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A New Approach for Filtering Nonlinear Systems

Simon J. Julier, Jeffrey K. Uhlmann and **Hugh F. Durrant-Whyte**
Robotics Research Group, Department of Engineering Science, University of Oxford,
Oxford, OX1 3PJ United Kingdom

Abstract—In this paper we describe a new recursive linear estimator for filtering systems with nonlinear process and observation models. This method uses a new parameterisation of the mean and covariance which can be transformed directly by the system equations to give predictions of the transformed mean and covariance. We show that this technique is more accurate and far easier to implement than an extended Kalman filter. Specifically, we present empirical results for the application of the new filter to the highly nonlinear kinematics of maneuvering vehicles.

I. Introduction

Filtering is one of the most pervasive tools of engineering. Whenever the state of a system must be estimated from noisy sensor information, some kind of state estimator is employed to fuse together the data from different sensors to produce an accurate estimate of the true system state. When the system dynamics and observation models are linear, the minimum mean squared error (MMSE) estimate may be computed using the *Kalman filter*. However, in most applications of interest the system dynamics and observation equations are nonlinear and suitable extensions to the Kalman filter have been sought. The optimal solution to the nonlinear filtering problem requires that a complete description of the conditional probability density is maintained. Unfortunately this exact description requires a potentially unbounded number of parameters (such as moments) and a number of suboptimal approximations have been proposed [5]. These usually employ unwieldy analytical approximations to probability distributions, derivatives of the state transition and observation equations, or Monte Carlo methods which require the use of many thousands of points to approximate the conditional density. In many high-dimensioned applications these methods are rarely practical. For these reasons the most widely used filter, the extended Kalman filter(EKF), is also the crudest generalisation. This filter closely resembles a Kalman filter except that each linear step is replaced by its linearised equivalent. Although the EKF is conceptually simple it has, in practice, three well-known drawbacks:

1. Linearisation can produce highly unstable filter performance if the timestep intervals are not sufficiently small.
2. The derivation of the Jacobian matrices are nontrivial in most applications and often lead to significant implementation difficulties.
3. Sufficiently small timestep intervals usually imply high computational overhead as the number of calculations demanded for the generation of the Jacobian and the predictions of state estimate and covariance are large.

In this paper we describe a new approach to generalising the Kalman filter to systems with nonlinear state transition and observation models. In Section 2 we describe the basic filtering problem and the notation used in this paper. In Section 3 we describe the new filter. The fourth section presents a summary of the theoretical analysis of the performance of the new filter against that of the EKF. In Section 5 we demonstrate the new filter in a highly nonlinear application and we conclude with a discussion of the implications of this new filter[1]

II. The Filtering Problem

The filtering problem of interest in this paper is to find the best (MMSE) linear estimate of the state vector $\mathbf{x}(k)$ of a system of interest which evolves according to the discrete-time non-linear (and possibly time varying) state transition equation [1]

$$\mathbf{x}(k+1) = \mathbf{f}[\mathbf{x}(k), \mathbf{u}(k+1), k+1] + \mathbf{v}(k), \qquad (1)$$

where $\mathbf{f}[\cdot,\cdot,\cdot]$ is the process model, $\mathbf{x}(k)$ is the state of the system at timestep k, $\mathbf{u}(k+1)$ is the input vector and $\mathbf{v}(k)$ is an additive process noise. It is assumed that the only information available about this system are its control inputs and a set of noisy observations $\mathbf{z}(k+1)$. These observations are related to the state vector by the nonlinear equation

$$\mathbf{z}(k+1) = \mathbf{h}[\mathbf{x}(k+1), \mathbf{u}(k+1), k+1] + \mathbf{w}(k+1), \qquad (2)$$

where $\mathbf{z}(k+1)$ is the observation vector, $\mathbf{h}[\cdot,\cdot,\cdot]$ is the observation model that transforms the state space vector into observation space and $\mathbf{w}(k)$ is an additive measurement noise.

