05 Generalizing from data

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Motivation

- Why we analyze data is about learning patterns in that data itself, among the observations it contains.
- Finding patterns from data
- Making generalization/inference to business/research questions
- Using statistical tools to quantify the uncertainty brought about by such a generalization
- External validity

Learning Outcomes

- Learning about generalization beyond the actual data
- Understanding the concept and logic of statistical inference and external validity
- Understanding the concepts of repeated samples and bootstrap
- Computing and using standard errors to create confidence intervals and use those for statistical inference
- Understanding whether and how additional data analysis can help assess external validity
- Using those analytical tools in machine learning and econometrics

Generalization

- Sometimes we analyze a dataset with the goal of learning about patterns in that dataset alone.
- In such cases there is no need to generalize our findings to other datasets.
- Example: We search for a good deal among offers of hotels, all we care about are the observations in our dataset.
- Often we analyze a dataset in order to learn about patterns that may be true in other situations.
- If findings from a single market is applicable to the general economic conditions!
- We are interested in finding the relationship between
  - Our dataset
  - The situation we care about

Generalization

- Generalize the results from a single dataset to other situations.
- The act of generalization is called *inference*:

we infer something from our data about a more general phenomenon because we want to use that knowledge in some other situation.

- Aspect 1: statistical inference
- Aspect 2: external validity

Statistical inference

- Aims at generalizing to the situation that our data represents, using statistical methods, to make inference ---> statistical inference
- The general pattern is an abstract thing that may or may not exist.
- If we can assume that the general pattern exists, the tools of statistical inference can be very helpful.

General patterns 1: Population and representative sample

- General pattern needs to be stable and consistent over time and space
- Simplest case: sample vs population: randomness and joint distribution
- A sample is representative of a population if the distribution of all variables is very similar in the sample and the population.
- Random sampling is the best way to achieve a representative sample.

General patterns 2: No population but general pattern

The concept of representation is less straightforward in other setups.

- Using data with observations from the past to uncover a pattern that may be true for the future.
- Generalizing patterns observed among some products to other, similar products.

There isn't necessarily a "population" from which a random sample was drawn on purpose. Instead, we should think of our data as one that represents a general pattern.

- There is a general pattern, each year is a random realization.
- There is a general pattern, each product is a random version, all represented by the same general pattern.

External validity

- Assessing whether our data represents the same general pattern that would be relevant for the situation we truly care about.
- Externally valid case: the situation we care about and the data we have represent the same general pattern
- With external validity, our data can tell what to expect.
- No external validity: whatever we learn from our data, may turn out to be not relevant at all.

The process of inference

The process of inference

1. Consider a statistic we may care about, such as the mean.
2. Compute its *estimated value* from a dataset
3. Infer the value in the population / in the general pattern that our data represents.

It is good practice to divide the inference problem into two.

1. Use statistical inference to learn about the population, or general pattern, that our data represents.
2. Assess external validity: define the population, or general pattern we are interested in and assess how it compares to the population, or general pattern, that our data represents.

Repeated samples

- Repeated samples - the conceptual background to statistical inference
- Our data - one example of many datasets that could have been observed
- Each datasets can be viewed as samples drawn from the population (general pattern)
- Easier concept: When our data is sample from a well-defined population - many other samples could have turned out instead of what we have
- Harder concept: No clear definition of population. We think of a general pattern we care about

Repeated samples

- The goal of statistical inference is learning the value of a statistic in the population, or general pattern, represented by our data.
- The statistic has a distribution: its value may differ from sample to sample.
- The distribution of the statistic of interest is called its sampling distribution

Repeated samples

- Standard deviation in this distribution: spread across repeated samples
- The standard error (SE) of the statistic = the standard deviation of the sampling distribution
- Any particular estimate is likely to be an erroneous estimate of the true value.
- The magnitude of that typical error is one SE.

Repeated samples properties

The sampling distribution of a statistic is the distribution of this statistic across repeated samples.

The sampling distribution has three important properties

1. Unbiasedness: The average of the values in repeated samples is equal to its true value (=the value in the entire population / general pattern).
2. Asymptotic normality: The sampling distribution is approximately normal. With large sample size, it is very very close.
3. Root-n convergence: The standard error (the standard deviation of the sampling distribution) is smaller the larger the samples, with a proportionality factor of the square root of the sample size.

