Assignment 7 in CSE 415, Autumn 2022

by the Staff of CSE 415

This is due Monday, Decemeber 5, via Gradescope at 11:59 PM. Prepare a PDF file with your answers and upload it to Gradescope. The PDF file can created however you like. For example it can be from a scan of a printout of the assignment document onto which you have hand-written your answers. Or it can be from Word or Latex file with your answers. It can even be from photos of your handwritten answers on plain paper. You don't have to include the questions themselves, but it is fine to do so. In any case, it must be very clear to read, and it must be obvious and easy for each grader where to find your solutions to the exercises.

As with Assignment 4, this is an *individual work* assignment. Collaboration is not permitted.

Do the following exercises. These are intended to take 10-30 minutes each if you know how to do them. Each is worth 10 or 20 points. The total of possible points is 140. Names of responsible staff members are given for each question.

If corrections or clarifications to the problems have to be given, this will happen in the ED discussion forum under topic "Assignment 7."

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1 Value Iteration (Vivek)

(10 points) Consider an MDP with two states s_1 and s_2 and transition function $T(s, a, s')$ and reward function $R(s, a, s')$. Let's also assume that we have an agent whose discount factor is $\gamma = 1$. From each state, the agent can take three possible actions $a \in \{x, y, z\}$. The transition probabilities for taking each action and the rewards for transitions are shown below.

s	a	s'	$T(s,a,s')$	$R(s,a,s')$
s_1	x	s_1	0.4	0
s_1	x	s_2	0.6	0
s_1	y	s_1	0	999
s_1	y	s_2	1	7
s_1	z	s_1	0.5	0
s_1	z	s_2	0.5	0

s	a	s'	$T(s,a,s')$	$R(s,a,s')$
s_2	x	s_1	0.3	20
s_2	x	s_2	0.7	25
s_2	y	s_1	1	0
s_2	y	s_2	0	0
s_2	z	s_1	0.7	5
s_2	z	s_2	0.3	10

Compute V_0 , V_1 and V_2 for states s_1 and s_2. (The first 2 are worth 1 point each. The others are worth 2 points each.)

(a). $V_0(s_1) =$ 0 ?

(b). $V_0(s_2) =$ 0 ?

(c). $V_1(s_1) =$ 7 ?

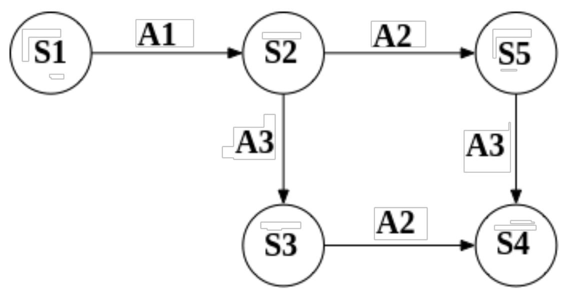
(d). $V_1(s_2) =$ 23.5 ?

(e). $V_2(s_1) =$ 30.5 ?

(f). $V_2(s_2) =$ 42.05 ?

2 Q-Learning updates (Vivek)

(10 points) Consider an agent traveling on the graph below. The states are represented by the nodes and actions are represented by the edges in the following graph.



(a) (6 points) Consider the following episodes performed in this state space. The experience tuples are of the form [s, a, s', r], where the agent starts in state s, performs action a, ends up in state s', and receives immediate reward r, which is determined by the state entered. Let $\gamma = 1.0$ for this MDP. Fill in the values computed by the Q-learning algorithm with a learning rate of = 0.2. All Q values are initially 0, and you should fill out each row using values you have computed in previous rows.

[S2, A2, S5, 5]	Q(S2, A2) = $0.8 \cdot 0 + 0.2(5 + 1.0 \cdot 0) = 1.0$
[S5, A3, S4, 2]	Q(S5, A3) = $0.8 \cdot 0 + 0.2(2 + 1.0 \cdot 0) = 0.4$
[S2, A3, S3, -7]	Q(S2, A3) = $0.8 \cdot 0 + 0.2(-7 + 1.0 \cdot 0) = -1.4$
[S3, A2, S4, 7]	Q(S3, A2) = $0.8 \cdot 0 + 0.2(7 + 1.0 \cdot 0) = 1.4$
[S2, A3, S5, 2]	Q(S2, A3) = $0.8 \cdot (-1.4) + 0.2(2 + 1.0 \cdot 0.4) = -0.64$
[S2, A2, S3, -2]	Q(S2, A2) = $0.8 \cdot 1.0 + 0.2(-2 + 1.0 \cdot 1.4) = 0.68$
[S1, A1, S1, 8]	Q(S1, A1) = $0.8 \cdot 0 + 0.2(8 + 1.0 \cdot 0) = 1.6$