It is assumed that the additive noise vectors, $\mathbf{v}(k)$ and $\mathbf{w}(k)$, are Gaussian, uncorrelated white sequences. $\mathrm{E}[\mathbf{v}(k)] = \mathrm{E}[\mathbf{w}(k)] = \mathbf{0}$, for all k, and their respective covariances are

$$\mathrm{E}\left[\mathbf{v}(i)\mathbf{v}^T(j)\right] = \delta_{ij}\mathbf{Q}(i), \qquad (3)$$

$$\mathrm{E}\left[\mathbf{w}(i)\mathbf{w}^T(j)\right] = \delta_{ij}\mathbf{R}(i), \qquad (4)$$

with cross covariance

$$\mathrm{E}\left[\mathbf{v}(i)\mathbf{w}^T(j)\right] = \mathbf{0}, \quad \forall i, j. \qquad (5)$$

The true states of the system are not known; they must be estimated and since the process and observation models

[1] A full discussion of the results outlined in this paper can be found on the WWW at **http://www.robots.ox.ac.uk.**

are disturbed by random processes, the estimation errors are random variables. For any distribution of these errors, the MMSE estimate coincides with the conditional mean. We let $\hat{\mathbf{x}}(i \mid j)$ be the conditional mean at time i conditioned on all observations up to time j,

$$\hat{\mathbf{x}}(i \mid j) = \mathrm{E}\left[\mathbf{x}(i)|\mathbf{Z}^j\right] \tag{6}$$

where $\mathbf{Z}^j = \{\mathbf{z}(1), \mathbf{z}(2), \ldots, \mathbf{z}(j)\}^T$. The estimated covariance is

$$\mathbf{P}(i \mid j) = \mathrm{E}\left[\{\mathbf{x}(i) - \hat{\mathbf{x}}(i \mid j)\}\{\mathbf{x}(i) - \hat{\mathbf{x}}(i \mid j)\}^T|\mathbf{Z}^j\right]. \tag{7}$$

Since these equations are difficult to calculate in practice, the class of *recursive linear estimators* is usually employed instead. These estimators apply the nonlinear state transition and observation equations to the current estimated state of the system, $\hat{\mathbf{x}}(k \mid k)$ with covariance $\mathbf{P}(k \mid k)$, to predict the state at a subsequent timestep, $\hat{\mathbf{x}}(k+1 \mid k)$ and $\mathbf{P}(k+1 \mid k)$. The system is observed at $k+1$ and the information is incorporated linearly into the state estimate, hence the name *linear* estimator. It can be shown [1] that the best linear MMSE estimate is such that the estimation error is both unbiased and orthogonal to the observation $\mathbf{z}(k+1)$. This necessitates a prediction of the observation at time $k+1$ and since $\hat{\mathbf{x}}(k+1 \mid k)$ is a random variable, the predicted observation is a random variable as well. It is distributed with conditional mean $\hat{\mathbf{z}}(k+1 \mid k)$ and covariance $\mathbf{P}_{zz}(k+1 \mid k)$. The linear update equations are

$$\begin{aligned} \hat{\mathbf{x}}(k+1 \mid k+1) &= \hat{\mathbf{x}}(k+1 \mid k) + \mathbf{W}(k)\nu(k) && (8) \\ \mathbf{P}(k+1 \mid k+1) &= \mathbf{P}(k+1 \mid k) \\ &- \mathbf{W}(k+1)\mathbf{P}_{\nu\nu}(k+1 \mid k)\mathbf{W}^T(k+1) && (9) \end{aligned}$$

The vector $\nu(k+1)$ is the *innovation*, which is equal to the difference between the actual observation and the predicted observation:

$$\nu(k+1) = \mathbf{z}(k+1) - \hat{\mathbf{z}}(k+1 \mid k). \tag{10}$$

The covariance of this quantity is

$$\mathbf{P}_{\nu\nu}(k+1 \mid k) = \mathbf{P}_{zz}(k+1 \mid k) + \mathbf{R}(k+1). \tag{11}$$

$\mathbf{W}(k+1)$ is the *Kalman gain* and its value is given by

$$\mathbf{W}(k+1) = \mathbf{P}_{xz}(k+1 \mid k)\mathbf{P}_{\nu\nu}^{-1}(k+1 \mid k), \tag{12}$$

where $\mathbf{P}_{xz}(k+1 \mid k)$ is the predicted cross-correlation matrix between $\hat{\mathbf{x}}(k+1 \mid k)$ and $\hat{\mathbf{z}}(k+1 \mid k)$.

