :

Here, we will discuss the working cycle of CBR.

- **Case retrieval –**

  After the issue result has been judged, the best coordinating case is explored in the case base and an estimated solution is retrieved.

- **Case adaptation –**
  The recovered result is adjusted to fit finer the new issue.

- **Solution evaluation –**
  The modified solution can be judged either before the solution is applied to the complication or after the solution has been applied, the modified solution must be adapted again or more cases should be modified.

- **Case- based updating –**
  If the solution was verified as correct the new case may be added to the case.

Knowledge in CBR :

Vocabulary includes the knowledge necessary for choosing the features utilized to describe the cases.

- Case features have to be specified so that they can be helpful in retrieving other cases, which contains useful solutions to similar problems.
- Similarity estimation includes the mastery about the similarity measure itself and the grip used to choose the most efficient firm of the employed case base and the most suitable case-retrieval method.
- Modification knowledge includes the knowledge necessary for executing the adaptation and evaluation phases of the CBR working cycle.
- Cases contain knowledge about solved problem instances and, in many CBR systems, this represents the knowledge that the system acquires during use.

Benefits of CBR :

Here, we will discuss the benefits of CBR.

- CBR supports ease of knowledge elicitation.

- CBR works efficiently in the absence of problem solving bias.
- It is suitable for multiplex and not completely formalized result positions.
- It holds up to ease of explanation.
- It carries ease of maintenance.

Limitations :

Here, we will discuss the limitations of CBR.

- CBR finds it complex to handle large case bases.
- It is almost impossible for CBR to solve dynamic domain problems
- CBR method is unable in handling noisy data

Statistical Reasoning(Bayesian Network)

- Bayes' theorem is also known as **Bayes' rule, Bayes' law**, or **Bayesian reasoning**, which determines the probability of an event with uncertain knowledge.
- In probability theory, it relates the conditional probability and marginal probabilities of two random events.
- Bayes' theorem was named after the British mathematician **Thomas Bayes**. The **Bayesian inference** is an application of Bayes' theorem, which is fundamental to Bayesian statistics.
- It is a way to calculate the value of P(B|A) with the knowledge of P(A|B).
- Bayes' theorem allows updating the probability prediction of an event by observing new information of the real world.

Example: If cancer corresponds to one's age then by using Bayes' theorem, we can determine the probability of cancer more accurately with the help of age.

Bayes' theorem can be derived using product rule and conditional probability of event A with known event B:

As from product rule we can write:

1. $P(A \wedge B) = P(A|B) P(B)$ or

Similarly, the probability of event B with known event A:

1. $P(A \wedge B) = P(B|A) P(A)$

Equating right hand side of both the equations, we will get:

$$P(A|B) = \frac{P(B|A)\, P(A)}{P(B)} \quad \text{....(a)}$$

- The above equation (a) is called Bayes' **rule** or **Bayes' theorem**. This equation is the basis of most modern AI systems for **probabilistic inference**.
- It shows the simple relationship between joint and conditional probabilities. Here,
- P(A|B) is known as **posterior**, which we need to calculate, and it will be read as Probability of hypothesis A when we have occurred evidence B.
- P(B|A) is called the likelihood, in which we consider that hypothesis is true, then we calculate the probability of evidence.
- P(A) is called the **prior probability**, probability of hypothesis before considering the evidence
- P(B) is called **marginal probability**, pure probability of an evidence.
- In the equation (a), in general, we can write P (B) = P(A)*P(B|Ai), hence the Bayes' rule can be written as:

$$P(A_i|B) = \frac{P(A_i) * P(B|A_i)}{\sum_{i=1}^{k} P(A_i) * P(B|A_i)}$$

Where $A_1, A_2, A_3, \ldots\ldots, A_n$ is a set of mutually exclusive and exhaustive events.

