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# Interpolate Points on a Shape

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## CONTENTS

### 1 Uniformly Spaced Interpolated Points

2

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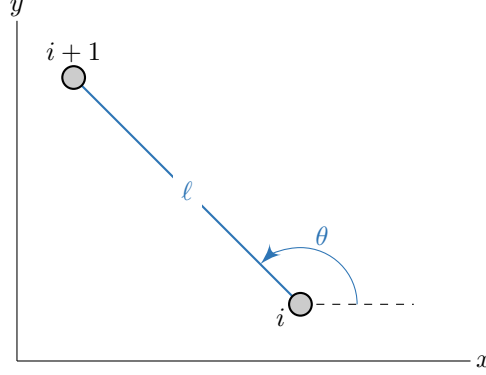
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# 1 UNIFORMLY SPACED INTERPOLATED POINTS

We assume we have two vectors,  $\mathbf{x}$  and  $\mathbf{y}$ , that store the  $N$  points  $\{x_i, y_i\}_{i=1}^N$  defining a shape. We want to find  $n$  evenly spaced points between every point defining the shape. Consider the line segment connecting the points  $(x_i, y_i)$  and  $(x_{i+1}, y_{i+1})$ , as shown in Fig. 1. The length,  $\ell$ , of this line segment (i.e. the distance between the two points) is



**Figure 1:** Line segment connecting points  $i$  and  $i + 1$ .

$$\ell = \sqrt{(x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2} \quad (1)$$

The angle (measured counter-clockwise) that the line segment forms with the horizontal line  $y = y_i$  can be found using the four-quadrant inverse tangent.

$$\tan \theta = \frac{y_{i+1} - y_i}{x_{i+1} - x_i} \rightarrow \theta = \arctan2(y_{i+1} - y_i, x_{i+1} - x_i) \quad (2)$$

We want to interpolate  $n$  uniformly spaced points between points  $i$  and  $i + 1$ , which means that the distance between each interpolated point is then  $\ell/n$ . It follows that the first interpolated point is a distance  $\ell/n$  from point  $i$ , the second interpolated point is a distance  $2\ell/n$  from point  $i$ , and so on. Thus, the  $j^{\text{th}}$  interpolated point is a distance  $j\ell/n$  from point  $i$ . Since all the interpolated points lie on the line segment connecting points  $i$  and  $i + 1$ , they all make the same angle  $\theta$  with the horizontal line emanating from point  $i$  (see Fig. 1). Thus, the  $x$ - and  $y$ - coordinates of the  $j^{\text{th}}$  interpolated point can be calculated as

$$x_{ji} = x_i + \frac{j\ell \cos \theta}{n} \quad (3)$$

$$y_{ji} = y_i + \frac{j\ell \sin \theta}{n} \quad (4)$$