Knowledge-Aided Direction Finding Based on Unitary ESPRIT

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***Abstract*—In certain applications involving direction finding, *a priori* knowledge of a subset of the directions to be estimated is sometimes available. Existing knowledge-aided (KA) methods apply projection and polynomial rooting techniques to exploit this information in order to improve the estimation accuracy of the unknown signal directions. In this paper, a new strategy for incorporating prior knowledge is developed for situations with a low signal-to-noise ratio (SNR) and a limited data record based on the Unitary ESPRIT algorithm. The proposed KA-Unitary ESPRIT algorithm processes an enhanced covariance matrix estimate obtained by applying a shrinkage covariance estimator, which linearly combines the sample covariance matrix and an *a priori* known covariance matrix in an automatic fashion. Simulations show that the derived algorithm achieves significant performance gains in estimating the unknown sources and additionally provides a high robustness in the case of inaccurate prior knowledge.**

***Index Terms*—Direction of arrival (DOA) estimation; prior knowledge; shrinkage covariance estimator; Unitary ESPRIT.**

I. Introduction

The performance improvement of direction of arrival (DOA) estimation algorithms, given *a priori* knowledge of a subset of the signal directions to be estimated, has attracted a lot of interest in recent years, due to its importance in practical applications, such as radar, sonar and wireless communications. Common scenarios, where such prior knowledge may be available are, for example, received reflections from stationary objects with known locations in a radar application or signals from base stations or static users in wireless communication systems. Among the existing methods for incorporating prior information to enhance the estimation accuracy of the unknown DOAs are the constrained MUSIC algorithm (C-MUSIC) [1], its extension [2], and the PLEDGE technique [3] based on the MODE algorithm [4]. In the constrained MUSIC method, the noise subspace is constrained to be orthogonal to the known directions. Thus, the column space of the array data is projected onto the orthogonal complement of the signal subspace spanned by the known DOAs. In [2], this idea was extended to oblique projections instead of orthogonal projections, which allows for a more accurate cancellation if the subspaces are not orthogonal. The PLEDGE algorithm incorporates the prior knowledge as known zeros in the rooting polynomial and is therefore restricted to uniform linear arrays (ULA). These existing methods provide significant performance gains if a large data record is available. However, in extreme conditions, such as a small sample size and a low signal-to-noise ratio (SNR), their efficacy to yield improved DOA estimates of the unknown sources degrades substantially. To overcome this problem, the concept of effectively exploiting prior knowledge for space-time adaptive processing [5], [6] and beamforming [7] is applied to DOA estimation.

Among the conventional algorithms developed for DOA estimation, Unitary ESPRIT [8] was shown to be one of the most powerful estimators, providing highly accurate DOA estimates while requiring only real-valued computations. The Unitary ESPRIT algorithm extracts the unknown parameters from an estimate of the true covariance matrix $\boldsymbol{R}$ of the recorded data by applying its sample-averaged version for each snapshot. However, in the aforementioned extreme conditions, the sample covariance matrix $\hat{\boldsymbol{R}}$ constitutes a poor estimator, implying a decreased performance. These covariance matrix estimates can be significantly improved by incorporating prior knowledge in the form of a known covariance matrix $\boldsymbol{C}$ to obtain an enhanced estimate $\tilde{\boldsymbol{R}}$.

In this paper, we propose a new knowledge-aided (KA) direction finding technique, termed KA-Unitary ESPRIT. It exploits the prior knowledge by including the known information via a shrinkage approach in the covariance matrix estimation procedure, yielding an enhanced estimate with an improved signal subspace. An extensive study of the proposed KA-Unitary ESPRIT algorithm is conducted, which demonstrates its superior estimation accuracy gain compared to the conventional Unitary ESPRIT algorithm and previously developed KA methods for direction finding. In addition, we illustrate the high robustness achieved in the case of imprecise *a priori* knowledge.