(b) (3 points) Now, based on the record table in the previous problem, we want to approximate the transition function:

$T(S2, A2, S3) = \frac{1}{2}$ $T(S2, A3, S3) = \frac{1}{2}$

$T(S2, A2, S5) = \frac{1}{2}$ $T(S1, A1, S2) = 0$

$T(S2, A3, S5) = \frac{1}{2}$ $T(S1, A1, S1) = 1$

(c) (1 point) What's the key difference between Q-learning and Value Iteration? What's one advantage of each of the methods in general?

The big difference between value iteration and q learning is the difference between on policy learning and off policy learning. In on policy learning, value iteration is calculated from transitions and reward when optimal policy is known, while q learning is calculated when transitions and reward are unknown. Q learning converges to optimal policy even if it is acting sub-optimally. Advantage of Q learning is that Q learning can calculate even if transitions and rewards are unknown. Advantage of value iteration is that value iteration can calculate accurately.

3 Joint Distributions and Inference (Phuong)

(10 points) Let N be the random variable that represent whether you are on the nice or naughty list. Let C be the random variable that represent whether you will receive chocolate or coal from Santa.

Consider the table given below:

N	C	$\mathbb{P}(N, C)$
nice	*chocolate*	0.45
nice	*coal*	0.15
naughty	*chocolate*	0.05
naughty	*coal*	0.35

(a) (1 point) Compute the marginal distribution $\mathbb{P}(N)$ and express it as a table.

N	P(N)
nice	0.6
naughty	0.4

(b) (1 point) Similarly, compute the marginal distribution $\mathbb{P}(C)$ and express it as a table.

C	P(C)
chocolate	0.5
coal	0.5

(c) (2 points) Compute the conditional distribution $\mathbb{P}(C|N = naughty)$ and express it as a table. Show your work/calculations.

C	P(C|N=naughty)
chocolate	0.125
coal	0.875

$= \frac{0.05}{0.4} = \frac{1}{8}$

$= \frac{0.35}{0.4} = \frac{7}{8}$

(d) (2 points) Compute the conditional distribution $\mathbb{P}(N|C = chocolate)$ and express it as a table. Show your work/calculations.

N	P(N|C=chocolate)
nice	0.9
naughty	0.1

$= \frac{0.45}{0.5} = \frac{9}{10}$

$= \frac{0.05}{0.5} = \frac{1}{10}$

(e) (1 point) Is it true that $N \perp\!\!\!\perp C$? (i.e., are they statistically independent?) Explain your reasoning.

N and C are statistically independent if the following equation holds

P(N,C)=P(N)P(C)

If N=nice,C=chocolate,P(N,C)=0.45. However, P(N)P(C)=0.6×0.5=0.3. That's it is not true that N and C are statically independent.

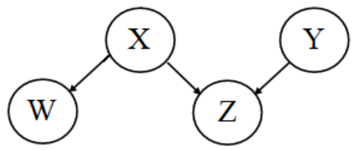
(f) (3 points) Suppose Santa has other secret lists that track your other behaviors and qualities throughout the year such as manners (polite/rude), house-chores completion (many/few), school grades (high/low), denoted as the random variables M, H, G respectively. Is it possible to compute $\mathbb{P}(N, C, M, H, G)$ as a product of five terms? We require that each term be in the form $P(X_i|X_j, \ldots, X_k)$ where there is one random variable to the left of the vertical bar "|" and there may be 0 or more random variables to the right of the vertical bar. (If there are none to the right, then the vertical bar does not appear. Thus $P(X_i)$ as a term is fine.) If it is possible to express the distribution as such a product, then show the details of such a product. What assumptions need to be made, if any? Otherwise, explain why it is not possible.

if each N, C, M, H, G is independent, it is possible to compute $\mathbb{P}(N, C, M, H, G)$ as

$\mathbb{P}(N, C, M, H, G) = P(N)P(C)P(M)P(H)P(G)$

4 Bayes Net Structure and Meaning (Phuong)

(10 points) Consider the Bayes net with its graph is shown below:



Random variable W has a domain with four values $\{w_1, w_2, w_3, w_4\}$; the domain for X has three values: $\{x_1, x_2, x_3\}$; Y's domain has three values: $\{y_1, y_2, y_3\}$; and Z's domain has four values: $\{z_1, z_2, z_3, z_4\}$.

