Integration Formulas

1. Common Integrals

Indefinite Integral

Method of substitution

$$\int f(g(x))g'(x)dx = \int f(u)du$$

Integration by parts

$$\int f(x)g'(x)dx = f(x)g(x) - \int g(x)f'(x)dx$$

Integrals of Rational and Irrational Functions

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

$$\int \frac{1}{x} dx = \ln|x| + C$$

$$\int c\, dx = cx + C$$

$$\int x dx = \frac{x^2}{2} + C$$

$$\int x^2 dx = \frac{x^3}{3} + C$$

$$\int \frac{1}{x^2} dx = -\frac{1}{x} + C$$

$$\int \sqrt{x} dx = \frac{2x\sqrt{x}}{3} + C$$

$$\int \frac{1}{1+x^2} dx = \arctan x + C$$

$$\int \frac{1}{\sqrt{1-x^2}} dx = \arcsin x + C$$

Integrals of Trigonometric Functions

$$\int \sin x\, dx = -\cos x + C$$

$$\int \cos x\, dx = \sin x + C$$

$$\int \tan x\, dx = \ln|\sec x| + C$$

$$\int \sec x\, dx = \ln|\tan x + \sec x| + C$$

$$\int \sin^2 x\, dx = \frac{1}{2}(x - \sin x \cos x) + C$$

$$\int \cos^2 x\, dx = \frac{1}{2}(x + \sin x \cos x) + C$$

$$\int \tan^2 x\, dx = \tan x - x + C$$

$$\int \sec^2 x\, dx = \tan x + C$$

Integrals of Exponential and Logarithmic Functions

$$\int \ln x\, dx = x \ln x - x + C$$

$$\int x^n \ln x\, dx = \frac{x^{n+1}}{n+1} \ln x - \frac{x^{n+1}}{(n+1)^2} + C$$

$$\int e^x\, dx = e^x + C$$

$$\int b^x\, dx = \frac{b^x}{\ln b} + C$$

$$\int \sinh x\, dx = \cosh x + C$$

$$\int \cosh x\, dx = \sinh x + C$$

2. Integrals of Rational Functions

Integrals involving ax + b

$$\int (ax+b)^n\,dx = \frac{(ax+b)^{n+1}}{a(n+1)} \qquad (for\ n \neq -1)$$

$$\int \frac{1}{ax+b}\,dx = \frac{1}{a}\ln|ax+b|$$

$$\int x(ax+b)^n\,dx = \frac{a(n+1)x-b}{a^2(n+1)(n+2)}(ax+b)^{n+1} \qquad (for\ n \neq -1, n \neq -2)$$

$$\int \frac{x}{ax+b}\,dx = \frac{x}{a} - \frac{b}{a^2}\ln|ax+b|$$

$$\int \frac{x}{(ax+b)^2}\,dx = \frac{b}{a^2(ax+b)} + \frac{1}{a^2}\ln|ax+b|$$

$$\int \frac{x}{(ax+b)^n}\,dx = \frac{a(1-n)x-b}{a^2(n-1)(n-2)(ax+b)^{n-1}} \qquad (for\ n \neq -1, n \neq -2)$$

$$\int \frac{x^2}{ax+b}\,dx = \frac{1}{a^3}\left(\frac{(ax+b)^2}{2} - 2b(ax+b) + b^2\ln|ax+b|\right)$$

$$\int \frac{x^2}{(ax+b)^2}\,dx = \frac{1}{a^3}\left(ax+b-2b\ln|ax+b| - \frac{b^2}{ax+b}\right)$$

$$\int \frac{x^2}{(ax+b)^3}\,dx = \frac{1}{a^3}\left(\ln|ax+b| + \frac{2b}{ax+b} - \frac{b^2}{2(ax+b)^2}\right)$$

$$\int \frac{x^2}{(ax+b)^n}\,dx = \frac{1}{a^3}\left(-\frac{(ax+b)^{3-n}}{n-3} + \frac{2b(a+b)^{2-n}}{n-2} - \frac{b^2(ax+b)^{1-n}}{n-1}\right) \qquad (for\ n \neq 1,2,3)$$

