

Gaussian Elimination

MATLAB Implementation

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Contents

gaussian_elimination	4
Syntax	4
Description	4
Examples	4
Links	5
Gaussian Elimination	6
References	8

gaussian_elimination

Solves the linear system $Ax = b$ for x using Gaussian elimination with partial pivoting.

Syntax

`x = gaussian_elimination(A,b)`

Description

`x = gaussian_elimination(A,b)` solves the linear system $Ax = b$ for the vector $x \in \mathbb{R}^n$, where $A \in \mathbb{R}^{n \times n}$ is a tridiagonal matrix and $b \in \mathbb{R}^n$ is a vector.

Examples

Example 1

Solve the linear system $Ax = b$ for x , where

$$A = \begin{bmatrix} 2 & -1 & 5 \\ 1 & 1 & -3 \\ 2 & 4 & 1 \end{bmatrix}, \quad b = \begin{bmatrix} 10 \\ -2 \\ 1 \end{bmatrix}$$

■ SOLUTION

Entering A and b into MATLAB,

```
% defines matrix A
A = [2,-1, 5;
     1, 1,-3;
     2, 4, 1];

% defines vector b
b = [10;
     -2;
     1];
```

To solve the linear system for x ,

```
x = gaussian_elimination(A,b)
```

This yields the result

```
x =
     2
    -1
     1
```

Links

MATLAB® Central's File Exchange:

https://www.mathworks.com/matlabcentral/fileexchange/89306-gaussian-elimination-gaussian_elimination

GitHub®:

https://github.com/tamaskis/gaussian_elimination-MATLAB

Gaussian Elimination

Gaussian elimination can be used to solve the linear system

$$A\mathbf{x} = \mathbf{b}$$

where $A \in \mathbb{R}^{n \times n}$ and $\mathbf{x}, \mathbf{b} \in \mathbb{R}^{n \times 1}$. In Algorithms 1 and 2 below, we will be referring to the rows and columns of A as well as to the elements of \mathbf{x} at different points. Here are the conventions we use:

$$A = \begin{bmatrix} \mathbf{a}_1 & \mathbf{a}_2 & \dots & \mathbf{a}_j & \dots & \mathbf{a}_{n-1} & \mathbf{a}_n \end{bmatrix} = \begin{bmatrix} \bar{a}_1 \\ \bar{a}_2 \\ \vdots \\ \bar{a}_i \\ \vdots \\ \bar{a}_{n-1} \\ \bar{a}_n \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_i \\ \vdots \\ x_{n-1} \\ x_n \end{bmatrix}$$

Algorithms 1 and 2¹ below implement Gaussian elimination with partial pivoting and are adapted from Algorithm 6.2 in [1, pp. 374–375]. Note that ε is used to denote the machine epsilon.

¹ Note that together, these algorithms represent a single algorithm. However, this single algorithm is split into two due to its length and the limitations of typesetting algorithms in L^AT_EX.

Algorithm 1: Gaussian elimination with partial pivoting (part 1).

```
1 Given:  $A, \mathbf{x}, \mathbf{b}$ 

   // determines  $n$  (where  $A \in \mathbb{R}^{n \times n}$ )
2  $n = \text{size}(A, 1)$ 

   // redefines  $A$  by augmenting the original  $A$  with  $\mathbf{b}$ 
3  $A = [A \ \mathbf{b}]$ 

   // initializes boolean variable to keep track if matrix is singular
4 singular = false

   // elimination process
5 for  $i = 1$  to  $n - 1$  do
    // determines pivot row
6    Define the vector  $\mathbf{p} = (i, i + 1, \dots, n - 1, n)^T$ .
7    Remove the elements  $p_k$  of  $\mathbf{p}$  where  $A_{k,i} = 0$ .
8     $p = \min(\mathbf{p})$ 

    // exit the loop if all possible pivots in the  $i^{\text{th}}$  column are zero (to machine
    // precision) because that would result in a singular matrix
9    if  $\max |\mathbf{a}_i| \leq \varepsilon$  then
10       singular = true
11       Exit loop – the matrix is singular and no unique solution exists.
12    end

    // switches pivot row with the  $i^{\text{th}}$  row if  $p \neq i$ 
13    if  $p \neq i$  then
14       Store the  $i^{\text{th}}$  row of  $A$  as  $\bar{a}_i$ .
15       Store the  $p^{\text{th}}$  row of  $A$  as  $\bar{a}_p$ .
16       Set the  $i^{\text{th}}$  row of  $A$  to  $\bar{a}_p$ .
17       Set the  $p^{\text{th}}$  row of  $A$  to  $\bar{a}_i$ .
18    end

    // elementary row operation
19    for  $j = i + 1$  to  $n$  do
20        $\bar{a}_j = \bar{a}_j - \left( \frac{A_{j,i}}{A_{i,i}} \right) \bar{a}_i$ 
21    end
22 end
```

Algorithm 2: Gaussian elimination with partial pivoting (part 2).

```
// determines if A is singular (if the bottom right element of A is 0 (to within machine  
precision), then the entire bottom row is 0 and A is singular)  
1 if  $|A_{n,n}| \leq \varepsilon$  then  
2   | singular = true  
3 end  
  
4 Preallocate/initialize the solution vector  $\mathbf{x} \in \mathbb{R}^{n \times 1}$ .  
  
// perform backward substitution to solve  $A\mathbf{x} = \mathbf{b}$  if A is nonsingular  
5 if !(singular) then  
6   |  $x_n = \frac{A_{n,n+1}}{A_{n,n}}$   
7   | for  $i = n - 1$  to 1 by -1 do  
8     |  $S = 0$  for  $i + 1$  to  $n$  do  
9       |  $S = S + A_{i,j}x_j$   
10    | end  
11    |  $x_i = \frac{A_{i,n+1} - S}{A_{i,i}}$   
12  | end  
13 end
```

References

- [1] Richard L. Burden and J. Douglas Faires. *Numerical Analysis*. 9th. Boston, MA: Brooks/Cole, Cengage Learning, 2011.