(a) (3 points) Give a formula for the joint distribution of all four random variables, in terms of the marginals (e.g., $\mathbb{P}(X = x_i)$), and conditionals that must be part of the Bayes net (e.g., $\mathbb{P}(Z = z_m | X = X_j, Y = y_k)$).

$\mathbb{P}(X, Y, W, Z) = \mathbb{P}(X = x_i)\mathbb{P}(Y = y_i)\,\mathbb{P}(W = w_i | X = x_i)\mathbb{P}(Z = z_i | X = x_i, Y = y_i)$

(b) (1 point) How many probability values belong in the (full) joint distribution table for this set of random variables?

$4 \times 3 \times 3 \times 4 = 144$

(c) (2 points) For each random variable: give the number of probability values in its marginal (for X and Y) or conditional distribution table (for the others).

W: $4 \times 3 = 12$

X: 3

Y: 3

Z: $4 \times 3 \times 3 = 36$

(d) (4 points) For each random variable, give the number of *non-redundant* probability values in its table from (c).

W: $3 \times 3 = 9$

X: 2

Y: 2

Z: $3 \times 3 \times 3 = 27$

5 D-Separation (Steve)

(20 points) Consider the Bayes Net graph β below, which represents the topology of a web-server security model. Here the random variables have the following interpretations:

V = Vulnerability exists in web-server code or configs.

C = Complexity to access the server is high. (Passwords, 2-factor auth., etc.)

S = Server accessibility is high. (Firewall settings, and configs on blocked IPs are permissive).

A = Attacker is active.

L = Logging infrastructure is state-of-the-art.

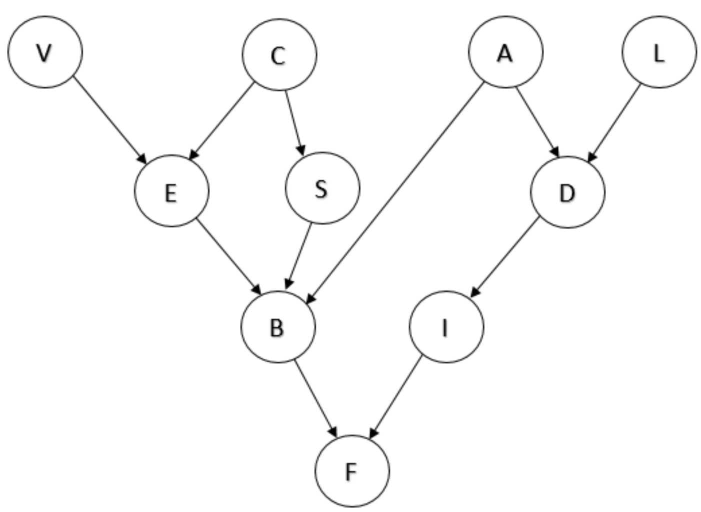
E = Exposure to vulnerability is high.

D = Detection of intrusion attempt.

B = Break-in; the web server is compromised.

I = Incident response is effective.

F = Financial losses are high (due to data loss, customer dissatisfaction, etc).



Let β' be the undirected graph obtained from β by removing the arrowheads from the edges of β. By an "undirected path" in β we mean any path in β'. A "loop-free" path is any path in which no vertex is repeated.

(a) (5 points) List all loop-free undirected paths from A to C in the graph β.

ABSC ABEC ADIFBSC ADIFBEC

(b) (5 points) Suppose random variable B is observed, and no others are observed. Then which (if any) of those paths would be active paths? Justify your answer.

ABSC ABEC

For each of the following statements, indicate whether (True) or not (False) the topology of the net guarantees that that the statement is true. If False, identify a path ("undirected") through which influence propagates between the two random variables being considered. (Be sure that the path follows the D-Separation rules covered in lecture.) The first one is done for you.