The prediction phase is vital for overall filter performance. However these equations do not specify how this process is to be carried out. The EKF assumes that the errors in the state estimates are small. As a consequence, the predicted mean is approximated by

$$\begin{aligned} \hat{\mathbf{x}}(k+1 \mid k) &= \mathrm{E}\left[\mathbf{f}[\mathbf{x}(k), \mathbf{u}(k+1), k+1]|\mathbf{Z}^k\right] \\ &\approx \mathbf{f}[\mathrm{E}\left[\mathbf{x}(k)|\mathbf{Z}^k\right], \mathbf{u}(k+1), k+1] \\ &= \mathbf{f}[\hat{\mathbf{x}}(k \mid k), \mathbf{u}(k+1), k+1]. \end{aligned} \tag{13}$$

That is, the predicted mean is equal to the prior mean projected through $\mathbf{f}[\cdot,\cdot,\cdot]$. This estimate does not consider the actual distribution of the errors on the state prediction. Further, it is assumed that the state errors propagate through a separate linearised system and the covariance of these errors evolves according to

$$\mathbf{P}(k+1 \mid k) = \mathcal{J}_f\mathbf{P}(k \mid k)\mathcal{J}_f^T + \mathbf{Q}(k), \tag{14}$$

where $\mathcal{J}_f$ is the Jacobian matrix of $\mathbf{f}[\cdot,\cdot,\cdot]$ evaluated about $\hat{\mathbf{x}}(k \mid k)$. Similar assumptions are made in predicting $\hat{\mathbf{z}}(k+1 \mid k)$ and $\mathbf{P}_{zz}(k+1 \mid k)$. However, the assumptions that (a) the mean can be accurately predicted without regard to the distribution of the errors and (b) that the state errors propagate through a separate linearised system rarely hold and this leads to the difficulties outlined in the introduction. In the light of these problems we have developed a new filter which uses the recursive linear estimator structure (Equations 8 to 12) but avoids the EKF's linearising assumptions.

III. The New Filter

We begin with the following intuition: *With a fixed number of parameters it should be easier to approximate a Gaussian distribution than it is to approximate an arbitrary nonlinear function* [3, 8]. Following this intuition we seek a parameterisation which captures the mean and covariance information while at the same time permitting the direct propagation of the information through an arbitrary set of nonlinear equations. We shall show that this can be accomplished by generating a discrete distribution composed of the minimum number of points which have the same first and second (and possibly higher) moments, where each point in the discrete approximation can be directly transformed. The mean and covariance of the transformed ensemble can then be computed as the estimate of the nonlinear transformation of the original distribution.

Given an n-dimensional Gaussian distribution having covariance $\mathbf{P}$, we generate a set of $O(n)$ points having the same covariance from the columns (or rows) of the matrices $\pm\sqrt{n\mathbf{P}}$ (the positive and negative roots). This set of points is zero mean, but if the original distribution has mean $\bar{\mathbf{x}}$, then simply adding $\bar{\mathbf{x}}$ to each of the points yields a symmetric set of $2n$ points having the desired mean and covariance. Because the set is symmetric its odd central moments are zero, so its first three moments are the same as the original Gaussian distribution. This is the *minimal* number of points capable of encoding this information. A random sampling of points from the distribution, on the other hand, will generally introduce spurious modes in the transformed distribution even if the set of sample points has the correct mean and covariance. In a filtering application these modes will take the form of high frequency noise that may completely obscure the signal.

