Derivation of the Differential Continuity Equation

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1 Continuity Equation

Starting with the first principle of conservation of mass:

$$\text{time rate change in mass} = \text{mass transfer in} - \text{mass transfer out} \tag{1}$$

This can be expressed mathematically as:

$$\frac{\partial m}{\partial t} = \dot{m}_{in} - \dot{m}_{out} \tag{2}$$

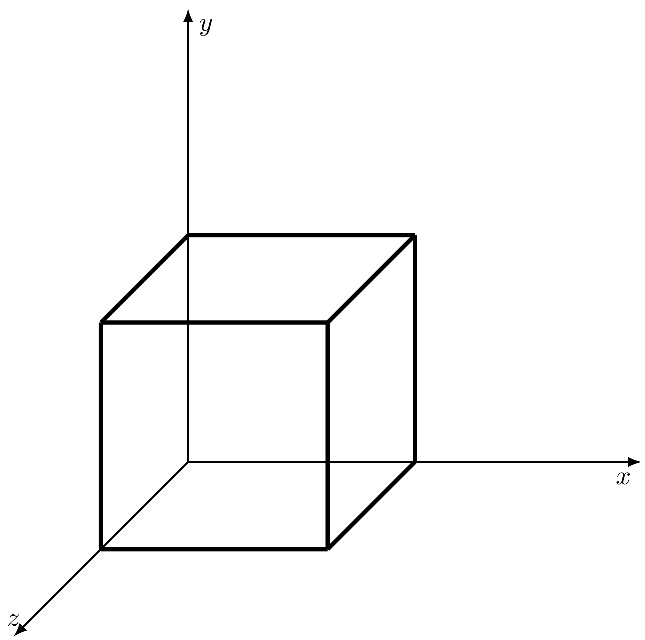
$$\frac{\partial m}{\partial t} = (\rho A u)_{in} - (\rho A u)_{out} \tag{3}$$

For the infinitesimal control volume defined by lengths $\mathrm{d}x$, $\mathrm{d}y$, and $\mathrm{d}z$, mass can be expressed in terms of the density, ρ, as:

$$m = \rho \, \mathrm{d}x \, \mathrm{d}y \, \mathrm{d}z \tag{4}$$

$$\frac{\partial \rho}{\partial t} \mathrm{d}x \, \mathrm{d}y \, \mathrm{d}z = \dot{m}_{in} - \dot{m}_{out} \tag{5}$$

$$\frac{\mathrm{D}\rho}{\mathrm{D}t} = -\rho \boldsymbol{\nabla} \cdot \vec{u} \tag{6}$$



y
x
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