Applying Bayes' rule:

Bayes' rule allows us to compute the single term P(B|A) in terms of P(A|B), P(*B*), and P(A). This is very useful in cases where we have a good probability of these three terms and want to determine the fourth one. Suppose we want to perceive the effect of some unknown cause, and want to compute that cause, then the Bayes' rule becomes:

$$P(cause|effect) = \frac{P(effect|cause)\, P(cause)}{P(effect)}$$

Question 1:

Three individuals A, B, and C have applied for a job at a private firm. Their chances of selection are in the ratio 1:2:4. The probabilities that A, B, and C can bring about changes to boost the company's profits are 0.8, 0.5, and 0.3, respectively. If the desired change doesn't occur, find the probability that it is due to the appointment of C.

Solution:

Let's denote the following events:

E1: A gets selected

E2: B gets selected

E3: C gets selected

A: Changes were introduced, but no profit occurred

Now, we can calculate the following probabilities:

P(E1) = 1/(1+2+4) = 1/7

P(E2) = 2/7 and P(E3) = 4/7

P(A|E1) = P(Profit didn't occur due to changes introduced by A) = 1 – P(Profit occurred due to changes introduced by A) = 1 – 0.8 = 0.2

P(A|E2) = P(Profit didn't occur due to changes introduced by B) = 1 – P(Profit occurred due to changes introduced by B) = 1 – 0.5 = 0.5

P(A|E3) = P(Profit didn't occur due to changes introduced by C) = 1 – P(Profit occurred due to changes introduced by C) = 1 – 0.3 = 0.7

We need to find the probability of profit not occurring due to the selection of C.

P(E3|A)=P(A|E3)P(E3)P(A|E1)P(E1)+P(A|E2)P(E2)+P(A|E3)P(E3)
P(E3|A)=0.7×470.2×17+0.5×27+0.7×47
= 7/10.

Thus, the required probability is 0.7.

Question 2:

A bag holds 4 balls. Two balls are drawn at random without replacement and both are found to be blue. What is the probability that all balls in the bag are blue?

Solution:

Let's denote the following events:

$E1$ = Bag contains two blue balls

$E2$ = Bag contains three blue balls

$E3$ = Bag contains four blue balls

A = Event of drawing two white balls

We can calculate the following probabilities:

$P(E1) = P(E2) = P(E3) = \frac{1}{3}$

$P(A|E1) = 2C2 / 4C2 = \frac{1}{6}$

$P(A|E2) = 3C2 / 4C2 = \frac{1}{2}$

$P(A|E3) = 4C2 / 4C2 = 1$

$P(E3|A)=P(A|E3)P(E3)P(A|E1)P(E1)+P(A|E2)P(E2)+P(A|E3)P(E3)$
$= [\frac{1}{3} \times 1]/[\frac{1}{3} \times \frac{1}{6} + \frac{1}{3} \times \frac{1}{2} + \frac{1}{3} \times 1]$

$= \frac{3}{5}$.

Verify your answers using a Bayes' theorem calculator .

Question 3:

In a certain neighborhood, 90% of children fell ill due to the flu and 10% due to measles, with no other diseases reported. The probability of observing rashes for measles is 0.95 and for the flu is 0.08. If a child develops rashes, find the probability of the child having the flu.

Solution:

Let's denote the following events:

F: Children with the flu

M: Children with measles

R: Children showing the symptom of a rash

We can calculate the following probabilities:

$P(F) = 90\% = 0.9$

$P(M) = 10\% = 0.1$

$P(R|F) = 0.08$

$P(R|M) = 0.95$

$P(F|R)=P(R|F)P(F)P(R|M)P(M)+P(R|F)P(F)$
$P(F|R)=0.08\times0.90.95\times0.1+0.08\times0.9$
$= 0.072/(0.095 + 0.072) = 0.072/0.167 \approx 0.43$

Therefore, $P(F|R) = 0.43$

Question 4:

There are three identical cards except that both sides of the first card are red, both sides of the second card are blue, and the third card is red on one side and blue on the other. One card is randomly selected from these three cards and placed down. The visible side of the card is red. What is the probability that the other side is blue?

