Concepts of Modern Physics

Sixth Edition

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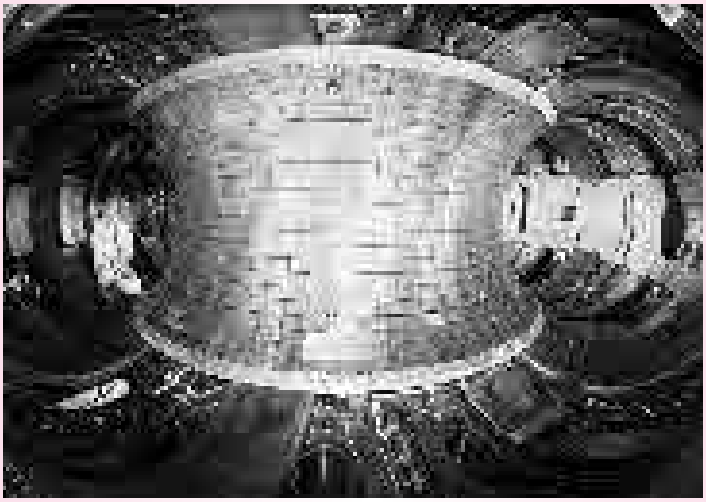
CHAPTER 12

Nuclear Transformations



Interior of the Tokamak Fusion Test Reactor at the Princeton Plasma Physics Laboratory. In December 1993 this reactor produced 6.2 MW of fusion power for 4 s from a deuterium-tritium plasma confined by strong magnetic fields.

12.1 RADIOACTIVE DECAY
Five kinds

12.2 HALF-LIFE
Less and less, but always some left

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APPENDIX: THEORY OF ALPHA DECAY

Despite the strength of the forces that hold nucleons together to form an atomic nucleus, many nuclides are unstable and spontaneously change into other nuclides by radioactive decay. And all nuclei can be transformed by reactions with nucleons or other nuclei that collide with them. In fact, all complex nuclei came into being in the first place through successive nuclear reactions, some in the first few minutes after the Big Bang and the rest in stellar interiors. The principal aspects of radioactivity and nuclear reactions are considered in this chapter.

12.1 RADIOACTIVE DECAY

Five kinds

No single phenomenon has played so significant a role in the development of nuclear physics as radioactivity, which was discovered in 1896 by Antoine Becquerel. Three features of radioactivity are extraordinary from the perspective of classical physics:

1 When a nucleus undergoes alpha or beta decay, its atomic number Z changes and it becomes the nucleus of a different element. Thus the elements are not immutable, although the mechanism of their transformation would hardly be recognized by an alchemist.
2 The energy liberated during radioactive decay comes from *within* individual nuclei without external excitation, unlike the case of atomic radiation. How can this happen? Not until Einstein proposed the equivalence of mass and energy could this puzzle be understood.
3 Radioactive decay is a statistical process that obeys the laws of chance. No cause-effect relationship is involved in the decay of a particular nucleus, only a certain probability per unit time. Classical physics cannot account for such behavior, although it fits naturally into the framework of quantum physics.

The radioactivity of an element arises from the radioactivity of one or more of its isotopes. Most elements in nature have no radioactive isotopes, although such isotopes can be prepared artificially and are useful in biological and medical research as "tracers." (The procedure is to incorporate a radionuclide in a chemical compound and follow what happens to the compound in a living organism by monitoring the radiation from the nuclide.) Other elements, such as potassium, have some stable isotopes and some radioactive ones; a few, such as uranium, have only radioactive isotopes.

