approximates the true covariance of y up to the third order (i.e., only terms to the fourth and higher powers are incorrect). This is the same approximation order as the linearization method, as seen on page 439. However, we would intuitively expect the magnitude of the error of the unscented approximation in Equation (14.43) to be smaller than the linear approximation HPH^T, because the unscented approximation at least contains correctly signed terms to the fourth power and higher, whereas the linear approximation does not contain any terms other than HPH^T.

The unscented transformation can be summarized as follows.

The unscented transformation

1. We begin with an n-element vector x with known mean $\bar{x}$ and covariance P. Given a known nonlinear transformation $y = h(x)$, we want to estimate the mean and covariance of y, denoted as $\bar{y}_u$ and P_u.

2. Form $2n$ sigma point vectors $x^{(i)}$ as follows:

$$\begin{aligned} x^{(i)} &= \bar{x} + \tilde{x}^{(i)} \quad i = 1, \cdots, 2n \\ \tilde{x}^{(i)} &= \left(\sqrt{nP}\right)_i^T \quad i = 1, \cdots, n \\ \tilde{x}^{(n+i)} &= -\left(\sqrt{nP}\right)_i^T \quad i = 1, \cdots, n \end{aligned} \tag{14.50}$$

where $\sqrt{nP}$ is the matrix square root of nP such that $(\sqrt{nP})^T\sqrt{nP} = nP$, and $(\sqrt{nP})_i$ is the ith row of $\sqrt{nP}$.

3. Transform the sigma points as follows:

$$y^{(i)} = h(x^{(i)}) \quad i = 1, \cdots, 2n \tag{14.51}$$

4. Approximate the mean and covariance of y as follows:

$$\begin{aligned} \bar{y}_u &= \frac{1}{2n}\sum_{i=1}^{2n} y^{(i)} \\ P_u &= \frac{1}{2n}\sum_{i=1}^{2n}\left(y^{(i)} - y_u\right)\left(y^{(i)} - y_u\right)^T \end{aligned} \tag{14.52}$$

■ **EXAMPLE 14.1**

To illustrate the unscented transformation, consider the nonlinear transformation shown in Equation (14.1). Since there are two independent variables (r and θ), we have $n = 2$. The covariance of P is given as $P = \text{diag}(\sigma_r^2, \sigma_\theta^2)$. Equation (14.32) shows that $W^{(i)} = 1/4$ for $i = 1, 2, 3, 4$. Equation (14.29) shows that the sigma points are determined as

$$\begin{aligned} x^{(1)} &= \bar{x} + \left(\sqrt{nP}\right)_1^T \\ &= \begin{bmatrix} 1 + \sigma_r\sqrt{2} \\ \pi/2 \end{bmatrix} \end{aligned}$$