70 3 Gaussian Filters

$$\begin{array}{lll} 1: & & \textbf{Algorithm Unscented_Kalman_filter}(\mu_{t-1}, \Sigma_{t-1}, u_t, z_t) \text{:} \\ 2: & & \mathcal{X}_{t-1} = (\mu_{t-1} \quad \mu_{t-1} + \gamma \sqrt{\Sigma_{t-1}} \quad \mu_{t-1} - \gamma \sqrt{\Sigma_{t-1}}) \\ 3: & & \bar{\mathcal{X}}_t^* = g(u_t, \mathcal{X}_{t-1}) \\ 4: & & \bar{\mu}_t = \sum_{i=0}^{2n} w_m^{[i]} \bar{\mathcal{X}}_t^{*[i]} \\ 5: & & \bar{\Sigma}_t = \sum_{i=0}^{2n} w_c^{[i]} (\bar{\mathcal{X}}_t^{*[i]} - \bar{\mu}_t) (\bar{\mathcal{X}}_t^{*[i]} - \bar{\mu}_t)^T + R_t \\ 6: & & \bar{\mathcal{X}}_t = (\bar{\mu}_t \quad \bar{\mu}_t + \gamma \sqrt{\bar{\Sigma}_t} \quad \bar{\mu}_t - \gamma \sqrt{\bar{\Sigma}_t}) \\ 7: & & \bar{\mathcal{Z}}_t = h(\bar{\mathcal{X}}_t) \\ 8: & & & \hat{z}_t = \sum_{i=0}^{2n} w_m^{[i]} \bar{\mathcal{Z}}_t^{[i]} \\ 9: & & & S_t = \sum_{i=0}^{2n} w_c^{[i]} (\bar{\mathcal{Z}}_t^{[i]} - \hat{z}_t) (\bar{\mathcal{Z}}_t^{[i]} - \hat{z}_t)^T + Q_t \\ 10: & & & \bar{\Sigma}_t^{x,z} = \sum_{i=0}^{2n} w_c^{[i]} (\bar{\mathcal{X}}_t^{[i]} - \bar{\mu}_t) (\bar{\mathcal{Z}}_t^{[i]} - \hat{z}_t)^T \\ 11: & & & K_t = \bar{\Sigma}_t^{x,z} S_t^{-1} \\ 12: & & \mu_t = \bar{\mu}_t + K_t(z_t - \hat{z}_t) \\ 13: & & \Sigma_t = \bar{\Sigma}_t - K_t S_t K_t^T \\ 14: & & \text{return } \mu_t, \Sigma_t \\ \end{array}$$

Table 3.4 The unscented Kalman filter algorithm. The variable n denotes the dimensionality of the state vector.

ear systems the UKF produces equal or better results than the EKF, where the improvement over the EKF depends on the nonlinearities and spread of the prior state uncertainty. In many practical applications, the difference between EKF and UKF is negligible.

Another advantage of the UKF is the fact that it does not require the computation of Jacobians, which are difficult to determine in some domains. The UKF is thus often referred to as a *derivative-free filter*.