II. System Model

Let a centro-symmetric M-element sensor array with invariance structure [8] receive narrowband signals originating from d $(d < M)$ far-field sources with the DOAs $\boldsymbol{\theta} = [\theta_1, \ldots, \theta_d]^T$.

The ith data snapshot of the $(M \times 1)$-dimensional array output vector can be modeled as

$$\boldsymbol{x}(i) = \boldsymbol{A}(\boldsymbol{\theta})\boldsymbol{s}(i) + \boldsymbol{n}(i), \qquad i = 1, 2, \ldots, N, \tag{1}$$

where $\boldsymbol{A}(\boldsymbol{\theta}) = [\boldsymbol{a}(\theta_1), \ldots, \boldsymbol{a}(\theta_d)] \in \mathbb{C}^{M \times d}$ is the array steering matrix, $\boldsymbol{s}(i) = [s_1(i), \ldots, s_d(i)]^T \in \mathbb{C}^{d \times 1}$ represents the signal waveforms, $\boldsymbol{n}(i) \in \mathbb{C}^{M \times 1}$ is the vector of white circularly symmetric complex Gaussian sensor noise with zero mean and variance σ_n^2, and N denotes the number of available snapshots.

Assuming a ULA as a representative of centro-symmetric invariance-structured sensor arrays for convenience, the $M \times 1$ steering vectors $\boldsymbol{a}(\theta_n)$ corresponding to the nth source, $n = 1, \ldots, d$, can be expressed as

$$\boldsymbol{a}(\theta_n) = [1, e^{\mathrm{j}2\pi \frac{\Delta}{\lambda_c} \sin\theta_n}, \ldots, e^{\mathrm{j}2\pi(M-1)\frac{\Delta}{\lambda_c} \sin\theta_n}]^T, \tag{2}$$

where Δ denotes the interelement spacing of the ULA and λ_c is the signal wavelength. Using the fact that $\boldsymbol{s}(i)$ and $\boldsymbol{n}(i)$ are modeled as uncorrelated random variables, the $M \times M$ spatial covariance matrix is given by

$$\boldsymbol{R} = \mathbb{E}\left\{\boldsymbol{x}(i)\boldsymbol{x}^H(i)\right\} = \boldsymbol{A}(\boldsymbol{\theta})\boldsymbol{R}_{ss}\boldsymbol{A}^H(\boldsymbol{\theta}) + \sigma_n^2 \boldsymbol{I}_M, \tag{3}$$

where $\boldsymbol{R}_{ss} = \mathbb{E}\{\boldsymbol{s}(i)\boldsymbol{s}^H(i)\}$, which is diagonal if the sources are uncorrelated, and $\boldsymbol{I}_M$ is the $M \times M$ identity matrix. In practice, the true covariance matrix $\boldsymbol{R}$ is not available, but can be estimated using its sample-averaged version given by

$$\hat{\boldsymbol{R}} = \frac{1}{N}\sum_{i=1}^{N} \boldsymbol{x}(i)\boldsymbol{x}^H(i). \tag{4}$$

III. Proposed KA-Unitary ESPRIT Algorithm

In this section, a novel way of exploiting *a priori* information for DOA estimation based on Unitary ESPRIT is derived and some important properties are highlighted.

A. Incorporation of the Prior Knowledge

In order to incorporate the prior knowledge of the DOAs and the signal powers of k uncorrelated sources into the covariance matrix estimation, the known covariance matrix $\boldsymbol{C}$ is formed as

$$\boldsymbol{C} = \sum_{l=1}^{k} \sigma_l^2 \boldsymbol{a}(\theta_l)\boldsymbol{a}^H(\theta_l), \tag{5}$$

where the steering vector $\boldsymbol{a}(\theta_l)$ and the power σ_l^2 of the lth signal, $l = 1, \ldots, k$, are known *a priori*. According to [5], the known matrix $\boldsymbol{C}$ is then processed as a weighted linear combination of $\boldsymbol{C}$ and the sample covariance matrix $\hat{\boldsymbol{R}}$ expressed by

$$\tilde{\boldsymbol{R}} = \alpha\boldsymbol{C} + \beta\hat{\boldsymbol{R}}, \tag{6}$$

where the weight factors are constrained to $\alpha > 0$ and $\beta > 0$. The objective is to find optimal estimates of the weight factors α and β, which automatically combine $\boldsymbol{C}$ and $\hat{\boldsymbol{R}}$ depending on the scenario.

