Chapter 1: The Foundations: Logic and Proofs

1.2 Applications of Propositional Logic

Applications Propositional Logic

- Can be used to design computer circuits, to construct computer programs, to verify the correctness of programs, and to build expert systems.

- Can be used to analyze and solve many familiar puzzles.

System Specifications

- System and Software engineers take requirements in English and express them in a precise specification language based on logic.

- Example: Express in propositional logic:

"The automated reply cannot be sent when the file system is full" (hint: replace 'when' with 'if')

Solution: One possible solution: Let r denote "The automated reply can be sent" and f denote "The file system is full."

$$f \rightarrow \neg r$$

Translating English Sentences

- Example: How can this English sentence be translated into a logical expression?

  "You can access the Internet from campus only if you are a computer science major or you are not a freshman."

Solution: Let *a*, *c*, and *f* represent "You can access the Internet from campus," "You are a computer science major," and "You are a freshman." The sentence can be translated into:

$$a \rightarrow (c \vee \neg f).$$

Boolean Searches

- Web Page Searches - Connective AND is used to match records that contain both of two search terms, the connective OR is used to match one or both of two search terms, and the connective NOT (sometimes written as AND NOT) is used to exclude a particular search term.

  Most Web search engines support Boolean searching techniques, which is useful for finding Web pages about particular subjects.

Boolean Searches

- To find Web pages about universities in New Mexico, we can look for pages matching NEW AND MEXICO AND UNIVERSITIES. – this will return all pages containing the three terms. Also, this will return the pages containing new universities in Mexico. To handle this, we can use quotation marks such as “New Mexico” and Universities.
- To find pages that contain universities in New Mexico or Arizona, we can search for pages matching (NEW AND MEXICO OR ARIZONA) AND UNIVERSITIES.
- To find Web pages that deal with universities in Mexico (and not New Mexico), we might first look for pages matching MEXICO AND UNIVERSITIES, but because the results of this search will include pages about universities in New Mexico, as well as universities in Mexico, it might be better to search for pages matching (MEXICO AND UNIVERSITIES) NOT NEW. The results of this search include pages that contain both the words MEXICO and UNIVERSITIES but do not contain the word NEW.

Logic Puzzles

- Puzzles that can be solved using logical reasoning are known as logic puzzles. Solving logic puzzles is an excellent way to practice working with the rules of logic. Also, computer programs designed to carry out logical reasoning often use well-known logic puzzles to illustrate their capabilities. Many people enjoy solving logic puzzles, published in periodicals, books, and on the Web, as a recreational activity.

Logic Circuits

- Propositional logic can be applied to the design of computer hardware. A logic circuit (or digital circuit) receives input signals p1, p2, … , pn, each a bit [either 0 (off) or 1 (on)], and produces a single output signal. Complicated digital circuits can be constructed from three basic circuits, called gates, shown below.

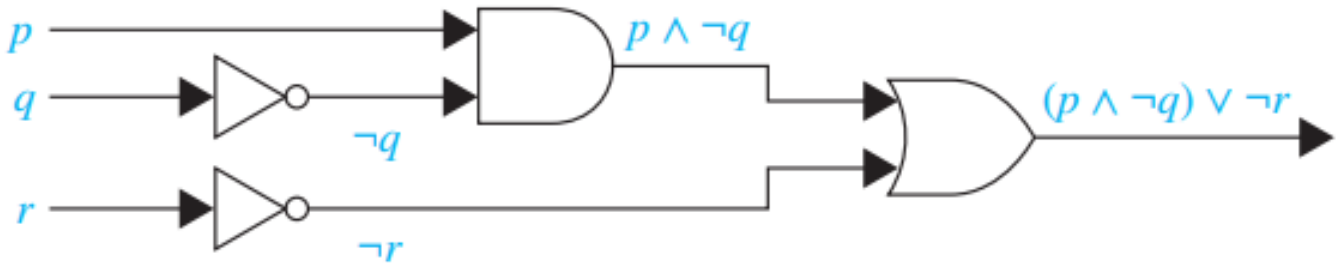
Inverter

OR gate

AND gate

Logic Circuits

- Determine the output for the following combinatorial circuit.



We display the output of each logic gate in the circuit. We see that the AND gate takes input of p and ¬q, the output of the inverter with input q, and produces p ∧ ¬q. Next, we note that the OR gate takes input p ∧ ¬q and ¬r, the output of the inverter with input r, and produces the final output (p ∧ ¬q) ∨ ¬r.