$$\int \frac{1}{x(ax+b)}\,dx = -\frac{1}{b}\ln\left|\frac{ax+b}{x}\right|$$

$$\int \frac{1}{x^2(ax+b)}\,dx = -\frac{1}{bx} + \frac{a}{b^2}\ln\left|\frac{ax+b}{x}\right|$$

$$\int \frac{1}{x^2(ax+b)^2}\,dx = -a\left(\frac{1}{b^2(a+xb)} + \frac{1}{ab^2x} - \frac{2}{b^3}\ln\left|\frac{ax+b}{x}\right|\right)$$

Integrals involving ax^2 + bx + c

$$\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}arctg\,\frac{x}{a}$$

$$\int \frac{1}{x^2-a^2}\,dx = \begin{cases} \dfrac{1}{2a}\ln\dfrac{a-x}{a+x} & for\ |x|<|a| \\ \dfrac{1}{2a}\ln\dfrac{x-a}{x+a} & for\ |x|>|a| \end{cases}$$

$$\int \frac{1}{ax^2+bx+c}dx = \begin{cases} \frac{2}{\sqrt{4ac-b^2}}\arctan\frac{2ax+b}{\sqrt{4ac-b^2}} & \text{for } 4ac-b^2>0 \\ \frac{2}{\sqrt{b^2-4ac}}\ln\left|\frac{2ax+b-\sqrt{b^2-4ac}}{2ax+b+\sqrt{b^2-4ac}}\right| & \text{for } 4ac-b^2<0 \\ -\frac{2}{2ax+b} & \text{for } 4ac-b^2=0 \end{cases}$$

$$\int \frac{x}{ax^2+bx+c}dx = \frac{1}{2a}\ln\left|ax^2+bx+c\right| - \frac{b}{2a}\int\frac{dx}{ax^2+bx+c}$$

$$\int \frac{mx+n}{ax^2+bx+c}dx = \begin{cases} \frac{m}{2a}\ln\left|ax^2+bx+c\right| + \frac{2an-bm}{a\sqrt{4ac-b^2}}\arctan\frac{2ax+b}{\sqrt{4ac-b^2}} & \text{for } 4ac-b^2>0 \\ \frac{m}{2a}\ln\left|ax^2+bx+c\right| + \frac{2an-bm}{a\sqrt{b^2-4ac}}\operatorname{arctanh}\frac{2ax+b}{\sqrt{b^2-4ac}} & \text{for } 4ac-b^2<0 \\ \frac{m}{2a}\ln\left|ax^2+bx+c\right| - \frac{2an-bm}{a(2ax+b)} & \text{for } 4ac-b^2=0 \end{cases}$$

$$\int \frac{1}{\left(ax^2+bx+c\right)^n}dx = \frac{2ax+b}{(n-1)\left(4ac-b^2\right)\left(ax^2+bx+c\right)^{n-1}} + \frac{(2n-3)2a}{(n-1)\left(4ac-b^2\right)}\int\frac{1}{\left(ax^2+bx+c\right)^{n-1}}dx$$

$$\int \frac{1}{x\left(ax^2+bx+c\right)}dx = \frac{1}{2c}\ln\left|\frac{x^2}{ax^2+bx+c}\right| - \frac{b}{2c}\int\frac{1}{ax^2+bx+c}dx$$