The early experimenters, among them Rutherford and his coworkers, distinguished three components in the radiations from radionuclides (Figs. 12.1 and 12.2). These components

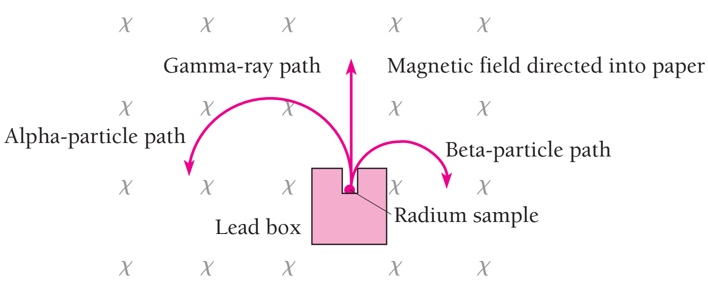
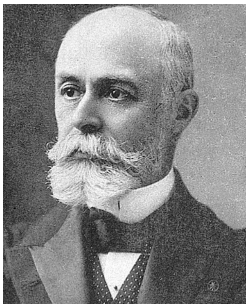


Figure 12.1 The radiations from a radium sample may be analyzed with the help of a magnetic field. Alpha particles are deflected to the left, hence they are positively charged; beta particles are deflected to the right, hence they are negatively charged; and gamma rays are not affected, hence they are unchanged.



Antoine-Henri Becquerel (1852–1908) was born and educated in Paris. His grandfather, father, and son were also physicists, all of them in turn professors at the Paris Museum of Natural History. Like his grandfather and father, Becquerel specialized in fluorescence and phosphorescence, phenomena in which a substance absorbs light at one frequency and reemits it at another, lower frequency.

In 1895 Roentgen had detected x-rays by the fluorescence they cause in an appropriate material. When he learned of this early in 1896, Becquerel wondered whether the reverse process might not occur, with intense light stimulating a fluorescent material to give off x-rays. He placed a fluorescent uranium salt on a photographic plate covered with black paper, exposed the arrangement to the sun, and indeed found the plate fogged when he had developed it. Becquerel then tried to repeat the experiment, but clouds obscured the sun for several days. He developed the plates anyway, expecting them to be clear, but to his surprise they were just as fogged as before. In a short time he had identified the source of the penetrating radiation as the uranium in the fluorescent salt. He was also able to show that the radiation ionized gases and that part of it consisted of fast charged particles.

Although Becquerel's discovery was accidental, he realized its importance at once and explored various aspects of the radioactivity of uranium for the rest of his life. He received the Nobel Prize in physics in 1903.

were called alpha, beta, and gamma, which were eventually identified as $^4_2$He nuclei, electrons, and high-energy photons respectively. Later, positron emission and electron capture were added to the list of decay modes. Figure 12.3 shows the five ways in which an unstable nucleus can decay, together with the reason for the instability. (The neutrinos given off when nuclei emit or absorb electrons are discussed in Sec. 12.5.) Examples of the nuclear transformations that accompany the various decays are given in Table 12.1.

Table 12.1 Radioactive Decay†

Decay	Transformation	Example
Alpha decay	$^A_ZX \rightarrow {}^{A-4}_{Z-2}Y + {}^4_2\text{He}$	$^{238}_{29}\text{U} \rightarrow {}^{234}_{90}\text{Th} + {}^4_2\text{He}$
Beta decay	$^A_ZX \rightarrow {}_{Z+1}^{\ A}Y + e^-$	$^{14}_6\text{C} \rightarrow {}^{14}_7\text{N} + e^-$
Positron emission	$^A_ZX \rightarrow {}_{Z-1}^{\ A}Y + e^+$	$^{64}_{29}\text{Cu} \rightarrow {}^{64}_{28}\text{Ni} + e^+$
Electron capture	$^A_ZX + e^- \rightarrow {}_{Z-1}^{\ A}Y$	$^{64}_{29}\text{Cu} + e^- \rightarrow {}^{64}_{28}\text{Ni}$
Gamma decay	$^A_ZX^* \rightarrow {}^A_ZX + \gamma$	$^{87}_{38}\text{Sr}^* \rightarrow {}^{87}_{38}\text{Sr} + \gamma$

†The * denotes an excited nuclear state and γ denotes a gamma-ray photon.

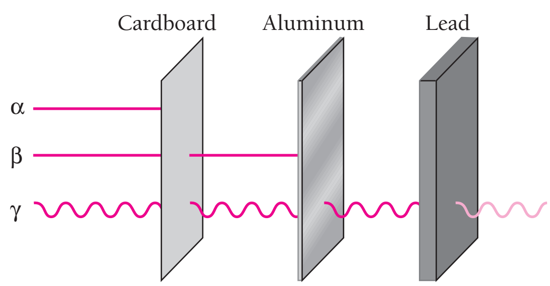


Figure 12.2 Alpha particles from radioactive materials are stopped by a piece of cardboard. Beta particles penetrate the cardboard but are stopped by a sheet of aluminum. Even a thick slab of lead may not stop all the gamma rays.