B. Shrinkage Covariance Estimator

The computation of the optimal weight factors is achieved via a shrinkage covariance estimator, i.e., by minimizing the difference between $\tilde{\boldsymbol{R}}$ and $\boldsymbol{R}$ in a mean squared error (MSE) sense [5], formulated as

$$\begin{aligned} \alpha, \beta = &\underset{\alpha,\beta}{\operatorname{argmin}} \ \mathrm{MSE} = \mathbb{E}\left\{\|\tilde{\boldsymbol{R}} - \boldsymbol{R}\|_F^2\right\} \\ &\text{s.t. } \tilde{\boldsymbol{R}} = \alpha\boldsymbol{C} + \beta\hat{\boldsymbol{R}}, \end{aligned} \tag{7}$$

where $\|\cdot\|_F$ denotes the Frobenius matrix norm. In order to solve this minimization problem, the expression in (7) is simplified using the fact that $\hat{\boldsymbol{R}}$ is an unbiased estimate of $\boldsymbol{R}$, i.e., $\mathbb{E}\{\hat{\boldsymbol{R}}\} = \boldsymbol{R}$ and that $\mathrm{Tr}\{\boldsymbol{A}^H\boldsymbol{B}\} > 0$ for any two positive semi-definite non-zero matrices $\boldsymbol{A}$ and $\boldsymbol{B}$, yielding

$$\begin{aligned} \mathrm{MSE}(\alpha,\beta) &= \mathbb{E}\left\{\|\tilde{\boldsymbol{R}} - \boldsymbol{R}\|_F^2\right\} = \mathbb{E}\left\{\|\alpha\boldsymbol{C} + \beta\hat{\boldsymbol{R}} - \boldsymbol{R}\|_F^2\right\} \\ &= \mathbb{E}\left\{\|\alpha\boldsymbol{C} - (1-\beta)\boldsymbol{R} + \beta(\hat{\boldsymbol{R}} - \boldsymbol{R})\|_F^2\right\} \\ &= \|\alpha\boldsymbol{C} - (1-\beta)\boldsymbol{R}\|_F^2 + \beta^2\mathbb{E}\left\{\|\hat{\boldsymbol{R}} - \boldsymbol{R}\|_F^2\right\} \\ &\quad + 2\beta\,\underbrace{\mathbb{E}\left\{\langle\hat{\boldsymbol{R}} - \boldsymbol{R}, \alpha\boldsymbol{C} - (1-\beta)\boldsymbol{R}\rangle_F\right\}}_{\underbrace{\langle\underbrace{\mathbb{E}\left\{\hat{\boldsymbol{R}} - \boldsymbol{R}\right\}}_{0}, \alpha\boldsymbol{C} - (1-\beta)\boldsymbol{R}\rangle_F}_{0}} \\ &= \alpha^2\|\boldsymbol{C}\|_F^2 - 2\alpha(1-\beta)\mathrm{Tr}\{\boldsymbol{C}^H\boldsymbol{R}\} \\ &\quad + (1-\beta)^2\|\boldsymbol{R}\|_F^2 + \beta^2\mathbb{E}\left\{\|\hat{\boldsymbol{R}} - \boldsymbol{R}\|_F^2\right\}, \end{aligned} \tag{8}$$

where $\langle\cdot,\cdot\rangle_F$ is the Frobenius norm inner product, which is defined as $\langle\boldsymbol{A},\boldsymbol{B}\rangle_F = \mathrm{Tr}\{\boldsymbol{A}^H\boldsymbol{B}\}$ for any two matrices $\boldsymbol{A}$ and $\boldsymbol{B}$.

