Essay

Kris Boudt

Bach Econometrics, VU

January 25, 2012

Rapport

- Kort wetenschappelijk artikel ("note") met volgende componenten:
  1. Onderzoeksvraag/motivatie
  2. Beschrijving datareeks (met figuur)
  3. Beschrijving regime switching model en schattingsmethode
  4. Output schatting regimemodel (geschatte parameters met standaardfouten, de geschatte kansen van in ieder regime te zijn)
  5. Conclusie met opmerkingen beperkingen van het gevoerde onderzoek en suggesties voor verder onderzoek.
  6. In appendix: code, voorzien van korte documentatie.

Voorbeeld

- AR(4) model voor trimestriële groei met regime switching in de locatieparameter:

$$
\begin{aligned}
y_t - \mu_{s_t} = {} & \phi_1(y_{t-1} - \mu_{s_{t-1}}) + \phi_2(y_{t-2} - \mu_{s_{t-2}}) \\
& + \phi_3(y_{t-3} - \mu_{s_{t-3}}) + \phi_4(y_{t-4} - \mu_{s_{t-4}}) + \epsilon_t,
\end{aligned}
$$

with $\epsilon_t \sim$ iid $N(0, \sigma^2)$.

TABLE 22.1
Maximum Likelihood Estimates of Parameters for Markov-Switching Model of U.S. GNP (Standard Errors in Parentheses)

$\hat{\mu}_1 = 1.16$ (0.07)	$\hat{\mu}_2 = -0.36$ (0.26)	$\hat{p}^*_{11} = 0.90$ (0.04)	$\hat{p}^*_{22} = 0.75$ (0.10)	$\hat{\sigma}^2 = 0.59$ (0.10)
$\hat{\phi}_1 = 0.01$ (0.12)	$\hat{\phi}_2 = -0.06$ (0.14)	$\hat{\phi}_3 = -0.25$ (0.11)	$\hat{\phi}_4 = -0.21$ (0.11)	

Maximum likelihood estimation

- The likelihood of the data is the joint conditional density:

$$\mathcal{L}(\theta) = \prod_{t=1}^{T} f(y_t|\Omega_{t-1};\theta)$$

- $\hat{\theta}_{ML}$ is the value for which $\mathcal{L}(\theta)$ is maximal, or equivalently, the value that maximizes the log-likelihood:

$$l(\theta) = \sum_{t=1}^{T} \log f(y_t|\Omega_{t-1};\theta),$$

or, for numerical reasons, the average log-likelihood:

$$\frac{1}{T} l(\theta) = \frac{1}{T}\sum_{t=1}^{T} \log f(y_t|\Omega_{t-1};\theta).$$

Maximum likelihood estimation

- Properties: Consistency

$$\operatorname{plim} \hat{\theta}_{ML} = \theta$$

- Asymptotic normality

$$\hat{\theta}_{ML} \overset{a}{\sim} N\left(\theta; I(\theta)^{-1}\right)$$

where

$$I(\theta) = -\left(\frac{\partial^2 l(\theta)}{\partial\theta\partial\theta'}\right)$$

is the Fisher information matrix.

Maximum likelihood estimation

- Numeric calculation of the hessian:

  Num2Derivative (LnLiklRegr , theta , &mH) ; I = - mH ;

  SE = sqrt(diag(invertgen(I)))

- If the average loglikelihood function is used:

  Num2Derivative (AvgLnLikl , theta , &mH) ; I = -mH*T

Maximum likelihood estimation

- Most of you have defined the loglikelihood function as θ_* a function g of the parameter of interest θ