The standard error and the confidence interval

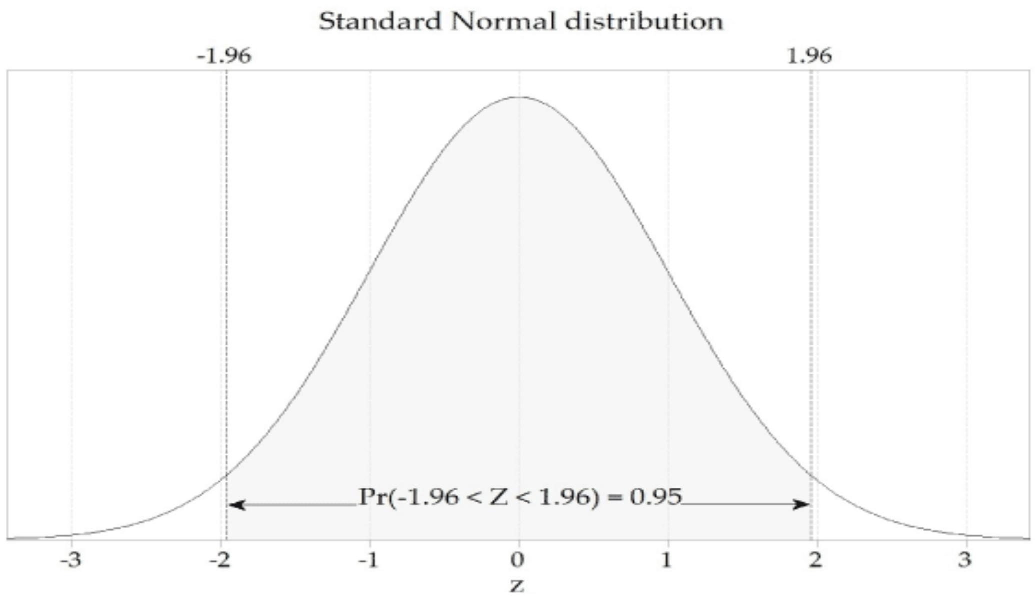
- Confidence interval (CI) - measure of statistical inference.
  - Recall: Statistical inference - we analyze a dataset to infer the true value of a statistic: its value in the population, or general pattern, represented by our data.
- The CI defines a range where we can expect the true value in the population, or the general pattern.
- CI gives a range for the true value with a probability
- Probability tells how likely it is that the true value is in that range
- Probability - data analysts need to pick it, such as 95%

The standard error and the confidence interval

- The "95 percent CI" gives the range of values where we think that true value falls with a 95 percent likelihood.
- Viewed from the perspective of a single sample, the chance (probability) that the truth is within the CI measured around the value estimated from that single sample is 95 percent.
- Also: we think that with 5 percent likelihood, the true value will fall outside the confidence interval.

The standard error and the confidence interval

- Confidence interval - symmetric range around the estimated value of the statistic in our dataset.
  - Get estimated value ---> Define probability ---> Calculate CI with the use of SE
  - In normal distribution, the scores ± 1.96 (2), ± 1.645 (1.6), and ± 2.576 (2.6) correspond to the standard deviations for the 95%, 90%, and 99% confidence intervals, respectively.



$$SE(\bar{x}) = \frac{1}{\sqrt{n}} Std[x]$$

Calculating the standard error

An important consequence of evidence from the repeated sample exercise:

- In reality, we don't get to observe the sampling distribution. Instead, we observe a single dataset
- That dataset is one of the many potential samples that could have been drawn from the population, or general pattern
- Good news: We can get a very good idea of how the sampling distribution would look like - good estimate of the standard error - even from a single sample.
- Getting SE – Option 1: Use a formula
- Getting SE – Option 2: Simulate by a new method, called 'bootstrapping'

Calculating the standard error

To calculate SE, consider the statistic of the sample mean.

- Assume the values of x are independent across observations in the dataset.
- $\bar{x}$ the estimate of the true mean value of x in the general pattern/population.
- Sampling distribution is approximately normal, with the true value as its mean.

$$SE(\bar{x}) = \frac{1}{\sqrt{n}} Std[x]$$

The standard error formula

- The standard error is larger...
  - the larger the standard deviation of the variable.
  - the smaller the sample and
- For intuition, consider $SE(\bar{x})$ vs $SE[x]$
- Think back to the repeated samples simulation exercise:
  - $SE(\bar{x}) =$ the standard error of $\bar{x}$ is the standard deviation of the various $\bar{x}$ estimates across repeated samples.
  - The larger the standard deviation of x itself, the more variation we can expect in $\bar{x}$ across repeated samples.