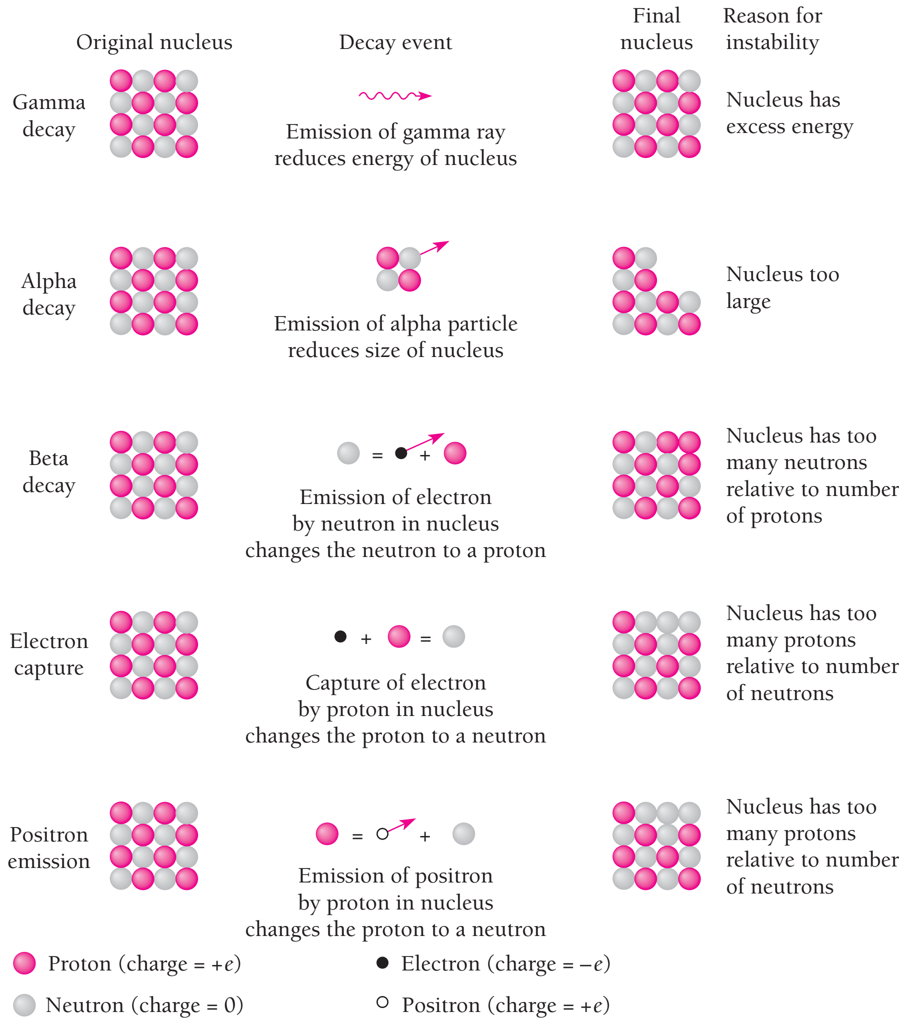


Figure 12.3 Five kinds of radioactive decay.

Example 12.1

The helium isotope $^6_2$He is unstable. What kind of decay would you expect it to undergo?

Solution

The most stable helium nucleus is $^4_2$He, all of whose neutrons and protons are in the lowest possible energy levels (see Sec. 11.3). Since $^6_2$He has four neutrons whereas $^4_2$He has only two, the instability of $^6_2$He must be due to an excess of neutrons. This suggests that $^6_2$He undergoes negative beta decay to become the lithium isotope $^6_3$Li whose neutron/proton ratio is more consistent with stability:

$$^6_2\text{He} \rightarrow {}^6_3\text{Li} + e^-$$

This is, in fact, the manner in which $^6_2$He decays.