$$\theta_* = \log\theta \;\; ; \;\; \theta_* = \log(\theta/(1-\theta))$$

$$\theta = \exp\theta_* \;\; ; \;\; \theta = \exp(\theta_*)/(1+\exp(\theta_*))$$

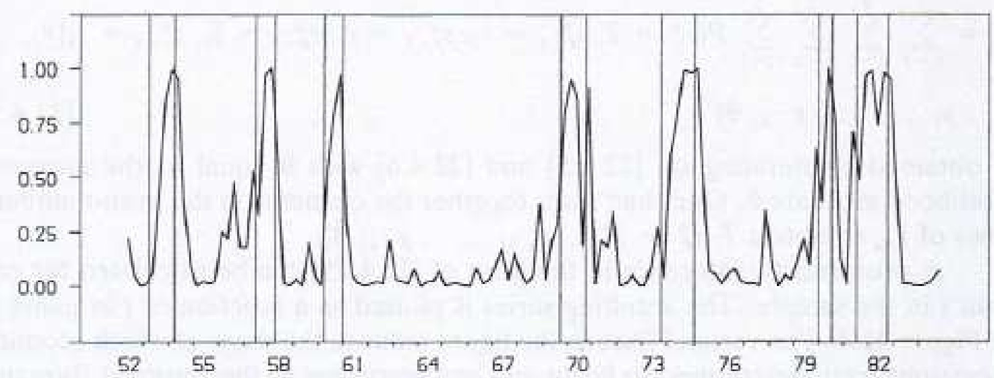
- First order Taylor approximation of $\hat{\theta} = g(\hat{\theta}_*)$ around $\theta = g(\theta_*)$:

$$\hat{\theta} \approx g(\theta_*) + \frac{dg(\theta_*)}{d\theta_*}(\hat{\theta}_* - \theta_*)$$

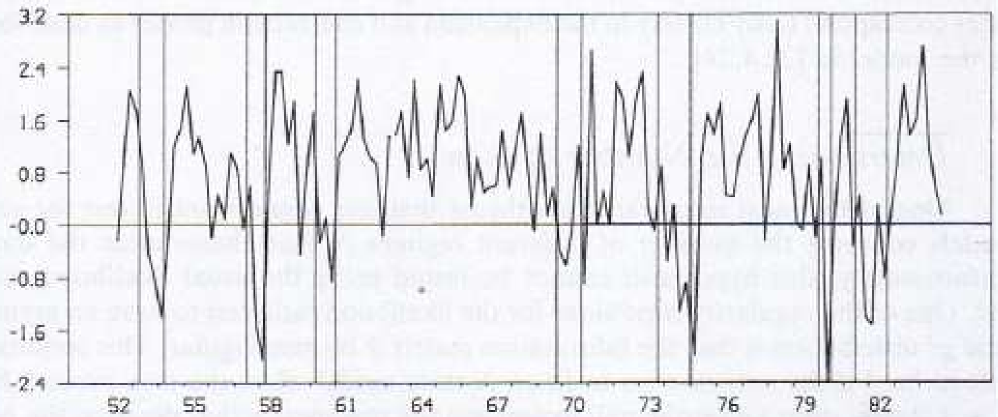
- Delta method:

$$\text{SE}_{\hat{\theta}} \approx \frac{dg(\theta_*)}{d\theta_*}\text{SE}_{\hat{\theta}_*}.$$

$$\frac{dg(\theta_*)}{d\theta_*} = \exp(\theta_*) \;\; ; \;\; \frac{dg(\theta_*)}{d\theta_*} = \exp(\theta_*)/(1+\exp(\theta_*))^2.$$



(a) Probability that economy is in contraction state, or $P\{s_t^* = 2 | y_t, y_{t-1}, \ldots, y_{-4}; \hat{\boldsymbol{\theta}}\}$ plotted as a function of t.



(b) Quarterly rate of growth of U.S. real GNP, 1952–84.

FIGURE 22.4 Output growth and recession probabilities.

Essay - RS model, Hamilton filter

$$\hat{\xi}_{j,t|t} = \text{Pr}(s_t = j|\Omega_t;\theta).$$

$$\hat{\xi}_{j,t+1|t} = \text{Pr}(s_{t+1} = j|\Omega_t;\theta).$$

- To estimate $\hat{\xi}_{t|t}$ uses only states $1,\ldots,t-1$. Improve using estimated states $1,\ldots,T$ by backward smoothing (starting at T-1):

$$\hat{\xi}_{t|T} = \hat{\xi}_{t|t}(*)\{P'(\hat{\xi}_{t+1|T}(:)\hat{\xi}_{t+1|t})\}$$

  [$(*)$ and $(:)$ stands for element-by-element multiplication and division ; $\iota' = (11)$]

Presentaties

- Elke groep: tussen de 10 en 15 minuten;
- Mondeling toelichten belangrijkste componenten van het ingediende onderzoeksrapport.