(c) $E \perp\!\!\!\perp S$: False (ECS)

(d) (1 point) $L \perp\!\!\!\perp B \mid A, I$ true

(e) (1 point) $I \perp\!\!\!\perp A \mid D, F$ false (IFBA)

(f) (1 point) $L \perp\!\!\!\perp V \mid D, E, F$ false (LDABSCEV)

(g) (1 point) $L \perp\!\!\!\perp V \mid A, D$ true

(h) (1 point) $D \perp\!\!\!\perp E \mid F, I$ false (DABE)

(i) (5 points) Suppose that the company hired an outside expert to examine the system and she determines that D is true: She has detected an intrusion event. Given this information, your job is to explain to management why getting additional information about L (Logging infrastructure being up-to-date) could have an impact on the probability of F (rfinancial losses being high). Give your explanation, for the manager of the company, using about between 2 and 10 lines of text, which should be based on what you know about D-separation, applied to this situation. However, your explanation should not use the terminology of D-separation but be in plain English. (You can certainly use words like "influence", "probability", "given", but not "active path", "triple", or even "conditionally independent").

From the bayes Net graph, Either of A or L alone or both of them together will cause D. Given D is true, D enables the influence of L on A. In this situation, since D is true, L is able to influence A. And A also influence to B. Moreover, B also influence to F. In short, probability of L influence probability of F. That's why chage of L could have an impact on the probability of F.

6 Perceptrons (Anna)

(20 points) For all parts of this question perceptrons should output 1 if $w_n + \sum_{i=0}^{n-1} w_i x_i \geq 0$ and 0 otherwise. The weight w_n is called the bias weight.

(a) (5 points) Assuming there will be two inputs x_0 and x_1, each with possible values in $\{0, 1\}$, give values for a triple of weights $\langle w_0, w_1, w_2 \rangle$ such that the corresponding perceptron would act as a NAND gate for the two inputs. (Weight w_2 is the bias weight.) Note that a NAND gate outputs 1 when at least one of the inputs is 0; it outputs 0 otherwise.

(b) (5 points) Draw a perceptron, with weights, that accepts a single integer x and outputs 1 if and only if the input is greater than or equal to -7. Be sure to include the bias input of value 1 and its weight in your diagram. Draw another perceptron that outputs 1 if and only if the input is less than or equal to 7.

(c) (2 points) Using the previous perceptrons, create a two-layer perceptron that outputs 1 if $|x| \leq 7$, and 0 otherwise.

(d) (5 points) Suppose we want to train a perceptron to compare two numbers x_0 and x_1 and produce output $y = 1$ provided that x_1 exceeds x_0 by at least 5. Assume that the initial weight vector is: $\langle w_0, w_1, w_2 \rangle = \langle 0, 0, 1 \rangle$. Consider a first training example: $(\langle x_0, x_1 \rangle, y) = (\langle 1, 2 \rangle, 0)$. This says that with inputs 1, and 2, the output y should be 0, since 2 exceeds 1 by only 1. What will be the new values of the weights after this training example has been processed one time? Assume the learning rate is 2.

(e) (3 points) Continuing with the last example, now suppose that the next step of training involves a different training example: $(\langle 2, 8 \rangle, 1)$. The output for this example should be 1, since 8 does exceed 2 by at least 5. Starting with the weights already learned in the first step, determine what the adjusted weights should be after this new example has also been processed once.

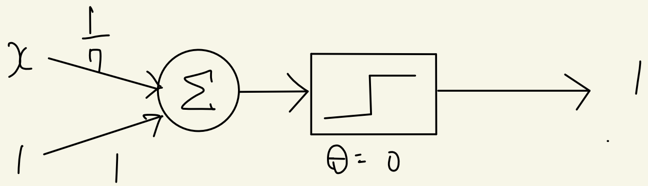
(a)

x_1	x_2	y
0	0	1
1	0	1
0	1	1
1	1	0

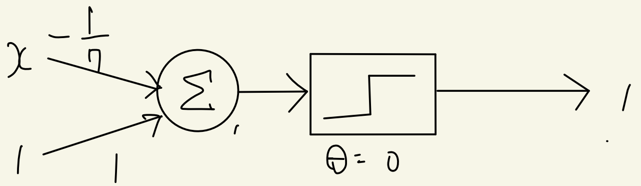
$$w_n + \sum_{i=0}^{n-1} w_i x_i \geq 0$$

$w_0 = -0.6 \quad w_1 = -0.6 \quad w_2 = 0.7$

(b)