The optimal parameters α_{opt} and β_{opt} of the obtained unconstrained optimization problem are dependent on each other, such that β is fixed to β_{opt} while solving for α. Taking the gradient of (8) with respect to α and equating it to zero gives

$$\alpha_{\mathrm{opt}} = \frac{\mathrm{Tr}\{\boldsymbol{C}^H\boldsymbol{R}\}}{\|\boldsymbol{C}\|_F^2}(1 - \beta_{\mathrm{opt}}). \tag{9}$$

Then, β_{opt} is determined by inserting (9) into (8) and by substituting β for β_{opt} as in

$$\mathrm{MSE}(\beta) = \frac{(1-\beta)^2(\|\boldsymbol{R}\|_F^2\|\boldsymbol{C}\|_F^2 - \mathrm{Tr}^2\{\boldsymbol{C}^H\boldsymbol{R}\})}{\|\boldsymbol{C}\|_F^2} + \beta^2\rho, \tag{10}$$

where $\rho = \mathbb{E}\{\|\hat{\boldsymbol{R}} - \boldsymbol{R}\|_F^2\}$. From the minimization of (10) with respect to β, the optimal parameter β_{opt} can be determined as

$$\beta_{\mathrm{opt}} = \frac{\gamma}{\gamma + \rho}, \tag{11}$$

where

$$\gamma = \frac{\|\boldsymbol{R}\|_F^2\|\boldsymbol{C}\|_F^2 - \mathrm{Tr}^2\{\boldsymbol{C}^H\boldsymbol{R}\}}{\|\boldsymbol{C}\|_F^2}. \tag{12}$$

The application of the Cauchy-Schwarz inequality to the numerator of γ proves that $\gamma > 0$ and thus the constraints $\alpha_{\text{opt}} > 0$ and $\beta_{\text{opt}} > 0$ on the weight factors are satisfied.

In practical applications, the optimal weights α_{opt} and β_{opt} need to be estimated. For convenience, we define

$$\nu = \frac{\text{Tr}\{\boldsymbol{C}^H \boldsymbol{R}\}}{\|\boldsymbol{C}\|_F^2}. \tag{13}$$

Furthermore, (12) can be more compactly expressed using (13) as

$$\gamma = \|\boldsymbol{R}\|_F^2 - \frac{\text{Tr}^2\{\boldsymbol{C}^H \boldsymbol{R}\}}{\|\boldsymbol{C}\|_F^2} = \|\nu \boldsymbol{C} - \boldsymbol{R}\|_F^2. \tag{14}$$

Applying (14) instead of (12) for γ and rewriting the expression in the denominator of (11), leads to

$$\begin{aligned}\gamma + \rho &= \|\nu \boldsymbol{C} - \boldsymbol{R}\|_F^2 + \mathbb{E}\left\{\|\hat{\boldsymbol{R}} - \boldsymbol{R}\|_F^2\right\} \\ &= \mathbb{E}\left\{\|\hat{\boldsymbol{R}} - \nu \boldsymbol{C}\|_F^2\right\},\end{aligned} \tag{15}$$

where again the property $\mathbb{E}\{\hat{\boldsymbol{R}}\} = \boldsymbol{R}$ is used. The identity in (15) enables us to estimate $\gamma + \rho$ by $\|\hat{\boldsymbol{R}} - \hat{\nu}\boldsymbol{C}\|_F^2$, such that the estimates $\hat{\alpha}_{\text{opt}}$ and $\hat{\beta}_{\text{opt}}$ can be obtained after some rearrangements as

$$\hat{\alpha}_{\text{opt}} = \frac{\hat{\nu}\hat{\rho}}{\|\hat{\boldsymbol{R}} - \hat{\nu}\boldsymbol{C}\|_F^2} \tag{16}$$

and

$$\hat{\beta}_{\text{opt}} = \max\left(\left(1 - \frac{\hat{\alpha}_{\text{opt}}}{\hat{\nu}}\right), 0\right), \tag{17}$$

