pdf of X - Gaussian

6.(a) $$\nabla_{\mu,\tau} \mathbb{E}_{X \sim \mathcal{N}(\mu, e^{2\tau})} [X \sin(X)] = \mathbb{E}_{X \sim \mathcal{N}(\mu, e^{2\tau})} [X \sin(X) \nabla_{\mu,\tau} \log(f_{\mu,\tau}(X))]$$

$$= \mathbb{E}_{X \sim \mathcal{N}(\mu, e^{2\tau})} \left[X \sin(X) \nabla_{\mu,\tau} \left(-\frac{1}{2}\left(\frac{X-\mu}{e^{\tau}}\right)^2 - \log(e^{\tau}\sqrt{2\pi})\right)\right]$$

$$\hookrightarrow -\frac{1}{2}(X-\mu)^2 e^{-2\tau}$$

$$= \mathbb{E}_{X \sim \mathcal{N}(\mu, e^{2\tau})} \left[X \sin(X) \left(\frac{X-\mu}{e^{2\tau}}, \frac{(X-\mu)^2}{e^{2\tau\cancel{+1}}} - 1\right)\right]$$

$$\approx \frac{1}{B} \sum_{i=1}^{B} \left(\frac{X_i \sin(X_i)(X_i-\mu)}{e^{2\tau}}, X_i \sin(X_i)\left(\frac{(X_i-\mu)^2}{e^{2\tau\cancel{+1}}} - 1\right)\right)$$

with $X_i \sim \mathcal{N}(\mu, e^{2\tau})$

(b) $$\nabla_{\mu,\tau} \mathbb{E}_{X \sim \mathcal{N}(\mu, e^{2\tau})} [X \sin(X)] = \mathbb{E}_{Y \sim \mathcal{N}(0,1)} [\nabla_{\mu,\tau} ((\mu + e^{\tau} Y) \sin(\mu + e^{\tau} Y))]$$

$$= \mathbb{E}_{Y \sim \mathcal{N}(0,1)} [(\sin(\mu+e^{\tau}Y) + (\mu+e^{\tau}Y)\cos(\mu+e^{\tau}Y), e^{\tau}Y \sin(\mu+e^{\tau}Y) + (\mu+e^{\tau}Y)e^{\tau}Y\cos(\mu+e^{\tau}Y))]$$

$$\approx \frac{1}{B} \sum_{i=1}^{B} (\sin(\mu+e^{\tau}Y_i) + (\mu+e^{\tau}Y_i)\cos(\mu+e^{\tau}Y_i), e^{\tau}Y_i \sin(\mu+e^{\tau}Y_i) + (\mu+e^{\tau}Y_i)e^{\tau}Y_i\cos(\mu+e^{\tau}Y_i))$$

with $Y_i \sim \mathcal{N}(0,1)$

$$\nabla_{\mu,\tau}\left(\frac{1}{2}(\mu-1)^2 + e^{\tau} - \log e^{\tau}\right) = \nabla_{\mu,\tau}\left(\frac{1}{2}(\mu-1)^2 + e^{\tau} - \tau\right)$$

$$= (\mu - 1, e^{\tau} - 1)$$