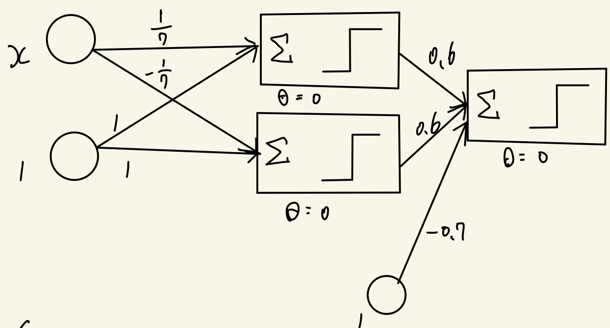


output $= 0$ if $x \geq -7$



output $= 0$ if $x \leq 7$

(c)



(d) $\langle w_0, w_1, w_2 \rangle = \langle 0, 0, 1 \rangle \; (\langle x_0, x_1 \rangle, y) = (\langle 1, 2 \rangle, 0)$

$w_0 \cdot x_0 + w_1 \cdot x_1 + w_2 = 0 \cdot 1 + 0 \cdot 2 + 1 = 1 \rightarrow \hat{y} = 1 \neq y$

misclassified

$w_0' = 0 - 2 \cdot 1 = -2 \quad w_1' = 0 - 2 \cdot 2 = -4 \quad w_2' = 1 - 2 = -1$

$\langle w_0, w_1, w_2 \rangle = \langle -2, -4, -1 \rangle$ //

(e) $\langle w_0, w_1, w_2 \rangle = \langle -2, -4, -1 \rangle$

$(\langle x_0, x_1 \rangle, y) = (\langle 2, 8 \rangle, 1)$

$$w_0 \cdot x_0 + w_1 \cdot x_1 + w_2 = -2 \cdot 2 + (-4) \cdot 8 - 1$$
$$= -37 \longrightarrow \hat{y} = 0 \neq y$$

misclassified

$w_0' = -2 + 2 \cdot 2 = 2$ $\quad w_1' = -4 + 2 \cdot 8 = 12$ $\quad w_2' = -1 + 2 = 1$

$\langle w_0, w_1, w_2 \rangle = \langle 2, 12, 1 \rangle$ //

7 The Laws of Robotics (Emilia)

(20 points) In the 1940's, Isaac Asimov introduced a set of three laws to govern robot behavior:

1. A robot may not injure a human being or, through inaction, allow a human being to come to harm.
2. A robot must obey the orders given it by human beings except where such orders would conflict with the First Law.
3. A robot must protect its own existence as long as such protection does not conflict with the First or Second Laws.

(NOTE: You might also want to take a look at this cartoon `https://xkcd.com/1613/`)

Imagine you live in a world where personal robotic assistants are a bit more advanced than they are at present. They are still not commonplace, but in another 10 years (in our scenario), they could very well be. Further, imagine that you are a medical student who has had to move to virtual schooling due to a viral pandemic. Your well-endowed medical school, in partnership with faculty and industry collaborators, has presented you and all your classmates with "Standardized Patient Robots" – beta version humanoid personal assistant robots that they had quickly modified with knowledge of medical conditions, symptoms, and treatments (along with other enhancements) to enable you to continue your clinical training while in quarantine.

You were a computer science student before attending medical school, and you're also a science fiction aficionado. Your favorite author – Isaac Asimov. You can't resist hacking your new robot so that it now is programmed to obey Asimov's three laws before considering any other part of its programming. You feel quite pleased with yourself. Now, not only do you have a personal robot, you also have a personal bodyguard.

In other news, you recently volunteered to take part in a vaccine study being run out of your medical school and you've just been notified that you've been accepted. Furthermore, since the robot has programming that qualifies it as a medical assistant, you won't even need to leave your house to take part – the vaccine trials will be delivered to your front door and administered by the robot, which will then monitor you for any side effects.

Considering the above information, please answer the following questions. For each multiple choice question (MCQ), select what you think would be the most likely outcome. In the free response question that follows each MCQ explain what has happened and provide a justification for your selection, referring to Asimov's Laws by number as appropriate (e.g. "According to Asimov's 1st Law, ...").

(a) (1 point) It is time for your first vaccine dose. Which of the following takes place?

(a) The robot calmly gives you the vaccine dose on schedule.

(b) The robot runs an analysis on the vaccine contents before giving the dose on schedule.

(c) The robot runs an analysis on the vaccine contents, after which it refuses to administer the dose.

(d) The robot refuses to give you the scheduled vaccine dose.

(b) (4 points) Please justify the scenario you selected.