Take a quick stop to summarize the idea of CI

- We are interested in generalizing from our data. Statistical inference.
- Consider a statistic such as the sample mean $\bar{x}$
- Take a 95% confidence interval - where we can expect to see the true value
- CI=statistic $+/-2SE$.
- We have a formula for the SE calculated from our data only using the standard deviation and sample size.
- Using the CI, we can now do statistical inference, generalize for the population / general pattern we care about.

The bootstrap

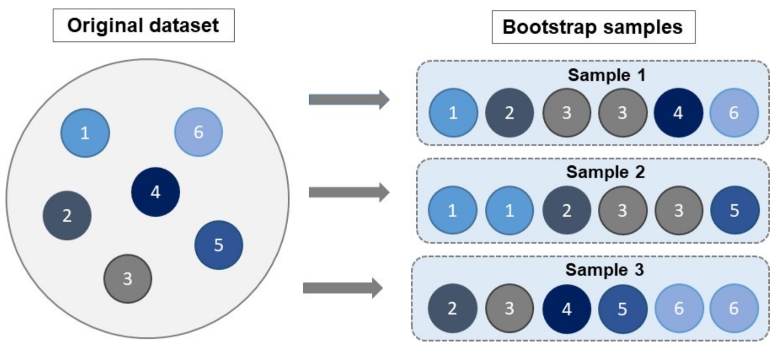
- Bootstrap is a method to create synthetic samples that are similar but different
- A method that is very useful in general.
- It is essential for many advanced statistics application such as machine learning

The bootstrap

- The bootstrap method takes the original dataset and draws many repeated samples of the size of that dataset.
- The trick is that the samples are drawn *with replacement*.
- The observations are drawn randomly one by one from the original dataset; once an observation is drawn it is "replaced" to the pool so that it can be drawn again, with the same probability as any other observation.
- The drawing stops when it reaches the size of the original dataset.
- The result is a sample of the same size as the original dataset, yielding a single *bootstrap sample*.