where $\boldsymbol{R}$ is simply replaced by $\hat{\boldsymbol{R}}$ in all the terms and $\hat{\beta}_{\text{opt}}$ is modified to fulfill $\hat{\beta}_{\text{opt}} > 0$. The estimate of the parameter ρ is computed according to the derivations in [5] as

$$\hat{\rho} = \frac{1}{N^2}\sum_{i=1}^{N} \|\boldsymbol{x}(i)\|_F^4 - \frac{1}{N}\|\hat{\boldsymbol{R}}\|_F. \tag{18}$$

Finally, we have the knowledge-aided enhanced covariance matrix $\tilde{\boldsymbol{R}}$, which yields an improved signal subspace estimate and is now processed by the KA-Unitary ESPRIT algorithm. The individual steps of the proposed algorithm are summarized in Table I. Further details about Unitary ESPRIT can be found in [8], [10].

TABLE I
THE PROPOSED KA-UNITARY ESPRIT ALGORITHM

1. Knowledge-Aided Processing:

- Compute the *a priori* covariance matrix $\boldsymbol{C} \in \mathbb{C}^{M\times M}$.
- Calculate the weight factors $\hat{\alpha}_{\text{opt}}$ and $\hat{\beta}_{\text{opt}}$ to obtain the enhanced covariance matrix estimate $\tilde{\boldsymbol{R}} \in \mathbb{C}^{M\times M}$.

2. Signal Subspace Estimation:

- Compute $\hat{\boldsymbol{E}}_s \in \mathbb{R}^{M\times d}$ as the d principal eigenvectors of the real-valued enhanced covariance matrix after the transformation $\text{Re}\{\boldsymbol{Q}_M^H \tilde{\boldsymbol{R}} \boldsymbol{Q}_M\} \in \mathbb{R}^{M\times M}$

3. Shift Invariance Equations:

- Solve the shift invariance equations $\boldsymbol{K}_1\hat{\boldsymbol{E}}_s\boldsymbol{\Upsilon} \approx \boldsymbol{K}_2\hat{\boldsymbol{E}}_s$ by using the LS, TLS or SLS algorithm

4. DOA Estimation:

- Calculate the d eigenvalues of the real-valued solution $\boldsymbol{\Upsilon} = \boldsymbol{T\Omega T}^{-1} \in \mathbb{R}^{d\times d}$ with $\boldsymbol{\Omega} = \text{diag}\{\omega_n\}_{n=1}^d$.
- Solve $\theta_n = \arcsin\left(\frac{\lambda_c}{\pi\Delta}\arctan(\omega_n)\right)$ to obtain the DOAs.

C. Key Properties of KA-Unitary ESPRIT

It is important to note that the KA-Unitary ESPRIT method estimates the DOAs of all the d signals in $\boldsymbol{\theta}$, including the known ones, as the incorporated knowledge is simply used to compensate the poor estimate of the sample covariance matrix in extreme conditions. Therefore, the rank is not reduced to $d - k$. However, the unknown and known sources are assumed distinguishable, so that the estimation error of the known DOAs can be omitted for the purpose of assessing the performance.

The computational complexity is slightly higher than the cost of the original Unitary ESPRIT algorithm. The additional complexity is due to the computation of the known covariance matrix $\boldsymbol{C}$, which is $\mathcal{O}(M^2)$ as $\boldsymbol{C}$ is the sum of rank-1 matrices and $k < M$.

