hw1_code_output

March 10, 2024

1 Problem 4

[ ]:
```
import sys

import numpy as np
import matplotlib
import matplotlib.pyplot as plt

print(f"Python version: {sys.version}\nNumpy version: {np.
 ↪__version__}\nMatplotlib version: {matplotlib.__version__}")
```

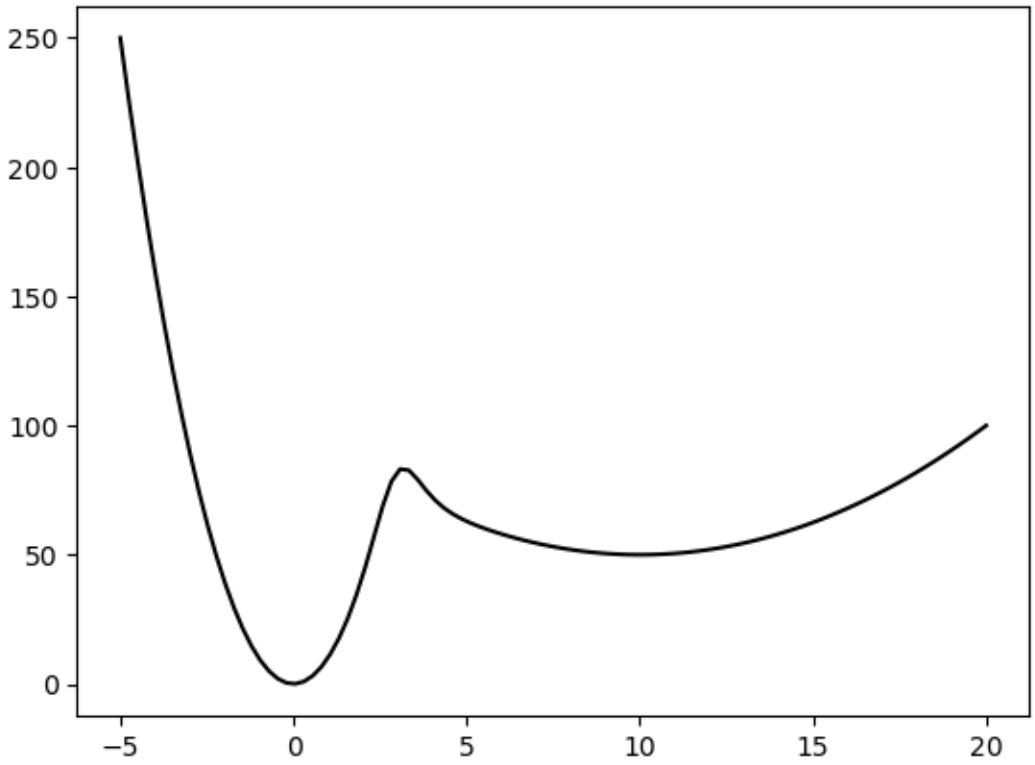
```
Python version: 3.11.8 (main, Feb 26 2024, 15:36:12) [Clang 14.0.6 ]
Numpy version: 1.26.4
Matplotlib version: 3.8.0
```

[ ]:
```
np.seterr(invalid='ignore', over='ignore')  # suppress warning caused by␣
 ↪division by inf

def f(x):
    return 1/(1 + np.exp(3*(x-3))) * 10 * x**2  + 1 / (1 + np.exp(-3*(x-3))) *␣
 ↪(0.5*(x-10)**2 + 50)

def fprime(x):
    return 1 / (1 + np.exp((-3)*(x-3))) * (x-10) + 1/(1 + np.exp(3*(x-3))) * 20␣
 ↪* x + (3* np.exp(9))/(np.exp(9-1.5*x) + np.exp(1.5*x))**2 * ((0.5*(x-10)**2␣
 ↪+ 50) - 10 * x**2)

x = np.linspace(-5,20,100)
plt.plot(x,f(x), 'k')
plt.show()
```



[ ]:
```
def gradient_descent(epochs, alpha):
    theta = np.random.uniform(x.min(), x.max())  # randomly initialises theta␣
 ↪in [x.min(), x.max())=[-5,20)

    for _ in range(epochs):  # performs GD for a given number of epochs
        theta = theta - alpha * fprime(theta)

    return theta  # returns final theta value after GD
```

