Algorithm 1.3 A Slightly Improved Extension of the Kalman Filter

$$\hat{\boldsymbol{X}}_i^- = \int_{t_{i-1}}^{t_i} \boldsymbol{f}(\boldsymbol{X}(\tau), \tau)\,\mathrm{d}\tau, \quad \boldsymbol{X}(t_{i-1}) = \hat{\boldsymbol{X}}_{i-1}^+, \quad \hat{\boldsymbol{X}}_0^+ = \bar{\boldsymbol{X}}_o \tag{1.45}$$

$$\mathbf{A}(t) = \left.\frac{\partial \boldsymbol{f}}{\partial \boldsymbol{X}}\right|_{\hat{\boldsymbol{X}}(t)}, \quad \mathbf{H}_i = \left.\frac{\partial \boldsymbol{h}_i}{\partial \boldsymbol{X}}\right|_{\hat{\boldsymbol{X}}_i^-} \tag{1.46}$$

$$\boldsymbol{\Phi}(t_i, t_{i-1}) = \text{a suitable approximation to (1.28)} \tag{1.47}$$

$$\mathbf{S}_i = \text{a suitable approximation to (1.42)} \tag{1.48}$$

$$\mathbf{P}_i^- = \boldsymbol{\Phi}(t_i, t_{i-1})\mathbf{P}_{i-1}^+\boldsymbol{\Phi}^\mathsf{T}(t_i, t_{i-1}) + \mathbf{S}_i, \quad \mathbf{P}_0^+ = \mathbf{P}_o \tag{1.49}$$

$$\mathbf{K}_i = \mathbf{P}_i^-\mathbf{H}_i^\mathsf{T}\left(\mathbf{H}_i\mathbf{P}_i^-\mathbf{H}_i^\mathsf{T} + \mathbf{R}_i\right)^{-1} \tag{1.50}$$

$$\hat{\boldsymbol{X}}_i^+ = \hat{\boldsymbol{X}}_i^- + \mathbf{K}_i\left(\boldsymbol{Y}_i - \boldsymbol{h}_i(\hat{\boldsymbol{X}}_i^-)\right) \tag{1.51}$$

$$\mathbf{P}_i^+ = (\mathbf{I} - \mathbf{K}_i\mathbf{H}_i)\,\mathbf{P}_i^-\,(\mathbf{I} - \mathbf{K}_i\mathbf{H}_i)^\mathsf{T} + \mathbf{K}_i\mathbf{R}_i\mathbf{K}_i^\mathsf{T} \tag{1.52}$$

1.2. The Multiplicative Extended Kalman Filter

An interesting variation on the EKF is possible in the context of estimating attitude parameters. An attitude correction may be viewed as a small-angle rotation from a frame associated with the previous estimate to a frame associated with a current estimate. In this context, one may use the previous attitude estimate as a linearization reference for a linearized Kalman Filter's Jacobian matrices, and estimate the small-angle correction as the filter state. After each state update, one performs a rectification of the attitude reference by applying the small-angle correction. Since for many attitude representations, a frame rotation is multiplicative operation, this procedure has become known as the multiplicative EKF. Chapter 8 covers this subject.