IV. Simulation Results

In this section, we evaluate the estimation capabilities of the proposed KA-Unitary ESPRIT algorithm for direction finding. Specifically, we compare its performance in terms of the root mean squared error (RMSE) in degrees to the deterministic KA Cramér-Rao lower bound (Det KA-CRB), which will be published in a journal version. For the simulations, we employ a ULA composed of $M = 30$ omnidirectional sensors with interelement spacing $\Delta = \lambda_c/2$ and a data record of $N = 10$ samples to simulate the extreme conditions. Now, we assume that $d = 5$ uncorrelated and equipowered signals with $\sigma^2 = 1$ from the directions $\boldsymbol{\theta} = [-50°, -10°, 10°, 20°, 70°]^T$ are captured by the array, whereas $k = 2$ sources at $\theta_1 = -50°$ and $\theta_5 = 70°$ are considered known. The source symbols are drawn from a complex Gaussian distribution and all the simulated curves are obtained by averaging a total of 3000 Monte Carlo trials.

In the first experiment, we assess the estimation accuracy of the KA-Unitary ESPRIT algorithm and compare it to the conventional Unitary ESPRIT method without *a priori* knowledge. In addition, for the sake of completeness, least squares (LS), total least squares (TLS), and structured least squares (SLS) [9] to solve the invariance equations are incorporated in the comparison. Fig. 1 shows the RMSE as a function of the SNR, where only the unknown $d - k = 3$ sources are included in the RMSE computation for all the methods. It is evident from Fig. 1 that incorporating prior knowledge using the proposed KA scheme provides a significant gain over the conventional Unitary ESPRIT-type algorithms. Also,

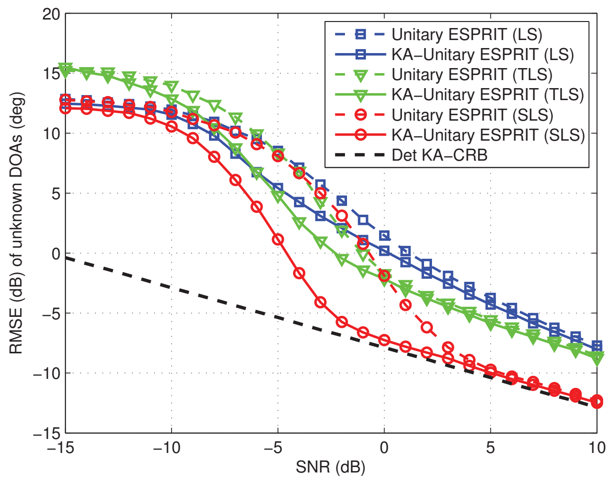

Fig. 1. RMSE versus SNR with $M = 30$, $N = 10$, $d = 5$ sources at $\boldsymbol{\theta} = [-50°, -10°, 10°, 20°, 70°]^T$, $k = 2$ sources at $\theta_1 = -50°$ and $\theta_5 = 70°$ considered known.

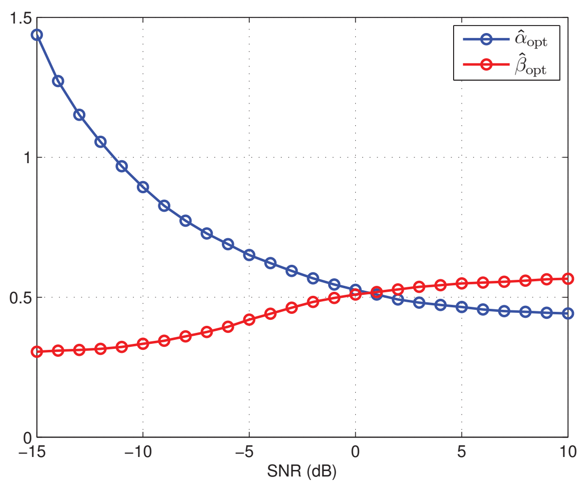

Fig. 2. Weight factors versus the SNR with $M = 30$, $N = 10$, $d = 5$ sources at $\boldsymbol{\theta} = [-50°, -10°, 10°, 20°, 70°]^T$, $k = 2$ sources at $\theta_1 = -50°$ and $\theta_5 = 70°$ considered known.

the gain is considerably higher for scenarios with a low SNR and decreases in the high SNR regime. However, the impact of the known information on the estimation accuracy is different for LS, TLS and SLS. While the gain for LS and TLS is rather moderate, SLS exhibits a more pronounced benefit of almost 10 dB and performs close to the asymptotic efficiency, leaving only a very small gap to the deterministic KA-CRB. Thus, we only present the results of KA-Unitary ESPRIT in combination with SLS in the following experiments.

