question 1 (4 pts): Let 10 seconds be the measured sequential running time, 40 seconds be the parallel running time using 10 processors for a parallel application. What would be the speedup prediction for 30 processors using Amdahl's Law? What would be the prediction using Gustafson's Law? Please show all your steps.

To calculate the speedup prediction using Amdahl's law first we need to find the percentage of time for the sequential part and percantage of time for the parallel part based on one processor then we could calculate the speedup. As we know, the total number of percentage is equal to one: $s + p = 1$ (s is the sequential running time while p is the parallel running time on single processor).

We need to calculate the time for a single processor, where N is the number of processors:
$s' = 10sec$ $p' = 40 * N = 40 * 10 = 400sec$

Therefore, normalizing the sequential and parallel running time we get:
$s = \frac{s'}{s'+p'} = \frac{10}{10+400} = 0.02439$
$p = \frac{p'}{s'+p'} = \frac{400}{10+400} = 0.97561$

Now, we could calculate Amdahl's law speedup for N=10:
$Speedup_a = \frac{1}{s+\frac{p}{N}} = \frac{1}{0.02439+\frac{0.97561}{10}} = 8.2$

In the same way, we could calculate the speedup for N=30 processors:
$s' = 10sec$ $p' = 40 * N = 40 * 30 = 1200sec$, normalizing:

$s = \frac{s'}{s'+p'} = \frac{10}{10+1200} = 0.00826$
$p = \frac{p'}{s'+p'} = \frac{1200}{10+1200} = 0.99174$

Amdahl's law speedup for N=30 is:
$Speedup_a = \frac{1}{s+\frac{p}{N}} = \frac{1}{0.00826+\frac{0.99174}{30}} = 24.2$

To calculate the Gustafson's law we also need to find the percentage of sequential and parallel running time and then to calculate the speedup.

$s' = 10sec$ $p' = 40sec$, normalizing:
$s = \frac{s'}{s'+p'} = \frac{10}{10+40} = 0.2$
$p = \frac{p'}{s'+p'} = \frac{40}{10+40} = 0.8$

Gustafson's law speedup for N=10:
$Speedup_g = s + p * N = 0.2 + 0.8 * 10 = 8.2$

Gustafson's law speedup for N=30:
$Speedup_g = s + p * N = 0.2 + 0.8 * 30 = 24.2$

question 2 (2pts): Explain the projection differences, if any.

As we could see from the results above there are no differences in speedup between Ambdahl's law Gustafson's law. The difference is only in calculation of those two laws. In Amdahl's law the serial percentage is calculated based on processing time on 1 processor. While in Gustafson's law the serial percentage is calculated according to total processing time using P processors. From both laws in our problem we could see that speedup is increased with number of processors (when N=10: Speedup=8.2, when N=30: Speedup = 24.2).

question 3 (3 pts): Using 100 processors, the sequential part is only 10 percent of the total 150 second running time of a parallel program. What would be the speedup prediction using Gustafson's Law? How would you predict for 200 processors using the Amdahl's formula? Show all your steps.

Using Gustafson's law we will calculate the speedup in the following way:
$s = 0.1$ $p = 0.9$ $N = 100$
$Speedup_g = s + p * N = 0.1 + 0.9 * 100 = 90.1$

when $N = 200$:
$Speedup_g = s + p * N = 0.1 + 0.9 * 200 = 180.1$

To calculate Amdahl's law we need to calculate running time of single processor first when $N = 100$:
$s' = 0.1 * 150 = 15sec$ $p' = 0.9 * 150 * 100 = 13500sec$

$s = \frac{s'}{s'+p'} = \frac{15}{15+13500} = 0.0011$
$p = \frac{p'}{s'+p'} = \frac{13500}{15+13500} = 0.9989$

The speedup of Amdahl's law for N=100:
$Speedup_a = \frac{1}{s+\frac{p}{N}} = \frac{1}{0.0011+\frac{0.9989}{100}} = 90.18$

when $N = 200$:
$s' = 0.1 * 150 = 15sec$ $p' = 0.9 * 150 * 200 = 27000sec$

$s = \frac{s'}{s'+p'} = \frac{15}{15+27000} = 0.0005$
$p = \frac{p'}{s'+p'} = \frac{27000}{15+27000} = 0.9995$

The speedup of Amdahl's law for N=200:
$Speedup_a = \frac{1}{s+\frac{p}{N}} = \frac{1}{0.0005+\frac{0.9995}{200}} = 181.9$

question 4 (1 pt): Explain how to quantify Amdahl's formula using Gustafson's formula. What is wrong if we take P->infinity using Gustafson's formula alone?

Gustafson's law become identical to Amdahl's law by following formula:
$s = s'$ and $p = N * p'$ then $p' = \frac{p}{N}$

As $s' + p' = 1 => s + \frac{p}{N}$
Scaled $Speedup_g$ becomes:
$\frac{Tseq}{Tpar} = \frac{s'+p'*N}{s'+p'} = \frac{s'+p'*N}{s+\frac{p}{N}} = \frac{s+p}{s+\frac{p}{N}} = \frac{1}{s+\frac{p}{N}}$ (identical to Amdahl's law with the same speedup bound)

If we take P -> infinity in Gustafson's law the speedup goes to infinity while Amdahl's law is bounded by $\frac{1}{s}$. From Gustafson's law term: $s + N * p$ we could see that p depends on N and when N goes to infinity p also changes. Meaning when we have more processors, the parallel time is increasing and p (percentage) is bigger while, on the other hand, s becomes smaller. The problem could be in wrongly use of percentage scale of Gustafson's law in place of non scaled percentage in Amdahl's law.