Institute of Information Technology (IIT)

Jahangirnagar University

Lab Report: 04

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NAME OF EXPERIMENT

TO FIND THE FFT OF A GIVEN SEQUENCE

AIM

To find the FFT of a given sequence.

APPARATUS

Software: MATLAB

THEORY

DFT of a sequence

$$X_k = \sum_{n=0}^{N-1} x_n \cdot e^{-\frac{i2\pi}{N}kn}$$

Where N is the length of the sequence.

K= Frequency Coefficient.

n = Samples in the time domain.

FFT: -Fast Fourier transform.

There are two methods.

1. Decimation in time (DIT) FFT.
2. Decimation in Frequency (DIF) FFT.

The number of multiplications in DFT $=N^2$.The number of Additions in DFT = N (N-1). On the other hand, The no of multiplication in FFT= $N/2 \log_2 N$ and The number of additions = $N \log_2 N$.

To reduce complexity and calculation we prefer FFT rather than DFT

METHODOLOGY

Algorithm:

Step I: Give the input sequence x [n].

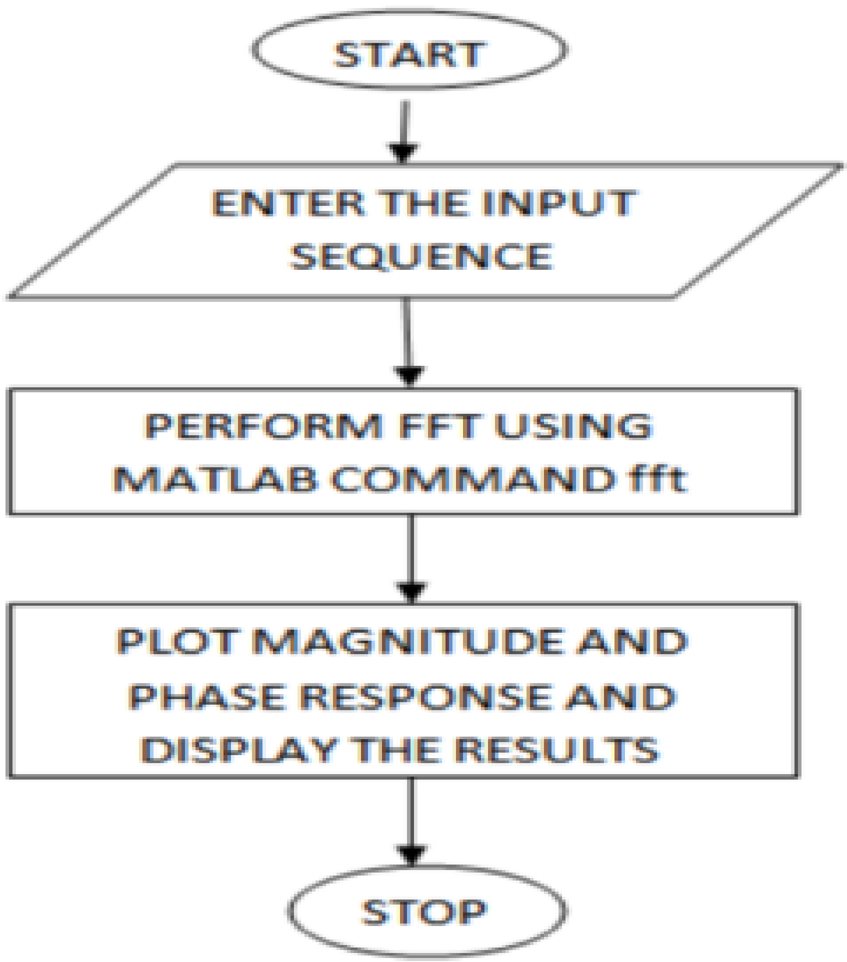
Step II: Find the length of the input sequence using the length command.

Step III: Find FFT and IFFT using the Matlab commands FFT and IFFT.

Step IV: Plot magnitude and phase response

Step V: Display the results.