In Fig. 2, we study the behavior of the estimated weight factors $\hat{\alpha}_{\text{opt}}$ and $\hat{\beta}_{\text{opt}}$ as a function of the SNR for the above experiment. The curves illustrate that the parameter $\hat{\alpha}_{\text{opt}}$ associated with the prior knowledge starts at a high value and decreases as the SNR increases, whereas $\hat{\beta}_{\text{opt}}$ linked with the sample covariance matrix grows slightly. This characteristic is completely in line with the stated theory that the prior knowledge is especially exploited in the low SNR regime when the sample covariance matrix provides a poor estimate.

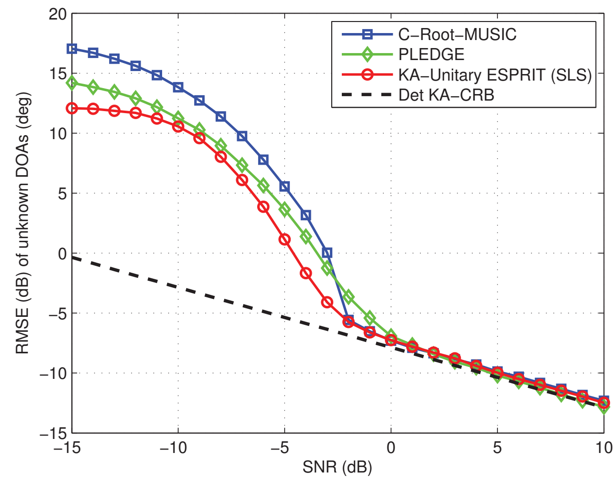

Fig. 3. RMSE versus SNR with accurate knowledge and $M = 30$, $N = 10$, $d = 5$ sources at $\boldsymbol{\theta} = [-50°, -10°, 10°, 20°, 70°]^T$, $k = 2$ sources at $\theta_1 = -50°$ and $\theta_5 = 70°$ considered known.

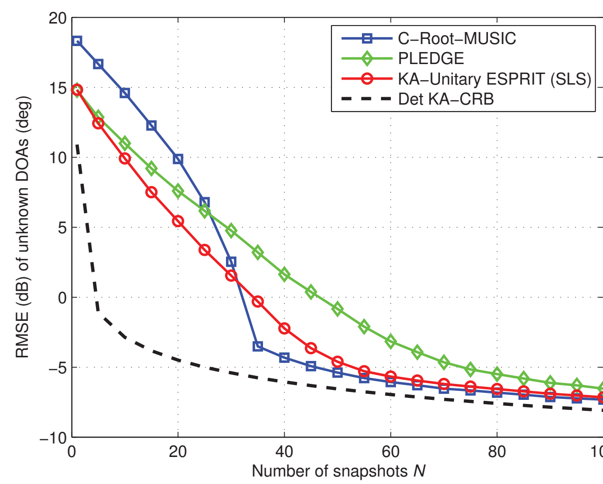

Fig. 4. RMSE versus snapshots N with accurate knowledge and $M = 30$, SNR $= -10$ dB, $d = 5$ sources at $\boldsymbol{\theta} = [-50°, -10°, 10°, 20°, 70°]^T$, $k = 2$ sources at $\theta_1 = -50°$ and $\theta_5 = 70°$ considered known.

