1.1 $\frac{y^{58}}{y^{4}\cdot y^{12}} = \frac{y^{58}}{y^{(4+12)}} = y^{58-16} = \boxed{y^{42}}$

1.2 $8^2 \cdot 2^x = 2^9$

$(2^3)^2 \cdot 2^x = 2^9$

$2^{6+x} = 2^9$

$6 + x = 9$

$\boxed{x = 3}$

1.3 $\frac{x}{y} = 3 \qquad x = 3y$

$x^{-2}\cdot y^2 = (3y)^{-2}\cdot y^2 = \frac{1}{9y^2}\cdot y^2 = \boxed{\frac{1}{9}}$

1.4 $\frac{\sqrt{2^{13}}}{\sqrt{8^3}} = \frac{\sqrt{2^{13}}}{\sqrt{(2^3)^3}} = \frac{\sqrt{2^{13}}}{\sqrt{2^9}} = \sqrt{2^{13-9}} = \sqrt{2^4} = \boxed{4}$

1.5 $x + y = y + x \rightarrow$ true

$x(y+z) = xy + xz \rightarrow$ true

$x^{y+z} = x^z x^y \rightarrow$ true

$\frac{x^y}{x^z} = x^{y-z} \rightarrow$ true

1.6 $\frac{x^2 - 25}{x - 5} = 3$

$\frac{\cancel{(x-5)}(x+5)}{\cancel{(x-5)}} = 3$

$x = \boxed{-2}$

2.1 $0\,K = -460\,°F$

$1000\,K = 1340\,°F$

$K = a + b\cdot F \Rightarrow$

$0 = a + b\cdot(-460) = a - 460b$

$1000 = a + b\cdot(1340)$

$1000 = a + 1340b - a - (-460)b$

$1000 = 1800b$

$b = \frac{5}{9}$

$0 = a + \frac{5}{9}\cdot(-460)$

$a = \frac{5\cdot 460}{9} = 255.\dot{5}$

$\Leftarrow$

$K = 255.5 + \frac{5}{9}\cdot F$

$F = -460 + \frac{9}{5}K$

$K = F$

$-460 + \frac{9}{5}K = K$

$\frac{4}{5}K = +460$ *

* itt folytatódik

$\boxed{K = +575}$

$\boxed{F = +575}$

2.2 $f(x) = 2x + 3$

$f(y) = 17$

$f(y) = 2y + 3 = 17$

$2y = 14$

$\boxed{y = 7}$

2.3 $3^{2x^2 - 4x + 3} = 27$

$3^{2x^2 - 4x + 3} = 3^3$

$2x^2 - 4x + 3 = 3$

$2x^2 - 4x = 0$

$2x \cdot (x - 2) = 0$

$\boxed{x = 0}$ $\boxed{x = 2}$

2.4 $1.01 = \text{growth}$

$1.01^x = 2 \rightarrow \boxed{x = 70}$

$1.01^{70} = 2$

Rule of 70

Number of years to double $= \dfrac{70}{\underset{\text{growth}}{1}} = \boxed{70}$

2.5 $\ln\left(\frac{e^2}{e^3}\right) = \ln\left(\frac{1}{e}\right) = \ln\left(e^{-1}\right) = \boxed{-1}$

3.1 $\sum_{i=0}^{\infty}\left(\frac{1}{6^i} + 0.25^i\right) = \sum_{i=0}^{\infty}\left(\frac{1}{6^i} + \left(\frac{1}{4}\right)^i\right) = \sum_{i=0}^{\infty}\left(\left(\frac{1}{6}\right)^i + \left(\frac{1}{4}\right)^i\right)$

$\sum_{i=0}^{\infty}$

3.2 $\lim_{x \to 3} \dfrac{x^2 - 9}{x - 3} = \lim_{x \to 3} \dfrac{\cancel{(x-3)}(x+3)}{\cancel{(x-3)}} = \lim_{x \to 3} x + 3 = \boxed{6}$

3.3 $f(x) = x^3 - 4$

$f'(x) = 3x^2$

$x = -1 \longrightarrow 3x^2 = \boxed{3}$

3.4 $\left(\dfrac{x^2+3}{x+2}\right)' = \dfrac{2x \cdot (x+2) - 1 \cdot (x^2+3)}{(x+2)^2} = \dfrac{2x^2 + 4x - x^2 + 3}{(x+2)^2} = \boxed{\dfrac{x^2 + 4x - 3}{(x+2)^2}}$

3.5 $f(x) = x^7 + 4x^2$

$f'(x) = 7x^6 + 8x$

$\boxed{f''(x) = 42x^5 + 8}$

3.6 $f(x) = \dfrac{x^4 + 4^x}{\ln(x)}$ $\begin{matrix} \to f \\ \to g \end{matrix}$

