Assignment 1 AI2

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I Counting and basic laws of probability

I.1 5-card Poker Hands

I.1.a

$$\binom{52}{5} = \frac{52!}{5!(52-5)!} = \frac{52!}{5!47!} = 2.598.960$$

There are 2.598.960 atomic events.

I.1.b

The probability of each atomic event is equal, thus the probability of one atomic event is

$$\frac{1}{2.598.960} = 0,000003848 = 0,0003848\%$$

I.1.c

A royal straight flush is one atomic event for each suit and because there are four different suits we get

$$\frac{4}{2.598.960} = 0,00000154 = 0,000154\%$$

To get the probability of getting a four of a kind hand we have $\binom{1}{13}$ options for what rank the card will be. Then the suits need to be all different and we have $\binom{4}{4} = 1$ of choosing the suits. The last card can be any of the 48 remaining cards.

$$\frac{\binom{13}{1} * \binom{4}{4} * \binom{48}{1}}{\binom{52}{5}} = \frac{624}{2.598.960} = 0,00024 = 0,024\%$$

I.2 Two cards in a deck

I.2.a

The probability that two randomly selected cards from a deck of 52 playing cards constituting a pair is

$$\frac{52}{52} * \frac{3}{51} = 1 * \frac{3}{51} = 1/17 = 0,0588 = 5,88\%$$

There is a $\frac{52}{52} = 100\%$ chance of drawing drawing the first card and a $\frac{3}{51}$ chance that the second card matches the first in rank.

I.2.b

The conditional probability the constitute a pair given they are of different suits is

$$\frac{52}{52} * \frac{3}{39} = \frac{1}{13} = 0,0769 = 7,69\%$$

The probability for the first card is the same, but for the second card, we can remove the possibilities matching the suit of the first card, and thus removing the 12 remaining cards of the same suit as the first one.

I.3 Conditional probability

I.3.1

From the formula for conditional probabilities, we know that

$$P(A|B) = \frac{P(A,B)}{P(B)}$$

We want to check that if

$$P(A|B) > P(A)$$

does the occurrence of A make B more likely

$$P(B|A) > P(B)?$$

We know that

$$P(A|B) > P(A)$$

This means:

$$\frac{P(A,B)}{P(B)} > P(A) \implies P(A,B) > P(A) * P(B)$$

Hence, from the formula for conditional probabilities we have:

$$\frac{P(B,A)}{P(A)} > P(B) \implies P(B|A) > P(B)$$

I.3.2

We are given the following probabilities:

$$P(S=0)=\frac{6}{10}$$

$$P(S=1)=\frac{4}{10}$$

$$P(R=1|S=0)=\frac{1}{3}$$

$$P(R=0|S=1)=\frac{1}{3}$$

From the given probabilities we can calculate

$$P(R=1|S=1)=1-P(R=1|S=0)=1-\frac{1}{3}=\frac{2}{3}$$

$$P(R=0|S=0)=1-P(R=0|S=1)=1-\frac{1}{3}=\frac{2}{3}$$

We want to find

$$P(S=0|R=0)$$

Using Bayes' theorem we get

$$P(S=0|R=0)=\frac{P(R=0|S=0)P(S=0)}{P(R=0)}$$

We need to find P(R=0)