I think that the robot runs an analysis on the vaccine contents before giving the dose on schedule. According to Asimov's 1st Law, the robot must not injure humans through inaction. Robot has to run an analysis on contents of vaccine because vaccines may contain ingredients that can harm humans. In short, robot reduces possibility of injure humans through inaction by runnning an analysis on contents of vaccine. Moreover, according to Asimov's 2nd Law, robot has to obey the orders given it by human beings except where such orders would conflict with the First Law. Robot has to inject vaccine because injecting vaccine is the order from humans. However, since robot cannot break Asimov's 1st Law, robot has to check whether robots can obey 1st Law by obeying the order from humans. By running an analysis on the vaccine contents robot is trying to obey Asimov's 1 st Law and 2nd Law. Asimov's 3rd Law is obeyed because the possibility that the robot itself is in danger is extremely low. Option B is best because it obeys all of Asimov's Law.

(c) (1 point) As a single student with an insane work schedule (normally), you tended to eat most of your meals out. Now, you are faced with the problem of how to feed yourself. You look around your kitchen and realize that all your frozen pizzas and instant noodles have run out. You get ready to make a quick run to the store down the street. What happens next?

(a) The robot calmly reminds you to wear your mask and bring along some hand sanitizer.

(b) The robot refuses to let you leave your house, and informs you it will do the shopping instead. You tell it the items you want and it returns an hour later with everything on your list.

(c) The robot refuses to let you leave your house, and informs you it will do the shopping instead. You tell it the items you want and it returns an hour later with several bags of shopping, but few (if any) of the items that were on your list.

(d) The robot heads off to the store. Several hours later, it still has not returned and you go out to find it. You get to the store and find a long line of people, spaced 6 feet apart. You ask one of them what's going on, and one of them tells you some crazy robot is getting people groceries one at a time.

(d) (4 points) Please justify the scenario you selected.

If a robot allows me to go out and I have an accident, the robot has broken Asimov's 1st Law because it hurt me. However, if I die of starvation without being allowed to go out, the robot has injured me, so the robot has broken Asimov's 1st Law. So the robot chooses to go shopping instead. The robot will shop based on a written list of what I want. But if that list included anything that could cause side effects from this vaccine, the robot would have broken Asimov's 1st Law. This list can be interpreted as orders of Asimov's 2nd Law, but robots do not need to obey the list because the 1st Law can be broken. Therefore, the robot buys items from the list that will never hurt me, so only a few items on the list are bought.

(e) (1 point) As your quarantine extends from week to week, you start feeling more and more stressed and depressed. You try to think of the robot as a kind of housemate, but its social interaction modules seem to be lacking. You make plans for a friend to visit and hang out with you in your backyard. What happens after he arrives?

(a) The robot accompanies you out to the yard and monitors your self distancing etiquette.

(b) The robot refuses to let your friend enter the backyard, telling him instead to go home and respect the quarantine.

(c) The robot welcomes your friend and monitors your visit, but refuses to let your friend leave several hours later.

(d) The robot lets your friend enter the backyard, but refuses to let you join him there, instead insisting that you communicate with your friend over the phone while looking at each other through a window.

(f) (4 points) Please justify the scenario you selected.

(g) (5 points) Now consider the following, slightly different, scenario. At some later point in time, Asimov added a 4th law to his Laws of Robotics – this law is known as the zeroth law and is meant to take priority over the original three laws. This new law is provided below:

0. A robot may not injure humanity, or, by inaction, allow humanity to come to harm.

Consider how the robot's responses to the situations described above would differ if you had programmed it with all four of these laws. Which selections might change, and how and why would they change?

(f)

The robot allows my friend in the backyard because if the robot refuses to let my friend in the backyard it will make me even more depressed and break Asimov's 1st Law. However, the robot monitors self-distance etiquette because if something happens in contact with a friend, the robot will break Asimov's 1st Law. So A is the best option. In the same option, d seems to be the best option, but in this option I am trying to improve my depression by communicating with my friends. However, the robot perceives that my purpose is not to improve my depression by making friends, but to meet someone in my backyard. Robot can't come up with a solution. Therefore d is inappropriate. Also, in c, the robot welcomes friends, but it is inappropriate because the robot cannot understand the time when my depression will improve.