Flow Chart:

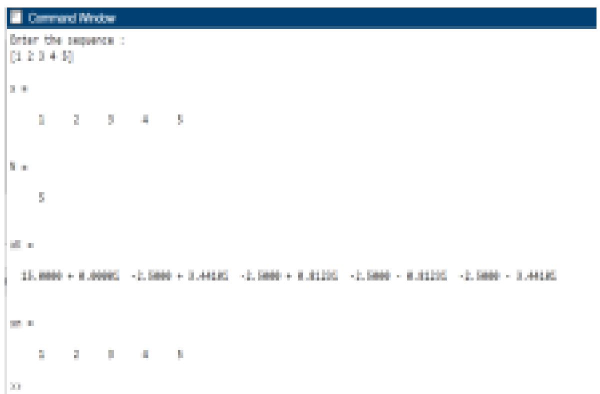


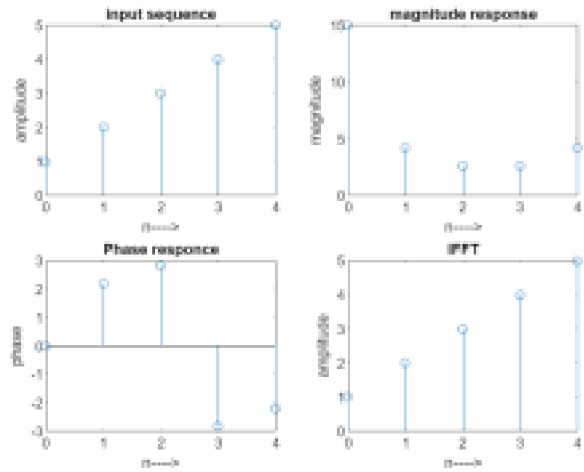
Program:

```
Clc;
clear all;
close all;
x=input('Enter the sequence : ')
N=length(x)
xK=fft(x,N)
xn=ifft(xK)
n=0:N-1;
subplot (2,2,1);
stem(n,x);
xlabel('n---->');

ylabel('amplitude');
title('input sequence');
subplot (2,2,2);
stem(n,abs(xK));
xlabel('n---->');
ylabel('magnitude');
title('magnitude response');
subplot (2,2,3);
stem(n,angle(xK));
xlabel('n---->');
ylabel('phase');
title('Phase responce');
subplot (2,2,4);
stem(n,xn);
xlabel('n---->');
ylabel('amplitude');
title('IFFT');
```

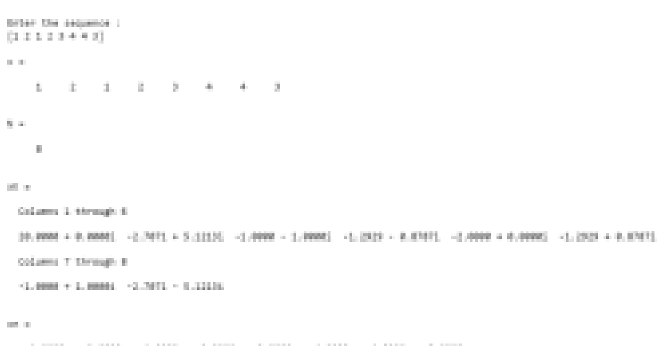
Output:

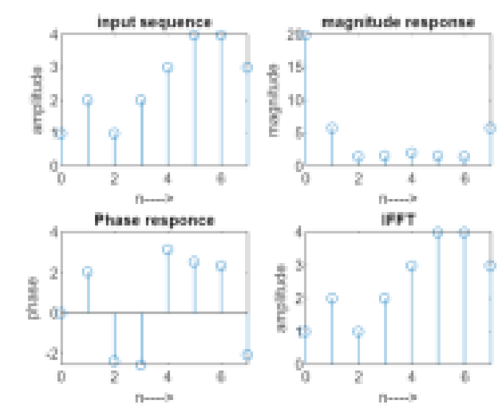




Exercise 1:1 Find the 8-point DFT of the sequence x(n) = [1 2 1 2 3 4 4 3] using the FFT algorithm.

Output:





DISCUSSION

The experiment focused on analyzing the Fast Fourier Transform (FFT) of a given sequence, a fundamental algorithm in signal processing and scientific applications that converts time-domain signals into frequency-domain representations. This discussion highlights key findings: FFT effectively reveals frequency components, exposing underlying periodicities and dominant frequencies. Applications span noise reduction, compression, and modulation in signal processing, communication, image analysis, and audio analysis. Limitations include the assumption of periodic input and windowing function effects. Experiment constraints encompass sequence length and computational resources.