3. Integrals of Exponential Functions

$$\int xe^{cx}dx = \frac{e^{cx}}{c^2}(cx-1)$$

$$\int x^2e^{cx}dx = e^{cx}\left(\frac{x^2}{c}-\frac{2x}{c^2}+\frac{2}{c^3}\right)$$

$$\int x^ne^{cx}dx = \frac{1}{c}x^ne^{cx} - \frac{n}{c}\int x^{n-1}e^{cx}dx$$

$$\int \frac{e^{cx}}{x}dx = \ln|x| + \sum_{i=1}^{\infty}\frac{(cx)^i}{i\cdot i!}$$

$$\int e^{cx}\ln x\,dx = \frac{1}{c}e^{cx}\ln|x| + E_i(cx)$$

$$\int e^{cx}\sin bx\,dx = \frac{e^{cx}}{c^2+b^2}(c\sin bx - b\cos bx)$$

$$\int e^{cx}\cos bx\,dx = \frac{e^{cx}}{c^2+b^2}(c\cos bx + b\sin bx)$$

$$\int e^{cx}\sin^n x\,dx = \frac{e^{cx}\sin^{n-1}x}{c^2+n^2}(c\sin x - n\cos bx) + \frac{n(n-1)}{c^2+n^2}\int e^{cx}\sin^{n-2}dx$$

4. Integrals of Logarithmic Functions

$$\int \ln cx dx = x \ln cx - x$$

$$\int \ln(ax+b)dx = x\ln(ax+b) - x + \frac{b}{a}\ln(ax+b)$$

$$\int (\ln x)^2 dx = x(\ln x)^2 - 2x\ln x + 2x$$

$$\int (\ln cx)^n dx = x(\ln cx)^n - n\int (\ln cx)^{n-1} dx$$

$$\int \frac{dx}{\ln x} = \ln|\ln x| + \ln x + \sum_{n=2}^{\infty} \frac{(\ln x)^i}{i \cdot i!}$$

$$\int \frac{dx}{(\ln x)^n} = -\frac{x}{(n-1)(\ln x)^{n-1}} + \frac{1}{n-1}\int \frac{dx}{(\ln x)^{n-1}} \qquad (\mathit{for}\ n \neq 1)$$

$$\int x^m \ln x dx = x^{m+1}\left(\frac{\ln x}{m+1} - \frac{1}{(m+1)^2}\right) \qquad (\mathit{for}\ m \neq 1)$$

$$\int x^m (\ln x)^n dx = \frac{x^{m+1}(\ln x)^n}{m+1} - \frac{n}{m+1}\int x^m (\ln x)^{n-1} dx \qquad (\mathit{for}\ m \neq 1)$$

$$\int \frac{(\ln x)^n}{x} dx = \frac{(\ln x)^{n+1}}{n+1} \qquad (\mathit{for}\ n \neq 1)$$

$$\int \frac{\ln x^n}{x} dx = \frac{(\ln x^n)^2}{2n} \qquad (\mathit{for}\ n \neq 0)$$

$$\int \frac{\ln x}{x^m} dx = -\frac{\ln x}{(m-1)x^{m-1}} - \frac{1}{(m-1)^2 x^{m-1}} \qquad (\mathit{for}\ m \neq 1)$$

$$\int \frac{(\ln x)^n}{x^m} dx = -\frac{(\ln x)^n}{(m-1)x^{m-1}} + \frac{n}{m-1}\int \frac{(\ln x)^{n-1}}{x^m} dx \qquad (\mathit{for}\ m \neq 1)$$

$$\int \frac{dx}{x \ln x} = \ln|\ln x|$$

$$\int \frac{dx}{x^n \ln x} = \ln|\ln x| + \sum_{i=1}^{\infty} (-1)^i \frac{(n-1)^i (\ln x)^i}{i \cdot i!}$$

$$\int \frac{dx}{x(\ln x)^n} = -\frac{1}{(n-1)(\ln x)^{n-1}} \qquad (\mathit{for}\ n \neq 1)$$