$$P(R=0)=P(R=0,S=0)+P(R=0,S=1)$$

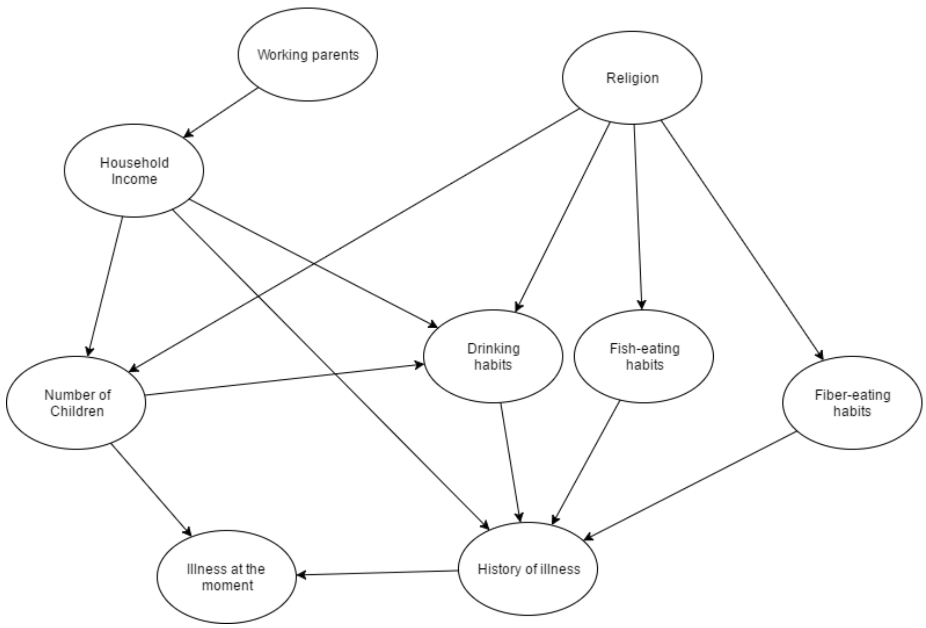
$$=P(R=0|S=0)P(S=0)+(R=0|S=1)P(S=1)$$

$$=\frac{2}{3}*\frac{6}{10}+\frac{1}{3}*\frac{4}{10}=\frac{8}{15}$$

Putting the results into the formula from Bayes' theorem we get

$$P(S=0|R=0)=\frac{\frac{2}{3}*\frac{6}{10}}{\frac{8}{15}}=\frac{3}{4}=75\%$$

II Bayesian Network Construction



I have put Working parents and Religion as the independent nodes. Fish-eating habits and Fiber-eating habits are children of Religion, as some religions can restrict what you eat. They could also be independent nodes, but I choose to place them under religion.

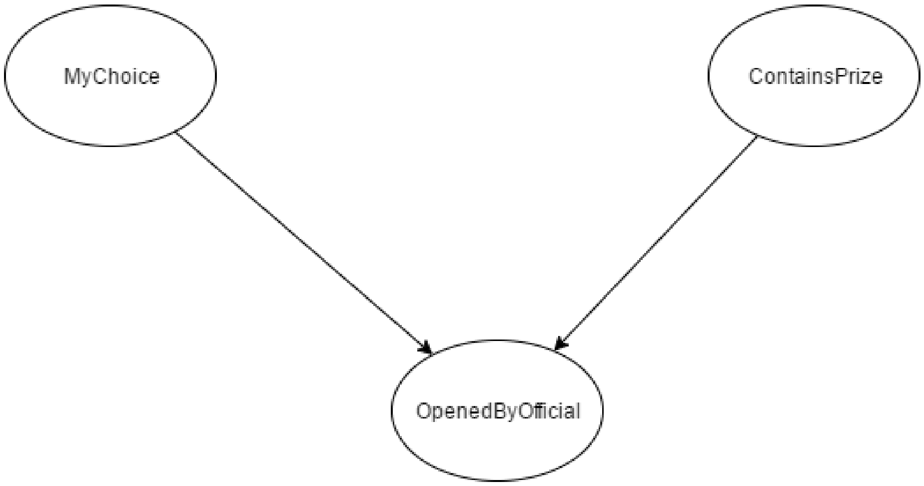
Household Income is directly affected by that the parents are working. Number of children is affected by the Household Income and the Religion. Parents with low income might not want/have children as they do not have the funds to raise them. Religion might have limitations on how many children parents should have.

Drinking habits depends on the Household Income, Number of Children and Religion. Some religions says you shouldn't drink alcohol. Parents with high Household Income Might drink more as they can afford more and parents with a low amount of children have more time to drink as they don't have to tend to the children.

History of Illness is dependant on drinking habits, Household income and eating habits. A family with high income can afford to go to the doctors more often and more easily prevent illness. The diet is also important for the history of illness and you are more susceptible for illness when drinking.

Illness at the Moment are influenced by the History of Illness and Number of children, the History of Illness tells something about how often you are sick and how likely you are to become sick. The number of children increases the chance of sickness as children are easily getting sick and they can infect each other.

III Bayesian Network Application



		OBO			
CP	MC	d_1	d_2	d_3	win by swapping
d_1	d_1	0	0.5	0.5	0%
	d_2	0	0	1	100%
	d_3	0	1	0	100%
d_2	d_1	0	0	1	100%
	d_2	0.5	0	0.5	0%
	d_3	1	0	0	100%
d_3	d_1	0	1	0	100%
	d_2	1	0	0	100%
	d_3	0.5	0.5	0	0%

CP: ContainsPrize
MC: MyChoice
OBO: OpenedByOfficial

As we can see from the table above the probability for the OBO opening a door is conditional on the CP and MC. We can also see from the table that we win $\frac{2}{3}$ of the time when swapping the door.