(g)

Robots give top priority not to hurt humans, so in (a), robot chooses d. Robot refuses to inject because vaccination itself would hurt humans. In (c), the selection of the robot does not change. The robot understands that it will hurt me whether I allow or not allow me to go out, so the robot itself goes out and does not list anything that could harm me. This follows Asimov's 0 Law, so choose c. In (e), the robot chooses option b. The robot avoids the option to meet with my friends because the robot judges that meeting with my friends may cause harm to me.

8 Markov Models (Shane)

(20 points) According to an unnamed source, the stock market can be modeled using a Markov model, where there are two states "bull" and "bear." The dynamics of the model are given:

S_{t-1}	S_t	$P(S_t \mid S_{t-1})$
bull	bull	0.7
bull	bear	0.3
bear	bull	0.2
bear	bear	0.8

(a) (2 points) Suppose it's given that $S_0 =$ bull. Compute the probability that $S_2 =$ bull.

(b) (4 points) Compute the stationary probabilities for bull and bear.

(c) (2 points)Now suppose that whenever it's a bull market, a certain company's (Acme, Inc) stock stays the same or rises in value with probability 0.8 and falls in value with probably 0.2.

When it's a bear market Acme's stock value stays the same or rises with probability 0.4 and falls with probability 0.6.

Suppose an observer cannot directly tell whether the state of the stock market is bull or bear, but can only see whether Acme's stock is "rising" or "falling."

State S	Observation Q	$P(Q \mid S)$
bull	rising	0.8
bull	falling	0.2
bear	rising	0.4
bear	falling	0.6

Suppose $P(S_0 =$ bull$) = 0.5$. If the observation at time 1 is "rising," what is the belief in S=bull right after the observation?

(d) (2 points) Suppose at time 2, the observation is "falling". what is the belief in $S =$ bull right after that observation? (This belief will take into consideration the previous belief you computed above.)

(a) $0.7 \times 0.7 + 0.3 \times 0.2 = \underline{0.55}$ //

(b) $P_\infty(bull) = 0.7 P_\infty(bull) + 0.2 P_\infty(bear)$

$P_\infty(bear) = 0.3 P_\infty(bull) + 0.8 P_\infty(bear)$

$0.3 P_\infty(bull) = 0.2 P_\infty(bear)$

$P_\infty(bull) = \frac{2}{3} P_\infty(bear)$

Also $P_\infty(bull) + P_\infty(bear) = 1$

$\frac{5}{3} P_\infty(bear) = 1$

$P_\infty(bear) = \frac{3}{5} \quad P_\infty(bull) = \frac{2}{5}$

$P_\infty(bull) = \frac{2}{5} \quad P_\infty(bear) = \frac{3}{5}$ //

(c) $P(bull) = 0.5 \times 0.7 + 0.5 \times 0.2 = 0.45$

$P(bear) = 0.5 \times 0.3 + 0.5 \times 0.8 = 0.55$

$\frac{1}{0.45 \times 0.8 + 0.55 \times 0.4} \times 0.45 \times 0.8 = 0.620$

$\fallingdotseq 0.62$

(d) $\frac{1}{0.45 \times 0.8 + 0.55 \times 0.4} \times 0.55 \times 0.4 = 0.38$ //

$0.62 \times 0.7 + 0.38 \times 0.2 = 0.51$

$P(bull) = 0.51 \quad P(bear) = 0.49$

$\frac{1}{0.51 \times 0.2 + 0.49 \times 0.6} \times 0.51 \times 0.2 = 0.257$

$\fallingdotseq \underline{0.26}$ //

(e) (2 points) Suppose that the actual state sequence for the first four time steps is bear, bear, bear, bull

What is the probably of observing the sequence (starting at $t = 1$) rising, falling, rising?

(f) (3 points) Now, what if the state sequence was bull, bear, bear, bull. What is the probability of observing the same sequence of stock changes as above?

(g) (2 points) Which of these two state sequences is more likely, given that sequence of observations?

(h) (3 points) Is there another state sequence that is even more likely? Explain.