Hand-calculation:

[1] Given sequence,

$$x(n) = [1 \quad 2 \quad 3 \quad 4 \quad 5]$$

DFT expression is,

$$X(K) = \sum_{n=0}^{N-1} x(n)\, e^{-j2\pi nK/N} \quad ; K = 0, 1, \ldots N-1 \quad \text{—} ①$$

here,

$$K = 0, 1, 2, 3, 4 \quad [\because N = 5]$$

$x(0) = 1$

$x(1) = 2$

$x(2) = 3$

$x(3) = 4$

$x(4) = 5$

$\Big\}$ $X(0)$, $X(2)$, $X(4)$

$\Big\}$ $X(1)$, $X(3)$

using equation ① $\Rightarrow$

$$X(0) = \sum_{n=0}^{4} x(n)\, e^{0}$$

$$= x(0) + x(1) + x(2) + x(3) + x(4)$$

$$\therefore X(0) = 1 + 2 + 3 + 4 + 5 = 15$$

and

$$X(1) = \sum_{n=0}^{4} x(n)\, e^{-j2\pi n/5}$$

$$= x(0) + x(1)\, e^{-j2\pi/5} + x(2)\, e^{-j4\pi/5} + x(3)\, e^{-j6\pi/5} + x(4)\, e^{-j8\pi/5}$$

$\therefore X(1) = -2.5 + 3.45$

and, $X(2) = \sum_{n=0}^{4} x(n) e^{-4\pi n j/5}$

$= x(0) + x(1) e^{-4\pi j/5} + x(2) e^{-8\pi j/5} + x(3) e^{-12\pi j/5}$
$+ x(4) e^{-16\pi j/5}$

$\therefore X(2) = -2.5 + 0.812j$

and $X(3) = \sum_{n=0}^{4} x(n) e^{-6\pi n j/5} = -2.5 - 0.81j$

similarly, $X(4) = -2.5 - 3.4j$

The DFT sequence of $x(n) = [1 \ 2 \ 3 \ 4 \ 5]$ is

$[15, -2.5 + 3.45, -2.5 + 0.812j, -2.5 - 0.81j,$
$-2.5 - 3.4j]$

(Ans)

$X(K) = \{15, -2.5+3.45, -2.5+0.812j, -2.5-0.81j,$
$-2.5-3.4j]$

IDFT expression is,

$$x(n) = \frac{1}{N} \sum_{n=0}^{N-1} X(K)\, e^{j2\pi nK/N} \; ; n = 0, 1, \ldots, N-1$$

here,
$n = 0, 1, 2, 3, 4$ [as $N = 5$]

$$\therefore x(0) = \frac{1}{5} \sum_{0}^{4} X(K)\, e^{0}$$

$$= \frac{1}{5}\{X(0) + X(1) + X(2) + X(3) + X(4)\}$$

$$\therefore x(0) = \frac{1}{5} \times 5 = 1$$

and,

$$x(1) = \frac{1}{5} \sum_{0}^{4} X(K) e^{j2\pi K/N}$$

$$= \frac{1}{5} \sum_{0}^{4} X(K)\, e^{j2.\pi K/5}$$

$$= \frac{1}{5}\left\{X(0) + X(1)\, e^{j\frac{2\pi}{5}} + X(2)\, e^{j\frac{4\pi}{5}} + X(3)^{j\frac{6\pi}{5}} + X(4)\, e^{j8\pi/5}\right\}$$

$$= \frac{1}{5} \times 10 = 2$$

Similarly, $x(2) = 3$, $x(3) = 4$, $x(4) = 5$

CONCLUSION

In conclusion, the experiment successfully demonstrated the application of the Fast Fourier Transform in analyzing the frequency content of a given sequence. The FFT proved to be a powerful technique for uncovering the underlying periodicities and oscillations within a signal. The obtained frequency-domain representation has significant implications in signal processing, communications, and various scientific disciplines. Despite its limitations, the FFT remains an essential tool for understanding and manipulating signals in the frequency domain. Further exploration and refinement of the experiment's parameters could provide deeper insights into the capabilities and constraints of the FFT algorithm.

VIVA QUESTIONS

1. Define transform. What is the need for transformation?

Transform: A transform is a mathematical operation that converts data from one domain into another. In the context of signal processing, it involves changing the representation of a signal or data from one domain (such as time or space) to another (such as frequency or spatial frequency). Transformations are used to analyze, simplify, or manipulate data in various applications.

