5. After the above recursion is complete we have the n-element vectors $\sigma_i^{(n)}$ $(i = 0, \cdots, n+1)$. We modify the unscented transformation of Equation (14.29) and obtain the sigma points for the unscented transformation as follows:

$$x^{(i)} = \bar{x} + \sqrt{P}\sigma_i^{(n)} \qquad (i = 0, \cdots, n+1) \tag{14.80}$$

We actually have $(n+2)$ sigma points instead of the $(n+1)$ sigma points as we claimed, but if we choose $W^{(0)} = 0$ then the $x^{(0)}$ sigma point can be ignored in the ensuing unscented transformation. The unscented Kalman filter algorithm in Section 14.3 is then modified in the obvious way based on this minimal set of sigma points.

The problem with the simplex UKF is that the ratio of $W^{(n)}$ to $W^{(1)}$ is equal to 2^{n-2}, where n is the dimension of the state vector x. As the dimension of the state increases, this ratio increases and can quickly cause numerical problems. The only reason for using the simplex UKF is the computational savings, and computational savings is an issue only for problems of high dimension (in general). This makes the simplex UKF of limited utility and leads to the spherical unscented transformation in the following section.

14.4.3 The spherical unscented transformation

The unscented transformation discussed in Section 14.2 is numerically stable. However, it requires $2n$ sigma points and may be too computationally expensive for some applications. The simplex unscented transformation discussed in Section 14.4.2 is the cheapest computational unscented transformation but loses numerical stability for problems with a moderately large number of dimensions. The spherical unscented transformation was developed with the goal of rearranging the sigma points of the simplex algorithm in order to obtain better numerical stability [Jul03, Jul04]. The spherical sigma points are chosen with the following algorithm.

The spherical sigma-point algorithm

1. Choose the weight $W^{(0)} \in [0, 1)$. The choice of $W^{(0)}$ affects only the fourth- and higher-order moments of the set of sigma points [Jul00, Jul02a].

2. Choose the rest of the weights as follows:

$$W^{(i)} = \frac{1 - W^{(0)}}{n+1} \qquad i = 1, \cdots, n+1 \tag{14.81}$$

Note that (in contrast to the simplex unscented transformation) all of the weights are identical except for $W^{(0)}$.

3. Initialize the following one-element vectors:

$$\begin{aligned} \sigma_0^{(1)} &= 0 \\ \sigma_1^{(1)} &= \frac{-1}{\sqrt{2W^{(1)}}} \\ \sigma_2^{(1)} &= \frac{1}{\sqrt{2W^{(1)}}} \end{aligned} \tag{14.82}$$