Radioactivity and the Earth

Most of the energy responsible for the geological history of the earth can be traced to the decay of the radioactive uranium, thorium, and potassium isotopes it contains. The earth is believed to have come into being perhaps 4.5 billion years ago as a cold aggregate of smaller bodies that consisted largely of metallic iron and silicate minerals that had been circling the sun. Heat of radioactive origin accumulated in the interior of the infant earth and in time led to partial melting. The influence of gravity then caused the iron to migrate inward to form the molten core of today's planet; the geomagnetic field comes from electric currents in this core. The lighter silicates rose to form the rocky mantle around the core that makes up about 80 percent of the earth's volume. Most of the earth's radioactivity is now concentrated in the upper mantle and the crust (the relatively thin outer shell), where the heat it produces escapes and cannot collect to remelt the earth. The steady stream of heat is more than enough to power the motions of the giant plates into which the earth's surface is divided and the mountain building, earthquakes, and volcanoes associated with these motions.

Activity

The **activity** of a sample of any radioactive nuclide is the rate at which the nuclei of its constituent atoms decay. If N is the number of nuclei present in the sample at a certain time, its activity R is given by

Activity

$$R = -\frac{dN}{dt} \tag{12.1}$$

The minus sign is used to make R a positive quantity since dN/dt is, of course, intrinsically negative. The SI unit of activity is named after Becquerel:

$$1 \text{ becquerel} = 1 \text{ Bq} = 1 \text{ decay/s}$$

The activities encountered in practice are usually so high that the megabecquerel ($1 \text{ MBq} = 10^6 \text{ Bq}$) and gigabecquerel ($1 \text{ GBq} = 10^9 \text{ Bq}$) are more often appropriate.

The traditional unit of activity is the **curie** (Ci), which was originally defined as the activity of 1 g of radium, ${}^{226}_{88}\text{Ra}$. Because the precise value of the curie changed as methods of measurement improved, it is now defined arbitrarily as

$$1 \text{ curie} = 1 \text{ Ci} = 3.70 \times 10^{10} \text{ decays/s} = 37 \text{ GBq}$$

The activity of 1 g of radium is a few percent smaller. Ordinary potassium has an activity of about 0.7 microcurie ($1\ \mu\text{Ci} = 10^{-6}$ Ci) per kilogram because it contains a small proportion of the radioisotope ${}^{40}_{19}\text{K}$.

Radiation Hazards

The various radiations from radionuclides ionize matter through which they pass. X-ray ionize matter, too. All ionizing radiation is harmful to living tissue, although if the damage is slight, the tissue can often repair itself with no permanent effect. Radiation hazards are easy to underestimate because there is usually a delay, sometimes of many years, between an exposure and some of its possible consequences. These consequences include cancer, leukemia, and changes in the DNA of reproductive cells that lead to children with physical deformities and mental handicaps.

Radiation dosage is measured in **sieverts** (Sv), where 1 Sv is the amount of any radiation that has the same biological effect as those produced when 1 kg of body tissue absorbs 1 joule of x-rays or gamma rays. Although radiobiologists disagree about the exact relationship between radiation exposure and the likelihood of developing cancer, there is no question that such a link exists. The International Commission on Radiation Protection estimates an average risk factor of 0.05 Sv^{-1}. This means that the chances of dying from cancer as a result of radiation are 1 in 20 for a dose of 1 Sv, 1 in 20,000 for a dose of 1 mSv (1 mSv = 0.001 Sv), and so on.

Figure 12.4 shows the chief sources of radiation dosage on a worldwide basis. The most important single source is the radioactive gas radon, a decay product of radium whose own origin traces back to the decay of uranium. Uranium is found in many common rocks, notably granite. Hense radon, colorless and odorless, is present nearly everywhere, though usually in amounts too small to endanger health. Problems arise when houses are built in uranium-rich regions, since it is impossible to prevent radon from entering such houses from the ground under them. Surveys show that millions of American homes have radon concentrations high enough to pose a nonneglible cancer risk. As a cause of lung cancer, radon is second only to cigarette smoking. The most effective method of reducing radon levels in an existing house in a hazardous region seems to be to extract air with fans from underneath the ground floor and disperse it into the atmosphere before it can enter the house.

