Week 1 Worksheet

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6/19/2022

1. A random variable X is defined to be the difference between the higher values and the lower value when two dice are thrown. If they have the same value, X is defined to be zero.

a) Find the probability distribution for X

b) Find the expected value of X

c) Calculate $E(X^2)$

d) Calculate the population variance and the standard deviation of X

e) Find the variance of the random variable X in (4) using the equation $\sigma_x^2 = E(X^2) - \mu_x^2$. Does it equal your answer from part (d)?

2. Prove that if Y=b, where b is a constant, COV(X,Y) = 0

3. Prove if Y=V+W, var(Y) = var(V) + var(W) + 2cov(V,W)

4. Suppose a variable Y is an exact linear function of X: Y = a + bX, where a and b are constants. Suppose Z is a third variable. Show that $\rho_{xz} = \rho_{yz}$

5. Consider data gathered on adult domestic cats' body weight and heart weight:

```
##      cat body_weight heart_weight
## [1,]  1         2.0          6.5
## [2,]  2         2.2          7.6
## [3,]  3         2.4          9.1
## [4,]  4         2.4          7.3
## [5,]  5         3.6         11.8
```

a) What is the sample mean of Body Weight
b) What is the sample variance of Heart Weight
c) What is the sample correlation coefficient between Body Weight and Heart Weight

6. Suppose a random variable X is assumed to have a normal distribution with a variance of 4. It is hypothesized that the unknown mean is equal to 10. Given a sample of 25 observations, suppose that we wish to determine the acceptance and rejection regions for $\bar{X}$ under $H_o : \mu = 10$ using (a) a 5 percent significance test and (b) using a 1 percent test.

7. A certain city abolishes its local sales tax on consumer expenditure. A survey of 20 households show that, in the following month, mean household expenditure increased by 160 and the standard error of the increase was 60.

a) Did abolishing have a significant effect on household expenditure? Use a 2 sided t-test at the 5 percent and 1 percent level.

b) Construct the 95% confidence interval and the 99% confidence interval