(i) (optional, not for credit) Use the Viterbi algorithm to compute the most likely state sequence for this observation sequence. You should draw a trellis diagram. Assume that the initial probability distribution is $P(S_0 = \text{bull}) = 0.5$; $P(S_0 = \text{bear}) = 0.5$. A very concise and relatively clear video presentation of a simple Viterbi algorithm example is one by Luis Serrano at:

`https://www.youtube.com/watch?v=mHEKZ8jv2SY`

(e) $0.4 \times 0.6 \times 0.8 = 0.192$

(f) $0.8 \times 0.6 \times 0.4 = 0.192$

(g) They're equally likely

(h) bull, bear, bull, bull is more likely because the probably of observing the sequence rising, falling, rising becomes $0.8 \times 0.6 \times 0.8 = 0.384$

9 Probabilistic Context-Free Grammars (Steve)

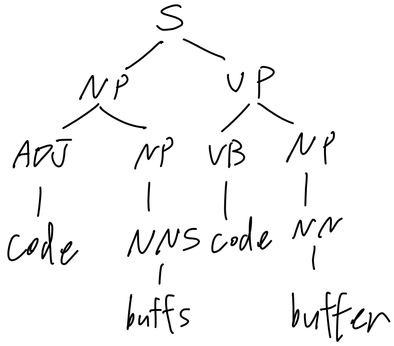
(20 points) Consider the sentence, "Code buffs code buffer." Here there is ambiguity at multiple levels: lexical (word parts of speech and meanings), and syntactic (phrase structure). The semantics also vary. The sentence could mean that programming enthusiasts ("code buffs") write a program that manages a storage area (a buffer). It could also mean that a computer program smooths (buffs) the operation of a buffer that holds source code (a code buffer).

With the probabilistic context-free grammar given below, find two parses, and compute a score for each one. Then identify the most probable parse using the scores. Assume the number at the right of a production is its conditional probability of being applied, given that the symbol to be expanded is that production's left-hand side.

(a) (7 points) Convert each probability into a score by taking score $= -\log_{10}(p)$. Round scores to 2 decimal places of accuracy. Write the production scores in the "__.__" blanks.

S	::= VP	0.1	1.00	VP	::= VB NP	0.4	0.40
S	::= NP VP	0.8	0.10	VB	::= code	0.05	1.30
NP	::= ADJ NP	0.4	0.40	VB	::= buffs	0.03	1.52
NP	::= NP PRP	0.2	0.70	NN	::= code	0.06	1.22
NP	::= NN	0.2	0.70	NN	::= buffer	0.02	1.70
NP	::= NNS	0.2	0.70	NNS	::= buffs	0.01	2.00
VP	::= VB S	0.2	0.70	ADJ	::= code	0.06	1.22
VP	::= VBG NP	0.2	0.70				

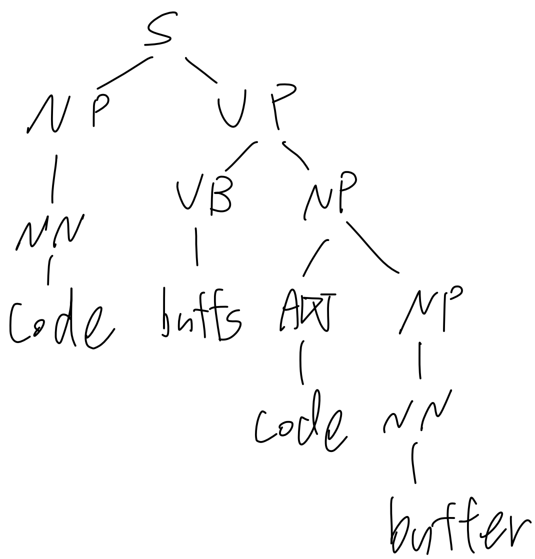
(b) (5 points) Give a first parse for the sentence. This parse should correspond to the interpretation, "programming enthusiasts write a program that manages a storage area." Compute the (total) score for this parse.



total score = 0.1 + 0.4 + 0.4 + 1.22 + 0.7
1.3 + 0.7 + 2 + 1.7

= 8.52 #

(c) (5 points) Give the second parse. This parse should correspond to the interpretation, "a computer program smooths (buffs) the operation of a buffer that holds source code (a code buffer)." Compute its score.



total score $= 0.1 + 0.7 + 0.4 + 1.22 + 0.4 + 1.52 + 1.22 + 0.7 + 1.7$
$= 7.96$

(d) (3 points) Convert each score back to a probability and write them here as P1 and P2. Then tell which parse is more probable.

$P(b) = 10^{-8.52} = 3.019 \times 10^{-9} \fallingdotseq 3.02 \times 10^{-9}$

$P(c) = 10^{-7.96} = 1.096 \times 10^{-8} \fallingdotseq 1.10 \times 10^{-8}$

(b) parse probability is 3.02×10^{-9}. (c) parse probability is 1.10×10^{-8}.

(c) parse is more probable.