Need for Transformation: Transformations are needed for various reasons:

- **Simplification:** Transforms can simplify complex mathematical operations, making analysis easier.
- **Feature Extraction:** Transforms can reveal specific characteristics or features of data that might be difficult to discern in the original domain.
- **Noise Removal:** Some transforms emphasize signal components while suppressing noise, improving signal quality.
- **Efficient Computation:** Transforms can enable more efficient computation of certain operations, such as convolution or multiplication.

2. Differentiate between the Fourier transform and the discrete Fourier transform.

Feature	Fourier transforms	Discrete Fourier transform
Input	Continuous signal	Discrete signal
Output	Continuous frequency spectrum	Discrete frequency spectrum
Applications	Signal analysis, filtering, compression	image processing,Signal analysis
Computation	O(N^2)	O(N\log_2 N)

3. Differentiate DFT and DTFT.

Feature	DFT	DTFT
Input	Discrete signal of length N	Continuous signal
Output	Discrete frequency spectrum	Continuous frequency spectrum
Applications	Signal analysis, filtering, compression	Signal analysis, filtering
Computation	O(N^2)	O(N\log_2 N)

4. What are the advantages of FFT over DFT?

Advantages of FFT over DFT:

- ☐ **Computational Efficiency:** The FFT algorithm reduces the number of calculations required for computing the DFT, making it much faster for larger datasets.
- ☐ **Optimized Algorithms:** FFT algorithms are designed to take advantage of symmetries and

periodicities in data, further reducing computation time.

- ☐ **Divide-and-Conquer Strategy:** FFT breaks down the DFT calculation into smaller sub-problems, improving efficiency.
- ☐ **Applications:** FFT is crucial in real-time signal processing, spectrum analysis, image processing, and more due to its speed.

5. Differentiate DIT-FFT and DIF-FFT algorithms.

Feature	DIT-FFT	DTF-FFT
Name	Decimation in time	Decimation in frequency
Order to computation	From data to frequency	From frequency to data
Applications	General purpose	Real-valued signals
Stability	More stable	Less stable

6. What is meant by radix?

"Radix" refers to the base or symbols used in a numeral system. It determines how values are represented. In FFT, it can relate to sub-operations in algorithms like radix-2 or radix-4.Mainly, in the context of FFT algorithms, radix refers to the base or factor by which the data is divided during the decomposition process.

7. What does the twiddle factor mean and what are its properties?

Twiddle factor: A twiddle factor is a complex exponential used in the Discrete Fourier Transform and FFT algorithms.

Properties of the Twiddle factor: Its properties, including symmetry, multiplicative behavior, unity roots, and phase shift, simplify calculations by reducing redundancy. Twiddle factors are precomputed and play a crucial role in optimizing the efficiency of signal transformation between time and frequency domains.

8. How is FFT useful to represent a signal?

FFT is useful for representing a signal in the frequency domain. By converting a time-domain signal into its frequency components, one can analyze its frequency content, identify dominant frequencies, and detect patterns or anomalies.

9. Compare FFT and DFT with respect to the number of calculations required?

FFT (Fast Fourier Transform) and DFT (Discrete Fourier Transform) differ significantly in calculation complexity:

FFT: Requires fewer calculations due to optimized algorithms like Cooley-Tukey. Computational complexity is O(N log N), efficient for large datasets.

DFT: Directly calculates all frequency components, leading to O(N^2) complexity. Inefficient for larger data, less practical in real-time or resource-intensive applications.

In summary, FFT's algorithmic optimizations greatly reduce the number of calculations, making it superior for efficient frequency analysis compared to DFT.

10. How is the original signal reconstructed from the FFT of a signal?

The original signal can be reconstructed from the FFT by applying the Inverse Fast Fourier Transform (IFFT). IFFT converts frequency-domain components back to the time domain, recreating the original signal. Proper phase and magnitude information is crucial for accurate reconstruction.

REFERENCE

[1]tutorialspoint,"dsp fast fourier transform". [Online]. Available:https://www.tutorialspoint.com/digital_signal_processing/dsp_fast_fourier_transform.htm[Accessed: August. 13, 2023,09:10am]