In the next experiment, we compare the proposed KA-Unitary ESPRIT algorithm using SLS to the previously developed methods for incorporating prior knowledge, i.e., constrained Root-MUSIC (C-Root-MUSIC) [1] and PLEDGE [3]. The simulation results depicted in Fig. 3 assess the RMSE performance versus the SNR, where the same scenario as in the first experiment is used. It can be seen that the proposed method provides the lowest estimation error at low

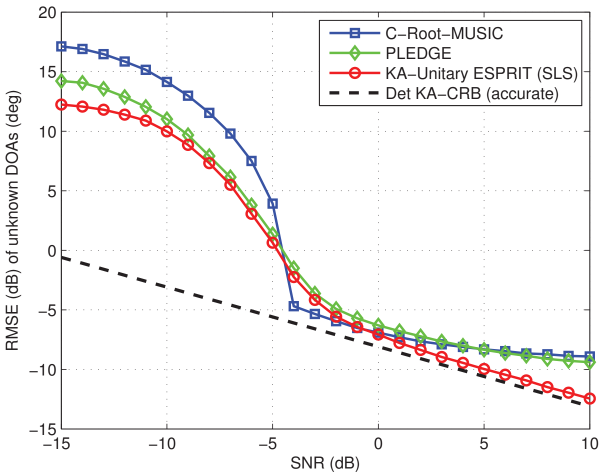

Fig. 5. RMSE versus SNR with inaccurate knowledge and $M = 30$, $N = 10$, $d = 5$ sources at $\boldsymbol{\theta} = [-50°, -10°, 10°, 20°, 70°]^T$, $k = 2$ sources at $\theta_1 = -53°$ and $\theta_5 = 67°$ considered known.

SNRs. However, all the analyzed algorithms attain a similar performance after the threshold region in the high SNR regime.

In the following experiment, we draw the same comparison but evaluate the RMSE against the number of snapshots N. Thus, the SNR is fixed at -10 dB while we again use the previous simulation setup. The curves shown in Fig. 4 demonstrate that the KA-Unitary ESPRIT algorithm outperforms C-Root-MUSIC and PLEDGE for a small data record $(N < M)$. When the sample size is increased, C-Root-MUSIC provides a slightly lower estimation error than the proposed method but both algorithms perform close to the deterministic KA-CRB.

In the last experiment, the realistic case of inaccurate *a priori* information of the known DOAs is investigated. Therefore, we change the locations of the two sources that are assumed known to $\theta_1 = -53°$ and $\theta_5 = 67°$ but keep all the other parameters fixed. Again, the RMSE versus the SNR is considered and the three analyzed algorithms are compared to the deterministic KA-CRB corresponding to the accurate positions of the imprecisely known sources. From Fig. 5, we see that the proposed KA-Unitary ESPRIT algorithm achieves the highest robustness against imprecise knowledge. The obtained gain over PLEDGE at low SNRs is not as substantial as for the accurate case, but the benefit remains considerable. As the SNR increases, the efficiency of C-Root-MUSIC and PLEDGE degrades significantly allowing a steady bias to the deterministic KA-CRB. This however, is not the case for KA-Unitary ESPRIT, which is still capable of performing close to the lower bound even for a large mismatch in the known DOAs. Further simulations have shown that the high robustness of KA-Unitary ESPRIT is only acquired in combination with SLS as it accounts for the specific structure between the noise-corrupted signal subspace on each side of the shift invariance equations.

V. Conclusion

A novel way of incorporating prior knowledge into the DOA estimation based on the Unitary ESPRIT algorithm has been proposed. The *a priori* information of a subset of the signal directions to be estimated is processed via a shrinkage covariance estimator in the MSE sense to obtain an enhanced covariance matrix, which yields a substantially improved signal subspace estimate. The proposed KA-Unitary ESPRIT algorithm provides significant performance gains in estimating the unknown sources and is designed for extreme conditions in the scenario. Simulation results show that KA-Unitary ESPRIT in combination with SLS achieves the largest gain and outperforms existing approaches for incorporating prior knowledge especially at low SNRs and for a limited data record. Also, it exhibits the highest robustness in the case of imprecise *a priori* information and performs close to asymptotic efficiency even for a large mismatch in the known DOAs. Note that the proposed KA scheme is applicable to any subspace-based high-resolution DOA estimation algorithm and extensions to the two- and multidimensional case are straightforward.

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