$f'(x) = \dfrac{f'g - g'f}{g^2} = \dfrac{(4x^3 + 4^x \cdot \ln(4)) \cdot \ln x - \frac{1}{x} \cdot (x^4 + 4^x)}{(\ln(x))^2}$

3.7 $3x^3 - 9x = f(x)$

$f'(x) = 9x^2 - 9 = 9(x^2 - 1) = 0$

$x_1 = 1 \quad x_2 = -1$

$f(1) = 3 \cdot 1^3 - 9 \cdot 1 = \boxed{-6}$ local minimum

$f(-1) = \underbrace{3 \cdot (-1)^3}_{-3} \underbrace{- 9 \cdot (-1)}_{+9} = \boxed{6}$ local maximum

3.8 $f(x, y) = x^2 + 2y^3$

$f(2, 3) = \underbrace{2^2}_{4} + \underbrace{2 \cdot 3^3}_{54} = \boxed{59}$

3.9

3.10 $f(x, y) = x^5 \cdot e^y + x^2 \cdot y^3$

$\dfrac{\partial f(x, y)}{\partial x} = \boxed{5x^4 \cdot e^y + 2x \cdot y^3}$

$\dfrac{\partial f(x, y)}{\partial y} = \boxed{x^5 \cdot e^y + 3y^2 \cdot x^2}$

3.11 $f(x, y) = (x \cdot y)^{\frac{1}{2}} - 0{,}7x - 0.7y$

$\dfrac{\partial f(x, y)}{\partial x} = \frac{1}{2} x^{-\frac{1}{2}} \cdot y - 0.7 - 0.7y$

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3.12 $f(x,y) = x^2 \cdot y^2$

s.t. $x + y = 10 \Rightarrow x + y - 10 = 0$

$\mathcal{L} = x^2 \cdot y^2 - \lambda \cdot (x + y - 10)$

$\frac{\partial \mathcal{L}}{\partial x} = 2x \cdot y^2 - \lambda$

$\frac{\partial \mathcal{L}}{\partial y} = 2y \cdot x^2 - \lambda$

$2x \cdot y^2 = 2y \cdot x^2$

$\boxed{y = x}$

$x + y - 10 = 0$

$2y = 10$

$\boxed{y = 5}$

$\boxed{x = 5}$

4.1 $\underline{A} = \begin{bmatrix} 2 & 3 \\ 4 & 1 \\ 1 & 2 \end{bmatrix}$ $\underline{B} = \begin{bmatrix} 1 & 4 & 1 \\ 2 & 1 & 2 \end{bmatrix}$

$\underline{A} \cdot \underline{B}$

$$\begin{array}{cc|ccc} & & 1 & 4 & 1 \\ & & 2 & 1 & 2 \\ \hline 2 & 3 & 8 & 11 & 8 \\ 4 & 1 & 6 & 17 & 6 \\ 1 & 2 & 5 & 6 & 5 \end{array} = \begin{bmatrix} 8 & 11 & 8 \\ 6 & 17 & 6 \\ 5 & 6 & 5 \end{bmatrix}$$

4.2 $\underline{A}$

$\underline{B}$

$\underline{B} \cdot \underline{A}$

$$\begin{array}{ccc|cc} & & & 2 & 3 \\ & & & 4 & 1 \\ & & & 1 & 2 \\ \hline 1 & 4 & 1 & 19 & 9 \\ 2 & 1 & 2 & 10 & 11 \end{array} = \begin{bmatrix} 19 & 9 \\ 10 & 11 \end{bmatrix}$$

4.3 $\begin{bmatrix} 3.3 & 5.1 \\ 6.1 & 1.23 \\ 45.76 & 0 \end{bmatrix} = A$

$A^T = \begin{bmatrix} 3.3 & 6.1 & 45.76 \\ 5.1 & 1.23 & 0 \end{bmatrix}$

4.4

$A = \begin{bmatrix} 2 & 3 & 0 \\ 4 & 5 & 2 \\ 2 & 5 & 3 \end{bmatrix}$

$\det(\underline{A}) = 2 \cdot 5 \cdot 3 + 3 \cdot 2 \cdot 2 + 0 \cdot 4 \cdot 5 - 0 \cdot 5 \cdot 2 - 2 \cdot 5 \cdot 2 - 3 \cdot 3 \cdot 4 = \boxed{-14}$

5.1 flip coin 2x

$\Omega = \{(H,H), (H,T), (T,H), (T,T)\}$

5.2 $30 = n$

$3 = k$

$\# P^n_k = \frac{30!}{(30-3)!} = \frac{30!}{27!} = 30 \cdot 29 \cdot 28 = \boxed{24{,}360}$

5.3 toss a dice 2x

both odd $\Rightarrow P = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

first odd, second even $\Rightarrow P = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

first even, second odd $\Rightarrow P = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

both even $\Rightarrow \frac{1}{4} = P$

$\Downarrow$

$P_{\text{at least one odd}} = 1 - \frac{1}{4} = \boxed{\frac{3}{4}}$

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