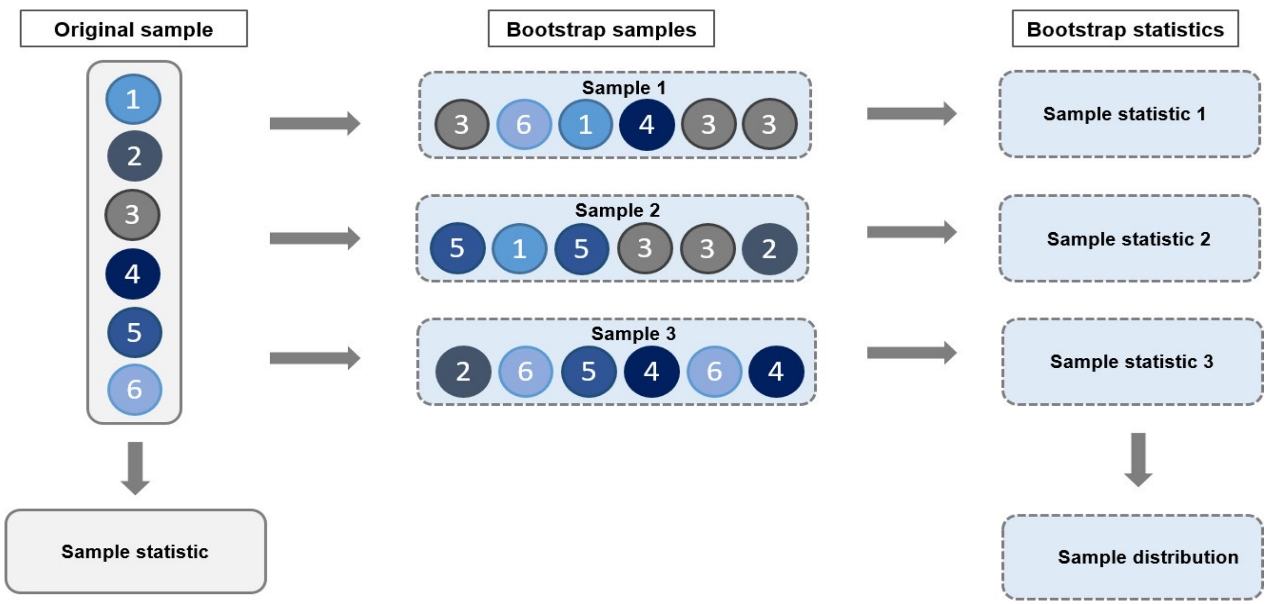
The bootstrap

- A bootstrap sample is always the same size the original
- it includes some of the original observations multiple times,
- it does not include some of other original observations.
- We typically create 500 - 10,000 samples
- Computationally intensive but feasible, relatively fast.



The bootstrap

- We have a dataset (the sample), can compute a statistic (e.g. mean)
- Create many bootstrap samples, and get a mean value for each sample
- Bootstrap estimate of SE = standard deviation of statistic based on bootstrap samples' estimates.



The bootstrap SE

- The bootstrap method creates many repeated samples that are different from each other, but each has the same size as the original dataset.

- Bootstrap gives a good approximation of the standard error, too.
- The bootstrap estimate (or the estimate from the bootstrap method) of the standard error is simply the standard deviation of the statistic across the bootstrap samples.

External validity

- We discussed statistical inference: CI - uncertainty about the true value of the statistic in the population / general pattern that our data represents.
- What is the population, or general pattern, we care about?
- How close is our data to this?
- External validity is the concept that captures the similarity of our data to the population/general pattern we care about.
- High external validity: if our data is close to the population or the general pattern we care about.
- External validity is as important as statistical inference.
- However, it is not a statistical question.

External validity

- The most important challenges to external validity may be collected in three groups:

- Time: we have data on the past, but we care about the future
- Space: our data is on one country, but interested how a pattern would hold elsewhere in the world
- Sub-groups: our data is on 25-30 year old people. Would a pattern hold on younger / older people?

External validity

- Daily 5%+ loss probability - 95 percent CI [0.2, 0.8] in our sample. This captures uncertainty for samples like ours.
- If the future one year will be like the past 11 years in terms of the general pattern that determines returns on our investment portfolio.
- However, external validity may not be high - not sure what the future holds.
- Our data: 2006-2016 dataset includes the financial crisis and great recession of 2008-2009. It does not include the dotcom boom and bust of 2000-2001. We have no way to know which crisis is representative to future crises to come.
- Hence, the real CI is likely to be substantially wider.

External validity in Big Data

- Big data: very large N
- Statistical inference not really important - CI becomes very narrow
- External validity remains as important

- 1.) Large sample DOES NOT mean representative sample
- 2.) Big data as result of actions - nature of things may change as people alter behavior, outside conditions change

Generalization - Summary

- The standard error of a statistic (like the mean) measures the variability of that statistic from sample to sample when drawing multiple samples from the same population.
- As you take more repeated samples, the means of these samples tend to be closer to the actual population mean, which decreases the standard error.
- This reflects the fact that you have more information about the population, which reduces the uncertainty around your estimate of the population mean.
- So, when you take more samples (increasing the sample size), you are more likely to get an accurate representation of the population, hence reducing the standard error.
- It's also important to note that the standard error depends not just on the number of samples, but also on the standard deviation of the population.
- If the population standard deviation is large, then the standard error will also be larger because the data points are more spread out from the mean, indicating more variability within the population.

Generalization - Summary

- In summary, the standard error for repeated samples is smaller because more data provides a more precise estimate of the population parameter.
- Generalization is a key task - finding beyond the actual dataset.
- This process is made up of discussing statistical inference and external validity.
- Statistical inference generalizes from our dataset to the population using a variety of statistical tools.
- External validity is the concept of discussing beyond the population for a general pattern we care about; an important but typically somewhat speculative process.

Stock market returns: The standard error formula

Let's consider our example of 11-years' of data on daily returns on the S&P 500 portfolio.