Other natural sources of radiation dosage include cosmic rays from space and radionuclides present in rocks, soil, and building materials. Food, water, and the human body itself contain small amounts of radionuclides of such elements as potassium and carbon.

Many useful processes involve ionizing radiation. Some employ such radiation directly, as in the x-rays and gamma rays used in medicine and industry. In other cases the radiation is an unwanted but inescapable byproduct, notably in the operation of nuclear reactors and in the disposal of their wastes. In many countries the dose limit for workers (about 9 million wordwide) whose jobs involve ionizing radiation is 20 mSv per year. For the general public, which has no choice in the matter, the dose limit for nonbackground radiation is 1 mSv per year.

An appropriate balance between risk and benefit is not always easy to find where radiation is concerned. This seems particularly true for medical x-ray exposures, many of which are made for no strong reason and do more harm than good. The once "routine" x-raying of symptomless young women to search for breast cancer is now generally believed to have increased, not decreased, the overall death rate due to cancer. Particularly dangerous is the x-raying of pregnant women, until not long ago another "routine" procedure, which dramatically increases the chance of cancer in their children. Of course, x-rays have many valuable applications in medicine. The

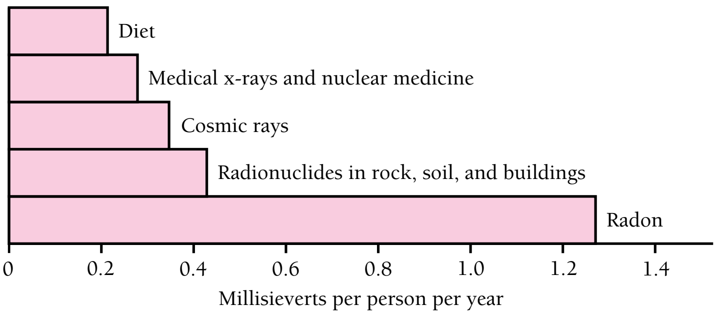


Figure 12.4 The chief sources of radiation dosage averaged around the world. The total is 2.7 mSv, but actual dosages vary widely. For instance, radon concentrations are not the same everywhere, some people receive much more medical radiation than others, cosmic rays are more intense at high altitudes (frequent fliers may get double the sea-level dose, residents of high-altitude cities up to five times as much), and so on. Nuclear power stations are responsible for less than 0.1 percent of the total, though accidents can raise the amount in affected areas to dangerous levels.

point is that every exposure should have a definite justification that outweights the risk involved. An ordinary chest x-ray using modern equipment involves a radiation dose of about 0.017 mSv, much less than in the past. However, a CT chest scan (Sec. 2.5) involves the considerable dose of 8 mSv. CT scans of children pose especially serious risks and need equally serious justification.

12.2 HALF-LIFE

Less and less, but always some left

Measurements of the activities of radioactive samples show that, in every case, they fall off exponentially with time. Figure 12.5 is a graph of R versus t for a typical radionuclide. We note that in every 5.00-h period, regardless of when the period starts, the activity drops to half of what it was at the start of the period. Accordingly the **half-life** $T_{1/2}$ of the nuclide is 5.00 h.

Every radionuclide has a characteristic half-life. Some half-lives are only a millionth of a second, others are billions of years. One of the major problems faced by nuclear power plants is the safe disposal of radioactive wastes since some of the nuclides present have long half-lives.

The behavior illustrated in Fig. 12.5 means that the time variation of activity follows the formula