Solution:

Let's denote the following events:

RR: The card is red on both sides

BB: The card is blue on both sides

RB: The card is red on one side and blue on the other

A: The event that the visible side of the chosen card is red

We can calculate the following probabilities:

$P(RR) = \frac{1}{3}$, $P(BB) = \frac{1}{3}$ and $P(RB) = \frac{1}{3}$

Applying the theorem of total probability , we get $P(A) = P(A|RR).P(RR) + P(A|BB).P(BB) + P(A|RB).P(RB)$

Hence, $P(A) = 1 \times \frac{1}{3} + 0 \times \frac{1}{3} + \frac{1}{2} \times \frac{1}{3} = \frac{1}{2}$

Now, we can calculate:

$P(RB|A)=P(RB\cap A)P(A)=P(A\cap RB)P(A)=P(A|RB)P(RB)P(A)$
$= \frac{1}{2} \times \frac{1}{3} \div \frac{1}{2} = \frac{1}{3}$.

Question 5:

There are three urns containing white and black balls. The first urn has 3 white and 2 black balls, the second urn has 2 white and 3 black balls, and the third urn has 4 white and 1 black ball. One urn is chosen at random, and a ball is selected from it, which turns out to be white. What is the probability that it came from the third urn?

Solution:

Let's denote the following events:

$E1$ = The ball is chosen from the first urn

$E2$ = The ball is chosen from the second urn

E3 = The ball is chosen from the third urn

A: The chosen ball is white

We can calculate the following probabilities:

$P(E1) = 1/6$, $P(E2) = 2/6 = ⅓$, $P(E3) = 3/6 = ½$

$P(A|E1) = 3/5$, $P(A|E2) = 2/5$, $P(A|E3) = 4/5$

Therefore,

P(E3|A)=P(A|E3)P(E3)P(A|E1)P(E1)+P(A|E2)P(E2)+P(A|E3)P(E3)
=4/5×1/23/5×1/6+2/5×1/3+4/5×1/2=4/101/10+2/15+4/10
= 4/9.

Question 6:

An insurance company has insured 4000 doctors, 8000 teachers, and 12000 businessmen. The probabilities of a doctor, teacher, and businessman dying before the age of 58 are 0.01, 0.03, and 0.05, respectively. If one of the insured individuals dies before 58, find the probability that he is a doctor.

Solution:

Let's denote the following events:

E1 = The person is a doctor

E2 = The person is a teacher

E3 = The person is a businessman

A = The death of an insured person

We can calculate the following probabilities:

P(E1) = 4000/(4000+8000+12000) = ⅙

$P(E2) = 8000/(4000+8000+12000) = ⅓$

$P(E3) = 12000/(4000+8000+12000) = ½$

$P(A|E1) = 0.01$, $P(A|E2) = 0.03$, $P(A|E3) = 0.05$

Therefore,

$P(E1|A)=P(A|E1)P(E1)P(A|E1)P(E1)+P(A|E2)P(E2)+P(A|E3)P(E3)$
$=0.01×1/60.01×1/6+0.03×1/3+0.05×1/2=122$
Thus, $P(E1|A) = 1/22$.

Question 7:

A card is lost from a deck of 52 cards. Two cards are drawn at random from the remaining cards and found to be both clubs. What is the probability that the lost card is also a club?

Solution:

Let's denote the following events:

E1 = The lost card is a club

E2 = The lost card is not a club

A = Both drawn cards are clubs

We can calculate the following probabilities:

$P(E1) = 13/52 = ¼$, $P(E2) = 39/52 = ¾$

$P(A|E1)$ = P(Drawing both club cards when the lost card is a club) = $12/51 × 11/50$

$P(A|E2)$ = P(Drawing both club cards when the lost card is not a club) = $13/51 × 12/50$

Now, we can calculate:

$P(E1|A)=P(A|E1)P(E1)P(A|E1)P(E1)+P(A|E2)P(E2)$

$=12/51\times11/50\times1/412/51\times11/50\times1/4+13/51\times12/50\times3/4=12\times1112\times11+3\times13\times12$
Therefore, $P(E1|A) = 11/50$.

