TCS II

Formal Languages and Computability
2020/21

1st Midterm

6th April 2021

Solutions

1. Assignment:

Question 1 (Is the language regular?)

- Language L_1 **is not regular**. (**1 point**)

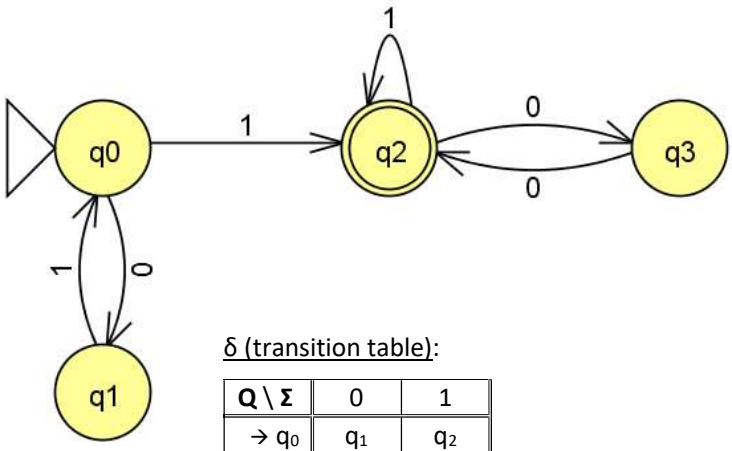
Proof by using the »Pumping lemma for regular languages«. (**9 points**)

- Language L_2 **is regular**. (**1 point**)

DFA for language L_2:

M = <Q, Σ, q_0, F, δ>

Q = {q_0, q_1, q_2, q_3} (**1 point**)
Σ = {0, 1} (**1 point**)
q_0 = q_0 (**1 point**)
F = {q_2} (**1 point**)



δ (transitions): (**5 points**)
δ(q_0, 0) = q_1
δ(q_0, 1) = q_2
δ(q_1, 1) = q_0
δ(q_2, 0) = q_3
δ(q_2, 1) = q_2
δ(q_3, 0) = q_2

δ (transition table):

Q \ Σ	0	1
→ q_0	q_1	q_2
q_1	/	q_0
* q_2	q_3	q_2
q_3	q_2	/

Question 2 (Grammars for the languages)

- CFG for language L_1 (list of productions only): (**5 points**)

S → **a**S**a** | **aba**

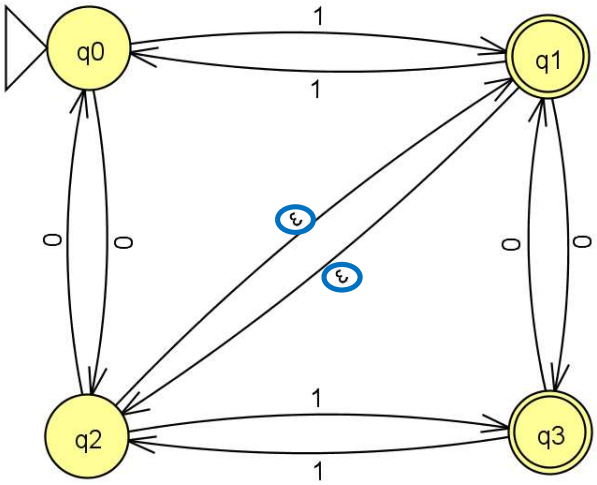
- CFG for language L_2 (list of productions only): (**5 points**)

S → A**1**X
A → **01**A | **ε**
X → **00**X | **1**X | **ε**

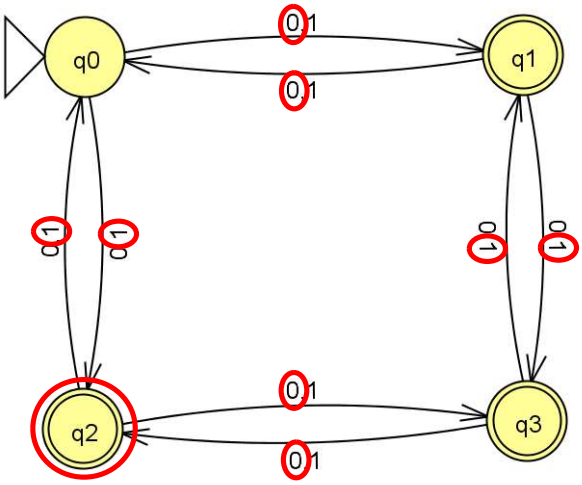
2. Assignment:

ε-NFA → NFA → DFA:

ε-NFA:



NFA: (**5 points**)



DFA: (**15 points**)

3. Assignment:

In the DFA (depicted below), **q5** is the "dead/trap state" and all states are "reachable":

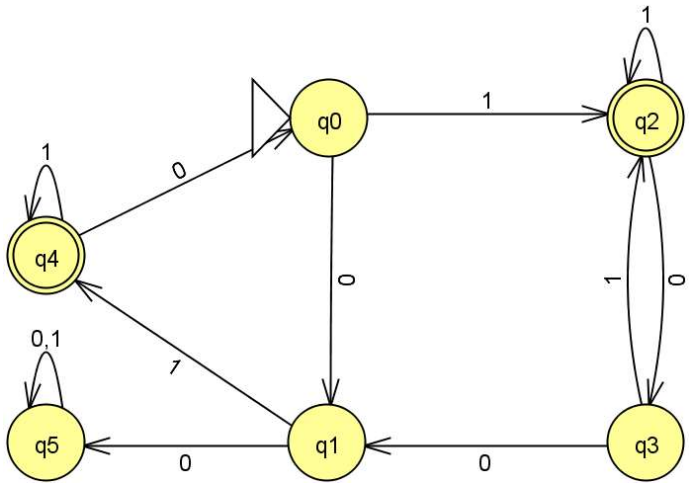


Table of distinguishable pairs of states: (**15 points** – pairs, **5 points** – δ function)

	q5	q4	q3	q2	q1
q0	X	X		X	X
q1	X	X	X	X	
q2	X		X		
q3	X	X			
q4	X				

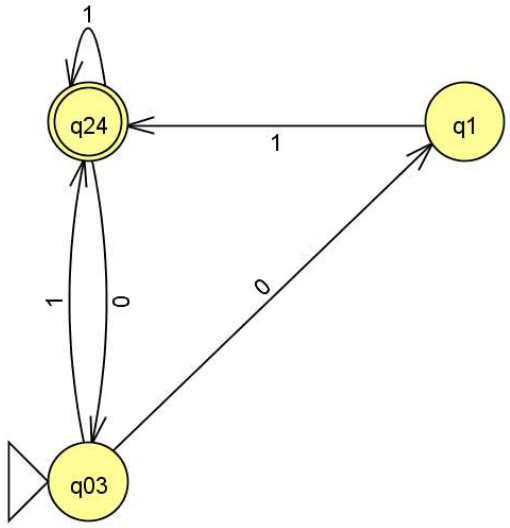
Q\Σ	0	1
→ q0	q1	q2
q1	q5	q4
* q2	q3	q2
q3	q1	q2
* q4	q0	q4
q5	q5	q5

X = pair final/non-final

X = distinguishable »on **1**«

X = distinguishable »on **0**«, after (q1, q5) marked as distinguishable on **1**.

Merge states q0 and q3, states q2 and q4, and remove »dead/trap state« (**q5**): (**5 points**)



4. Assignment:

Given the CFG:

S → X | Y | XY | YX
X → 01 | 0Y1 | ε
Y → 2 | X2
Z → 12 | 1XX2

1. step – remove ε-productions: (**5 points**)

S → X | Y | XY | YX
X → 01 | 0Y1
Y → 2 | X2
Z → 12 | 1XX2 | 1X2

2. step – remove unit productions: (**5 points**)

S → XY | YX | 01 | 0Y1 | 2 | X2
X → 01 | 0Y1
Y → 2 | X2
Z → 12 | 1XX2 | 1X2

3. step – remove variables that don't produce strings of all terminals: there are no such variables.

4. step – remove »unreachable« variables: variable Z is »unreachable« (**5 points**)

S → XY | YX | 01 | 0Y1 | 2 | X2
X → 01 | 0Y1
Y → 2 | X2

5. step – Chomsky Normal Form (CNF):

a) Terminals → Variables: (**5 points**)

S → XY | YX | ZO | ZYO | 2 | XT
X → ZO | ZYO
Y → 2 | XT
Z → 0
O → 1
T → 2

b) »Shortening«: (**5 points**)

S → XY | YX | ZO | ZW | 2 | XT
X → ZO | ZW
Y → 2 | XT
Z → 0
O → 1
T → 2
W → YO

CNF