To restate the general problem, we have the mean $\hat{\mathbf{x}}(k \mid k)$ and covariance $\mathbf{P}(k|k)$ of the state at time k and would like to predict $\hat{\mathbf{x}}(k+1 \mid k)$ and $\mathbf{P}(k+1|k)$ through the nonlinear function $\mathbf{f}[\cdot,\cdot,\cdot]$. We summarise the basic method as follows:

1. Compute the set $\sigma_i(k|k)$ of $2n$ points from the columns of the matrices $\pm\sqrt{n\mathbf{P}(k|k)}$. This set is zero mean with covariance $\mathbf{P}(k|k)$. Compute a set of points with the

same covariance, but with mean $\hat{\mathbf{x}}(k \mid k)$, by translating each of the points as $\mathcal{X}_i(k \mid k) = \sigma_i(k|k) + \hat{\mathbf{x}}(k \mid k)$.

2. Transform each point through the state dynamics equations as $\mathcal{X}_i(k+1 \mid k) = \mathbf{f}[\mathcal{X}_i(k \mid k), \mathbf{u}(k+1), k+1]$.

3. Compute $\hat{\mathbf{x}}(k+1 \mid k)$ and $\mathbf{P}(k+1|k)$ by computing the mean and covariance of the $2n$ points in the set $\mathcal{X}_i(k+1 \mid k)$.

Process noise is injected into the state transition model by adding a dynamic noise covariance matrix $\mathbf{Q}(k)$ to $\mathbf{P}(k \mid k)$ before the sigma points are calculated. To predict $\hat{\mathbf{z}}(k+1 \mid k)$ and $\mathbf{P}_{zz}(k+1 \mid k)$ we apply the same intuition to $\mathbf{h}[\cdot,\cdot,\cdot]$ using the set of projected sigma points $\mathcal{X}_i(k+1 \mid k)$.

The basic method is generalised in two ways. First, any of the infinite number of (not necessarily square) matrix square roots can be chosen. If the orthogonal matrix square root is chosen, then the sigma points lie along the eigenvectors of the covariance matrix. Second, κ copies of the prior mean $\hat{\mathbf{x}}(k \mid k)$ can be included in the set of sigma points. Although the mean of the sigma points is unaffected, the distribution of the points is scaled (since they are now found from $\pm\sqrt{(n+\kappa)\mathbf{P}(k|k)}$). In certain circumstances this scaling leads to improvements in performance, which is discussed in more detail in the next section.

We now summarise the general formulation [3] of the new filter:

1. The set of translated sigma points is computed from the $n \times n$ matrix $\mathbf{P}(k|k)$ as

$$\begin{aligned} \sigma(k|k) &\longleftarrow \quad 2n \text{ columns from } \pm\sqrt{(n+\kappa)\mathbf{P}(k \mid k)} \\ \mathcal{X}_0(k \mid k) &= \hat{\mathbf{x}}(k \mid k), \\ \mathcal{X}_i(k \mid k) &= \sigma_i(k|k) + \hat{\mathbf{x}}(k \mid k). \end{aligned}$$

2. The predicted mean is computed as

$$\hat{\mathbf{x}}(k+1 \mid k) = \frac{1}{n+\kappa}\left\{\kappa\mathcal{X}_0(k+1 \mid k) + \frac{1}{2}\sum_{i=1}^{2n}\mathcal{X}_i(k+1 \mid k)\right\}.$$

3. And the predicted covariance is computed as

$$\begin{aligned} \mathbf{P}(k+1|k) = \frac{1}{n+\kappa}\Bigg\{&\kappa[\mathcal{X}_0(k+1 \mid k) - \hat{\mathbf{x}}(k+1 \mid k)][\mathcal{X}_0(k+1 \mid k) - \hat{\mathbf{x}}(k+1 \mid k)]^T \\ &+\frac{1}{2}\sum_{i=1}^{2n}[\mathcal{X}_i(k+1 \mid k) - \hat{\mathbf{x}}(k+1 \mid k)][\mathcal{X}_i(k+1 \mid k) - \hat{\mathbf{x}}(k+1 \mid k)]^T\Bigg\} \end{aligned}$$