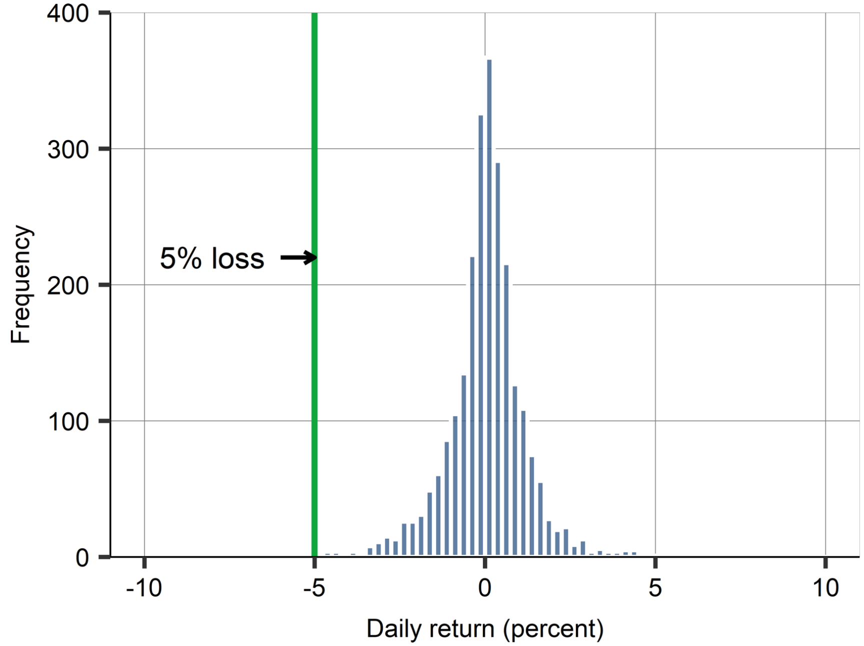
- The calculated statistics, $P(loss > 5\%) = 0.5\%$
- The $SE[P(loss > 5\%)]$ is calculated by,
  - The size of the sample is n = 2,519 so that $1/\sqrt{n} = 0.02$.
  - The standard deviation of the fraction of $SD[P(loss > 5\%)] = 0.07$.
  - So the $SE = 0.07 * 0.02 = 0.0014$ (0.14 percent).
- Can calculate the 95 percent CI:
  - $CI = [0.5 - 2 * SE, 0.5 + 2 * SE] = [0.22, 0.78]$
- This means that in the general pattern represented by the 11-year history of returns in our data, we can be 95 percent confident that daily losses of more than 5 percent occur with a 0.2 to 0.8 percent chance.

Stock market returns: Inference

- Task: Assess the likelihood of experiencing a loss of certain magnitude on an investment portfolio from one day to the next day
- Predict the frequency of a loss of certain magnitude for the coming calendar year

- The investment portfolio is the S&P 500, a US stock market index
- Data: day-to-day returns on the S&P 500, defined as percentage changes in the closing price of the index between two consecutive days
- 11 years: 25 August 2006 to 26 August 2016. It includes 2,519 days.

Histogram of daily returns



Note: *S&P 500 market index. Day to day (gaps ignored) changes, in percentage. From August 25 2006 to August 26*

Stock market returns: Inference

- To define "loss", we take a day-to-day loss exceeding 5 percent.
- "loss" is a binary variable, taking 1 when the day-to-day loss exceeds 5 percent and zero otherwise.
- The statistic in the data is the proportion of days with such losses.
- It is 0.5 percent in this dataset
  - the S&P500 portfolio lost more than 5 percent of its value on 0.5 percent of the days between August 25 2006 and August 26 2016.
- Inference problem: How can we generalize this finding? What can we infer from this 0.5 percent chance for the next calendar year?