Activity law
$$R = R_0 e^{-\lambda t} \tag{12.2}$$

where λ, called the **decay constant**, has a different value for each radionuclide. The connection between decay constant λ and half-life $T_{1/2}$ is easy to find. After a half-life has elapsed, that is, when $t = T_{1/2}$, the activity R drops to $\frac{1}{2}R_0$ by definition. Hence

$$\tfrac{1}{2}R_0 = R_0 e^{-\lambda T_{1/2}}$$
$$e^{\lambda T_{1/2}} = 2$$

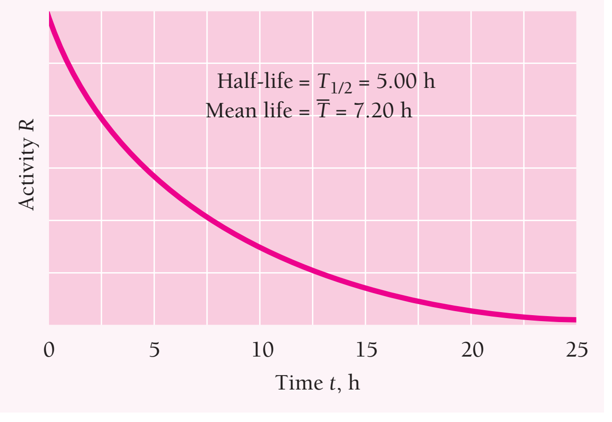


Figure 12.5 The activity of a radionuclide decreases exponentially with time. The half-life is the time needed for an initial activity to drop by half. The mean life of a radionuclide is 1.44 times its half-life [Eq. (12.7)].

Taking natural logarithms of both sides of this equation,

$$\lambda T_{1/2} = \ln 2$$

Half-life

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda} \tag{12.3}$$

The decay constant of the radionuclide whose half-life is 5.00 h is therefore

$$\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{(5.00\text{ h})(3600\text{ s/h})} = 3.85 \times 10^{-5}\text{ s}^{-1}$$

The larger the decay constant, the greater the chance a given nucleus will decay in a certain period of time.

The activity law of Eq. (12.2) follows if we assume a constant probability λ per unit time for the decay of each nucleus of a given nuclide. With λ as the probability per unit time, $\lambda\, dt$ is the probability that any nucleus will undergo decay in a time interval dt. If a sample contains N undecayed nuclei, the number dN that decay in a time dt is the product of the number of nuclei N and the probability $\lambda\, dt$ that each will decay in dt. That is,

$$dN = -N\lambda\, dt \tag{12.4}$$

where the minus sign is needed because N decreases with increasing t.

Equation (12.4) can be rewritten

$$\frac{dN}{N} = -\lambda\, dt$$

and each side can now be integrated:

$$\int_{N_0}^{N} \frac{dN}{N} = -\lambda \int_0^t dt$$

$$\ln N - \ln N_0 = -\lambda t$$

Radioactive decay

$$N = N_0 e^{-\lambda t} \tag{12.5}$$

This formula gives the number N of undecayed nuclei at the time t in terms of the decay probability per unit time λ of the nuclide involved and the number N_0 of undecayed nuclei at $t = 0$.

Figure 12.6 illustrates the alpha decay of the gas radon, $^{222}_{86}\text{Rn}$, whose half-life is 3.82 days, to the polonium isotope $^{218}_{84}\text{Po}$. If we start with 1.00 mg of radon in a closed container, 0.50 mg will remain after 3.82 days, 0.25 mg will remain after 7.64 days, and so on.

Example 12.2

How long does it take for 60.0 percent of a sample of radon to decay?

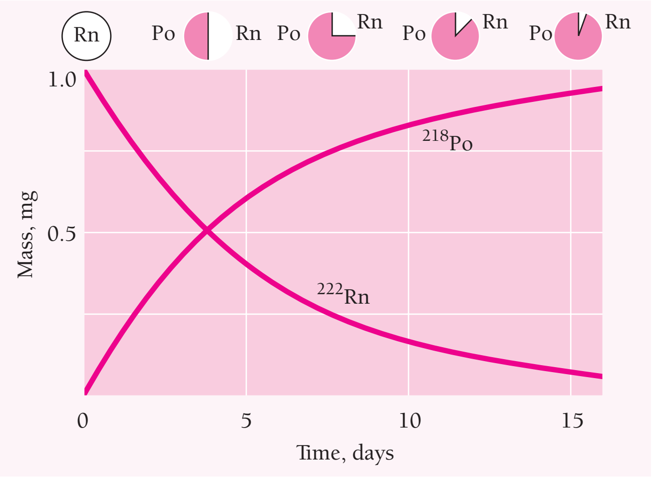


Figure 12.6 The alpha decay of ^{222}Rn to ^{218}Po has a half-life of 3.8 d. The sample of radon whose decay is graphed here had an initial mass of 1.0 mg.