4. The predicted observation is calculated by

$$\hat{\mathbf{z}}(k+1 \mid k) = \frac{1}{n+\kappa}\left\{\kappa\mathcal{Z}_0(k+1 \mid k) + \frac{1}{2}\sum_{i=1}^{2n}\mathcal{Z}_i(k+1 \mid k)\right\}.$$

5. And the covariance is determined by

$$\begin{aligned} \mathbf{P}_{zz}(k+1 \mid k) = \frac{1}{n+\kappa}\Bigg\{&\kappa[\mathcal{Z}_0(k+1 \mid k) - \hat{\mathbf{z}}(k+1 \mid k)][\mathcal{Z}_0(k+1 \mid k) - \hat{\mathbf{z}}(k+1 \mid k)]^T \\ &+\frac{1}{2}\sum_{i=1}^{2n}[\mathcal{Z}_i(k \mid k-1) - \mathcal{Z}_0(k+1 \mid k)]\,[\mathcal{Z}_i(k \mid k-1) - \mathcal{Z}_0(k+1 \mid k)]^T\Bigg\} \end{aligned}$$

where $\mathbf{P}_{\nu\nu}(k+1 \mid k) = \mathbf{P}_{zz}(k+1 \mid k) + \mathbf{R}(k+1)$.

6. Finally the cross correlation matrix is determined by

$$\begin{aligned} \mathbf{P}_{xz}(k+1 \mid k) = \frac{1}{n+\kappa}\Bigg\{&\kappa\,[\mathcal{X}_0(k+1 \mid k) - \hat{\mathbf{x}}(k+1 \mid k)]\,[\mathcal{Z}_0(k+1 \mid k) - \hat{\mathbf{z}}(k+1 \mid k)]^T \\ &+\frac{1}{2}\sum_{i=1}^{2n}[\mathcal{X}_i(k+1 \mid k) - \mathcal{X}_0(k+1 \mid k)]\,[\mathcal{Z}_i(k+1 \mid k) - \mathcal{Z}_0(k+1 \mid k)]^T\Bigg\}. \end{aligned}$$

IV. The Performance of the New Filter

In [3, 4] we have analysed the performance of the new filter and the EKF in terms of the accuracy of predicting the mean and covariance of the states of the system given that the current distribution is Gaussian. It is not possible to present this analysis here and so we provide a brief summary of the results.

Assuming that the state transition equations can be expressed in terms of a unique Taylor series, it can be shown that the new filter predicts the mean and covariance correctly up to the fourth term in the Taylor series. That is, the error is of the order of the product of the fourth moments of the state errors and the fourth order derivatives of the state transition equations. As a contrast, the EKF predicts the mean correctly up to the second order and the covariance up to the fourth order. Therefore the new filter predicts the mean more accurately than the EKF and its estimate incorporates the so-called second order "bias" terms which are normally considered to be the principal advantage of using the modified second order Gaussian filter [5]. Further, the new filter predicts the covariance at least as accurately as the EKF but without the need to undertake the difficult and time consuming task of determining the Jacobian matrices. By an appropriate choice of κ we can strengthen these statements further since the magnitudes of the higher order errors can be reduced.

For any choice of κ the first, second and third order moments of the sigma point distribution are unchanged. However the fourth order moment or the *kurtosis* is scaled by $n+\kappa$ and all higher order (even) moments scale geometrically with $n+\kappa$. All odd ordered moments are zero. Therefore κ can be adjusted according to the higher order moments of the prior distribution to reduce the errors in these terms[2]. For example consider the case when the prior distribution is Gaussian. By linearly transforming the state space using $\sqrt{(n+\kappa)(\mathbf{P}(k \mid k)+\mathbf{Q}(k))}$ this distribution is equivalent to n independent, Gaussian, zero-mean random variables each with covariance of one. From the moment generating function it can be shown that all odd moments of these independent distributions are zero and that the kurtosis of the ith state of these independent distributions is given by $\mathrm{E}\left[x_i^4\right] = 3$. Higher order even moments scale factorially. By a similar transformation the distribution of the sigma points can be expressed as n independent discrete distributions each with a covariance of one. The kurtosis of the ith state of these distributions is $\mathrm{E}\left[x_i^4\right] = n+\kappa$. The higher moments scale geometrically with $n+\kappa$. Therefore, if κ is chosen such that $n+\kappa = 3$ then the kurtosis of one state of the sigma points agrees with that of the Gaussian distribution. Further, the higher order moments of the sigma points scale geometrically with common ratio 3, which is a slower growth than the higher moments of the Gaussian distribution. The EKF, however, effectively assumes that all of these higher order terms are zero.