$$\int \ln(x^2 + a^2) dx = x\ln(x^2 + a^2) - 2x + 2a\tan^{-1}\frac{x}{a}$$

$$\int \sin(\ln x) dx = \frac{x}{2}(\sin(\ln x) - \cos(\ln x))$$

$$\int \cos(\ln x) dx = \frac{x}{2}(\sin(\ln x) + \cos(\ln x))$$

5. Integrals of Trig. Functions

$$\int \sin x dx = -\cos x$$

$$\int \cos x dx = -\sin x$$

$$\int \sin^2 x dx = \frac{x}{2} - \frac{1}{4}\sin 2x$$

$$\int \cos^2 x dx = \frac{x}{2} + \frac{1}{4}\sin 2x$$

$$\int \sin^3 x dx = \frac{1}{3}\cos^3 x - \cos x$$

$$\int \cos^3 x dx = \sin x - \frac{1}{3}\sin^3 x$$

$$\int \frac{dx}{\sin x} xdx = \ln\left|\tan\frac{x}{2}\right|$$

$$\int \frac{dx}{\cos x} xdx = \ln\left|\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)\right|$$

$$\int \frac{dx}{\sin^2 x} xdx = -\cot x$$

$$\int \frac{dx}{\cos^2 x} xdx = \tan x$$

$$\int \frac{dx}{\sin^3 x} = -\frac{\cos x}{2\sin^2 x} + \frac{1}{2}\ln\left|\tan\frac{x}{2}\right|$$

$$\int \frac{dx}{\cos^3 x} = \frac{\sin x}{2\cos^2 x} + \frac{1}{2}\ln\left|\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)\right|$$

$$\int \sin x \cos x dx = -\frac{1}{4}\cos 2x$$

$$\int \sin^2 x \cos x dx = \frac{1}{3}\sin^3 x$$

$$\int \sin x \cos^2 x dx = -\frac{1}{3}\cos^3 x$$

$$\int \sin^2 x \cos^2 x dx = \frac{x}{8} - \frac{1}{32}\sin 4x$$

$$\int \tan x dx = -\ln|\cos x|$$

$$\int \frac{\sin x}{\cos^2 x} dx = \frac{1}{\cos x}$$

$$\int \frac{\sin^2 x}{\cos x} dx = \ln\left|\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)\right| - \sin x$$

$$\int \tan^2 x dx = \tan x - x$$

$$\int \cot x dx = \ln|\sin x|$$

$$\int \frac{\cos x}{\sin^2 x} dx = -\frac{1}{\sin x}$$

$$\int \frac{\cos^2 x}{\sin x} dx = \ln\left|\tan\frac{x}{2}\right| + \cos x$$

$$\int \cot^2 x dx = -\cot x - x$$

$$\int \frac{dx}{\sin x \cos x} = \ln|\tan x|$$

$$\int \frac{dx}{\sin^2 x \cos x} = -\frac{1}{\sin x} + \ln\left|\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)\right|$$

$$\int \frac{dx}{\sin x \cos^2 x} = \frac{1}{\cos x} + \ln\left|\tan\frac{x}{2}\right|$$

$$\int \frac{dx}{\sin^2 x \cos^2 x} = \tan x - \cot x$$

$$\int \sin mx \sin nx dx = -\frac{\sin(m+n)x}{2(m+n)} + \frac{\sin(m-n)x}{2(m-n)} \qquad m^2 \neq n^2$$

$$\int \sin mx \cos nx dx = -\frac{\cos(m+n)x}{2(m+n)} - \frac{\cos(m-n)x}{2(m-n)} \qquad m^2 \neq n^2$$

$$\int \cos mx \cos nx dx = \frac{\sin(m+n)x}{2(m+n)} + \frac{\sin(m-n)x}{2(m-n)} \qquad m^2 \neq n^2$$

$$\int \sin x \cos^n x dx = -\frac{\cos^{n+1} x}{n+1}$$

$$\int \sin^n x \cos x dx = \frac{\sin^{n+1} x}{n+1}$$

$$\int \arcsin x dx = x \arcsin x + \sqrt{1-x^2}$$

$$\int \arccos x dx = x \arccos x - \sqrt{1-x^2}$$

$$\int \arctan x dx = x \arctan x - \frac{1}{2}\ln\left(x^2+1\right)$$

$$\int \operatorname{arc\,cot} x dx = x \operatorname{arc\,cot} x + \frac{1}{2}\ln\left(x^2+1\right)$$