Solution

From Eq. (12.5)

$$\frac{N}{N_0} = e^{-\lambda t} \qquad -\lambda t = \ln \frac{N}{N_0} \qquad \lambda t = \ln \frac{N_0}{N}$$

$$t = \frac{1}{\lambda} \ln \frac{N_0}{N}$$

Here $\lambda = 0.693/T_{1/2} = 0.693/3.82$ d and $N = (1 - 0.600)\ N_0 = 0.400 N_0$, so that

$$t = \frac{3.82 \text{ d}}{0.693} \ln \frac{1}{0.400} = 5.05 \text{ d}$$

The fact that radioactive decay follows the exponential law of Eq. (12.2) implies that this phenomenon is statistical in nature. Every nucleus in a sample of a radionuclide has a certain probability of decaying, but there is no way to know in advance *which* nuclei will actually decay in a particular time span. If the sample is large enough—that is, if many nuclei are present—the actual fraction of it that decays in a certain time span will be very close to the probability for any individual nucleus to decay.

To say that a certain radioisotope has a half-life of 5 h, then, signifies that every nucleus of this isotope has a 50 percent change of decaying in every 5-h period. This does *not* mean a 100 percent probability of decaying in 10 h. A nucleus does not have a memory, and its decay probability per unit time is constant until it actually does decay. A half-life of 5 h implies a 75 percent probability of decay in 10 h, which increases to 87.5 percent in 15 h, to 93.75 percent in 20 h, and so on, because in every 5-h interval the probability is 50 percent.

It is worth keeping in mind that the half-life of a radionuclide is not the same as its **mean lifetime** $\overline{T}$. The mean lifetime of a nuclide is the reciprocal of its decay probability per unit time:

$$\overline{T} = \frac{1}{\lambda} \tag{12.6}$$

Hence

Mean lifetime

$$\overline{T} = \frac{1}{\lambda} = \frac{T_{1/2}}{0.693} = 1.44T_{1/2} \tag{12.7}$$

$\overline{T}$ is nearly half again more than $T_{1/2}$. The mean lifetime of a radionuclide whose half-life is 5.00 h is

$$\overline{T} = 1.44T_{1/2} = (1.44)(5.00 \text{ h}) = 7.20 \text{ h}$$

Since the activity of a radioactive sample is defined as

$$R = -\frac{dN}{dt}$$

we see that, from Eq. (12.5),

$$R = \lambda N_0 e^{-\lambda t}$$

This agrees with the activity law of Eq. (12.2) if $R_0 = \lambda N_0$, or, in general, if

Activity

$$R = \lambda N \tag{12.8}$$

Example 12.3

Find the activity of 1.00 mg of radon, ^{222}Rn, whose atomic mass is 222 u.

Solution

The decay constant of radon is

$$\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{(3.8 \text{ d})(86{,}400 \text{ s/d})} = 2.11 \times 10^{-6} \text{ s}^{-1}$$

The number N of atoms in 1.00 mg of ^{222}Rn is

$$N = \frac{1.00 \times 10^{-6} \text{ kg}}{(222 \text{ u})(1.66 \times 10^{-27} \text{ kg/u})} = 2.71 \times 10^{18} \text{ atoms}$$

Hence

$$R = \lambda N = (2.11 \times 10^{-6} \text{ s}^{-1})(2.71 \times 10^{18} \text{ nuclei})$$
$$= 5.72 \times 10^{12} \text{ decays/s} = 5.72 \text{ TBq} = 155 \text{ Ci}$$

Example 12.4

What will the activity of the above radon sample be exactly one week later?

Solution

The activity of the sample decays according to Eq. (12.2). Since $R_0 = 155$ Ci here and

$$\lambda t = (2.11 \times 10^{-6} \text{ s}^{-1})(7.00 \text{ d})(86{,}400 \text{ s/d}) = 1.28$$

we find that

$$R = R_0 e^{-\lambda t} = (155 \text{ Ci})e^{-1.28} = 43 \text{ Ci}$$