These results imply that for *any* symmetric prior distribution with kurtosis k, if κ is chosen such that $0 < n+\kappa \leq k$ then the predictions of mean and covariance are more accurate than those made by the EKF. Errors are injected into the fourth and higher orders. By choosing κ such that $n+\kappa = k$, then the error in the kurtosis is minimised.

V. Empirical Results

We illustrate the effectiveness of the new filter against that of the EKF for a real application which is currently under development at Oxford. We are designing a navigation system to estimate the position of a conventional road vehicle with submetre accuracy. The vehicle will operate in an unstructured environment (the road surrounding Oxford) and at typical driving speeds (up to $25\mathrm{ms}^{-1}$.) As explained in [2], the process models for vehicle motion become complicated by the action of appreciable nonlinear dynamic effects. As an example, we present an eight-state model which describes the motion of a conventional road vehicle. The control inputs are vehicle steer angle $\delta(k)$ and the angular speed of the front drive wheel $\omega(k)$. These states are: the position and course angle of the centre of mass of the vehicle $[x_G(k), y_G(k), \psi(k)]^T$, mean tyre radius $R(k)$, front and rear tyre forces $F_{yf}(k)$, $F_{yr}(k)$ and front and rear tyre stiffness parameters $C_{\alpha_f}(k), C_{\alpha_r}(k)$. The discrete state transition equation of this system is:

$$\mathbf{f}[\mathbf{x}(k), \mathbf{u}(k+1), k+1] = \begin{bmatrix} x_G(k) + \left(\frac{\sin[\nu(k)+\rho(k)V_G(k)\Delta T]-\sin\nu(k)}{\rho(k)}\right) \\ y_G(k) + \left(\frac{\cos[\nu(k)+\rho(k)V_G(k)\Delta T]-\cos\nu(k)}{\rho(k)}\right) \\ \nu(k) + \rho(k)V_G(k)\Delta T \\ R(k) \\ F_{yf}(k) + \Delta T\Omega\left\{\rho(k)mV_G^2(k) \right. \\ \quad \left. + \cos[\delta(k+1)-\beta(k)]F_{yf}(k) - \cos\beta(k)F_{yr}(k)\right\} \\ F_{yr}(k) + \Delta T\Omega\left\{a\cos\delta(k+1)F_{yf}(k) - bF_{yr}(k)\right\} \\ C_{\alpha_f}(k) \\ C_{\alpha_r}(k) \end{bmatrix}$$

where $\rho(k)$ is the radius of curvature of the motion, $\beta(k)$ is the angle between the course taken by the vehicle and its orientation, and $\alpha_f(k)$ and $\alpha_r(k)$ are the side slip angles. These intermediate quantities are calculated from

$$\begin{aligned} \beta(k) &= \tan^{-1}\left(\frac{b\tan[\delta(k+1)-\alpha_f(k)] - a\tan\alpha_r(k)}{a+b}\right), \\ \rho(k) &= \frac{\sin\beta(k)}{a+b}\left\{\tan[\delta(k+1)-\alpha_f(k)] + \tan\alpha_r(k)\right\}, \\ V_G(k) &= R(k)\omega(k+1)\frac{\cos\delta(k+1)}{\cos\beta(k)}, \\ \alpha_f(k) &= \frac{F_{yf}(k)}{C_{\alpha_f}(k)}, \\ \alpha_r(k) &= \frac{F_{yr}(k)}{C_{\alpha_r}(k)}. \end{aligned}$$