Stock market returns: A simulation

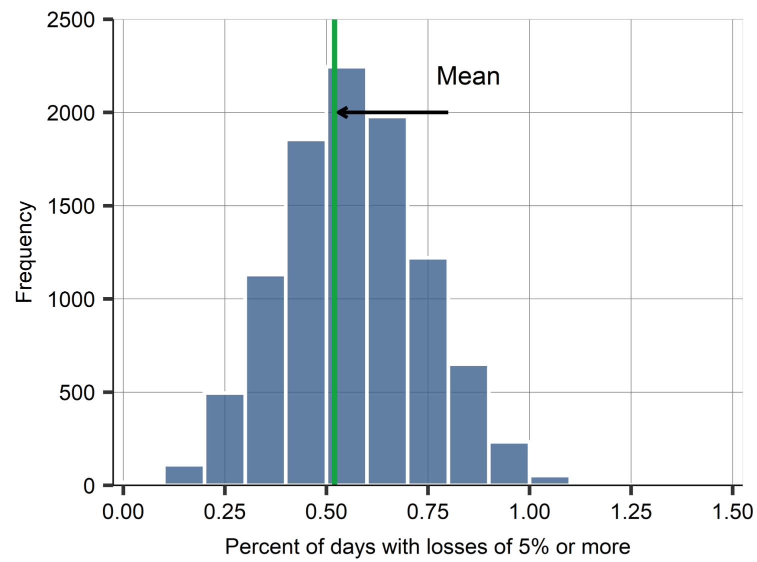
- We can not rerun history many many times...
- Simulation exercise - to better understand how repeated samples work
- Suppose the 11-year dataset is *the* population - the fraction of days with 5%+ losses is 0.5% in the entire 11 years' data. That's the true value.
- Assume we have only three years (900 days) of daily returns in our dataset.
- Task: estimate the true value of the fraction in the 11-year period from the data we have using a simulation exercise.
  1. many data table with three years' worth of observations may be created from the 11 years' worth of data,
  2. compute the fraction of days with 5%+ losses in data tables
  3. learn about the true value

Stock market returns: A simulation

- Do simple random sampling: days are considered one after the other and are selected or not selected in an independent random fashion.
  - This sampling destroys the time series nature
  - This is OK because daily returns are (almost) independent across days in the original dataset
- We do this 10,000 times....

Stock market returns: A simulation

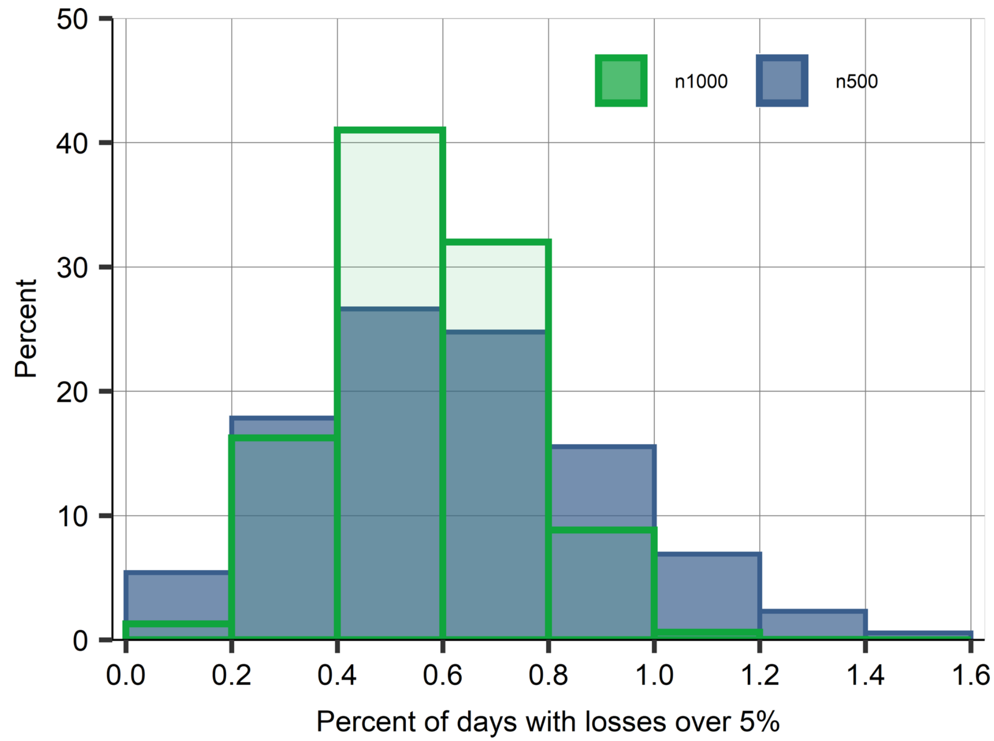
- percent of days with losses of 5% of more.
- histogram created from the 10,000 random samples, each w/ 900 obs, drawn from entire dataset
- distribution has some spread: smallest realization is 0.1 %, while the largest is smaller than 1.25 %



Histogram of the proportion of days with losses of 5 percent or more, across repeated samples of size n=900. 10,000 random samples. Source: sandp-stocks data. S&P 500 market index.