Question 8:

Shop A has 30 tins of pure ghee and 40 tins of adulterated ghee for sale, while shop B has 50 tins of pure ghee and 60 tins of adulterated ghee. One tin of ghee is randomly purchased from one of the shops and found to be adulterated. What is the probability that it was purchased from shop B?

Solution:

Let's denote the following events:

E1 = The ghee is purchased from shop A

E2 = The ghee is purchased from shop B

A = The purchased ghee is adulterated

We can calculate the following probabilities:

$P(E1) = ½$ and $P(E2) = ½$

$P(A|E1)$ = P(Purchasing adulterated ghee from shop A) = 40/70 = 4/7

$P(A|E2)$ = P(Purchasing adulterated ghee from shop B) = 60/110 = 6/11

Therefore,

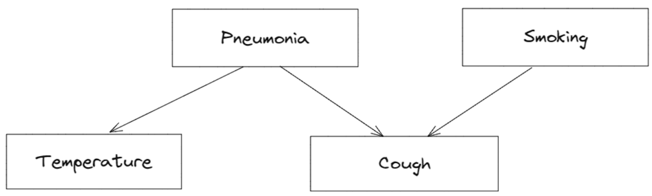
$P(E2|A)=P(A|E2)P(E2)P(A|E1)P(E1)+P(A|E2)P(E2)$
$=6/11\times1/24/7\times1/2+6/11\times1/2=2143$
Hence, $P(E2|A) = 21/43$.

Q. From the figure and the data given below, using Bayes Rule and calculate:

		Cough	
Pneumonia	**Smoking**	**True**	**False**
True	**Yes**	0.95	0.05
True	**No**	0.8	0.2
False	**Yes**	0.6	0.4
False	**No**	0.05	0.95

Pneumonia			**Smoking**		
True	0.1			**Yes**	0.2
False	0.9			**No**	0.8

	Temperature	
Pneumonia	**Yes**	**No**
Yes	0.9	0.1
No	0.2	0.8



Solution:

Let c, be the event for the cough. *(Here, $\underline{c}$ represents to not cough)*

Let p, be the event for pneumonia. *(Here, $\underline{p}$ represents to not pneumonia)*

Let s, be the event for smoking. *(Here, $\underline{s}$ represents to not smoking)*

i. P (c/ (p & s)) = 0.95
ii. P ($\underline{c}$/ (p & s)) = 0.05
iii. P (c/ ($\underline{p}$ & s)) = 0.6
iv. P (c/ (p & $\underline{s}$)) = 0.8

$$P(c) = [P(c/p \cap s)] * P(p) * P(s) + [P(c/\underline{p} \cap s)] * P(\underline{p}) * P(s)$$

$$+ [P(c/p \cap \underline{s})] * P(p) * P(\underline{s}) + [P(c/\underline{p} \cap \underline{s})] * P(\underline{p}) * P(\underline{s})$$

$$= 0.95 * 0.1 * 0.2 + 0.6 * 0.9 * 0.2 + 0.8 * 0.1 * 0.8 + 0.05 * 0.9 * 0.8$$

$$= 0.019 + 0.108 + 0.064 + 0.036$$

$$= 0.227$$

$$P(c/s) = [P(c/p \cap s)] * P(p) + [P(c/\underline{p} \cap s)] * P(\underline{p})$$

$$= 0.95 * 0.1 + 0.6 * 0.9$$

$$= 0.095 + 0.54$$

$$= 0.635$$

$$P(s/c) = [P(c/s) * P(s)] / P(c)$$

$$= [0.635 * 0.2] / 0.227$$

$$= 0.55$$

$$P(c/\underline{s}) = [P(c/p \cap \underline{s})] * P(p) * 1 + [P(c/\underline{p} \cap s)] * P(\underline{p}) * 1$$

$$= 0.8 * 0.1 + 0.05 * 0.9$$

$$= 0.08 * 0.045$$

$$= 0.125$$