[ ]:
```
def determine_minima(theta, tolerance=1e-2):
    if abs(theta - 0) < tolerance:
        return "sharp"
    elif abs(theta - 10) < tolerance:
        return "wide"
    else:
        raise ValueError(f"theta did not converge to a minimum (within␣
 ↪tolerance={1e-2})")
```

[ ]:
```
def run_experiment(num_trials, epochs, learning_rate):
    results = {"sharp": 0, "wide": 0, "failed": 0}

    digits = np.floor(np.log10(num_trials)).astype(int) + 1
```

```
    for i in range(num_trials):
        print_i = str(i).zfill(digits)
        print(f"Trial num: {print_i}/{num_trials}", end="\r")
        gd_theta = gradient_descent(epochs, learning_rate)
        try:
            results[determine_minima(gd_theta)] += 1
        except ValueError:
            results["failed"] += 1

    # results = {k: str(v).zfill(digits) for k, v in results.items()}
    results = {k: f"{(v/num_trials) * 100:6.2f}" for k, v in results.items()} ␣
↪# convert results to a percentage

    print(f"""In {num_trials} trials (with {epochs} epochs each) using learning␣
↪rate alpha = {learning_rate}:
   Convergence to sharp minimum: {results['sharp']}%
    Convergence to wide minimum: {results['wide']}%
Failed to converge to a minimum: {results['failed']}%
""")
```

[ ]:
```
num_trials = 150
epochs = 5000

for alpha in (0.01, 0.30, 4.00):
    run_experiment(num_trials, epochs, alpha)
```

```
In 150 trials (with 5000 epochs each) using learning rate alpha = 0.01:
   Convergence to sharp minimum:  33.33%
    Convergence to wide minimum:  66.67%
Failed to converge to a minimum:   0.00%

In 150 trials (with 5000 epochs each) using learning rate alpha = 0.3:
   Convergence to sharp minimum:   0.00%
    Convergence to wide minimum: 100.00%
Failed to converge to a minimum:   0.00%

In 150 trials (with 5000 epochs each) using learning rate alpha = 4.0:
   Convergence to sharp minimum:   0.00%
    Convergence to wide minimum:   0.00%
Failed to converge to a minimum: 100.00%
```

2 Problem 5

[ ]:
```
import numpy as np

class Convolution1d :
    def __init__(self, filt) :
        self.__filt = filt
        self.__r = filt.size
        self.T = TransposedConvolution1d(self.__filt)

    def __matmul__(self, vector) :
        r, n = self.__r, vector.size

        return np.asarray([np.dot(
            self.__filt,
            vector[i:i+r]
        ) for i in range(n-r+1)])  # IMPLEMENTATION

class TransposedConvolution1d :
    '''
    Transpose of 1-dimensional convolution operator used for the
    transpose-convolution operation A.T@(...)
    '''
    def __init__(self, filt) :
        self.__filt = filt
        self.__r = filt.size

    def __matmul__(self, vector) :
        r = self.__r
        n = vector.size + r - 1

        return np.asarray([np.dot(
            np.flip(self.__filt),
            np.concatenate((np.zeros(r-1), vector, np.zeros(r-1)))[i:i+r]
        ) for i in range(n)])  # IMPLEMENTATION

def huber_loss(x) :
    return np.sum( (1/2)*(x**2)*(np.abs(x)<=1) + (np.sign(x)*x-1/2)*(np.
 ↪abs(x)>1) )
def huber_grad(x) :
    return x*(np.abs(x)<=1) + np.sign(x)*(np.abs(x)>1)


r, n, lam = 3, 20, 0.1

np.random.seed(0)
k = np.random.randn(r)
```

```
b = np.random.randn(n-r+1)
A = Convolution1d(k)
#from scipy.linalg import circulant
#A = circulant(np.concatenate((np.flip(k),np.zeros(n-r))))[r-1:,:]


x = np.zeros(n)
alpha = 0.01
for _ in range(100) :
    x = x - alpha*(A.T@(huber_grad(A@x-b))+lam*x)

print(huber_loss(A@x-b)+0.5*lam*np.linalg.norm(x)**2)
```

```
0.4587586843129764
```