

Derivation of the Differential Continuity Equation

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1 Continuity Equation

Starting with the first principle of conservation of mass:

$$\text{time rate change in mass} = \text{mass transfer in} - \text{mass transfer out} \quad (1)$$

This can be expressed mathematically as:

$$\frac{\partial m}{\partial t} = \dot{m}_{in} - \dot{m}_{out} \quad (2)$$

$$\frac{\partial m}{\partial t} = (\rho A u)_{in} - (\rho A u)_{out} \quad (3)$$

For the infinitesimal control volume defined by lengths dx , dy , and dz , mass can be expressed in terms of the density, ρ , as:

$$m = \rho \, dx \, dy \, dz \quad (4)$$

$$\frac{\partial \rho}{\partial t} \, dx \, dy \, dz = \dot{m}_{in} - \dot{m}_{out} \quad (5)$$

$$\frac{D\rho}{Dt} = -\rho \nabla \cdot \mathbf{u} \quad (6)$$