Stock market returns: Sampling distributions

- Proportion of days with losses of 5 percent or more
- Repeated samples in two simulation exercises, with n=500 and n=1,000. (10,000 random samples)
- Kernel density (goes to minus / can cut it at 0)
- Role of sample size: smaller sample: skewed; higher standard deviation



Stock market returns: The Bootstrap standard error

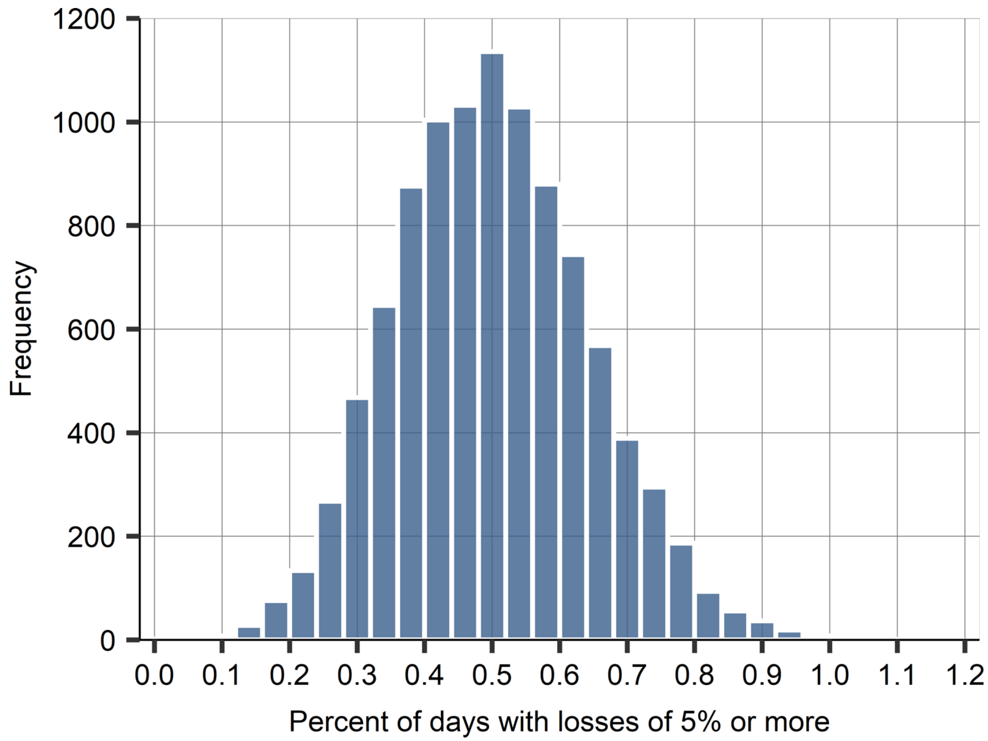
- We estimate the standard error by bootstrap.
- Let's consider our example of 11-years' of data on daily returns on the S&P 500 portfolio.
- Do the process ————————>
- End up with a new a dataset: one observations / bootstrap sample. Only variable is the estimated proportion in a sample
- **The SE is simply the standard deviation of those estimated values in this new dataset.**

The process

1. Take the original dataset and draw a bootstrap sample.
2. Calculate the proportions of days with 5%+ loss in that sample.
3. Save that value.
4. Then go back to the original dataset and take another bootstrap sample.
5. Calculate the proportion of days with 5%+ loss and save that value, too.
6. And so on, repeated many times.

Stock market returns: The Bootstrap standard error

- 10,000 bootstrap samples with 2,519 observations
- The proportion of days with 5+ percent loss.
- Varied 0.1 percent to 1.2 percent. Mean=Median= 0.5
- Standard deviation across the bootstrap samples = 0.14
- CI: the 95 percent CI is [0.22, 0.78].



Stock market returns: The Bootstrap standard error

- This means that in the general pattern represented by the 11-year history of returns in our data, we can be 95 percent confident that daily losses of more than 5 percent occur with a 0.22 to 0.78 percent chance.
- SE formula and bootstrap gave the **same** exact answer
- Under some conditions, this is what we expect
  - Large enough sample size
  - Observations independent
  - ... (other we overlook now)