

Problem 1

An MLP has two input nodes, one hidden layer, and two outputs. The two sets of weights and biases are given by:

$$W_1 = \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix} \quad b_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$W_2 = \begin{bmatrix} 2 & 2 \\ 2 & -3 \end{bmatrix} \quad b_2 = \begin{bmatrix} 0 \\ -4 \end{bmatrix}$$

The non-linear activation for the hidden layer is ReLU (rectified linear unit) – that is $h(x) = \max(x, 0)$. The output layer is linear (i.e., identity activation function). The output for layer l is given by $a^{(l)} = h_l(W_l a^{(l-1)} + b_l)$.

What is the output activation for input $x = [+1; -1]^T$?

$$1(x_1) + (-2)(x_2) + 1 = z_1$$

$$1 + (2) + 1 = z_1 = 4 \text{ hidden neuron 1}$$

$$3(x_1) + (4)(x_2) + 0 = 3 + (-4) + 0 = -1$$

$$z_2 = -1 \text{ hidden neuron 2}$$

$$\text{hidden output before activation} = (4, -1)$$

$$h_1 = \text{RELU}(x_1) = 4$$

$$h_2 = \text{RELU}(x_2) = 0$$

$$\text{hidden layer is } (4, 0)$$

$$\text{output 1 } y_1 = 2(x_1) + 2(x_2) + 0$$

$$y_1 = 2(4) + 2(0) + 0 = 8$$

$$y_2 = 2(x_1) + (-3)(x_2) + (-4)$$

$$y_2 = 2(4) + (-3)(0) + (-4) = 4$$

$$\text{Output } \mathbf{a} = (8, 4)$$