The position and orientation of the vehicle is observed. The nonlinear observation equation is

$$\mathbf{z}(k) = \begin{bmatrix} x_G(k) \\ y_G(k) \\ \psi(k) \end{bmatrix} = \begin{bmatrix} x_G(k) \\ y_G(k) \\ \nu(k) - \beta(k) \end{bmatrix},$$

[2]The matrix square root should be able to handle non-positive semidefinite matrices in a robust fashion.

From the analysis we expect to see significant differences in performance between the two filters if (a) the kurtosis and higher order moments in the state errors distributions are significant and/or (b) the fourth and higher order differentials of the state transition and observation equations are appreciable. However in this application the magnitudes of the covariances are significantly less than unity and so the kurtosis is small. Further, to ensure good performance for the EKF the time steps are of short duration, and the system behaves in a quasi-linear fashion. The principle benefit of the new filter in this case is that performance comparable to that of the EKF is achieved without the need to evaluate the Jacobians or, indeed, any partial derivatives of the above equations.

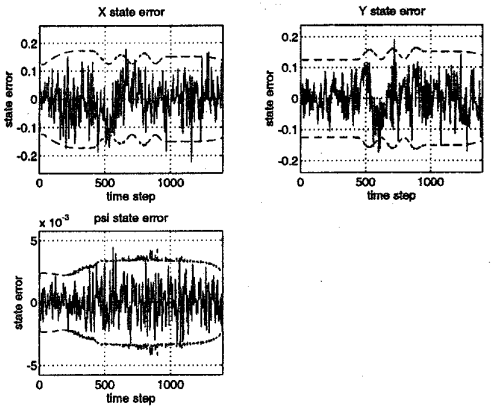

Fig.1: State errors in position and heading estimates using the EKF. Position estimates in metres, ψ estimate in radians.

VI. Discussion

In this paper we have described a new method for filtering nonlinear systems. This method uses a recursive linear estimator structure. However, unlike the EKF we do not make crude linearising assumptions in order to predict the new state of the system. Rather, we approximate the prior distribution using the minimum set of points that capture the first three moments of the prior distribution. Our new filter produces predictions of state and covariance which are provably more accurate as those of the EKF but without the need to calculate Jacobian matrices[3].

For these reasons we believe that the new filter should be preferred to the EKF in all nonlinear applications where linear estimators are employed.

To test this hypothesis the ADAMS mechanical systems simulation software package was used to simulated a vehicle as it drove around a test track. Both the EKF and new filter were incorporated into navigation systems to estimate position and orientation of the vehicle. Figure 1 shows the errors in the estimates of position and course angle along with the two standard deviation bounds. The plots for the new filter are given in Figure 2. As can be seen, there is little difference in the magnitude of the state errors committed by each filter. Similar results have been found in other practical applications such as satellite tracking [6] and map-based position determination [8].

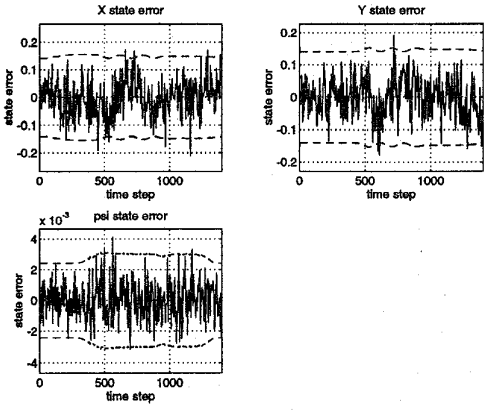

Fig.2: State errors in position and heading estimates using the new filter. Position estimates in metres, ψ estimates in radians.

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[3] An alternative parameterisation of the basic principle is presented in [7]. This parameterisation converges to the EKF but without the need to explicitly calculate the Jacobian matrices.